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**FUNDAMENTAL STUDY OF CRACK
INITIATION AND PROPAGATION
ANNUAL PROGRESS REPORT
March 1976 - March 1977**

MASTER

D. M. Norris, Jr., Ed.

and

M. I. Wilkins, J. E. Reaugh, B. Moran,

J. K. Scudder, D. F. Quiñones, M. E. Prado

June 1, 1977

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Foreword

We have completed the first year of a 2-year program on ductile fracture in nuclear pressure vessels. The work is supported by the Electric Power Research Institute (EPRI) under Contract RP603-1 and RP603-2. M. L. Wilkins is Principal Investigator; D. M. Norris, Jr. is Program Manager. The EPRI Project Manager is T. U. Marston.

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Symbols

a	neck radius
a_0	initial neck radius
A	neck area
A_0	initial neck area
A_D	damage ratio
B	specimen thickness
α	damage exponent
β	damage constant
b	damage constant
c	triaxiality constant
C	elastic compliance
CVN	Charpy energy
D_0	damage constant
D	damage
D_c	critical damage
D_s	damage constant
d	diameter
ds	element of arc length
δ	load point displacement
E	Young's modulus
$\bar{\epsilon}^p$	equivalent plastic strain
ϵ_r^p	radial plastic strain
ϵ_θ^p	hoop plastic strain
$\dot{\epsilon}_{yy}$	strain rate
F	fracture fraction
H	notch opening
H_0	initial notch opening
γ	damage constant
J_2	scalar invariant of stress-deviator tensor
J	J-integral
$\bar{J}$	J-integral (non plane strain)
J_{Ic}	critical value of J
λ	damage exponent
L_0	bar 1/2 length

M1	heat-treated material
NDTT	nil ductility transition temperature
ω_1	damage weighting term
ω_2	damage weighting term
P	load
r	radial coordinate, notch root radius
R	radius of curvature
$\bar{R}$	$\epsilon_r^P / \epsilon_\theta^P$
R_C	Rockwell hardness
R_B	Brinell hardness
RNDT	reference nil ductility temperature
σ_z	axial stress
σ_r	radial stress
σ_θ	hoop stress
σ_m	mean stress
$\bar{\sigma}$	equivalent stress
s_1	principal stress deviator
$\bar{T}$	traction vector
$\bar{u}$	displacement vector
u_z	axial displacement at end of bar
V	notch height
V_0	initial notch height
v	pull velocity
W	strain energy density
Y_0	initial yield stress
z	axial coordinate

FUNDAMENTAL STUDY OF CRACK INITIATION AND PROPAGATION

Abstract

We have characterized ductile fracture in nuclear pressure vessel steel using a computer model of material damage. The model predicts crack initiation and growth and contains constants that are set by computer simulation of the following fracture tests: the simple tension test, the circumferentially notched,

round tension test, the blunt-notched compact tension test, and the Charpy V-notch test. The simulations provide the stress and strain states of these tests at fracture. The major goal of our characterization program is to determine the correlation between Charpy toughness and fracture toughness.

Introduction and Summary

The purpose of the fracture program is to characterize ductile fracture, initiation, growth, and arrest in a large pressure vessel. The method of characterization differs from conventional approaches in that it uses a macroscopic computer model of damage accumulation to simulate quasi-static and dynamic crack initiation and growth in SA-533 Grade B Class 1 steel. The undetermined constants of the damage model are evaluated by simulating tests of different specimen geometries on the computer and adjusting the constants so that the model predicts both crack

initiation and growth in all geometries. The calibrated model and necessary coordinate generation routines will be included in the EPRI two- and three-dimensional computer programs for transfer to the pressure vessel industry.

In September 1976, the research described above was redirected at the request of EPRI to concentrate on irradiation damage surveillance. The goal now is to determine the correlation between Charpy V-notch test data and fracture toughness. A second objective is to ascertain if the correlation is satisfactory for

practical applications. A suite of experiments that examines the Charpy test will be performed, supported by computer simulations using a damage model of material failure. The parameters of the damage model will provide the means to connect Charpy data with fracture toughness.

The decision to concentrate on the Charpy V-notch specimen is due to its role as the primary toughness diagnostic specimen in nuclear pressure vessel capsules that monitor radiation damage. Operating reactors are faced with power reduction or shutdowns for thermal annealing due to the reduction in toughness caused by neutron embrittlement. Most of the embrittlement data, the majority of the in-reactor surveillance fracture specimens, and the embrittlement prediction methodology are based upon the standard Charpy V-notch impact test. These Charpy correlations are empirical and only used as bases of comparison. There is no procedure by which Charpy data can be used directly in analysis of crack initiation, propagation, or arrest. Correlations between Charpy and fracture toughness (see, for example, Marandet and Sanz, 1976, Corten and Sailors, 1971, Rolf and Novak, 1970) are empirically derived and are only valid for certain sections of the transition region.

The following highlights summarize the work performed:

- A computer model of fracture initiation accounts for material size effects. The gradient of the damage serves as a measure of the spatial extent of material damage in a manner that minimizes the influence of zone size.
- Compact tension specimen simulations show that load vs load-point displacement is insensitive to details of the near-notch deformation field.
- The Bridgman correction method for smooth tension data overpredicts flow stress at an equivalent plastic strain of 100% by 10%. His formulae underestimate the mean stress at failure by 23% and the hoop and radial stresses by 52%.
- Only 10% of the CVN energy needed to fracture the standard Charpy specimen is required for crack initiation. Fracture initiates at about 40% of the maximum load time of the instrumented striker. Adiabatic heating raises the notch temperature more than 66°C at the time of crack initiation.
- Charpy simulations and tests show that either the crack-opening angle or crack-opening displacement is a good measure of notch equivalent plastic strain.

Methodology

INTRODUCTION

Our goal is to relate irradiated standard Charpy V-notch energy (CVN) to a measure of fracture toughness that can be used to design and analyze nuclear pressure vessels. The measure of fracture toughness selected for this study is the J-integral (Rice, 1968), suggested as a "characterizing parameter" for onset of crack growth by Begley and Landes (1972). For this study J_{Ic} is used only as a correlation; it is not required to be geometry independent.

We relate J_{Ic} to CVN by establishing a relation between J_{Ic} and a critical damage for crack initiation, D_c , and a similar relation between CVN and D_c . This provides the relationship between J_{Ic} and CVN.

The parameter used to establish this relationship is the damage, D , proposed by Wilkins (1977). D is the integrated equivalent plastic strain weighted by the local mean tensile stress and the asymmetry of the deviatoric stress field. Crack initiation takes place (or an existing crack elongates) when $D > D_c$ over some characteristic length associated with the microstructure of the material.

The methodology tested here uses three materials prepared from a plate

of SA-533 Grade B Class 1 steel. The three materials were prepared by heat treating pieces of the plate to different degrees of hardness to obtain a range of toughness similar to that experienced by Charpy surveillance specimens. The CVN- J_{Ic} relationship is to be established for all three materials. The methodology may then be applied to irradiated materials to evaluate reactor irradiation damage.

DETAILS

The program is divided into three separate research tasks repeated for each of the three materials. The objective is a CVN- J_{Ic} relationship over a range of toughness for A533 steel similar to that experienced by in-service irradiated reactor material. The relationship will only be established at 100°C. These three tasks are:

- A. Calibrate the Wilkins damage model - Test four specimens of different geometries to macroscopic failure initiation. Simulate these tests on the computer. By adjusting constants of a damage model obtain a critical damage D_c that predicts onset of fracture in all four geometries.

- B. Relate D_c to CVN - Test to obtain Charpy toughness CVN, and plot CVN vs the critical damage D_c for the three materials. Corrections to CVN may be required.
- C. Relate D_c to $J_{Ic} - J_{Ic}$ is obtained by computer simulation of a test of a precracked, 25.4-mm-thick, compact tension specimen. The critical damage D_c is plotted vs J_{Ic} for the three materials. A second objective of this task is to determine the relationship between J obtained in plane strain from a 2-D compact tension specimen calculation and $\bar{J}$ obtained from a 3-D computer simulation of a 25.4-mm-thick specimen.

CVN and J_{Ic} may then be related by elimination of the parameter D_c . Figure 1 shows the research plan.

The four specimen geometries we have chosen to calibrate the damage model are a 12.7-mm-diam smooth tension specimen, a 13-mm-diam 60°-notched tension specimen, a 27-mm-

diam 60°-notched tension specimen scaled up by two from the smaller specimen, and a 25.4-mm blunt-notched compact tension specimen. The notches all have radii of 0.25 mm. The smooth tension specimen is also used to obtain the strain-hardening law for these materials. Three specimens of each geometry will be tested for each of the three materials.

In addition to the computer simulations required to calibrate the damage model, we are also simulating the Charpy V-notch test in 2- and 3-D to compare the D_c value obtained from the calibration tests with that obtained with those same calibration constants in the Charpy simulation. These calculations also describe the physics of the deformation process of the Charpy V-notch test, and may lead to corrections to CVN and a better estimate of energy for initiation. The J-integral work requires tests of 25.4-mm-thick precracked compact tension specimens to serve as a check on the 2- and 3-D computer simulations.

^a $\bar{J}$ may be obtained from a 1T CT test. J is obtained from a ∞T CT test.
For the materials used here, $\bar{J}$ may equal J .

Fig. 1. CVN/ J_{Ic} research program.

Computer Model for Ductile Fracture

INTRODUCTION

Fracture is usually described under two broad headings: brittle and ductile. Here, brittle fracture refers to a separation of material during elastic deformation, where the separation is normal to the maximum principal stress. Ductile fracture refers to a separation that occurs in the plastic state when certain imperfections in the material have been amplified to a point where a crack starts to propagate.

Loading can have an important effect on whether a material fails in a ductile or brittle fashion. For example, loading the material dynamically (as in the case of dynamic spall) can cause a one-dimensional strain state that is not possible with static loading, other than in large precracked specimens. Under such a load, the material at failure is subjected to high stress and low strain. Because high stress is largely an elastic stress, spall is considered brittle fracture. However, some material subjected to increasingly large tensile stresses in spall-recovery experiments evidence ductile failure. Thus, to adequately describe the spectrum of spall fracture to ductile fracture, a fracture model must recognize the

influence of triaxial tensile stress on shear strain.

The mechanics of plastic hole growth have been studied by McClintock (1971). He emphasizes the role of triaxially elevated stress states in increasing the incremental void growth that results from an increment of plastic strain. These stresses also serve to more readily nucleate voids from inclusions. The combined effects contribute to the possibility of both brittle and ductile crack behavior.

This hole-growth concept can be applied to such diverse fracture phenomena as large deformations at stresses that are low compared to stress required for spall (as in the static tensile test), and very small deformations at high hydrodynamic tension (as in spall). The concepts of ductile fracture by hole growth are convenient in calculating all forms of fracture, with brittle fracture a limiting case.

In an attempt to develop a methodology for ductile fracture, we used the computer to simulate the stress and strain fields at experimentally determined fracture strains for four aluminum tensile specimens. We simulated smooth and circumferentially notched, round tension specimens, a flat-plate specimen, and a

prenecked specimen. Experimental data and information from these calculations were used to establish a damage model for the initiation of fracture. The details of this work are discussed by Wilkins (1977). The results suggest neither mean nor axial stress is important as a fracture criterion. Mean stress is important for describing spall when fracture occurs at tensions several times greater and strain several times smaller than those shown in the above examples. A fracture model based on plastic strain modified by mean stress and by the asymmetry of the loading history might correlate all of the data, including that on spall.

DAMAGE MODEL

In the model proposed here, fracture is assumed to be a result of the history of the strain damage to the material. The strain damage is accelerated when

- A hydrostatic tension field is present
- Stress loading is asymmetric

The first damage-enhancement factor can account for fracture by spallation when the loading conditions are large triaxial stress and small strain. The second damage-enhancement factor can account for

the observation that the elongation before failure decreases as the shear load increases for fracture tests with combined-stress loads. This result was also noted in the work of Mogi (1973), who studied the effect of the intermediate stress on the fracture of rocks. Mogi's experimental data show that elongation decreases when tensile fracture occurs with confining stresses that produce asymmetric strains.

The simplest expression for incorporating the concepts described above is given by:

$$D = \int \omega_1 \omega_2 d\epsilon^p, \quad (1)$$

where

D = damage

ω_1 = triaxial stress weighting term; i.e., $\left(\frac{1}{1 - \gamma \sigma_m}\right)^\alpha$

ω_2 = asymmetric-strain weighting term; i.e., $(2 - A_D)^\beta$.

The term A_D can be set equal to

$$\min \left\{ \left| \frac{\dot{\epsilon}_2}{\dot{\epsilon}_3} \right|, \left| \frac{\dot{\epsilon}_3}{\dot{\epsilon}_2} \right| \right\},$$

or from the associated flow law

$$A_D = \min \left\{ \left| \frac{s_2}{s_3} \right|, \left| \frac{s_3}{s_2} \right| \right\},$$

where α , β , and γ are constants determined by simulating tests on the computer.

CHARACTERISTIC LENGTH

Ductile crack growth takes place over the dimension of the material microstructure. An objective of experimental measurements in the study of fracture is to relate large scale observation to nonmeasurable small scale processes. Obviously, the experimental measurement should be made over the smallest length possible at the position where the fracture is to occur. Crack opening displacement measurements and tension test strain measurements at the neck are required to correlate experimental data with calculations.

Linear fracture mechanics has successfully correlated large scale observations with implied, unobservably small stress and strain conditions. Computer programs that simulate large plastic deformation phenomena can extend the analysis provided by linear fracture mechanics to plastic work-hardened states. Thus, large-scale observations can be related to stress and strain conditions on a scale closer to that at which the physical phenomena are taking place. Models for ductile fracture can then be developed from the calculated stress and plastic-strain states near the fracture. The practical modeling approach is to consider the material a continuum and to calculate implied microstructural

behavior from calculated stress and strain histories. It is not the intent here to model the microstructure of the material.

The observed behavior attributed to a material size effect can be conveniently modeled by using the magnitude of the gradient of the damage parameter associated with fracture. The gradient serves as a measure of the spatial extent of the damage parameter. The critical magnitude of the damage required for fracture is assumed to increase as the magnitude of the gradient of the parameter increases. The effect is to require the damage to reach some limiting value over some characteristic length in the material.

FRACTURE INITIATION

Fracture is initiated when the strain history of an incremental volume of material results in an accumulation of damage such that

$$D/D_c > 1. \quad (2)$$

Here, D is the damage calculated from Eq. (1) and D_c is the critical damage:

$$D_c = D_0(1 + b|\nabla D|^\lambda). \quad (3)$$

In Eq. (3) $|\nabla D|$ is the magnitude of the gradient of D , and D_0 , b , and λ are constants. The constant D_0 is obtained from experiments where

fracture occurs with $\nabla D = 0$, such as the smooth tension test. The constants b and λ may be obtained from experiments where fracture occurs with a large gradient, such as in sharp-notched tensile tests.

The fracture condition given by the inequality (2) and Eq. (3) minimizes the effect of finite zone sizes in the finite-difference programs for geometries where fracture occurs with large gradients in the damage D . Equation (3) by itself does not give a critical length over which damage must occur to cause fracture, but the combination of the inequality (2) and Eq. (3) implies that damage must extend over some finite length before fracture can occur.

FRACTURE PROPAGATION*

When damage exceeds a critical value, material parameters such as flow stress and elastic moduli become affected. As damage further increases, the material loses load-bearing ability across a section as though voids were present until it ultimately separates completely and loses all load-bearing ability. It is convenient to define a fracture fraction F to represent the degree

that damage in excess of some critical damage modifies the zone material properties:

$$F = (D - D_c)/D_s,$$

where

$$0 \leq F \leq 1 \text{ and } D_s = \text{constant.}$$

To reduce the flow stress, elastic moduli, and the stress normal to the fracture in the program, we use F as the multiplier $(1 - F)$. When fracture occurs in an incremental volume, the reduction of stress by the multiplier permits the surrounding stress/strain field to further damage the volume that has exceeded the critical damage. Thus the local stress/strain field supplies the energy to complete the fracture of a given volume.

The direction of the fracture is considered to be governed by the local state of stress. This state is determined by the triaxiality, the ratio of $\sigma_m/\bar{\sigma}$. If the local state of stress is tension, the fracture is assumed to be perpendicular to the maximum principal stress. If the local state of stress at fracture is shear, the fracture is assumed to be at an angle to the maximum principal stress. (Since there are two possible shear angles, the computer uses the sign of a round-off number to decide.) That is, when $\sigma_m/\bar{\sigma} > c$, fracture is perpendicular to the

* Fracture propagation is included here for completeness. No use is made of the method in this report.

maximum principal stress, and when $\sigma_m/\bar{\sigma} < c$ the fracture is at an angle to the maximum principal stress. The term c is a constant.

The definition of a crack is:

- Normal and shear stresses are zero for tension
- Shear stress increases with increasing compressive normal stress across the crack (Coulomb-friction)
- Tangential stress is modified by a Poisson effect.

These boundary conditions are introduced into the computer simulation program in three ways:

- Ignore the direction of the crack at the level of the zone size and reduce the stresses to zero by the multiplier $(1 - F)$. This corresponds to isotropically pulverizing a local region of the material and, with the use of very fine zones, gives a good physical model.
- If the direction of the crack is known in advance, provide a double set of grid points in the fracture plane. The grid can then be opened and a free-surface boundary condition applied. The resulting acceleration is then multiplied by F .
- After the crack direction has been determined, rotate the

stress tensor each time step into the coordinate system of the crack. The boundary conditions outlined above can then be applied by way of the multiplier $(1 - F)$. Then rotate the stresses back to the calculational coordinate system to advance the material motion in time. When $F = 1$, the stress state at the zone will have the crack-boundary conditions described above in terms of the calculational coordinate system. Reducing the stress normal to the crack to zero implies the creation of a void. The zone volume used in the equation of state to calculate the pressure must be reduced accordingly.

We plan to use a combination of the first two methods to simulate shear lip failure in three dimensions.

COMPUTER PROGRAMS

The computer programs HEMP and HEMP3D (see Wilkins, 1969, 1975) are used to analyze stress and strain fields in specimens during fracture. The programs use explicit finite-difference methods with second-order accuracy to solve the equations of continuum mechanics in two and three space dimensions and time. The programs incorporate nonlinear models

of material behavior ranging from linear elasticity to large deformation plasticity to calculate strain field. The plasticity obeys J_2 flow

theory. Isotropic work hardening is described by an effective stress-equivalent plastic strain curve obtained from experiment.

Material Description

NORMAL-STATE MATERIAL (M1)

We have tested and simulated SA-533 Grade B Class 1 steel plate, a manganese-molybdenum-nickel alloy steel used for nuclear pressure vessels. All specimens were machined from a nozzle dropout of a nuclear pressure vessel fabricated by Combustion Engineering Corporation (CE). The material has archival designation CDB. The properties are described in a CE report by Marston, Borden, Fox, and Reardon (1975).

The CE tensile data give a yield strength of 441 MPa (64,000 psi), an ultimate strength of 602 MPa (87,300 psi), 26% elongation, and 65% reduction in area. Figure 2 shows the sensitivity of these data to temperature. Properties determined at LLL are in agreement with the results of CE.

The fracture toughness properties are $NDTT^* = -18^\circ\text{C}$ (0°F) and $RNDTT = -1^\circ\text{C}$ (30°F). Upper-shelf Charpy energy is about 136 J (100 ft-lb) in the transverse orientation and 190 J (140 ft-lb) in the longitudinal direction. CE reports upper-shelf fracture toughness, obtained from instrumented precracked Charpy tests, of about $283 \text{ MPa} \sqrt{\text{m}}$ ($260 \text{ ksi} \sqrt{\text{in}}$).

* See Symbols.

Fig. 2. Variance of A533B tensile behavior with temperature [Marston et al. (1975)].

The computer simulations used a Young's modulus of 211 GPa (30×10^6 psi), a Poisson's ratio of 0.29, and a yield stress of 458 MPa (66,400 psi). The flow law assumed isotropic hardening with a universal stress-plastic strain curve relating the effective stress (measuring the size of the yield surface) and the integral of the equivalent plastic strain increment. These data were obtained by testing to failure smooth tensile specimens with the

above geometry. Figure 3 gives the flow curves used for the calculations. The chemistry of heat CDB was established by Marston et al. (Table 1).

We found the material contained many millimeter-size inclusions with some up to a centimeter in length. Before sectioning, we subjected the archival plate to x-ray and ultra-

sonic inspection. A large seam of foreign material was detected in a face-parallel plane midway through the thickness. This material was discarded. A number of specimens were rejected after ultrasonic testing and radiography showed unacceptably large flaws relative to specimen size.

Table 1. Chemical analysis of heat CDB for SA-533 Grade B Class 1 steel plate.

Element	ASME requirements	wt%
C	0.25 max	0.22
Mn	1.10 - 1.55	1.37
P	0.035 max	0.009
S	0.040 max	0.015
Si	0.13 - 0.32	0.23
Ni	0.37 - 0.73	0.59
Cr		0.18
Mo	0.41 - 0.64	0.60
Al(soluble)		0.35
Al(insoluble)		0.002
As		0.011
B		0.0008
Cb		<0.01
Co		0.008
Cu		0.12
N		0.008
O		0.003
Pb		<0.001
Sb		0.0018
Sn		0.008
Ti		<0.01
V		0.004
W		<0.01
Zr		<0.001

Fig. 3. Comparison of effective stress-equivalent plastic strain curves used for the computer simulations.

HEAT-TREATED MATERIALS (M2 AND M3)

We have finished testing the A533 material in its normal state (M1). To determine the tensile properties of the material in its heat-treated state (M2 and M3), we have tested smooth, round, 12.7-mm-diam tensile specimens. The M2 heat treatment brought the material to a Rockwell hardness (R_C) of 34 and the M3 heat treatment resulted in a R_C of 46; the virgin material had a Brinell hardness (R_B) of 90.

The materials M2 and M3 were heated to $1600^\circ\text{F} \pm 10^\circ$ for 1 hr in a neutral atmosphere and water quenched. M3 was tempered at $400^\circ\text{F} \pm 10^\circ\text{F}$ for 4 hr and water quenched. M2 was

tempered at $1000^\circ\text{F} \pm 10^\circ\text{F}$ for 4 hr and water quenched. All the specimen blanks were heat treated before final machining.

Figure 4 compares the load-extension (50.8-mm gauge length) behavior of the original material (state M1) with the two heat treatment states (M2 and M3). Table 2 summarizes the data.

SPECIMEN PREPARATION

All specimens in this program were prepared from a SA-533 Grade B Class 1 steel plate (Fig. 5) supplied by Combustion Engineering. The material properties of the specimens cut from the plate were evaluated as a function of plate thickness and orientation. The specimens used in the simulation portion of the program were also cut from the plate.

Fig. 4. Effect of heat treatment on tensile properties.

All simulation specimens were cut so that the notch (or crack) was loaded in the short transverse direction. The exception was smooth tension specimen 2499, whose sym-

metry axis was in the rolled direction. The detail drawings of specimen location in the archival plate will be included in the final report.

Table 2. Tensile behavior of SA-533 Grade B Class 1 steel for three heat treatments.

Specimen	Yield stress, GPa (ksi)	Ultimate stress, GPa (ksi)	Reduction in area, %
2499 (M1)	0.458 (66.4)	0.621 (90)	64
2614 (M2)	0.941 (136)	1.06 (153)	28
2587 (M3)	1.190 (172)	1.50 (218)	2

Fig. 5. Archival plate.

Computer Simulation of the Tension Test

SUMMARY

We calculated the necking deformation of a round tensile bar of SA-533 Grade B Class 1 nuclear pressure-vessel steel. Results compare favorably with experiment up to the time of fracture. The computer analysis gives the macroscopic stress-strain state in this material at fracture and provides an accurate method for obtaining a plastic flow curve at large strain. The stress-strain state is compared with the results obtained by others.

INTRODUCTION

We computed the necking deformation of a smooth tensile specimen with the following goals in mind:

- Obtain the stress-strain state at failure
- Calibrate a computer model for ductile fracture
- Obtain an improved flow-stress curve that is valid for large-deformation, computer simulations.

Such data are unavailable from direct experimental measurements but may be obtained by simulating the test on a computer. The insight gained from the computer calculation allows corrections to the measured flow

stress curve. In all simulations we assumed that the material properties were the same as those of SA-533 Grade B Class 1 nuclear pressure-vessel steel.

The tension test is a convenient way to obtain mechanical properties of a material. The test is characterized by an instability that occurs at the limit load and that results in localized necking. This happens when the rate of increase of stress resulting from strain hardening is less than the rate of decrease of neck area. It is this necking instability that complicates analysis of test results and calculation of the stress/strain state.

The calculational problem is non-linear, both in the constitutive equation and in the large deformation and rotation that occur at necking. Elongation of an element at the neck may reach 400% at failure; rigid-body zone rotation approaches 30°.

Chen (1971) and Needleman (1972) made calculations similar to those presented here. To start necking, Chen used a tapered bar, as we do, and Needleman used a uniform bar and solved an eigenvalue problem, starting from a bifurcation from a state of uniform stress. Our calculations, which were successfully carried to a greater neck deformation

than those above, result in stress distributions that differ from these earlier results.

Our calculations show that at failure the stress/strain state for the minimum cross section of the neck varies substantially over the radius of the bar. The axial stress on axis reaches three times the initial yield stress and is 37% lower at the neck surface. This difference increased with elongation. We find all stresses and strains at the minimum cross section of the neck increase smoothly from the surface to the axis of symmetry of the specimen. Axial and mean stress decrease along the axis of symmetry and away from the neck to a compression of $0.458 Y_0$ and $0.926 Y_0$, respectively. The effective plastic strain at failure is 100% on axis and 19% lower on the surface. Our results differ by as much as 52% from those given by Bridgman (1964), who assumed (with some experimental justification) that the axial strain

was constant across the minimum section.

In this work, we discuss the computer model in more detail, compare the calculated results with experiment, and present the stress/strain state in the bar at failure. Comparisons are made with solutions given by Bridgman (1964), Davidenkov and Spiridonova (1946), Chen (1971), and Needleman (1972). Based on computer results, we suggest an improved method for correcting uniaxial test data to obtain a universal flow stress curve that is valid for large strains.

COMPUTER MODEL

The zoning for the simulation, after necking has taken place, is shown in Fig. 6. Only one-half of the axially symmetric geometry is used, with appropriate boundary conditions on axis and across the center of the specimen. The initial shape

Fig. 6. Calculated neck at 59% reduction in area. Note the variety of zones used in the simulation. The units are in cm.

is a tapered cylinder with a maximum diameter of 12.83 mm and a minimum diameter of 12.70 mm. The total length of the model is 53.34 mm, but only one-half is considered in the calculations. We used 255 zones and a 2 to 1 zone-size change at 7.62 mm from the symmetry plane. Some calculations were stopped near the ultimate strength, and additional zones were added in the neck region. The material description is given in the next section.

We used an iterative calculational procedure to convert the test data for average neck stress to point data for true stress at strains greater than 10%. The Bridgman (1964) correction was applied to test data for the first calculation. An improved correction for flow stress test data is then available from the computer calculation. The average axial stress at the cross section of the neck is converted to equivalent stress at the neck surface by using the calculated ratio of $\bar{\sigma}$ at the surface to average axial stress.

The equivalent plastic strain at the neck surface is obtained by measuring neck surface hoop strain and again using computer results to convert to equivalent plastic strain. The final iteration shows that the corrected flow stress at a natural strain of 1.0 is approximately 10% lower than that obtained by the

Bridgman (1964) correction procedure. Details of the correction are given in Appendix B.

The resulting flow curve is approximately a bilinear function of natural strain and is accurately fitted by using a ratio of a cubic to a quadratic polynomial in equivalent plastic strain. This material is not well represented by a power-law fit.

Specimen loading was simulated by moving the end of the computer model at a constant velocity of 3 or 6 m/s. We chose the higher velocity because the two calculations showed only small differences in the stress/strain and deformation states. A high pull velocity is necessary to reduce computer run time. These rates are five orders-of-magnitude higher than those used in conventional quasi-static testing. For a plane wave in a bar, 6 m/s corresponds to a stress of about 53% of yield for this material.

Two damping viscosities were included in all calculations to assure stability of the differencing scheme and to inhibit a keystone grid distortion. To check the effect of the first and larger of these two viscosities (CANS = 0.125),* we also ran one final calculation with a damping viscosity that was 60% lower. The effect was to increase the

* See Giroux (1973).

Fig. 7. Comparison of calculation and experiment: (a) neck radius vs elongation for a 50.8-mm (2-in.) gauge length (points 1 and 2 refer to computer edits discussed in text), and (b) normalized load vs neck radius (maximum load = 79 180 N).

necking action. At 950 μ s and with the higher damping, the calculated neck radius was the same as that obtained with the lower damping at 925 μ s. At 950 and 925 μ s, neck stress and strain differed by less than 1.7%. The neck profiles at these two times were identical. The keystone control used was $CAQ = 0.001$.

COMPARISON WITH EXPERIMENTS

The computer results are compared with the experiments in Figs. 7 through 9. In Appendix B, calculated ratios of neck radius, a , to neck radius of curvature, R , are compared to experimental data at various stages of necking. Test temperature

Fig. 8. Comparison of calculated and experimental load vs engineering strain for a 50.8-mm (2-in.) gauge length.

Fig. 9. Calculated grid superposed on photograph of specimen 2499R; $a/a_0 = 0.646$.

and crosshead speed are given in Table 3. Flow stress data from specimen 2499R were used for all calculations reported here.

We found the experimental material contained many millimeter-long inclusions, some being up to a centimeter in length. A number of speci-

mens were rejected because ultrasonic testing and radiography showed unacceptably large flaws relative to specimen size. Table 3 data are representative of a large number of tests performed on this material. However, because of the inhomogeneity of the material, more testing would

Table 3. Tension test experiments.

Specimen *	Crosshead rate, mm/s (in./min)	Specimen temperature,	
		°C	(°F)
2499R	0.0212 (0.050)	21	(70)
2515ST	0.0212 (0.050)	100	(212)
2501R	0.0212 (0.050)	71	(160)
2502R	0.212 (0.500)	71	(160)

* R is the rolled direction; ST is the short transverse (plate thickness) direction.

be required to obtain a valid statistical sample.

In Fig. 7a, the calculated and experimental results for neck radius vs elongation at a 50.8-mm gauge length are compared. Relatively small variations were obtained in this curve, whether calculations were done at a pull rate of 3 or 6 m/s. The curve was also insensitive to the different flow laws used in the iterative procedure discussed above. The curve is largely determined by the incompressibility of the plastic flow. Load vs neck radius (Fig. 7b) is perhaps a more sensitive measure of the accuracy of the simulation. The left portion of this curve represents the unloading portion of the engineering stress/strain curve.

The simulation of the necking instability is shown in Fig. 8, a plot of load vs elongation. To simulate this turn-over, the slope of the true stress/natural strain curve must be equal to the true stress at maximum load. For a power-law fit^{*} the strain-hardening exponent must be equal to strain at maximum load. The load-elongation curve at large strains is sensitive to this hardening exponent. The flow-stress curve used for the simulation is a monotonically increasing function of

equivalent plastic strain. Figure 9 shows the calculated grid superposed on the photograph of the deformed test specimen.

STRESS/STRAIN STATE AT FAILURE

We selected a time both before and after breakage of specimen 2499R and plotted the computed stress/strain contours and profiles at the minimum cross section of the neck. These two times, labeled "1" and "2" in Fig. 7a correspond to the times of the stress profiles of Figs. 10a and b, respectively, and to the strain profiles of Figs. 10c and d, respectively. Rezoning, which was done early in these calculations, made three zones out of the zone closest to the minimum cross section of the neck. The calculated, zone-centered, stress/strain data were extrapolated smoothly to the axis and to the surface. We did not extrapolate the data axially.

The axial stress distribution near the neck is shown in the contours of Fig. 11a for $a/a_0 = 0.540$. An almost spherical reduction of axial stress exists at the neck. A tear-drop-shaped volume adjacent to the minimum cross section of the neck is in compression, causing the tensile loads to be carried by the outer region of the specimen. The mean stress distribution is similar (see

* A polynomial ratio was used in these calculations.

Fig. 10. Calculated neck stress and strain profiles: (a) stress at the neck before fracture of the specimen, $a/a_0 = 0.627$; (b) stress at the neck beyond time of experimental fracture, $a/\epsilon_0 = 0.540$; (c) strain across the neck before time of experimental fracture, $a/a_0 = 0.627$; and (d) strain across the neck beyond time of experimental fracture, $a/a_0 = 0.540$, $Y_0 = 0.458$ GPa.

Fig. 11. Contours for stress and strain: (a) calculated contours of axial stress, (b) contours of mean tensile stress, and (c) contours of equivalent plastic strain. In all cases, $a/a_0 = 0.540$. In (a) and (b), contour interval = 0.1 GPa, $Y_0 = 0.458$ GPa, and the lowest contour defines the compressive region.

Fig. 11b). The compression zone for axial stress starts at $z = 5.96$ mm, with a minimum stress at $z = 8.32$ mm and with a value of $0.458 Y_0$. The mean stress goes compressive at $z = 3.75$ mm and is minimum at $z = 7.76$ mm, with a value of $0.926 Y_0$. Contours of equivalent plastic strain are given in Fig. 11c).

Profiles of axial stress across different sections of the bar are given in Fig. 12. The stress at approximately 10 mm above the neck varies from 40% of yield in compression on axis to 163% of yield in tension on the surface. This compressive zone has been discussed earlier by Argon, Im, and Needleman (1975).

DISCUSSION

We performed these calculations to obtain an accurate stress/strain state at the time of fracture of this widely used, nuclear pressure vessel steel. The tension-test geometry is particularly suitable because of the absence of sharp notches (such

Fig. 12. Calculated profiles of axial stress at different positions in the bar: at the neck, and at 2.6, 9.3, and 25.0 mm above the neck. For these, $a/a_0 = 0.540$.

notches require small zones to capture the large strain gradients). All stresses and strains have low gradients on axis, where they are maximum and where the material starts to come apart.

The mechanisms of this fracture process have their origins at the microscopic level. Voids initiate on inclusions in the triaxial tensile field and grow and coalesce. The local stress concentrations associated with these minute discontinuities are responsible for the local failure. At large strains, the center of the specimen becomes pock-marked with holes as seen, for example, in photomicrographs by Puttick (1959).

In calculations presented here, we ignored the growth of these holes and assumed continuum behavior. The flow law represents a combined response of the continuum and the damage associated with growth and coalescence of these holes. The calculated response is that of the composite material. This in no way detracts from the usefulness of the flow-stress curve, as shown by the agreement of the tensile calculations with experiment.

We axially sectioned specimen 2499R after failure. Holes and inclusions represent less than 2% of a transverse line across the neck of the specimen. We found no obvious

correlation of void density with the stress contours of Figs. 11a and b or with similar contours of strain, although we have not studied the sectioned specimen in detail.

The calculations reported here have been carried out to a radius ratio (a/a_0) of 0.540, which represents a state just past the fracture of specimen 2499R. No difficulties were experienced, although zones had to be added as the neck developed to obtain accurate zone-centered data and to continue to satisfy the zero-normal-stress boundary condition on the specimen surface. The problem arose from the large aspect ratio that developed in the neck zones. Chen (1971) experienced similar difficulties but continued his calculation to $a/a_0 = 0.467$. Needleman (1972) stopped his calculation much earlier at $a/a_0 = 0.769$ when the calculational procedure broke down.

Our results are generally similar to those of Chen (1971) and Needleman (1972) but differ in detail. The differences between the three calculations are shown in Fig. 13. We selected published data at approximately the same neck radius ($a/a_0 \approx 0.77$) and normalized the axial stress by the yield stress. The curves are remarkably similar although the shapes are distinctly different. For radius ratios greater than 0.89, the plot of the data of

Fig. 13. Comparison of our calculated axial stress profiles with earlier calculations of Chen (1971) and Needleman (1972).

Chen and Needleman exhibits the same smooth* nature as our results but gradually develops the shapes of Fig. 13. In Needleman's calculation, the development of the sharp stress peak on the axis of the bar and, in Chen's calculation, the shifting of the peak stress off the axis of the bar both increase during the latter stages of necking. All stress components behaved in this way, but the strains have the same smooth form as in our calculation.

The general agreement of the three calculations is surprising in view of the differences in the constitutive laws employed. The Chen calculation used a Ramberg-Osgood hardening law; Needleman used a piece-wise power law. A hardening exponent of eight

*Careful inspection of our stress profiles show a minute dip at about one-half the radius of the neck.

was used in both calculations. Our flow curve employed data fitted to a polynomial, but when the data were

Fig. 14. Comparison of our calculated stress data with those obtained by the formulae of Bridgman (1964) and of Davidenkov and Spiridonova (1946). The a/a_0 ratio for (a) is 0.627 and for (b) is 0.540. In both cases, the yield stress is 0.458 GPa.

approximately fitted to a power law, a similar exponent was obtained.

The results of Bridgman (1964) and of Davidenkov and Spiridonova (1946) are displayed together with our results in Figs. 14a and b. Their formulae require a knowledge of the neck radius, the neck radius of curvature, and the load at each stage of the deformation history. These factors are supplied by our calculations.

A summary of calculated stress values on axis for four deformation states are compared in Table 4 with the Bridgman results. His formulae underestimate by 23% the triaxial tension in the neck at specimen failure. Our calculated hoop and

radial stresses are 52% higher than Bridgman's values on axis. However, if we apply the Bridgman correction procedure and also use measured values of a/R , the flow curve is only 10% lower, at 100% strain, than our calculated flow curve.

In Appendix B, the calculated values of a/R (given in Table 4) are compared with experimental data.

It appears that at 20°C this steel fails under some combination of the following:

- Macroscopic equivalent plastic strain of 1.0
- Axial stress of $3 Y_0$
- Triaxial tensile stress of $1.5 Y_0$.

Table 4. Ratio of calculated to Bridgman formulae stress values on axis.*

u_z/L_0	$\ln(A_0/A)$	a/a_0	a/R	$\sigma_z/\sigma_z^\dagger$	$\sigma_m/\sigma_m^\dagger$	$\sigma_r/\sigma_r^\dagger$	$\bar{\sigma}/\sigma^\dagger$
0.180	0.511	0.775	0.273	1.09	1.24	1.92	0.980
0.203	0.699	0.706	0.324	1.11	1.30	2.07	0.957
0.225	0.933	0.627	0.490	1.10	1.28	1.78	0.950**
0.248	1.23	0.540	0.755	1.09	1.23	1.52	0.955**

* Bridgman formulae evaluated using our calculated values of a/R .

† Primed values refer to Bridgman formulae.

** Surface value is 0.908.

Round, Notched Tension Bar Simulations

SUMMARY

We have computer simulated tests of round, notched tension bars in two diameters. The simulated specimens were fabricated from SA-533 Grade B Class 1 nuclear pressure vessel steel. The specimens were geometrically similar, except for their notch root radii, which were identical. The simulations showed that the plastic stress/strain state at failure was similar at the two notches: In the larger specimen, which had a greater r/d ratio, the maximum equivalent plastic strain at the notch root was 0.87 ± 0.10 and the maximum principal stress below the notch surface was 1.35 ± 0.05 GPa. For the smaller specimen, the maximum equivalent plastic strain was 0.93 ± 0.10 and the maximum principal stress was 1.30 ± 0.05 GPa.

INTRODUCTION

We have simulated two round-notched tension tests to set constants in the Wilkin's damage model. The tests were chosen for simulation because they provide the time of crack initiation and a continuous record of the applied loads and deformation state. These records are a check on the accuracy of the com-

puter simulations. In turn, the simulations provide the stress and strain state at fracture initiation. These states are used with a post-processor to set the damage model constants so that the critical damage, corresponding to crack initiation, agrees with the notch experiments.

Round-notched tension tests have proved useful for qualitatively screening material toughness, as have the Charpy V-notch and Izod tests. Failure cases reported by Sachs (1954) illustrate the value of the test. He attributed the brittle fracture of B-36 landing gear to the use of steel that had high unnotched tensile properties but whose notched tensile strength was low.

Round-notched tension tests have also been investigated by Sachs and Lubahn (1945), Neuber (1946), and Ripling (1956). These earlier investigators described the effects of geometry on the notch strength (average stress at failure) and determined the stress and strain distribution in the plane of the minimum cross section for an elastic bar. Clausing (1969) measured the plastic strain distribution and presented an approximate stress distribution based on Bridgman's analysis. The latter is valid for

notches with a large profile radius. Earl and Brown (1976), using notches of various profiles, found that the strain distribution depends, to some extent, on the degree of straining and strongly depends on notch geometry. This distribution cannot be described by the ratio of the minimum radius to the profile radius alone. Thus, in the next section, we calculate notches with a much lower notch radius to diameter ratio than those described above. We then compare the results with experiment, and present the stress/strain state in the bar at failure.

COMPUTER MODEL

Figure 15 shows the zoning for the two simulations. Only one-half of the axisymmetric geometry is used, with appropriate boundary conditions on axis and across the symmetry plane. The initial shape is a notch with an angle of 60° ; the notch depth results in a 50% reduction in area in both simulations. The outside diameter of the large specimen is 27 mm and its notch ratio is 0.013; for the small specimen the outside diameter is 13 mm and its notch ratio is 0.028. The notch root radius is 0.25 mm in both simulations and is represented with three zones. We used 3800 zones for the large specimen and 1700 for the small one. A

transition line 9 mm from the center of both specimens allowed a 2:1 zone size change, with a finer zoning at the notch root.

The large specimen was simulated on the CDC STAR computer with a table-look-up for the plastic flow curve. The small specimen was simulated on a CDC 7600 computer and had a continuous piece-wise smooth flow curve closely approximating the table-look-up (see Fig. 3). The flow stress curve was obtained from corrected tensile test data for the same material.

Specimen loading was simulated by moving the end of the computer model at a constant velocity. We chose a velocity of 6 m/s. Calculations at 3 m/s showed only small differences in the stress/strain and deformation state.

COMPARISON WITH EXPERIMENTS

We used a standard load cell, extensometers, and clip gauges as well as high-resolution photography to record the notch opening, depth, and general deformed shape. Four specimens were tested at 100°C , with a crosshead speed of 0.0212 mm/s. Table 5 summarizes the results.

The load vs elongation curves for the two small specimens are compared to the computer calculations in Fig. 16a. The oscillations of the

(a) Large specimen; $r/d = 0.013$.

(b) Small specimen; $r/d = 0.023$.

(c) Large specimen

(d) Small specimen

(e) Large specimen

(f) Small specimen

Fig. 15. Zoning for the computer-notched-tension simulations:

- a) + b) large + small specimen zoning (cm)
- c) + d) large + small specimen initial notch zoning (cm)
- e) + f) large + small rezoned specimen deformation state at notch (cm).

Table 5. Round-notched tension tests.

Specimen	Outside diameter, mm	r/d	Ultimate load, N
2417 A	27	0.013	2.33×10^5
2417 B	27	0.013	2.33×10^5
2417 D	13	0.028	5.4×10^4
2417 F	13	0.028	5.4×10^4

Fig. 16. Load vs elongation: (a) Small specimen, gauge length = 50.8 mm; (b) Large specimen, gauge length = 50.8 mm.

calculated load at early times represent a dynamic transient response which is caused by the high simulated pull rate. The frequency of these oscillations coincides with the natural frequency of the specimen. We have checked the effect of these oscillations by halving the 6 m/s

pull rate; we find no effect on the residual plastic field, although the amplitude of the oscillations decreases. The calculated plastic strain at the notch (Fig. 17) is a linear function of bar-end displacements and suggests these early oscillations do not lock spurious plastic

Fig. 17. Equivalent plastic strain and maximum principal stress at the notch surface of the small specimen vs bar end displacement.

deformation into the specimen. The calculated load for the small specimen is 10% higher than the experiment.

A comparison of load vs elongation for the larger specimen is shown in Fig. 16b. There is good agreement between the measured and the calculated load. We see no evidence of necking and attribute the decrease in experimentally measured load to the existence of cracks.

We find the average value of the calculated load for both specimens is monotonically increasing. This is in contrast with the experiment, which shows a decrease in load before separation. A reduction in load with elongation can be caused by necking, as is observed in a uniaxial tension test, or by cracking that can relieve the load. We believe the decrease is due to cracking because any significant necking would be pre-

dicted by the simulation and would reveal itself as a decrease in the calculated load.

Plots of notch depth vs notch width (Fig. 18) were obtained by time sequence photography during the tests. The calculation predicts that the small specimen will show more reduction in area for the same notch opening. More constraint in the larger specimen limits area reduction. The notch opening vs notch width tracks the small specimen simulation. The agreement between the measured and calculated depth vs width for the large specimen is poor. Figure 19b is a photograph of the large diameter specimen after failure. A magnified section of the notch (Fig. 19a) is shown just before separation.

RESULTS

The computer edit that best matches the deformation state of failure corresponds to a time of 100 μ s for both simulations. Figure 16 compares the simulated elongation at 85, 100, and 115 μ s with the small specimen tests. If failure starts at maximum load then the two small specimens failed at 85 and 115 μ s. The failure initiation estimate of 100 μ s yields a calculated notch strain that differs from those based on the experimental fracture times obtained from Fig. 16.

Fig. 18. Notch depth vs notch width.

(a) Surface crack at notch root.

(b) Fracture surface

Fig. 19. Failure of specimen 2417A, 27 mm diam.

Fig. 20. Notched tension stress profiles across the radius: (a) 13 mm diam; (b) 27 mm diam.

The surface strains at 85, 95, and 110 μ s are 0.77, 0.83, and 0.90, respectively. These strains were computed by calculating the natural strain of the line at the notch root surface.

Stress profiles at failure in the plane of minimum cross section for the two simulations are shown in Fig. 20. All components of stress have their maximum value at a distance 0.6 mm below the surface, approximately twice the current root radius. The radial and hoop stresses are equal in the notch plane from the axis-of-symmetry to 70% of the radius. This is in contrast to results reported for blunt notches by Earl and Brown (1976). The plastic strain gradient is large near the notch root

Fig. 21. Equivalent plastic strain profile near the notch.

(Fig. 21). Contours of stress and strain near the notch at failure are shown in Fig. 22.

In Fig. 23 we plot the axial stress vs normalized radius for the

(a) Small specimen

(b) Large specimen

(c) Small specimen

(d) Large specimen

(e) Small specimen

(f) Large specimen

Fig. 22. Comparison of stress and strain state in 13-mm diam (left) and 27-mm diam (right) round-notched tension specimens at failure: (a,b) Equivalent plastic strain; (c,d) Axial stress; (e,f) Mean stress. Stress units are GPa.

two simulations. The small specimen, having a relatively blunter notch, shows less stress concentration and a larger average stress. This is in agreement with experimental results published by Wundt (1965) on the decrease of strength with increasing size for circumferentially notched specimens. Notched strength is de-

fined as average stress at failure. The simulation predicts a notch strength of 1.03 GPa (150 ksi) for the small specimen vs 0.883 GPa (128 ksi) for the large one, a 17% increase in notch strength for a decrease in size by a factor of two. Table 6 summarizes the stress state at failure in both specimens.

Fig. 23. Axial stress vs normalized radius at failure.

FAILURE COMPARISONS

Crack Initiation

Photographs taken during the large-specimen experiment show that a sharp crack develops at the notch root and grows approximately 0.3 mm deep before the specimen breaks. As noted in the experiment, the surface of the specimen begins to crack at the maximum load of Fig. 20. In comparison, the small specimen fails with no evidence of surface cracking. The experiments also showed that the

Table 6. Stress state at failure of round-notched tension specimens.

Specimen		$\bar{\epsilon}^P$	σ_m/Y_0	$\sigma_m/\bar{\sigma}$	σ_x/Y_0
Large	extrapolation to notch surface	0.87	1.07	0.55	2.18
	0.6 mm below notch surface	0.07	2.10	1.5	2.95
Small	extrapolation to notch surface	0.93	1.00	0.50	2.07
	0.6 mm below notch surface	0.08	1.90	1.3	2.84

smaller specimen is more susceptible to failure from a given size crack than the larger specimen. Proof of this can be seen in Fig. 21, which shows that the notch root plastic strain is a function of the root radius and is independent of the size of the specimen (see also Chippenfield et al., 1973). This means that a crack moving through the equal regions of intense plastic strain would cause the smaller specimen to have a larger reduction in cross-sectional area. A large reduction in area would cause the load to be carried by the small remaining ligament, which would further enhance failure and make the crack unstable for the small specimen.

It is important to determine the precise time of crack initiation so that the stress/strain state may then be determined from the computer simulation. The larger specimen begins to crack externally when the experimental load reaches a maximum. The three photographs of the smaller specimen, taken at different elongations after maximum load, showed no external cracking. Thus, we have assumed that cracks start in the smaller diameter specimen when it is subjected to maximum load. Interrupted elongation tests should be done to describe the phenomenology.

Plastic Deformation

The computer simulations show that the plastic deformation history of the two specimens differ. The plastic strain starts at the notch root of the small specimen and progresses toward the axis at an angle of approximately 45° . The large specimen forms two plastic nuclei, one at the notch root, the other on axis. Both nuclei grow and merge (Fig. 24). Southwell and Allen (1950), using a relaxation method in a plane-strain notched geometry, observed a similar merging of plastic enclaves originating on axis and at the notch root. This behavior is discussed by Hill (1950).

Fig. 24. Comparison of plastic region development (ϵ^p) in round-notched tension bars. Both bars have root radii equal to 0.25 mm.

CONCLUSIONS

The stress/strain state at failure is found to be similar near identical notches cut in round bars of different diameters. At failure, the equivalent plastic strain at the notch root is about 0.9, and the

maximum principal stress peaks two root radii below the notch surface at about 1.3 GPa. Surface stress is 1 GPa. The stress distribution across the plane of minimum area explains the increase in notch strength with decrease in size.

Compact Specimen Simulations

SUMMARY

We tested blunt-notched compact specimens made of SA-533 Grade B Class 1 steel plate and then simulated these tests in plane strain with the HEMP computer code. The purpose of the tests and calculations was to

- Establish an appropriate method of applying load for plane strain and three-dimensional compact simulations
- Provide the stress and strain fields at fracture initiation. The fields are used to calibrate the damage model.

The results so far show that the computed load vs load-point displacement is insensitive to details of the near-notch deformation field.

INTRODUCTION

The precracked compact specimen is one of the two ASTM test geometries used to measure fracture toughness.* The geometry of the specimens used for the computer simulations is shown in Fig. 25. We decided to use compact specimens without fatigue cracks, but with 0.25-mm notch radii, to calibrate the damage model because:

* ASTM Standard E399.

- Less computer time was required for the coarser zoning
- The local deformation field would be analogous to the deformation field in the presence of a fatigue crack; thus, the general features of the notched and the precracked damage model functions might be similar.

In the experiments, the load is applied by pulling at constant velocity on hardened pins that are slipped into the holes drilled through the specimen. It is convenient in the plane-strain simulations (and required in the three-dimensional simulations) that the pin not be considered explicitly, but rather be replaced by a constant velocity condition near the pin-specimen contact region. An elastic beam analysis indicated that the pins exhibit little distortion as they are pulled. It is therefore assumed that the velocity does not change through the thickness. The appropriate velocity was determined by computer experiment. We are currently simulating the baseline heat of A533 steel for the compact specimen.

METHOD OF LOAD APPLICATION

We have studied the plane-strain compact specimen to determine an

Fig. 26. Prescribed velocity field for welded pin calculations.

(a) 9 m/s pull velocity

(b) 6 m/s pull velocity

(c) 3 m/s pull velocity

Fig. 27. Contours of equivalent plastic strain for three different pull velocities. Contour levels are $A = 0.1\%$, $B = 0.5\%$, and $C = 1.0\%$. Pin displacement is 0.54 mm.

Fig. 28. Development of plastic strain at the notch for different pulling velocities.

of the three indicates that 9 m/s is probably too fast. In Fig. 28 the plastic strain at the notch root is plotted against the notch opening displacement as measured under the point of load application (NODA). In Fig. 29 the load as measured by the stress acting across the plane of symmetry is plotted against NODA. Although differences between the three calculations are not large, the 6 m/s results seem to agree more precisely with the 3 m/s results (which are assumed correct) than do the 9 m/s results. Thus, 6 m/s was chosen for all further compact specimen calculations.

The second set of calculations was designed to investigate how much detail was necessary to adequately describe the geometry of the pin and

Fig. 29. Effect of pulling velocity on compact specimen simulations, welded-pin model.

specimen. Comparison was therefore made between the welded-pin simulation, a simulation in which a semi-circular pin was welded to the top of a circular hole, and one in which the pin was free to slip in a circular hole. The geometry near the pin and the imposed velocity fields of the three simulations are shown in Fig. 30, together with the resulting plastic strain field at a NODA of 0.5 mm. Note that the significantly increased plastic strain for the welded pin calculations is paralleled by increased load for the same NODA (Fig. 31). It is apparent that additional torque is supplied to keep the welded pin moving vertically as the specimen

tries to rotate around the pin. Ultimately, the shear strength is reached, and plastic strain develops near the pin. Although the frictional restraint between the hardened steel pin and the deforming specimen is unknown, it is unlikely that the two dissimilar metal surfaces will weld spontaneously. Therefore, the circular cutout was selected for all future simulations.

The six calculations reported above were needed as a basis for simulating the compact specimen with and without fatigue cracks. Results available from these simulations are presented in the following section.

RESULTS OF COMPACT SIMULATIONS

Table 7 lists the simulations that have been completed and those in progress. Figure 32 shows the effect of mesh refinement on the plastic strain near the notch. The two calculations exhibit the same distant strain field, with the finer mesh calculation exhibiting a higher plastic strain near the notch tip. Figure 33 compares the plastic strain fields at 340 μ s. The load-displacement plots for the two calculations are exhibited with the experimental values in Fig. 34.

It is apparent that changing mesh size has little effect on

Welded pin calculation

Semicircular welded pin calculation

Slippery pin calculation

Fig. 30. Velocity boundary conditions and near-pin geometry (left), together with equivalent plastic strain field (right) at a time when NODA is 0.54 mm.

load-displacement results. The corollary, however, is perhaps more important: load-displacement meas-

urements are insensitive indicators of the deformation field near the notch of uncracked compact specimens.

Fig. 31. Effect of pin geometry on compact specimen simulations.

Fig. 32. Plastic strain field near the notch at 340 μ s for two different mesh sizes.

(a) 1.5-mm zones near the notch.

(b) 0.5-mm zones near the notch.

Fig. 33. Equivalent plastic strain field at 340 μ s for two different zone sizes near the notch. Contour levels: 1 = 10%, 2 = 5%, 3 = 1%, 4 = 0.5%, 5 = 0.1%.

Table 7. Compact specimen simulations.

Specimen	Zone size around notch, mm	Geometry *	Stop time, μ s	Description
Preliminary calculations				
CFT4.1	1.5	2-D	100	Welded pin, 6 m/s
CFT4.2	1.5	2-D	33	Welded pin, 9 m/s
CFT4.3	1.5	2-D	100	Welded pin, 3 m/s
CFT4.15	1.5	2-D	100	Welded, semicircular pin, 6 m/s
CFT4.14	1.5	2-D	100	Circular cutout, 6 m/s
A533 simulations				
CT2DE1	1.5	2-D	340	Baseline, 1.5-mm zones everywhere
CT2DE1A1	1.5	2-D	340	0.3-cm zones around pin to save computer time
CT01A	0.5	3-D	G [†]	Same zoning as above
CT2DE2A	0.5	2-D	P(340 μ s)	Mesh refined only near notch
CT2DE3A	0.15	2-D	U	Mesh refined only near notch

* 2-D = plane strain (HEMP2D)

3-D = HEMP3D.

† G = generated but not run

P = in progress

U = being generated.

Fig. 34. Load per cm thickness vs. displacement from experiment and plane-strain simulations.

Charpy-V-Notched Computer Simulation

SUMMARY

We have calculated the dynamic Charpy V-notch test. The calculational models used both a rounded and a flat-tipped striker. We found that the notch stress/strain state was independent of the three-point loading type and was most strongly correlated with notch-opening displacement. The dynamic stress/strain state at the time of fracture initiation was obtained by comparing the calculated deformed shape with that obtained in interrupted Charpy V-notch tests where cracking had started. The calculation was also compared with stress/strain states calculated in other geometries at failure. We have also calculated the distribution and partition of specimen energy. Adiabatic heating and strain rate are discussed.

INTRODUCTION

We have computer simulated a Charpy V-notch impact test using the ASTM Type A specimen geometry and striker. Calculations show a distribution of plasticity in the specimen, at general yield, similar to the slip-line solutions; the plastic zones grow from the notch and the impact surface and merge as plastic

hinges. These bands then grow to the specimen notch-side surface. Although the hinges have the same general shape as predicted by the slip-line solutions, they differ from a rigid-perfectly plastic material in that the slip lines become wide bands of plastic flow. Thus, while the general picture is similar to the slip-line field, the details of the dynamic stress and strain state are different.

We estimated the time of crack initiation by comparing the stress/strain state of the Charpy calculation with results of computer simulations of other specimen geometries at failure. This estimate agrees with crack initiation times found in interrupted Charpy tests. Determination of the time of fracture allows the energy required to initiate fracture to be calculated and related to measures of fracture toughness appropriate for fracture initiation.

We find that fracture initiates at about 230 μ s, well before the maximum load found in the instrumented test. The stress/strain field at the notch tip is essentially independent of striker tip shape and is most closely correlated with the notch opening displacement. This in turn is an accurate measure of the notch root strain. This strain, at

failure, is 100%. The maximum stress occurs at 1.6 notch radii below the notch surface and is $2.9 Y_0$. The maximum mean stress at this point is $2 Y_0$. The analysis provides information on the distribution and partition of kinetic and internal energy, and adiabatic heating.

BACKGROUND

The first slip-line solutions for a rigid-perfectly plastic material in three- and four-point bending were found by Green (1953) and Green and Hundy (1955). Their experiments with partially deformed specimens, which were sectioned and etched to color the regions of plastic flow, showed remarkable similarity to their calculated plastic strain field. These solutions assumed a sharp V-notch. The three-point bending solution for the sharp V-notch specimen was extended to a specimen with a finite root radius by Alexander and Komoly (1962). Their solutions assumed yielding was localized in plastic hinges around the minimum section. It was later learned that this is true only if the notches are sufficiently deep.

Ewing (1968) showed that the Type A specimen had insufficient notch depth for localized yielding, and that the plastic flow would spread away from the minimum cross

section to the notch-side surface. Ewing then suggested his own solution, in which he first determined the slip-line field for a flat-tipped indenter and then obtained the general yield load, i.e., the indenter force consistent with the assumed slip-line field. This general yield load, which may be related to the yield stress of the material, may be obtained from instrumented Charpy V-notch tests, and has been used to estimate the dynamic yield strength of the material. The calculated state is called general yield, for which a consistent set of external loads and internal stresses are calculated.

It is questionable whether these perfectly plastic slip-line solutions can be applied to work-hardening materials. Numerical work-hardening solutions show that the plastic flow is distributed in wide bands centered approximately on the slip lines. At general yield the strain in these bands ranges from 35% at the notch to 1% at the point of the hinge farthest from the symmetry axis. Maximum stresses are developed well inside the notch plastic region and not at the elastic-plastic interface. The load-shear yield stress relation of Green and Hundy and Ewing is inaccurate in view of the varied work hardening in the plastic hinges at general yield. Further, since crack

initiation in a ductile material is associated with large geometry changes, the initial geometry slip-line solutions fail to satisfy the boundary conditions in the deformed state. The solution does not track the changing stress state with the deformation.

Computer solutions have contributed to the understanding of notched bending. Griffiths and Owen (1971) used a finite-element, elastic-plastic, small deformation program to calculate a static Charpy bar in pure bending. Their calculation used the Type A notch, but assumed a notch depth of 4.23 mm, a minimum cross section of 8.47 mm, and a support span of 38.1 mm (compared to the Type A dimensions of 2, 8, and 40 mm, respectively). Their zone size at the notch radius (r) was about $r/4$.

Their calculations, which were made on 3% silicon steel, showed that:

- Notch surface strain is constant over a distance of 1.2 times the root radius
- Stress intensification at progressively larger loads differs significantly from the slip-line field predictions
- Local fracture stress is independent of temperature for fractures preceding general yield.

Their calculations were only taken to a notch strain of about 8%, cor-

responding to general yield for this material.

COMPUTER MODEL

Figure 35 shows the meshes for the simulations. The geometry is plane strain, with only one-half of the striker and specimen included in the calculation. A boundary support point at $x = 0$, $y = 20$ mm allows a vertical (y) velocity, but not a horizontal (x) velocity. The striker was given an initial velocity of 4.91 m/s (16.1 ft/s), corresponding to a typical 1.22-m (4-ft) striker drop height. The mass of the three zones on the back end was increased to give one-half of the striker a total mass of 13.6 kg (30 lb). The initial energy in the (one-half) calculation was 163 J (120 ft-lb).

The material properties have been described earlier. The plastic flow law is shown in the chapter on Material Description and compared with those used in other simulations. No damping was used in the simulation.

We simulated the impact of a rounded striker tip to see if the phenomenology differed from a flat-tip simulation. The computer program allowed the gap between the rounded striker and the specimen to close as the specimen deformed; it also allowed interface slip. The flat-tipped striker-specimen interface was

Fig. 35. Zoning for Charpy V-notch computer simulations: (a) Flat-tip striker, (b) Zoning detail for round-tip striker; (c) Zoning detail for flat-tip striker.

modeled with a rigid connection. The data from this latter calculation are reported below.

CALCULATED STRESS/STRAIN STATE

We have simulated the deformation caused by the rounded-tip striker for 100 μ s and the flat-tip striker for 400 μ s. Figure 36 compares specimen deformation at three different times for both striker tips. Note that the plastic flow originates at the point where the tip of the rounded striker hits the specimen. For the flat-tip striker, flow originates at the corner areas of high shear strain. The specimen has a large elastic region between the notch and impact surface.

The contours of equivalent plastic strain (Fig. 36) show that there are differences in the strain fields of the two calculations (round and flat tip), even at the notch. These apparent differences at the notch are more associated with differences in the two mesh sizes than in a real difference in the state at the notch. We have plotted profiles of stress and strain across the ligament for both calculations that are in agreement near the notch root. While the general stress state differs for the two calculations, the stress/strain states at the notch are similar.

Another calculation, using pressure to drive the specimen, was compared with the flat-tip striker calculation to a time of 230 μ s. The specimen had the same zoning in both calculations. For the same notch opening displacement, we found that the notch stress/strain state differed by only a few percent. Therefore, we have concluded that the notch stress/strain state is not sensitive to the driving force model.

At 35 μ s, the growing plastic hinge segments of the flat-tip calculation join at a point in the specimen originally 4 mm from the notch root and 3 mm above the center plane of the specimen. The time history of equivalent plastic strain at this point is shown in Fig. 37a; at 100 μ s, the strain reaches 1%. The spatial distribution of plastic strain at 100 μ s is shown in Fig. 37b. The corresponding notch strain at this time is 35%, and the average notch strain rate is about 3000 s^{-1} . This is about the time when instrumented striker experiments show a sharp reduction in the slope of the load-vs-time record. This is referred to in the instrumented Charpy literature as general yield and corresponds to the general yield of the slip-line solutions.

An elastic finite-element program was used to obtain the natural

Fig. 36. Equivalent plastic strain distribution for rounded-tip (left) and flat-tip (right) striker at three times. Contour levels are 1 = 10%, 2 = 1.0%, 3 = 0.1%.

Fig. 37. Plastic hinge growth from the notch and the impact surface: (a) $\bar{\epsilon}^p$ vs time at the location where the plastic zones merge; (b) $\bar{\epsilon}^p$ contour levels at general yield, $t = 100 \mu s$.

frequencies of the plane-strain Charpy V-notch specimen and the displacement required for yielding to start in the notch. The calculation used 730 zones, with 60 zones across the 8-mm ligament. Fewer zones were used away from the notch. The displacement for notch yielding is 0.0611 mm at a load of 5.55 kN (1248 lb). The nondimensional compliance ECB is 23.22. The dynamic elastic-plastic calculation shows an equivalent plastic strain at the notch for this displacement of 1.5%. The periods of the first three vibrational modes of the specimen were calculated to be 84.1, 23.5, and 20.8 μs . These

calculations all assumed that the specimen was free to rotate and translate on the support.

Figure 38 shows the distribution of principal stress, mean stress, and equivalent plastic strain at the notch. The figure compares the states at 230 and 330 μs . The stress and mean stress appear to be relatively stationary with time; the straining varies strongly with time as the notch opens. Profiles of stress perpendicular to the notch, mean stress, and equivalent plastic strain below the notch at these two times are shown in Fig. 39.

Fig. 38. Stress (GPa) and strain contours at 230 μs (left) and 330 μs (right):
 (a) Principal stress; (b) Mean stress; (c) Equivalent plastic strain.

CHARPY CRACK INITIATION

We have estimated the time of crack initiation* by calculating the stress/strain states at the notch and then comparing them with similar states in other calculated geometries at times when fracture initiation is known to occur. Next, we compared the estimated time with the crack initiation times of deformed Charpy specimens from interrupted dynamic tests. Based on these comparisons we estimated that fracture starts at the notch surface between 230 and 340 μ s for a 4.9 m/s (16.1 ft/s) impact velocity. Instrumented Charpy data shows that the maximum load for this material, at 100°C and this impact velocity, occurs at about 700 μ s.

The specimens from the interrupted tests were heat tinted, cooled in liquid nitrogen, and broken to expose the fracture surface. These surfaces were inspected at 30 $\times$ magnification to locate the specimen that had the least amount of deflection, but still exhibited cracking. The procedure is not precise; the crack initiation is camouflaged by the blunting that occurs in this tough steel.

The results are shown in Table 8. The specimen displacement was assumed

*We defined a crack as a new surface (disregarding blunting) greater than approximately 0.25 $\times$ 0.25 mm.

Fig. 39. Stress and strain distribution at failure in four specimen geometries. The origin for distance is taken at the notch for the notched specimens and at the symmetry axis for the smooth tension specimen.

Table 8. Crack initiation in A533 B, Class 1 Charpy V-Notch specimens.

Temperature, °C (°F)	Displacement,* mm	Time, [†] μs
28 (82)	0.762-1.12	155-230
79 (175)	1.12-1.63	230-334
177 (350)	1.63-1.80	334-369

* Striker displacement below specimen initial impact surface.

[†] Based on a constant 4.91 m/s impact velocity.

to be equal to the prescribed striker displacement. The times associated with this deformation were determined by dividing these displacements by an impact velocity* of 4.9 m/s. The data show that, at 100°C, evidence of macroscopic initiation is seen at about 340 μs. This is well before the maximum load from an instrumented Charpy test.

Four additional specimens were deformed at 93°C (200°F) in the interrupted tests. These unbroken specimens were cut across the notch in the long dimension and polished and inspected. The first crack occurred at a deformation between 1.63 and 2.16 mm. A study of the surface (see Fig. 40) suggested that the crack started at the notch surface and grew inward. These data show that the crack started between 332 and 440 μs.

Estimated time of crack initiation may be obtained by comparing the calculated Charpy stress/strain state with known stress/strain states at failure. The time history of equivalent plastic strain in five zones below the Charpy notch is shown in Fig. 41. Blunting of the notch root radius results in intense straining and a high strain gradient. We have extrapolated the zone-centered equivalent plastic strain to the notch surface and have also computed the natural strain of the surface line closest to the notch root. The results are also shown in Fig. 41. Failure in the smooth tensile specimen occurs at an equivalent plastic strain of 100%, shown by the horizontal dashed line in Fig. 41a. It is seen that, based on strain alone, the specimen cracks between 230 and 290 μs.

The time history of stress normal to the notch is shown in Fig. 41b for four zones below the notch. The smooth tension test calculation

* The experiments were conducted at an impact velocity of 0.91 m/s (3 ft/s).

Fig. 40. Polished middle cross section of a Charpy V-notch specimen at 100°C. The specimen is from interrupted dynamic tests: (a) Notch root, 125x; (b) Magnified notch root, 650x; (c) Magnified view of fracture damage at base of notch for a specimen with greater interrupted deformation. Circular depressions are MnS inclusion sites.

Fig. 41. Strain and stress time histories for zones below the notch: (a) Equivalent plastic strain - zone 1 is immediately below notch; (b) Principal stress σ_1 vs time. Initial notch zone size is 0.133 mm.

indicates failure at an axial stress of 1.28 GPa. This stress is not reached in the Charpy surface zone

until much later; the surface stress normal to the notch at 400 μ s is about 1.1 GPa. The increase of this stress with time from 100 μ s to 400 μ s is small, but strain at 400 μ s has increased to about 150%. The surface stress at 230 μ s is about 0.95 GPa. A more precise estimate of the fracture initiation time will be available after calibration of the damage model.

ENERGY

The energy in a small process zone near the notch root at 230 μ s is only 12% of the energy in the specimen. At this time the local root temperature has risen over 150°C. Kinetic energy equals internal energy at 20 μ s.

The distribution of internal energy near the notch is shown in Fig. 42. These energy contours were converted to temperature contours using a specific heat of 0.106 cal/g - °C. Note that a small volume of material at the root of the notch experiences a temperature rise in excess of 150°C. The material in a process zone of about two root radii has a temperature rise greater than 50°C. The temperature rise of this material is of the same order as the transition temperature range. NDTT for A533 is about -18°C (0°F) and uppershell is 93°C (200°F).

Fig. 42. Temperature distribution at Charpy notch at 230 μ s, °C.

Since this dynamic test is nearly adiabatic,* this temperature rise should result in a considerable enhancement of toughness in the transition region, a result that is not obtained dynamically. For ductile rupture associated with high straining, the crack originates in the hottest material.

The notch energy at 230 μ s is only 12% of the total energy in the specimen. This number was obtained by editing the energy near the notch root into a process zone with dimensions 1. deep and 2.6 mm perpendicular to the notch. Notch radius is 0.25 mm. Most of the energy is

* The characteristic thermal diffusion time is about 1 ms.

distributed away from the notch (i.e., in the plastic hinges, at the support, and below the striker), where it has little effect on fracture toughness.

The partition of energy in the specimen is shown in Fig. 43. The dashed horizontal line in the figure is the specimen kinetic energy, assuming a notch velocity of 4.9 m/s and rotation around the anvil support. The calculation simulated a 27.3-kg (60-lb) striker impacting the specimen at 4.9 m/s. At 17 μ s, specimen kinetic energy is 2.4 times greater than internal energy, but they become

Fig. 43. Calculated Charpy V-notch specimen energy partition. Initial striker energy is 325 J (240 ft-lb). The dashed line is the rigid body striker KE, assuming the notch root moves relative to the support at 4.91 m/s (16 ft/s).

equal at 20 μ s. At 100 μ s, corresponding to general yield, kinetic energy is 3.5% of the energy in the specimen. At 230 μ s, the ratio is 1.2%. The specimen contains only 4.3% of initial energy at this time and only 10.3% of the 136 J (100 ft-lb) of CVN energy required for complete fracture.

Estimates of CVN kinetic energy corrections may be obtained from Fig. 43. It is obvious that corrections must be made for fractures that occur before 100 μ s and that meaningful data for fractures that occur before 20 μ s are not possible at this impact velocity. Because of the periodicity of kinetic energy associated with the natural frequency of the specimen, more of a correction may be required as time increases. For example, no correction is required at 35 μ s, but at 55 μ s, 27% of the total specimen energy is kinetic. The kinetic energy is a spurious measure of fracture toughness and should be removed in evaluating toughness. The damping of the kinetic energy in this simulation relies entirely on the energy lost in plastic deformation. We have included no viscous damping in the calculation.

STRAIN RATE

The average strain rate at the notch is about 3000 s^{-1} for the

4.91 m/s impact velocity. Wilshaw (1966) reports a range of 2000 to 5000 s^{-1} . Our strain-rate calculations show considerable oscillations for the first 120 μ s of impact, with a peak rate of 6700 s^{-1} . Strain rate vs time is shown in Fig. 44a for the zone closest to the notch root.

The stress/strain curve for these calculations was obtained experimentally at a strain rate of approximately seven orders-of-magnitude lower than the maximum calculated notch rate. To determine the effect of the high rate on the flow-stress curve, we made high-rate tensile tests. The results gave us the stress/strain relationship at a rate of 3400 s^{-1} . We found that ultimate strength increased 13% in the A533 steel due to the high rate.

These tests used a split Hopkinson bar in tension (Hauser, 1966). Three specimens were tested at a strain rate of 3400 s^{-1} , and one quasi-statically at a crosshead rate of 1.3 mm/min (0.05 in./min) (see Fig. 44b). All tests were made at 20°C. LLL Standard T-2 Tab-2 tensile specimens were used. The overall specimen length of 38.1 mm included the enlarged threaded ends. The center-section diameter was 4.06 mm for a length of 9.53 mm. It is doubtful that the small increase in ultimate strength can account for the reduction in toughness seen in the

Fig. 44. Strain-rate behavior:
(a) Calculated strain rate in the Charpy V-notch specimen at the notch root; (b) Comparison of Hopkinson tensile bar data at $\dot{\epsilon} = 3400 \text{ s}^{-1}$ with static data.

dynamic Charpy test in the presence of the adiabatic heating.

STRIKER RESPONSE

Striker force vs time is shown in Fig. 45. Two of these curves compare the average calculated cross-section force from a single zone at the striker midsection (the normal strain-gauge location) with the force

obtained by integrating the axial stress over the tip of the striker. A third curve is instrumented Charpy experimental data at this striker velocity and temperature.

The oscillations are associated with the complex wave interactions between the specimen and the striker. The fundamental period of the elastic specimen is $84.1 \mu\text{s}$.^{*} The specimen leaves the striker at about $25 \mu\text{s}$. Analysis of the velocity fields of the striker and the specimen clearly shows the interactive wave effects that result in separation. Specimen kinetic energy has reached a maximum at $17 \mu\text{s}$. The high amplitude of the calculated striker oscillations results from a lack of damping in the simulation that is probably present in the actual test. The magnitude of the oscillations appears to have little effect on the plastic deformation history of the specimen, as may be seen by the linear relationship between energy and time in Fig. 43.

The axial stress distribution across the striker is shown in Fig. 46 at 60 and $70 \mu\text{s}$. The distribution is shown at the striker tip and at a point 16 mm behind the tip, corresponding to the instrumented Charpy strain-gauge location. There is considerable variation across the

^{*}Support friction lowers this period to $58.4 \mu\text{s}$.

Fig. 45. Comparison of calculated striker with instrumented standard Charpy data: (a) Specimen 2430, $T = 100^\circ\text{C}$; (b) Calculated data from striker tip and strain gauge location; (c) Superposition of calculated and experimental data.

tip, but at the striker midsection the stress is uniform.

To interpret these results for the striker, we must weigh the relation between the computer striker model and the real structure. The actual striker is a small cantilever plate that impacts the specimen on the plate edge. The plate mass is

negligible compared to the massive open-bottomed steel box on which it is mounted. Our striker model uses only the cross-section dimensions of the striker, with the rear-end density increased to model the box mass. The agreement of experimental data with the calculation shown in Fig. 45 suggests the model may be adequate.

Fig. 46. Stress distribution across striker at impact end and at strain-gauge location.

RESULTS

- The calculations show the general plasticity features of the slip-line field solutions.
- Fracture initiates at about 40% of the maximum load time of the instrumented standard Charpy V-notch test.
- There is no theoretical basis for applying the three-point (or four-point) load/dynamic yield

stress relation of rigid-plastic slip-line theory to this material.

- A process zone contains less than 12% of the specimen energy at fracture.
- Specimen energy at fracture initiation is only 10% of measured CVN.
- Specimen internal energy becomes equal to kinetic energy at 20 μ s. At 100 μ s, KE is less than 4% of IE.
- Notch adiabatic heating raises the notch temperature more than 150°C at time of fracture.
- Notch average strain rate is about 3000 s^{-1} . Ultimate strength of this material is increased 10% at 3400 s^{-1} .
- The notch stress/strain state is essentially independent of three-point loading method, but the state correlates well with the notch opening displacement.

Conclusions

CALIBRATION

Table 9 summarizes the stress state at fracture for the four damage model calibration specimens: the smooth tension specimen, two notched tension specimens scaled by two

(except for the 0.25-mm root radii), and a blunt-notched ($r = 0.25$ mm) compact tension specimen.

Data for the notched specimens are given both at the notch root, where the strain is maximum, and approximately 0.5 mm inside the specimen,

where the stress is maximum. Origin of failure in the smooth tension specimen is unambiguous; stress and strain are maximum on axis at the neck minimum cross section, and fracture starts there. The uniformity of the high stress and strain states across the neck suggests prompt failure. Photographs of surface fracture initiation in the 27-mm-diam notched tension specimen show ductile tearing starts at the notch surface and proceeds inward. The time of initiation in the blunt-notched compact tension specimen and

the 13-mm-diam notched tension specimen is unknown, so we assumed that initiation was coincident with maximum load.

Table 9 allows speculation on the prospects of calibrating the Wilkins damage model for macroscopic crack initiation. The ratio of the standard deviation to the arithmetic mean of those criteria displayed in Table 9 is a measure of consistency of a stress or strain as a failure criterion. The surface data combined with the smooth tension data suggest that the four specimens, when heated

Table 9. Stress state at fracture, A533 Grade B Class 1, 100°C.

Calculation	$\bar{\epsilon}^P$	σ_m/Y_0	$\sigma_m/\bar{\sigma}$	σ_1/Y_0	$\min(s_2/s_3 , s_3/s_2)$
Smooth tension	0.97	1.42	0.77	2.79	1.00
13-mm-diam notched tension ^{*,†}	0.93	1.0	0.50	2.07	0.12
27-mm-diam notched tension [*]	0.87	1.07	0.55	2.18	0.05
Compact tension ^{*,†}	1.12	1.22	0.73	2.27	0.00
Standard deviation/mean	0.11	0.16	0.20	0.14	
13 mm diam notched tension ^{†, **}	0.08	1.90	1.32	2.82	0.27
27 mm diam notched tension ^{**}	0.07	2.10	1.50	2.95	0.11
Compact tension ^{†, **}	0.05	2.16	1.55	2.97	0.00
Standard deviation/mean	1.5	0.18	0.28	0.03	

* Notch surface values, extrapolated from zone centers; all notch radii = 0.25 mm.

† Fracture assumed to initiate at time of maximum load.

** Measurement taken inside specimen below notch, where σ is maximum.

to 100°C, start to fail at a strain of about 100%. Since the proposed Wilkins critical damage is essentially an integrated weighted history of the equivalent plastic strain increment, the prognosis for determining a unique critical damage is good.

The subsurface data combined with the smooth tension data of Table 9 show that the best criterion for fracture initiation may be principal stress. Although principal stress appears to give excellent correlation with crack initiation, it is not a good predictor for ductile fracture. The stress in these simulated test specimens quickly reaches and maintains a nearly constant stress level, whereas the strain varies strongly with the deformation state. For brittle fracture, where stress and strain are proportional, principal stress is an excellent predictor.

The data of Table 9, together with their time histories, show that we can obtain a critical damage that predicts crack initiation in all four calibration specimens. The damage D is probably an insufficient measure because of the high strain gradients in the notched tests. Thus, the ratio of D/D_c of Eq. (2) is important. This is equivalent to saying that for this material and the tests selected, a single "characterizing parameter" is insufficient to predict

damage; in addition, a length parameter over which the damage exceeds a critical value is important.

The calculational results of Table 9 used three different effective stress vs equivalent plastic strain flow curves to describe the plasticity. These were identical to ultimate strength and used the same yield strength, but they differed at a strain of 100% by 10% (see Fig. 3). All these flow stress curves were higher than the correct curve (Fig. B2). The latter curve was predicted by the final computer iteration described in the chapter on the smooth tension simulation. The principal stress reported in Table 9 is composed of the mean stress plus the deviator stress. Since the mean stress is unaffected by the flow law and represents over half the critical stresses of Table 9, these different flow stress definitions are not thought to introduce large errors. Different curves were needed to start the simulation calculations and the tests simultaneously.

The tests selected for this calibration procedure do not load the material in a manner to cause all types of failure initiation; e.g., the shear failure associated with shear lips. As expected, the three notched specimens with the same notch root radius give similar stress/strain

fields at the notch at failure. The smooth tensile specimen is, however, different. The specimen selection may be adequate for the Charpy V-notch orientation of this research program, but for more general use, additional specimen geometries must be included in the calibration procedure.

CHARPY V-NOTCH SIMULATIONS

Four calculations described in previous sections were done to set the constants of the Wilkins damage model. The Charpy computer simulations were done to gain insight into the mechanics of the deformation, to obtain the material state at the time of crack initiation, and to discriminate between the energy required to start the crack and that needed to continue the rupture process. The goal is to prescribe modifications to measured CVN toughness so as to better effect a correlation with fracture toughness.

The Charpy computer simulations have provided an estimate of fracture initiation time. This estimate was obtained by comparing the computation with data from interrupted Charpy V-notch tests as well as with stress states at failure in other geometries. The specimen starts to fail at the notch in the symmetry plane at about 300 μ s. for a 4.91 m/s impact velocity.

The notch strain at initiation is about 100%. The notch surface stress is 1.1 GPa.

Since there is less certainty about the time of crack initiation in the Charpy specimen and, hence, about the calculated stress state at that time, we have listed the Charpy stress/strain data at fracture initiation in Table 10. These data are preliminary and will probably change when we are better able to define the time of Charpy crack initiation.

A most important outcome of these calculations is the unique relation between the notch displacement and notch opening angle and the stress and strain states at the notch. This relationship, first pointed out by Wilshaw (1966), is a striking feature of the calculations. It appears that the notch opening angle is an excellent measure of the notch root strain, the essential ingredient of the damage model. The work in the coming year will consider this relationship more closely, with the goal of using a static three-point bend test and crack-opening displacement or crack-opening angle as a measure of fracture toughness.

COMPACT TENSION SIMULATIONS AND THE J-INTEGRAL

We have computer simulated the deformation of the compact specimen

Table 10. Calculated stress state at fracture, A533 Grade B Class 1, 100°C.

Calculation	$\bar{\epsilon} P$	σ_m/Y_0	$\sigma_m/\bar{\sigma}$	σ_1/Y_0	$\min(s_2/s_3 , s_3/s_2)$
Charpy V-notch*	0.25	2.00	1.19	2.97	0.00
Charpy V-notch†	1.00	1.43	0.72	2.40	0.00

* Data taken inside specimen below notch, where stress is maximum.

† Notch surface values, extrapolated from zone centers; all notch radii = 0.25 mm.

for the blunt-notch damage calibration. Test problems to determine loading method and zoning effects were also done. Work has just started on the precracked specimens.

The third main task of this research program is to correlate Wilkins damage with a measure of fracture toughness. On the recommendation of EPRI, we have selected the J-integral as a suitable correlating parameter. It is not crucial to the program to determine whether the critical J (the value of the J-integral at fracture initiation) is a material property (hence geometry independent) or not. The prominent role it plays here in the correlation of fracture toughness with Charpy energy does not depend on the uniqueness of two different quantities, which are given the name J-integral.

The original definition of J is the two-dimensional contour integral given by

$$J = \int \left[W dy - \bar{T} \cdot \frac{\partial \bar{u}}{\partial x} ds \right] \quad (4)$$

The contour surrounds the tip of the notch parallel to the x axis (Rice, 1968). For conservative systems it has been demonstrated that this integral has a value independent of contour path. More recently (Rice, Paris, and Merkle, 1972), it was established that for deeply notched, three-point-bend specimens and for compact specimens, the J-integral is also given by

$$J = \frac{2}{A} \int_0^\delta P d\delta, \quad (5)$$

where P is the load, δ the load point displacement due to the crack, and A is the ligament area. Corrections have been suggested to Eq. (5).

For the plastically deforming compact specimen, following J_2 - flow theory, there is no *a priori* reason to expect that the integral of Eq. (4) is even approximately independent of path, or that the integral of Eq. (5) is equal to the integral of Eq. (4) when taken on any contour. We will correlate J with $\bar{J}$

by using the Eq. (5) definition of J . As a side issue, J , as given by Eq. (5), will be compared with J given by Eq. (4) on several contours.

PROGNOSIS

The work has proceeded without major problems. The codes appear able to handle the specimen geometries and constitutive properties well, and computer calculational time, while long, is not unusual for this Laboratory. Some concessions have been made in zoning to minimize computation time, but our zoning size near the blunt notches appears adequate to capture the strain gradients. Zone size at the notches are about one-half the notch radius.

The fatigue crack simulations required for the compact tension J calculations will probably not cause difficulties because the details of the process zone are not required in the J calculation. Further, we find that calculating the ratio D/D_c [see Eq. (2)], minimizes the effect of finite zone sizes for geometries where fracture occurs with large gradients in the damage D . This will allow us to check the calibrated damage in the more severe cases of crack geometry.

So far, we have only considered a single material. The J vs D_c and the J vs $\bar{J}$ relationships must still be established. We see no problems here.

Work on the other two materials duplicates that on the virgin material; the zoning for the simulations is identical, and those calculations should go rapidly. We expect to complete the Charpy surveillance work in the contract period.

There are, however, areas of concern. Our major concern is to correlate Charpy toughness with a measure of fracture initiation, D , in a useful way. The Charpy calculations show that little of the specimen energy at fracture initiation contributes to fracture in the process zone, and only 10% of measured CVN at this temperature contributes to fracture initiation. The correlation of test and calculated data are weaker than we would have hoped for in the case of the blunt-notched compact tension specimen. The potential for difficulties with sharp crack damage calculations exists.

The concept of what constitutes fracture initiation, referred to extensively in this report, has not been clearly defined. To define an initiated crack larger than one zone size, we have to use the propagation techniques discussed in the chapter on the computer model. A crack length less than one zone size ($\Delta x \approx 0.13$ mm) may not be large enough to guarantee that the crack has spanned several 20- μ m inclusion sites for A533 steel.

The research in the final year of
this program will address these

problems, as well as those more
plentiful tasks that appear routine.

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References

- J. M. Alexander and T. J. Komoly, "On The Yielding of a Rigid/Plastic Bar with an Izod Notch," *J. Mech. Phys. Solids* 10, 265 (1962).
- A. S. Argon, J. Im, and A. Needleman, "Distribution of Plastic Strain and Negative Pressure in Necked Steel and Copper Bars," *Met. Trans. A.* 6A, 815 (1975).
- J. A. Begley and J. D. Landes, "J-Integral as a Fracture Criterion," *Proc. 1971 National Symp. Fracture Mechanics, Part II*, pp. 1-23, ASTM STP-514 (1972).
- P. W. Bridgman, *Studies in Large Plastic Flow and Fracture* (Harvard University Press, Cambridge, Mass. 1964).
- W. H. Chen, "Necking of a Bar," *Int. J. Sol. Struct.* 7, 685 (1971).
- C. G. Chippenfield, J. F. Knott, and R. F. Smith, "Critical Crack Opening Displacement in Low Strength Steels," in *Third Inter. Cong. Fracture* (German Steel Institute, Dusseldorf, West Germany, 1973).
- D. P. Clausing, "Stress and Strain Distribution in a Tension Specimen with a Circumferential Notch," *J. Mater.* 4, 566-582 (1969).
- H. T. Corten and R. H. Sailors, *Relationship Between Material Fracture Toughness Using Fracture Mechanics and Transition Temperature Tests*, University of Illinois, Urbana, Rept. UILU-ENG71-60010 (1971).
- N. N. Davidenkov and N. I. Spridonova, "Analysis of the State of Stress in the Neck of a Tension Test Specimen," *Proc. Am. Soc. Test. Mater.* 46, 1147 (1946).
- C. J. Earl and D. K. Brown, "Distributions of Stress and Plastic Strain in Circumferentially Notched Tension Specimens," *Eng. Fract. Mech.* 8, 599-611 (1976).
- D. J. F. Ewing, "Calculations on the Bending of Rigid/Plastic Notched Bars," *J. Mech. Phys. Solids* 16, 205-213 (1968).
- E. D. Giroux, *HEMP User's Manual*, Lawrence Livermore Laboratory, Rept. UCRL-51079 Rev. 1 (1973).
- A. P. Green, "The Plastic Yielding of Notched Bars Due to Bending," *Quart. Journ. Mech. and Applied Math.* VI, Pt. 2, p. 223 (1953).
- A. P. Green and B. B. Hundy, "Inelastic Plastic Yielding in Notch Bend Test," *J. Mech. Phys. Solids* 4, No. 3, p. 128 (1956).
- J. R. Griffiths and D. R. J. Owen, "An Elastic Plastic Stress Analysis for a Notched Bar in Plane Strain Bending," *J. Mech. Phys. Solids* 19, 419-431 (1971).

- F. E. Hauser, "Techniques for Measuring Stress-Strain Relations at High Strain Rates," *Exp. Mech.* 6, 395 (1966).
- R. Hill, *The Mathematical Theory of Plasticity* (Clarendon Press, Oxford, 1950).
- B. Marandet and G. Sanz, "Evaluation of the Toughness of Thick Medium Strength Steels by Using Linear Elastic Fracture Mechanics and Correlations Between K_{IC} and CVN" (Tenth Symposium on Fracture Mechanics, Philadelphia, August 1976).
- T. U. Marston, M. P. Borden, J. H. Fox, and L. D. Reardon, Combustion Engineering, Chattanooga, Tenn., Rept. MML 75-152 (1975).
- F. A. McClintock, "Plasticity Aspects of Fractures," in *Fracture* Vol. III, H. Liebowitz, Ed. (Academic Press, New York, 1971), pp. 47-225.
- K. Mogi, *Rock Fracture, Annual Review of Earth and Planetary Science*, Vol. I., A. Donath, Ed. (Annual Review, Palo Alto, 1973), pp. 63-84.
- A. Needleman, "A Numerical Study of Necking in Circular Cylindrical Bars," *J. Mech. Phys. Solids* 20, 11 (1972).
- H. Neuber, *Theory of Notch Stresses* (J. W. Edwards, transl.) (Edward Brothers, Ann Arbor, Michigan, 1946).
- K. E. Puttick, "Ductile Fracture of Metals," *Phil. Mag.* 4, 8, 964 (1959).
- J. R. Rice, "A Path Independent Integral and the Approximate Analysis of Strain Concentration by Notches and Cracks," *J. Applied Mechanics* 35, 386 (1968).
- E. J. Ripling, "Materials Evaluation in Relation to Component Behavior," *Proc. Third Sagamore Ordnance Materials Research Conf.* (Duke University, Durham, N.C., 1956), pp. 207-233.
- S. T. Rolfe and S. R. Novak, "Slow Bend K_{IC} Testing of Medium Strength, High Toughness Steels," *ASTM STP-463*, 124 (1970).
- G. Sachs, "Survey of Low-Alloy Aircraft Steels Heat Treated to High Strength Levels, Part 3: Failure Cases," WADC TR 53-254 (1954).
- G. Sachs and J. D. Lubahn, "The Affect of Triaxiality on the Technical Cohesive Strength of Steel," *J. Applied Mechanics* 67, A-241 (1945).
- R. Southwell and D. N. de G. Allen "Relaxation Methods Applied to Engineering Problems XIV, Plastic Straining in Two-Dimensional Stress-Systems," *Phil. Trans. Roy. Soc.* A242, 379-444 (1950).
- M. L. Wilkins, *A Method for Computer Simulation of Problems in Solid Mechanics and Gas Dynamics in Three Dimensions and Time*, Lawrence Livermore Laboratory, Rept. UCRL-51574, Rev. 1 (1975).
- M. L. Wilkins, *Calculation of Elastic-Plastic Flow*, Lawrence Livermore Laboratory Rept. UCRL-7322 Rev. 1 (1969).

- M. L. Wilkins, *Fracture Studies with Two- and Three-Dimensional Computer Simulation Programs*, Lawrence Livermore Laboratory, Rept. UCRL-78376 (1977).
- T. R. Wilshaw, "Deformation and Fracture of Mild Steel Specimens," *J. Iron and Steel Inst.* 204, Pt. 9, p. 936 (1966).

Appendix A

Experiment Diagnostics

Our testing methods followed approved ASTM procedures and used load-cell-equipped tension machines as well as extensometers and clip gauges. We supplemented these standard measurement methods with high-resolution photography.

During the tension test the specimen was repeatedly photographed to record necking, elongation, and general distortion. The developed film was read with a scanning microdensitometer to convert the film images into tables of spatially resolved film density values. Later, computer routines operated on the numerical tables to produce dimensionally calibrated enlarged pictures of the evolving specimen shape.

The principal advantage of this photographic supplement is that each picture can be used to simultaneously measure many different specimen dimensions — far more than are measured with clip gauges alone. We have found that a comparison of the actual size and shape of the specimen with that predicted through calculation provides a sensitive test of the fidelity of the entire simulation procedure.

We used a standard 35-mm SLR camera, equipped with a motor drive

to advance the film and reset the shutter. The camera viewed the specimen through a special flat-field lens at a magnification of approximately $1/2\times$. We used KODAK type 5062 (PLUS-X) film with a 4-min processing step in an Acufine developer at 22°C.

Before loading the specimen, a photograph is taken and is used later, together with the specimen inspection records, to calculate the magnification factor of the lens. During the experiment, the tensile specimen is repeatedly photographed, while the load and elongation of the specimen are monitored with a load cell, an extensometer, and an X-Y pen recorder.

Each time the camera is triggered, an external circuit generates a small single pulse that is mixed with the load cell signal. The result is a series of fiducial marks (one for each picture) on the load-elongation record. These marks are numbered and later read to determine the specimen load and extension corresponding to each frame. Figure A1 shows the measurement method and Fig. A2 is a typical load-elongation record with the small fiducial marks visible.

Fig. A1. Schematic diagram of measurement method.

Fig. A2. Typical load elongation record with fiducial marks.

After the experiment, the film is developed and machine-digitized (film density as a function of X,Y) to create a computer data file. Handling codes convert these density data into calibrated specimen silhouettes, which are then compared to HEMP generated profiles. Figure 9, shown earlier, is a composite picture made from the last picture taken prior to fracture of a smooth round tensile specimen and from the zone grid generated by HEMP. Both the

Fig. A3. Silhouette of left side of notched tensile specimen; photographic method resolves slight imperfections in notch machining process.

experimental load and that in the simulation were 55.6 kN (12,500 lb).

As a demonstration of the sensitivity of our method, Fig. A3 shows the left-hand side of the silhouette of the central region in a 13-mm-diam notched-round tensile specimen. The actual distance across the notch opening is 2.60 mm. The 0.34-mm (nominal) root radius is clearly seen. We are confident our measurements have an absolute accuracy of $\pm 3.8 \times 10^{-2}$ mm or better.

Appendix B

Flow-Stress Data Corrections

The tension test provides load and neck radii, from which average stress and neck-surface hoop strain may be calculated. The relation between average axial stress and surface equivalent stress is obtained from iterative computer calculations. The Bridgman (1964) flow-stress correction is suitable for the first trial approximation.

The surface effective plastic strain $\bar{\epsilon}^P$ is calculated from the measured neck hoop strain, $\epsilon_\theta^P = \ln a/a_0$, by the relation

$$\bar{\epsilon}^P = \frac{2}{\sqrt{3}} \left[1 + \bar{R} + \bar{R}^2 \right]^{1/2} \ln \frac{a_0}{a}. \quad (B1)$$

The computer calculations provide ratios of $\bar{R} = \epsilon_r^P / \epsilon_\theta^P$. The test provides a_0/a . Equation (B1) is a statement of the incompressibility of the plastic strains which, for $\bar{R} = 1$, reduces to

$$\bar{\epsilon}^P = 2 \ln \frac{a_0}{a} = \ln \frac{A_0}{A}. \quad (B2)$$

Calculated ratios of equivalent stress to average stress and $\bar{R}$ as a function of neck area ratio are given in Fig. B1. These curves together with Eq. (B1) provide a method for converting average stress and neck hoop strain to point data. The results are for the A533 Grade B Class 1 steel studied here.

Fig. B1. Calculated stress and strain ratios as a function of neck-area ratio: (a) Ratio of equivalent stress at neck surface to average stress at neck vs $\ln A_0/A$; (b) Ratio of radial strain to hoop strain at neck surface vs $\ln A_0/A$.

Fig. B2. Comparison of corrected and normalized flow-stress values. Yield stress is 0.458 GPa.

We have compared our corrected data with that of Bridgman (1964) and Davidenkov and Spiridonova (1946) (Fig. B2). Results are identical up to the start of necking and then diverge linearly. The Bridgman-corrected data are higher by about 10% at $\bar{\epsilon}^P = 1$.

Calculated ratios of neck radius to neck radius of curvature are compared to data at various stages of necking in Fig. B3. These data are compared with Bridgman's composite a/R curve, representing an average of his data on many different metals. His data for steel falls below his average curve. This average curve

Fig. B3. Comparison of calculated and experimental values for ratio of neck radius to neck-profile curvature. The average values obtained by Bridgman (1964) for many different metals are represented by the solid line.

is not suitable when correcting tensile data on A533 steel.

The corrected-flow curve (the lower circles in Fig. B3) may be accurately fitted to a strain of 100% (using the test data of specimen 2499R), by the formula

$$\bar{\sigma} = \frac{c_1 + c_2 x + c_3 x^2 + c_4 x^3}{1 + c_5 x + c_6 x^2}, \quad (B3)$$

where x is the equivalent plastic strain and $c_1 = 0.458$, $c_2 = 5.74$, $c_3 = 484$, $c_4 = 140$, $c_5 = 8.50$, and $c_6 = 695$. Units for $\bar{\sigma}$ are in GPa.