STABILITY LIMITATIONS ON
HIGH-BETA TOKAMAKS

MASTER

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ABSTRACT

The MHD equilibrium and stability limitations on the maximum pressure achievable by means of the flux-conserving tokamak concept are investigated. Stability criteria associated with ballooning effects appear to provide the most stringent limitation.

The flux-conserving tokamak concept¹ has provided a prescription for choosing achievable plasma parameters when rapidly heating a plasma from an easily formed low-pressure configuration. It has been shown that equilibrium considerations do not provide an absolute limit to the maximum pressure that can be contained. MHD stability considerations can provide such a limit. We study this problem numerically, using the PEST package,² by treating a series of equilibria that could be associated with the TFTR device³ being built at the Princeton University Plasma Physics Laboratory.

We choose as our starting point the equilibrium of Fig. 1. The pressure is proportional to $(\psi_b - \psi)^2$ with ψ_b the poloidal flux at the plasma boundary. The safety factor $q \equiv d\phi/d\psi$, with ϕ the toroidal flux, increases from 1.04 at the axis to 3.77 at the boundary. The plasma, with aspect ratio $R/a = 4.6$ where R is the distance from the major axis to the magnetic axis and πa^2 the cross-sectional area of the plasma surface, is supported by currents in external poloidal field coils that create a favorable field index throughout the plasma. The fusion-power averaged pressure is

$$\beta^* \equiv 2\mu_0 \left(\int p^2 dV/V \right)^{1/2} / B_0^2 = 1.6\% \quad (1)$$

for this configuration. Other commonly used pressure averages include $\bar{\beta} \equiv 2\mu_0 \int p dV / B_0^2 V = 1.0\%$ and $\beta_p \equiv 4 \int p dV / \int (I' V / V') d\psi = 2.1$, with I the toroidal current, V the volume, and B_0

the toroidal field. The difference between β^* and $\bar{\beta}$ depends strongly on the shape of the pressure distribution.

We consider the effect on this equilibrium of increasing the material pressure, keeping the shape of the pressure distribution, the magnetic fluxes, and the position of the magnetic axis fixed. To prevent the outward shift associated with a pressure increase, we raise the current in the poloidal field coils, maintaining the shape of the field. The series of equilibria is summarized in Fig. 2. The aspect ratio decreases slightly since the plasma must expand to maintain the toroidal flux. The poloidal beta tends to saturate near the aspect ratio. As shown in Fig. 3, a plot of the flux surfaces when $\beta^* = 6.0\%$, the separatrix in the vacuum region approaches the plasma as the pressure is increased. As it comes close, the change in shape of the magnetic surfaces modifies their inductance and thus induces a large current in the plasma close to the surface. Our numerical scheme breaks down when the separatrix gets too close to the surface.

We now consider the stability problem. Evaluation of the usual localized instability criteria⁴ shows that both ideal and resistive interchange modes should be stable. We investigate nonlocal instabilities using a Galerkin method² to find the eigenmodes ξ . We use finite elements in the Ψ -direction, Fourier-analyze in the poloidal and toroidal angles θ and ϕ , $\xi = \sum_{m,\ell} \xi_{m,\ell,n} \Phi_m(\Psi) \exp i(\ell\theta - n\phi)$, and treat the different n -modes separately.

The axisymmetric modes with $n = 0$ are stable. This should be expected since the shape of the externally imposed field was tailored carefully to avoid such instabilities.

Instabilities with $n = 1$ set in for $\beta^* > 1.9\%$ as shown in Fig. 4. The growth rate is normalized to the poloidal Alfvén time. The projection of the displacement vector onto a constant ϕ plane is shown in Fig. 5. The $\ell = 1$ component dominates at the axis; higher ℓ 's emerge near the rational surfaces where their perturbed fields are small, i.e., where $|\ell - nq| < 1$, and thus the $\ell = 4$ mode is largest at the surface. The modes twist with the field lines in going around the torus with largest amplitude on the outside. The system is found to be stable to this mode when the plasma surface is held fixed. Over all, this mode has the character of an $\ell = 4$ kink with an enhanced and deepened vortex on the outer side of the plasma column.

Higher- n modes set in for somewhat large β^* 's as shown for $n = 3$ in Fig. 4. Holding the plasma boundary fixed decreases the growth rate but does not eliminate the instability. A typical displacement vector is shown in Fig. 6. The instability assumes the form of vortices which are much stronger on the outside of the plasma column. This can be seen clearly in Fig. 7, where the normal component of ξ is plotted as a function of θ for several surfaces. The apparent decrease in growth rate as β^* is increased in the fixed-surface $n = 3$ case in Fig. 4 is due to the difficulty in representing

the increasing localization of the perturbation with a finite set of expansion functions.

To understand the importance of these modes, it is useful to consider the potential energy associated with small distortions from equilibrium²

$$2\delta W = \int d\tau \left[\underline{Q}_\perp^2 + \left(\underline{Q}_\parallel - \underline{B} \frac{\underline{\xi} \cdot \underline{\nabla} p}{B^2} \right)^2 + \gamma p (\underline{\nabla} \cdot \underline{\xi})^2 + \frac{\underline{J} \cdot \underline{B}}{B^2} \underline{B} \times \underline{\xi} \cdot \underline{Q} - 2 \underline{\xi} \cdot \underline{\nabla} p \underline{\xi} \cdot \underline{\kappa} \right] \quad (2)$$

with $\underline{Q} \equiv \underline{\nabla} \times (\underline{\xi} \times \underline{B})$ the perturbed field and $\underline{\kappa}$ the field-line curvature. The first term in Eq. (2) is stabilizing. The unstable modes have $\underline{\nabla} \cdot \underline{\xi}_\perp = -2 \underline{\xi} \cdot \underline{\kappa}$ to eliminate the second term and $\xi_\parallel$ picked to make $\underline{\nabla} \cdot \underline{\xi} = 0$, thereby removing the third term. Kink modes⁵ are driven primarily by the interaction of the perturbed field with the force-free part of the current, the fourth term. The toroidal field provides a large curvature of order $1/R$, unfavorable on the outside of the plasma, and favorable on the inside, such that the average over the surface is stabilizing. Then interchange modes⁶ with $\underline{\xi}$ constant along field lines are stable. Relaxing this constraint on $\underline{\xi}$ allows the unfavorable curvature to drive the modes unstable despite the increase in the magnitude of the $\underline{Q}_\perp^2$ term. The shear term, $\underline{Q}^2 \approx (\underline{B} \cdot \underline{\nabla} \underline{\xi}_\perp)^2 \approx (B/qR)^2 \xi^2$ with qR the connection length, and the curvature term, $2 \underline{\xi} \cdot \underline{\nabla} p \underline{\xi} \cdot \underline{\kappa} \approx 2(p/a) \xi^2/R$, balance when $\beta^* \lesssim a/q^2 R \approx 1.5\%$ for our parameters. These ballooning effects augment the driving forces for kink modes to establish an upper limit on β^* .⁷

The ballooning modes, illustrated by the fixed-wall $n = 3$ case, provide a mechanism for moving plasma from the center to the outer surface. Since they cannot be cured by tailoring the current distribution,⁸ these modes pose a severe limitation on the achievable beta. They appear to be described by a quasimode formulation.⁹ Thus it is reasonable to expect that the instability will develop non-linearly on a resistive instability time scale.

The low β^* 's found here are due to the large values of the aspect ratio and safety factor, chosen to ensure stability with respect to axisymmetric and kink modes. With careful design it should be possible to increase this limit significantly. Clearly a better understanding of these instabilities is essential.

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Fig. 1. Flux surfaces for a TFTR configuration with $\beta^* = 1.6\%$. The heavy solid curve is the plasma surface.

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Fig. 2. Properties of the flux-conserved equilibria.

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Fig. 3. Flux surfaces with $\beta^* = 6.0\%$

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Fig. 4. Instability growth rates on a $\sinh^{-1} 100 \Omega^2$ scale for $n = 1$ and $n = 3$ modes with a free boundary (solid curves) and for $n = 3$ modes with a fixed boundary (dashed curve). The fixed-boundary $n = 1$ mode is stable. These calculations used twelve finite elements in ψ and twenty-one Fourier components in θ . Extrapolation from convergence studies results in the values shown by the dots.

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Fig. 5. Projection of the displacement vector for the free-boundary $n = 1$ mode at $\beta^* = 3.0\%$.

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Fig. 6. Displacement vector for the fixed-boundary $n = 3$ mode at $\beta^* = 3.0\%$.

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Fig. 7. The normal component of ξ as a function of the poloidal angle on surfaces with $\psi/\psi_b = 1/12$ (dashed curve), $\psi/\psi_b = 1/2$, (broken curve), and $\psi/\psi_b = 11/12$ (solid curve) at $\beta^* = 3.0\%$.