**GEOSTATISTICAL ORE RESERVE ESTIMATION
FOR
A ROLL-FRONT TYPE URANIUM DEPOSIT
(PRACTITIONER'S GUIDE)**

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MASTER

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OBJECTIVE AND SCOPE OF THIS REPORT

1.1 OBJECTIVE OF THIS REPORT

This report is prepared for the U. S. Energy Research and Development Administration (ERDA) as part of contractual research entitled "A Case Study of the Geostatistical Ore Reserve Estimation for a Roll-Front Type Uranium Deposit."

The primary objective of this report is to provide a useful guide to implementing geostatistics into the ore reserve estimation procedures for a roll-front type uranium deposit.

In order to accomplish this objective, the report has been prepared in a textbook format, comprehensible to the practitioners in ore reserve estimation. As such, this report can be used as a guide not only by the ERDA personnel but also by others involved in ore reserve estimation.

In the past, the advantages of using geostatistics in ore reserve estimation have been sufficiently emphasized in the literature. However, the comparative study performed by the authors^{1,2} in 1975 under the sponsorship of ERDA was the first published report of its kind. In this comparative study, the practicality as well as the superiority of geostatistics were compared over the conventional methods using real life data. Obviously, the outcome of the study was in favor of geostatistics.

The theoretical basis of geostatistics has been fairly well documented, although there is only one text by Matheron³ which contains most of the theory. Unfortunately, this text is not readily comprehensible to the average practitioners in ore reserve estimation. Consequently, this difficulty in understanding has served as one of the obstacles in implementing

this new methodology to ore reserve estimation.

In other words, what has been missing to date is a practical comprehensible text that gives illustrative examples of each phase of a geostatistical study and that provides comprehensive user's manuals for the computer programs necessary to make a practical geostatistical study. This report, therefore, is intended to provide the above missing link, thus paving the way to future implementation of geostatistics.

1.2 SCOPE OF THIS REPORT

This report essentially consists of two parts. Part I contains illustrative examples of each phase of a geostatistical study using a roll-front type uranium deposit. Part II contains five (5) computer programs and comprehensive user's manuals for these programs which are necessary to make a practical geostatistical study.

In developing these examples and computer programs, emphasis has been placed naturally on the feasibility of implementing geostatistics into ERDA reserve estimation procedures. However, the developed illustrations as well as the tools are general enough to be used by all practitioners in ore reserve estimation.

The remaining Part I portion of this report consists of Chapters 2 through Chapters 5. In Chapter 2, a capsule view of the geostatistical ore reserve estimation method is presented in which the reader is exposed to; a) the basic difference between classical statistics and Matheron's geostatistics, b) variogram, and c) the role of "kriging" in geostatistics. These topics are subsequently expanded in Chapters 3 through Chapters 5, using illustrative examples and a sufficient amount of mathematical finesse.

Specifically, Chapter 3 discusses a formal definition, procedures and examples of variogram modeling. In Chapter 4, the Lagrange multiplier method of converting a constrained optimization problem into an equivalent form of unconstrained optimization method is discussed. Understanding of the Lagrange multiplier method is mandatory for technical comprehension of the kriging procedure. Chapter 5 provides a concise explanation and a mathematical derivation of the kriging process and presents two examples that explain in detail the calculations involved in kriging.

Part II of this report which contains five (5) computer program user's manuals complete with program listings, consists of five appendices. These five computer programs are:

- a) Variogram computation program called UGAMM.
- b) Calcomp plotter variogram display program called VARPLOT.
- c) Printer map creation program called MAP.
- e) Estimation variance computation program called ESTVAR.

Each manual describes in detail all the program capabilities, limitations, and sample inputs and outputs.

Program UGAMM is designed to conveniently accept data from ERDA's SCOR program output in its variogram computation. The SCOR program, which is not included in this report, calculates the grade, thickness and grade thickness product for the combined total of all ore zones encountered in each drill hole. Program VARPLOT provides a plot of both the variogram calculated by UGAMM and, if specified, the theoretical variogram model that best describes the calculated variogram.

The kriging program UKRIG accepts data from ERDA's SCORE program. The output from UKRIG consists of a file containing the grade, thickness

and grade thickness product of the ore within each block of the deposit.

Program MAP prints a plan map of the block values computed by program UKRIG.

Program ESTVAR calculates the estimation variance due to giving the grade of a set of samples to some volume. An example of its use would be to calculate the probable error in an ore reserve estimate made by assigning the average grade of a group of samples within a deposit to the deposit.

A CAPSULE VIEW OF GEOSTATISTICAL ORE RESERVE ESTIMATION METHOD

2.1 INTRODUCTION

The geostatistical ore reserve estimation method (or geostatistics in short) is a relatively new method developed by G. Matheron in France during the early 1960's. The underlying mathematical basis of the method is called "The Theory of Regionalized Variables," and the method somewhat resembles the classical statistical approach.

However, there exists one fundamental difference between geostatistics and the classical statistics. In the latter, the samples taken to infer the unknown population are always assumed to be random samples. Furthermore, they are assumed to be independent of each other; i.e., obtaining one sample does not affect the outcome of the next sample.

Since the above conditions are rarely satisfied when one takes drill hole assay samples, geostatistics does not demand that samples be independent. On the contrary, this method assumes that adjoining samples are correlated to each other spatially (or in space). It also assumes that the particular relationship expressing the extent of the above correlation can be analytically and statistically captured in a function known as "variogram function" or "variogram" in short.

Geostatistics, therefore, has a special appeal to a geologist or to a mining engineer, not only because of the above explicit recognition of the spatial relationship existing between adjoining samples, but also due to its ability to provide this relationship in quantitative terms.

2.2 VARIOGRAM

The variogram is used to describe the spatial correlation between grades (or any other characteristics such as the thickness of

mineralization) within an ore deposit. Thus, it is the basic tool of geostatistics. To a mining engineer or geologist, the variogram is of interest since it incorporates several geological features that are important in ore deposit evaluation, such as the continuity of mineralization, zones of influence, and whether the zones of influence are different along different directions.

These interesting features are captured by analyzing the similarity or dissimilarity between grades for some distance (h) apart. Specifically, the squared difference between two grades (h) meters apart is calculated. To get a generalized relationship applicable to the area of interest, the calculation is repeated for all the samples that are (h) distance apart and the average squared difference obtained. This is the definition of the variogram.

Figure 2.1 shows an example of a variogram calculated for a roll-front type uranium deposit. On this graph, distance (h) is on the X axis and the average squared difference of the grade in $(\%)^2$, which is called $\gamma(h)$, is plotted on the Y axis. As can be seen from this variogram, the values of $\gamma(h)$ increase with increasing distance until it levels off at a certain distance. This tells us what we intuitively knew; i.e., the samples nearby do have similar values and that samples farther away are likely to have quite different values. The variogram thus numerically describes the continuity of mineralization. Figure 2.2 shows several examples of different degrees of continuity.

The distance at which the variogram levels off is called the range and is simply the traditional geological notion of range of influence. It means that beyond the range the samples are no longer correlated; i.e., they are independent of one another.

The value of the variogram where it levels off is called the sill value. In Figure 2.1, this sill value of $0.0043 (\%)^2$ is also the variance of all the

URANIUM PROJECT GRADE VARIOGRAM

DIRECTION = 0. WINDOW = 90.

CLASS SIZE = 100.00

LOGARITHMS -

MEAN =

VARIANCE = 0.0043

NO. OF SAMPLES = 770

Figure 2.1

SAMPLE VARIOGRAM FOR A ROLL-FRONT TYPE URANIUM DEPOSIT

High degree of continuity. Can be approximated by parabola near the origin. Bed thickness is an example.

Average continuity. Almost linear near origin. Typical of many metal deposits.

Nugget effect. Discontinuity at the origin and thereafter much like above variogram.

Purely random.

Figure 2.2

DEGREES OF CONTINUITY EXPRESSED BY VARIOGRAM

samples used in variogram development. This is no accident because the computation of the average squared difference between samples (h) distance apart is similar to calculating a variance in classical statistics.

Just as the statistical variance of an estimate is frequently viewed as the uncertainty associated with the given estimate, the variogram can be viewed as the measurement of uncertainty that exists when a sample assay is extended some distance. This uncertainty exists mainly due to the variability between grades, as has been captured in the variogram.

The value of the variogram at distance $(h) = 0$ is called the nugget value. In Figure 2.1, the estimated nugget value is approximately $0.002 (\%)^2$ which is a relatively high value in relation to the sill value of $0.0043 (\%)^2$. Ideally, this nugget value should be zero simply because two samples from the same point (i.e., $h = 0$) should have the same grade. A positive nugget value, on the other hand, can have real physical meaning or be due to errors in sampling or assaying. In general the nugget should be relatively small if correct sampling and assaying procedures have been used. A relatively high nugget value is very common in the uranium deposits, thus indicating the difficulty of obtaining accurate assays from the present sampling procedures.

A third item of geological interest that can be obtained from the variogram is whether there is anisotropy in the mineralization; i.e., is the mineralization more continuous in one direction than another? Anisotropy is determined by comparing the range of influence of the variograms, computed along different directions.

It is hoped that the above brief description of the variogram gives at least some feeling of how the variogram captures and displays several of the features of the mineralization. Some examples on the process of variogram

modeling is given in Chapter 3 of this report to further illustrate the nature of a variogram.

2.3 GEOSTATISTICAL ORE RESERVE ESTIMATION METHOD

The ultimate objective of any geostatistical study is to obtain an ore reserve estimate and related confidence levels using geostatistics, thus reaping all the associated benefits of this new methodology. Two basic steps must be performed for any full-fledged geostatistical ore reserve estimation. These two basic steps are: (1) developing the representative variogram model(s) of the deposit and (2) assigning block grades using an inference (or weighting) scheme known as "kriging."

The first step is mandatory because it is a basic tool of geostatistics. Performing the first step is sufficient for obtaining a global estimate and associated confidence limits of a deposit, i.e., total tonnage and its average grades. On the other hand, the second step of kriging individual blocks within an orebody is needed if one is interested in knowing the specific ore locations in the deposit.

The attractiveness of the second step, kriging, compared to other methods of assigning block grades is that kriging provides an unbiased estimate having minimum estimation error.* It does this by utilizing the variogram to assign weights to each sample grade in such a manner that the sample information is used to best advantage.

Another tangible benefit of the geostatistical ore reserve estimation method is that it can provide statistical confidence limits around the obtained grade estimate. For this purpose, the variations indicated by the

*Estimation error refers to the error one incurs when the sample grade from a drill hole is extended to the surrounding area for reserve estimation purposes.

variogram are used. Consequently, the task of performing the first step, i.e., variogram development, is the most important one in any geostatistical study. Yet, it is a step requiring a considerable amount of subjective interpretation of computation results. On the other hand, performing the second task, i.e., kriging the blocks, is quite straightforward although extremely time-consuming.

Although the task of performing kriging is straightforward, technical understanding of the kriging procedures requires a familiarity with the method of Lagrange multipliers.⁵ It is a method of converting a constrained optimization problem into an equivalent form of unconstrained optimization problem, so that the classical calculus method can be applied to obtain the desired solution.

Consequently, the Lagrange multiplier method is explained in Chapter 4 of this report whereas techniques and examples of kriging are provided in Chapter 5.

EXAMPLES OF VARIOGRAM DEVELOPMENT

Obtaining good and representative variograms involves a considerable amount of computations due to the empirical as well as subjective nature of the work. The magnitude of the task increases as the complexity of the deposit increases. For instance, variograms may have to be calculated for 1) each variable of interest (grade, thickness, etc.), 2) various directions, 3) different geologic or mineralogic zones within the deposit and 4) different types of sampling or assaying procedures (chemical or radio-metric analyses).

In this chapter procedures involved in variogram calculation will be illustrated by two examples. The first example simply illustrates the computations involved in the calculation of a variogram, whereas the second example illustrates the sequence of steps leading to the discovery of a representative variogram for a roll front uranium deposit.

3.1 EXAMPLE OF COMPUTATIONS

Figure 3.1 shows a small section of an uranium deposit. This section will be used to briefly illustrate the actual computations involved in calculating a variogram. The holes are on a nearly uniform grid of 100 feet, but for this illustration we will assume the grid spacing is exactly 100 feet.

The equation for the variogram is given by Equation 3.1.

$$\gamma(h) = \frac{1}{2N} \sum_{i=1}^N [f(x_i) - f(x_i + h)]^2 \quad (3.1)$$

where N = number of sample pairs with the samples $f(x_i)$ and $f(x_i + h)$ being h distance apart.

Note: N Hole Number
 + Hole Location
 VAL Value of Interest such as Grade

Figure 3.1

SMALL SECTION OF AN URANIUM DEPOSIT

To illustrate its use a variogram will be calculated for the E-W direction. This is done by picking all the sample pairs lying E-W, squaring their difference, and grouping them by distance.

For example, for $h = 100$.

$$\begin{aligned} \gamma(100) = \frac{1}{2(20)} & [(X_1 - X_2)^2 + (X_2 - X_3)^2 + (X_3 - X_4)^2 + (X_4 - X_5)^2 \\ & + (X_6 - X_7)^2 + (X_7 - X_8)^2 + (X_8 - X_9)^2 + (X_9 - X_{10})^2 \\ & + (X_{11} - X_{12})^2 + (X_{12} - X_{13})^2 + (X_{13} - X_{14})^2 + (X_{14} - X_{15})^2 \\ & + (X_{16} - X_{17})^2 + (X_{17} - X_{18})^2 + (X_{18} - X_{19})^2 + (X_{19} - X_{20})^2 \\ & + (X_{21} - X_{22})^2 + (X_{22} - X_{23})^2 + (X_{23} - X_{24})^2 + (X_{24} - X_{25})^2] \end{aligned}$$

$$\begin{aligned} \gamma(100) = \frac{1}{40} & [(.14 - .28)^2 + (.28 - .10)^2 + (.10 - .19)^2 + (.19 - .09)^2 \\ & (.08 - .14)^2 + \dots \\ & \dots \dots \dots (.07 - .17)^2] \end{aligned}$$

$$\gamma(100) = \frac{1}{40} [(.14)^2 + (.18)^2 + (.09)^2 + (.10)^2 \dots (.10)^2]$$

$$\gamma(100) = \frac{1}{40} [.1533] = .0038$$

For $h = 200$ feet we take all sample pairs lying E-W and separated by 200 feet.

$$\begin{aligned} \gamma(200) &= \frac{1}{2(12)} [(X_1 - X_3)^2 + (X_2 - X_4)^2 + (X_3 - X_5)^2 \\ &+ (X_6 - X_8)^2 + (X_7 - X_9)^2 + (X_8 - X_{10})^2 \\ &+ \dots \dots \dots (X_{23} - X_{25})^2] \\ &= \frac{1}{24} [(.04)^2 + (.09)^2 + (.01)^2 \dots (.04)^2] \\ &= \frac{1}{24} [.0806] = .00335 \end{aligned}$$

Similar calculations can be made for $h = 300$ feet and $h = 400$ feet. Note, however, that as h gets larger there are fewer and fewer sample pairs that are h feet apart, and thus the reliability of the variogram decreases. Also note that there is no pair of samples for $h = 500$ ft. or larger, at least in the above example problem.

3.2 EXAMPLE OF VARIOGRAM DEVELOPMENT

The procedure of variogram calculation will be illustrated by the calculation of variograms for a roll front type uranium deposit located in Wyoming. Roll front uranium deposits are usually fairly complex due to their often having several mineralized zones and the mineralization being quite variable in both grade and thickness. A cross section through part of the deposit used for this example is shown in Figure 3.2. Note the variation in grade and thickness for each of the horizons and the poor horizontal continuity of the zones.

The first step in variogram modeling is to clearly define the purpose of the study in order to learn which variables are of interest in the study. For example, is it to calculate global ore reserves, or to calculate ore reserves for mine planning purposes? In this study, global ore reserves are of most interest. Thus, the interest focuses on how much uranium is present in a deposit, but not necessarily where it is located within the deposit.

The variables of interest are the grade of uranium, the thickness of uranium ore within the deposit and the grade thickness product (GT). Thus, to determine the global ore reserves it is only necessary to calculate the

Figure 3.2

CROSS SECTION OF URANIUM DEPOSIT

average grade of the deposit and multiply this by the area of the deposit and the average thickness of ore. For each drill hole, the grade, thickness, and GT are calculated for each mineralized zone intersected by the drill hole. These are totaled to give the total thickness of mineralization, the average grade and the GT within each hole.

The cumulative total thickness, the average grade and the GT are chosen as the variables of interest for this study. It should be noted that the vertical distribution of ore zones is lost by this procedure, but the quantity of uranium per unit area is of course the same.

The deposit is examined to determine if there are any areas that should be analyzed separately due to mineralogical or geological differences. None are found. Since the variogram measures the degree of continuity between sample grades (or thickness, GT, etc.) within the deposit, it is necessary to delete from the variogram calculations the drill holes that are obviously not within the deposit. This is especially necessary in uranium deposits where a single project may have several thousand drill holes of which quite a few may be outside the deposit.

Finally, actual variogram calculations begin. Variograms are calculated for the grade, the thickness and GT.

It should be mentioned here that the natural values of the variables are used in all the calculations. Optionally the variables could be transformed by taking their logarithms. Initial variograms are calculated along the E-W direction but with a window of 90° . This is referred to as an average variogram for all directions and mainly serves to show the general characteristics of the variogram. Figures 3.3, 3.4, and 3.5 show the average variogram for grade, thickness and G.T.

URANIUM PROJECT GRADE VARIOGRAM

DIRECTION = 0. WINDOW = 90.

CLASS SIZE = 100.00

LOGARITHMS -

MEAN =

VARIANCE = 0.0043

NO. OF SAMPLES = 771

Figure 3.3

AVERAGE VARIOGRAM FOR GRADE

URANIUM PROJECT THICKNESS VARIOGRAM

DIRECTION = 0. WINDOW = 90.

CLASS SIZE = 100.00

LOGARITHMS -

MEAN =

VARIANCE = 74.4000

NO. OF SAMPLES = 771

Figure 3.4

AVERAGE VARIOGRAM FOR THICKNESS

URANIUM PROJECT GRADE THICKNESS VARIOGRAM
 DIRECTION = 0. WINDOW = 90. MEAN =
 CLASS SIZE = 100.00 VARIANCE = 3.1100
 LOGARITHMS - NO. OF SAMPLES = 771

Figure 3.5

AVERAGE VARIOGRAM FOR GRADE THICKNESS

The average variogram for grade is a good example of a spherical variogram with a range about 300 feet and a relatively high nugget value. The average variograms for thickness and GT are very similar in shape. They do not resemble the spherical variogram model as closely as the grade variogram does. Their range is about 700 feet and they also have a relatively large nugget effect.

Next, variograms are calculated along the directions of N-S, E-W, NE-SW and NW-SE to test for anisotropy. Sometimes the directions of anisotropy can be deduced apriori from the geology of the deposit, thus saving a lengthy search for anisotropy. For example, one might think that the continuity of grade (or thickness, or GT) is greater along the axis of the roll front than across it. The roll fronts in the deposit studied are quite variable, perhaps much like a meandering stream, but the average axis direction is probably NW-SE.

The variograms for each of the directions are shown in Figures 3.6, 3.7, and 3.8. Examination of the variograms for thickness and GT show no readily apparent geometric anisotropy. The variograms for each direction are quite similar to each other and their differences could be due to fluctuations in the variograms. It should be remembered that as the variograms are calculated using a narrower window they become less smooth due to the decrease in the number of sample pairs used to calculate each point on the variogram.

The variograms for grade reveal a noticeable difference between the variograms of the NW-SE direction and the NE-SW direction, whereas the variograms N-S and E-W are nearly the same.

The difference between the NW-SE and NE-SW variograms is probably not

URANIUM PROJECT GRADE VARIOGRAM
 DIRECTION = WINDOW = 30.
 CLASS SIZE = 100.00
 LOGARITHMS -

MEAN =
 VARIANCE = 0.0043
 NO. OF SAMPLES = 771

Figure 3.6

VARIOGRAMS FOR GRADE

Figure 3.7

VARIOGRAMS FOR THICKNESS

Figure 3.8

VARIOGRAMS FOR GRADE THICKNESS

a simple geometric anisotropy, since both variograms appear to have the same range. The difference is due to the NE-SW variogram overriding the sill value between 200 and 300 feet and then dropping down to the sill value. This effect is not uncommon, but has not been really adequately explained. One theoretical model called the hole-effect model, which is illustrated in Figure 3.9, has a similar behavior. This model has been used to describe a deposit where a series of rich and poor zones exist. While the NE-SW and NW-SE have obvious differences, they both can be modeled by the same spherical variogram.

The final step in the analysis of the variograms is to find the theoretical variogram that best describes the experimental variograms. Both choosing the model and estimating its parameters is quite subjective. Fortunately, the selection of a model can often be narrowed to the choice of either a spherical model or a DeWijsian model, with the spherical model being the more common.

The experimental variograms calculated for this example are best described by the spherical model. The variograms for grade are especially good examples of spherical type variograms. A simple procedure is available to fit the spherical model and will be illustrated with the average variogram for grade, shown in Figure 3.3.

The spherical variogram is defined by Equation 3.2

$$\begin{aligned} \gamma(h) &= C_0 + C \left[\left(1.5 \cdot \left(\frac{h}{a} \right) - .5 \left(\frac{h}{a} \right)^3 \right) \right] & \text{for } h < a \\ &= C_0 + C & h \geq a \end{aligned} \quad (3.2)$$

The nugget effect is C_0 , the range a , and the sill value $C + C_0$.

Figure 3.9

PLOT OF THEORETICAL "HOLE-EFFECT" MODEL

The first step is to draw a tangent line through the first two or three points as shown on Figure 3.10. The y intercept of this line is an estimate of the nugget effect, C_0 (.0027). Next, the sill is drawn. Remember the sill is equal to the variance of samples (.0043). Thus $C + C_0 = .0043$ and $C = 0.0016$. The intersection of the tangent and the sill is an estimate of $2/3a$, or $a = 210'$. The fit can be checked by plotting the fitted curve and comparing it with the experimental variogram. As can be seen on Figure 3.11 the fit is quite good. If necessary the parameters can be modified to give a better fit.

Although this procedure is subjective, it usually leads to good fits and is very quick. It should be noted that the nugget value, C_0 , should be estimated most closely, while errors in C and a are less important. This is because the values of the variogram near the origin are of most importance in estimation problems.

URANIUM PROJECT GRADE VARIOGRAM
 DIRECTION = 0. WINDOW = 90.
 CLASS SIZE = 100.00
 LOGARITHMS -

MEAN =
 VARIANCE = 0.0043
 NO. OF SAMPLES = 770

Figure 3.10

FITTING A SPHERICAL MODEL

URANIUM PROJECT GRADE VARIOGRAM
 DIRECTION = 0. WINDOW = 90.
 CLASS SIZE = 100.00
 LOGARITHMS -

MEAN =
 VARIANCE = 0.0043
 NO. OF SAMPLES = 770

Figure 3.11

FITTED SPHERICAL MODEL

TECHNIQUE OF LAGRANGE MULTIPLIERS

The method of Lagrange multipliers is a technique for converting a constrained optimization problem into an unconstrained problem so that the classical calculus method can be applied to obtain the desired answer. Although this method has been known to mathematicians for quite some time, the practical significance of the Lagrange multiplier has not been appreciated until very recently by engineers, scientists, and economists.

4.1 UNCONSTRAINED VS CONSTRAINED OPTIMIZATION

From calculus, one is familiar with an unconstrained optimization problem. The simplest form of such a problem is to find either the maximum or the minimum point of some function such as $f(x) = x^2 - 2x + 4$. Mathematically, this problem can be stated as follows:

$$\begin{array}{ll} \text{Minimize} & f(x) = x^2 - 2x + 4 \\ \text{subject to} & -\infty \leq x \leq \infty \end{array} \quad (4.1)$$

The above problem is an unconstrained optimization problem because the variable x is not confined to within some specified range. In other words, the desired point can be sought within all possible ranges of x . In an unconstrained optimization problem, the solution can be always found at one of the stationary points, i.e., where the slope (or the derivative) is zero. Consequently, one attempts to obtain the solution to the above problem by first locating one or more stationary points through differentiating the function with respect to the variable x and then equating to zero.

Unfortunately, however, the approach may not always work for a constrained optimization problem, i.e., the range of variable is restricted. In the above problem, suppose one restricts the value of x to within the range of 5 to 10 instead of $-\infty$ to $+\infty$. Then, the problem becomes:

$$\begin{array}{ll} \text{Minimize} & f(x) = x^2 - 2x + 4 \\ \text{subject to} & 5 \leq x \leq 10 \end{array} \quad (4.2)$$

The resulting solution to the above constrained problem is $x = 5$ which is not the stationary point of the given function. (Note that $x = 1$ is the stationary point.) As can be seen from this example, solving the constrained optimization problems by the classical calculus approach is in most instances quite difficult and sometimes impossible. To overcome this difficulty, Lagrange devised an ingenious technique about 1797 which, since then, became known as the method of Lagrange multipliers. However, he gave no explanation beyond the statement that the rules he gave worked.

4.2 THE CLASSICAL CALCULUS METHOD FOR A FUNCTION OF MANY VARIABLES

Most real life problems including kriging problems possess many variables. Furthermore, most of us are less familiar with the technique of locating the stationary points when the function contains many variables. Hence, this method is reviewed here prior to the discussion of the Lagrange multiplier method.

Suppose a function F with many variables can be expressed as in Equation 4.3:

$$F = f(x_1, x_2, x_3, \dots, x_n) \quad (4.3)$$

Theorem 1: The necessary condition that this function has a stationary point at the point $X^0 = (x_1^0, x_2^0, x_3^0, \dots, x_n^0)$ is as follows:

$$f_i(x_1^0, x_2^0, x_3^0, \dots, x_n^0) = 0 \text{ for } i = 1, 2, \dots, n \quad (4.4)$$

where $f_i(x_1^0, x_2^0, x_3^0, \dots, x_n^0)$ is the partial derivative function of F with respect x_i evaluated at the point X^0 .

It should be noted that Equation 4.4 above consists of a set of n -simultaneous equations having n -variables. Therefore, this set of simultaneous equations must be solved in order to locate the point $X^0 = (x_1^0, x_2^0, \dots, x_n^0)$, where the solution to this set of simultaneous equations provides values for $x_1^0, x_2^0, \dots, x_n^0$.

Locating the stationary point X^0 , however, is not sufficient to know whether that point is the maximum (or the minimum) of the function F in Equation 4.3. For this purpose, the sufficient condition as given in Theorem 2 below must be satisfied by that point X^0 .

Theorem 2: The sufficient condition that a function F is strictly maximum at the point $X^0 = (x_1^0, x_2^0, x_3^0, \dots, x_n^0)$ is to satisfy the following conditions:⁶

$$f_{11}(X^0) < 0$$

$$\begin{vmatrix} f_{11}(X^0) & f_{12}(X^0) \\ f_{21}(X^0) & f_{22}(X^0) \end{vmatrix} > 0 \quad (4.5)$$

$$\begin{vmatrix} f_{11}(X^0) & f_{12}(X^0) & f_{13}(X^0) \\ f_{21}(X^0) & f_{22}(X^0) & f_{23}(X^0) \\ f_{31}(X^0) & f_{32}(X^0) & f_{33}(X^0) \end{vmatrix} < 0$$

etc.

In Equation 4.5 above, $f_{ij}(x^0)$ is second-order partial derivative evaluated at that point.

If the indicated determinants in Equation 4.5 alternate in sign as shown, then the function F is strictly maximum at that point. If the indicated determinants are all positive, it is strictly minimum. The word "strict" implies that there exists no doubt regarding the point X^0 . However, if the indicated determinants are all zeros or if they fail to satisfy the condition strictly, then there exists doubt, i.e., the sufficiency test is inconclusive.

Because of the rather involved nature, the testing of sufficient condition is frequently by-passed in favor of some empirical procedure such as viewing the nature of this function F . Such is the case with kriging.

4.3 CONSTRAINED OPTIMIZATION PROBLEMS AND THEIR SOLUTION METHODS

Most real life problems fall under the category of constrained optimization problems because a problem without any constraint is really not a problem. There are several solution methods for the constrained optimization problem. Some of these are:

- 1) The substitution method.
- 2) The method of Lagrange multiplier.
- 3) Mathematical programming.

The subject matter of mathematical programming is too broad to be discussed in this report. Therefore, the substitution method is first illustrated here using an example problem, followed by the Lagrange multiplier method.

Illustrative Example Problem:

A lot has the form of a right triangle with the two legs equal to 60 ft. and 100 ft., respectively. A rectangular building is to be erected on the lot. Determine its dimensions for maximum floor area.

Solution:

This problem is obviously a constrained optimization problem in which we want to maximize the floor space of the building subject to the geometry constraints of the lot as well as the building. The size of lot and the proposed rectangular building are shown in Figure 4.1 below.

If one defines the dimensions of this building along x direction to be x_1 and along y direction to be x_2 , the geometry constraint of the problem can be specified using the relationship of two similar triangles $\triangle ABC$ and $\triangle DEC$ in Figure 4.1. That is:

$$\frac{100}{60} = \frac{x_2}{60-x_1}$$

$$(60-x_1) 100 = 60x_2 \quad (4.6)$$

$$5x_1 + 3x_2 - 300 = 0$$

Consequently, the constrained optimization problem can now be stated formally as follows:

$$\begin{array}{ll} \text{Maximize} & F = x_1 x_2 \\ \text{subject to} & 5x_1 + 3x_2 - 300 = 0 \end{array} \quad (4.7)$$

Figure 4.1

CONFIGURATION OF THE EXISTING LOT & PROPOSED BUILDING

Initial Attempt: (Classical Calculus Method of Section 4.2 Ignoring Constraint)

This problem contains two variables, x_1 and x_2 , and one equality constraint. Attempting to locate the stationary point of the objective function, i.e., $F = x_1 x_2$, by performing the partial differentiation of the function F with respect to each variable while ignoring the constraint equation provides the following answer:

$$\begin{aligned}x_1 &= 0 \\x_2 &= 0 \\ \text{Area} &= 0\end{aligned}\tag{4.8}$$

Although the stationary point of the above function F is readily found, it is not the answer to the problem. The sufficiency condition testing using Equation 4.5 also provides an inconclusive answer because the second-order partial derivatives are both zero which results in the determinant being zero. Thus, this is a good example in which the direct application of the classical calculus method fails to provide the answer.

The Substitution Method

In this method, the constraint equation of Equation 4.7 is incorporated into the objective function through direct substitution of one variable in terms of the other variable, using the constraint equation. Afterwards, the previous attempt is repeated to locate the stationary point of the revised objective function, and subsequently the optimum solution to the problem.

$$\begin{aligned}\text{From Equation 4.7, one has } 5x_1 + 3x_2 - 300 &= 0 \\ \text{or } x_2 &= \frac{5}{3} (60 - x_1)\end{aligned}\tag{4.9}$$

The revised objective function is:

$$F = \frac{5}{3} (60 - x_1) x_1 = 100 x_1 - \frac{5}{3} x_1^2$$

$$f_1 = \frac{dF}{dx_1} = 100 - \frac{10}{3} x_1 \quad (4.10)$$

$$f_{11} = \frac{d^2F}{dx_1^2} = -\frac{10}{3}$$

By setting $f_1 = 0$, we obtain the desired stationary point $X^0 = (30, 50)$, i.e., $x_1 = 30$ ft. and $x_2 = 50$ ft. The floor area is consequently 1500 sq. ft. which is the maximum attainable. The fact that the answer X^0 is the maximum point of the revised objective function can be easily demonstrated through the sufficiency condition given earlier, i.e., f_{11} is negative.

4.4 THE METHOD OF LAGRANGE MULTIPLIER

This procedure will now be explained using the above example problem. Although the substitution method can be used for a simple constrained optimization problem such as the example problem, it is not always possible to make the required substitution particularly when the constraint equations have product terms, i.e., the variables are not separable. Fortunately, the method of Lagrange multiplier does not possess this drawback. Even when the substitution is possible, the Lagrange multiplier method is frequently chosen for the additional information that is obtained.

Procedure:

First, the optimization problem of Equation 4.7 is repeated below:

$$\begin{array}{ll} \text{Maximize} & F = x_1 x_2 \\ \text{subject to} & 5x_1 + 3x_2 = 300 \end{array} \quad (4.7)$$

or

$$\begin{array}{ll} \text{Maximize} & f(x_1, x_2) \\ \text{subject to} & g(x_1, x_2) = b \end{array} \quad (4.7')$$

The procedure begins by converting the above constrained optimization problem into an unconstrained optimization of a new function called Lagrangean function. To do this, the following steps must be performed.

- i) Convert all constraint equations into equality constraints of the form given above.
- ii) Multiply each equality constraint by a new variable λ_i , where λ_i is the Lagrange's undetermined multiplier for the i -th constraint.
- iii) Simply add (or subtract) the resulting constraint equations to the original objective function f to obtain the Lagrangean function.

In the example problem, we can start from step ii). Since there is only one constraint equation, we need only one λ . Consequently, the obtained Lagrangean function has three variables, x_1 , x_2 , and λ .

$$\begin{aligned} L(x_1, x_2, \lambda) &= f(x_1, x_2) - \lambda [g(x_1, x_2) - b] \\ &= x_1 x_2 - \lambda (5x_1 + 3x_2 - 300) \end{aligned} \quad (4.11)$$

Equation 4.11 can now be treated as an unconstrained optimization problem. Thus, the method of Section 4.2 can be applied directly to obtain

the solution. Differentiating the Lagrangean function with respect to each variable and setting the resulting equations to zero, the stationary point is obtained after solving Equation 4.12.

$$\begin{aligned}\frac{\partial L}{\partial x_1} &= x_2 - 5\lambda = 0 \\ \frac{\partial L}{\partial x_2} &= x_1 - 3\lambda = 0 \\ \frac{\partial L}{\partial \lambda} &= -(5x_1 + 3x_2 - 300) = 0\end{aligned}\tag{4.12}$$

Equation 4.12 is a set of simultaneous equations having three variables and three equations. Also, note that the original constraint reappears and becomes part of the set of equations. Solving Equation 4.12, one obtains the following answer.

$$\begin{aligned}x_1 &= 30 \text{ ft.} \\ x_2 &= 50 \text{ ft.} \\ \lambda &= 10 \\ \text{Area} &= 1500 \text{ sq. ft.}\end{aligned}$$

Testing the sufficiency condition for this situation gives an inconclusive answer due to the vanishing of all second-order partial derivatives. However, one can conclude that the obtained solution is the maximum point of the function f , simply from the nature of the function which can only increase in value (See Figure 4.2).

Figure 4.2

LOCATION OF THE OPTIMAL SOLUTION

Justification and Significance of the Lagrange Multiplier:

The development and an interpretation of the Lagrange multiplier method can be made by utilizing Figure 4.2. It shows the ranges of x_1 and x_2 , the constraint equation of $5x_1 + 3x_2 = 300$, and some iso-F curves. An iso-F curve is equivalent to a contour line having the same objective function value.

As one searches for the x_1 and x_2 values that maximize $F = f(x_1, x_2)$, the constraint equation restricts us to the set of points defined by $g(x_1, x_2) = b$. In other words, as we move from one point of this set to another point, the value of g is not permitted to change. Or, one must search the maximizing point only along the constraint equation.

From Figure 4.2, one notes that the desired solution is at point X^0 which lies along the constraint equation ($5x_1 + 3x_2 = 500$) and which touches one of the iso-F curves. Other points such as A, B, C, D, all of which also lie on the constraint equation, can not be the maximum point because these points intersect inferior iso-F curves. Thus, the Lagrange multiplier method expresses the condition that the best iso-F curve attainable is the one which is just tangent to the constraint curve.

In a more general situation in which the constraint equation is not linear, the condition for the maximizing point is that the tangent gradient of the constraint equation g must be equal to that of the objective function F . In other words, we need only to look at those points where two tangent gradients are equal. This fact naturally eliminates much of needless searching along the points of $g(x_1, x_2) = b$, such as points A, B, C, and D in Figure 4.2.

The above requirement can be expressed mathematically as

$$\frac{\partial F/\partial x_1}{\partial F/\partial x_2} = \frac{\partial g/\partial x_1}{\partial g/\partial x_2}$$

or

$$\frac{\partial F/\partial x_1}{\partial g/\partial x_1} = \frac{\partial F/\partial x_2}{\partial g/\partial x_2} = -\lambda \quad (4.13)$$

The proportionality factor expressed as $-\lambda$ in Equation 4.13 is the Lagrange multiplier. It can also be shown that $-\lambda$ has a special significance in economics. That is, it gives the "shadow price" of the constraint, i.e., $\partial F/\partial b$. For example, $\lambda = 10$ in the above example problem indicates that one can increase the floor space by 10 sq. ft., if the constraint value of $b=300$ is relaxed by one unit, i.e., $b = 301$.

In case of a function of more than two variables and (for simplicity) one constraint equation, the partial derivatives of f and g with respect to each variable must be proportional at the maximizing point $(x_1^0, x_2^0, x_3^0, \dots, x_n^0)$.

$$\frac{\partial F/\partial x_1}{\partial g/\partial x_1} = \frac{\partial F/\partial x_2}{\partial g/\partial x_2} = \dots = \frac{\partial F/\partial x_n}{\partial g/\partial x_n} = -\lambda \quad (4.14)$$

Finally, the generalized Lagrangean function involving n variables and m constraints are:

$$L(x_1, x_2, \dots, x_n; \lambda_1, \lambda_2, \dots, \lambda_m) = f(x_1, x_2, \dots, x_n) + \sum_{i=1}^m \lambda_i g_i(x_1, x_2, \dots, x_n) \quad (4.15)$$

Then the necessary condition is comprised of a system of $(n+m)$ equations that can (at least in principle) be solved for the $(n+m)$ unknowns. The practical aspects of solving such a system of simultaneous equations is discussed in the following chapter on kriging.

ILLUSTRATION OF KRIGING PROCEDURE AND EXAMPLES

In order to comprehend the theoretical basis of the kriging procedure, one must be familiar with the following concepts from Matheron's geostatistics (or geostatistics in short).

1. The variogram function (or variogram).
2. Variance of point samples o within any size volume v , given as $\sigma^2_{(o/v)}$.
3. Block variance in a deposit, given as $\sigma^2_{(v/D)}$.
4. Krige's Relationship.
5. Extension (or Estimation) variance, given as σ^2_E .

In addition, one must be familiar with the method of Lagrange multiplier. Since a detailed exposition on all of the above concepts is clearly beyond the scope of this report, only a brief review will be given here. The reader is referred to other sources of references.^{1,2,3,4}

5.1 VARIANCE OF POINT SAMPLES WITHIN ANY SIZE VOLUME V.

Bypassing a review on the variogram function, the variance of point samples o within any size volume can be defined as:

$$\sigma^2_{(o/v)} = \frac{1}{v^2} \int_V dx \int_V \gamma(x-y) dy \quad (5.1)$$

In Equation 5.1 above, the variables x and y are two dummy variables used for integration of the variogram function values over the volume of interest. Strictly speaking, the equation corresponds to sextuple integrals,

if v is a volume in a 3-dimensional space. On the other hand, it reduces to quadruple integrals when v is reduced to an area.

As an illustration, suppose v is a square area of 10' by 10'. Further, suppose the point (0,0) is the south-west corner of the area as shown in Figure 5.1a. Then, for a given variogram function, the variance of point samples (such as DDH assays) within this 10' x 10' area can be obtained by performing the following integration.

$$\sigma^2_{(o/v)} = \frac{1}{(10 \times 10)^2} \int_0^{10} \int_0^{10} dx \, dy \int_0^{10} \int_0^{10} \gamma(\sqrt{x^2+y^2}) \, dx \, dy \quad (5.2)$$

Evaluation of the above integral is both time consuming as well as difficult. Therefore, a numerical approximation is used to obtain the answer. Specifically, some discrete number of points are located in the area v as shown in Figure 5.1b.

Next, the distances between the first point and the remaining points are computed, thus obtaining eight distinct distances in Figure 5.1b. Afterwards, variogram values corresponding to these distances are obtained from the given variogram function and these values are summed up.

The above steps are repeated for each and every point in the block, i.e., 9 holes in Figure 5.1b. Finally, the cumulated variogram value is divided by the total number of points in the block. The resulting value, therefore, represents the average value of the variogram in the block. In other words, Equation 5.1 can be viewed as the average value of the variogram within a given size volume v .

Figure 5.1a

SAMPLE AREA OF INTEGRATION

Figure 5.1b

NINE (9) POINT NUMERICAL APPROXIMATION FOR INTEGRATION

If this volume v is enlarged to coincide with the deposit under consideration, the variance of point samples in a deposit can be given as Equation 5.3 below and simply represents the sample variance.

$$\begin{aligned}\sigma^2_{(o/D)} &= \frac{1}{D^2} \int_D dx \int_D \gamma(x-y) dy \\ &= \text{Sample variance in the deposit} \\ &= \text{Sill value (in a spherical variogram model)}\end{aligned}\tag{5.3}$$

Consequently, one can also view the sample variance in the deposit as being equivalent to the average value of the variogram function in the deposit (D).

5.2 BLOCK VARIANCE IN A DEPOSIT

Using a similar notation given in the previous section, the block variance in a deposit can be written as $\sigma^2_{(v/D)}$. If one divides the entire deposit into blocks of a given size v , then the block variance is the variance of these block grades. For computational purposes, it can be computed using Krige's relationship as follows:

$$\sigma^2_{(v/D)} = \sigma^2_{(o/D)} - \sigma^2_{(o/v)}\tag{5.4}$$

Equation 5.4 states that the block variance is simply the difference between the "average value" of the variogram in the deposit minus the "average value" of the variogram in a given size block. Since the first term on the right hand side of Equation 5.4 is also the sample variance in the deposit

(or the sill value) and since the second term can be easily computed for a given variogram function, the desired block variance can also be computed readily.

5.3 EXTENSION (OR ESTIMATION) VARIANCE (σ_E^2).

Extension variance refers to the variance (or error) one incurs when the grade of a known volume v is extended to infer the grade of another volume V . If a small known volume v is a DDH sample and a larger volume V is a block in a mine, then the extension variance σ_E^2 can be defined as:

$$\begin{aligned}\sigma_E^2 &= \sigma_{(v/D)}^2 - 2\sigma_{(v,V/D)} + \sigma_{(V/D)}^2 \\ &= \sigma_{(o/D)}^2 - 2\sigma_{(o,V/D)} + \sigma_{(V/D)}^2\end{aligned}\tag{5.5}$$

Equation 5.5 says that the extension variance is equal to the variance of the sample minus twice the covariance between the sample and the block plus the variance of the block. It was shown in the preceding section that the sample variance and the block variance can be readily computed. Similarly, the covariance between the sample and the block can be computed using the numerical approximation technique shown in Figure 5.1b. Consequently, the desired extension error can be computed for a given size block and a given variogram function.

In estimating the block grade, it is conceivable that more than one DDH samples are used to improve estimation. If a number of sample grades are extended to a block, the error one incurs is called "Estimation Variance" instead of "Extension Variance." When a number of sample grades are used

to estimate the block grade, one obvious way for estimation is to use the average grade of all N samples. If such mode of estimation is used, the estimation variance (σ_E^2) can be given by the following equation:

$$\sigma_E^2 = \frac{2}{Nv} \sum_{i=1}^N \int_V \gamma(x-x_i) dx - \frac{1}{v^2} \int_V dx \int_V \gamma(x-y) dy - \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N \gamma(x_i-x_j) \quad (5.6)$$

Equation 5.6 is similar to Equation 5.5 with one exception. That is, a sample grade is replaced by the average grade of N samples. Consequently, the estimation variance is now equal to twice the "average value" of the variogram between samples and the block minus the variance of a point within the block minus the "average value" of the variogram between the samples.

Using the average value of N samples is equivalent to assigning an equal weight to each sample grade, where this equal weight is given as $1/N$. However, one can conceivably assign different weights to different holes for obvious reasons. For instance, holes closer to the block can be given greater weights (or influence) than those that are farther away. Kriging is the procedure which attempts to do just this.

If different weights λ_i 's (where λ_i is the weight to be assigned to the i-th hole) are to be used instead of the equal weight of $1/N$, then Equation 5.6 can be rewritten as follows to obtain the desired "Estimation Variance," or more appropriately the "Kriging Variance."

$$\sigma_K^2 = \sigma_E^2 = \sigma_v^2 - 2 \sum_{i=1}^N \lambda_i \sigma_{vx_i} + \sum_{i=1}^N \sum_{j=1}^N \lambda_i \lambda_j \sigma_{x_i x_j} \quad (5.7)$$

Equation 5.7 can be obtained directly from Equation 5.6 by substituting λ_i for $1/N$ and then by expressing the resulting equation in terms of the covariogram instead of the variogram function. The above equation says that the "Kriging variance" is equal to the block variance minus twice the weighted covariance function values between the samples and the block plus the weighted covariance function values between samples.

Since there exists a direct relationship between the variogram and the covariogram functions, the kriging variance as given by Equation 5.7 can be readily computed for a given variogram function and for a given set of weights, i.e., $\lambda_1, \lambda_2, \dots, \lambda_n$.*

5.4 EXPLANATION OF KRIGING PROCEDURE

In estimating block grades by kriging, one is interested in minimizing the estimation variance σ_E^2 which is synonymous with the kriging variance σ_K^2 . This task of minimization is accomplished through a proper choice of the weights, i.e., λ_i 's in Equation 5.7, with one constraint. The constraint is that the sum of the weights must be equal to 1, as any weighting scheme should be. In other words, the weighting process of kriging is equivalent to solving a constrained optimization problem, where the objective function is given by Equation 5.7 and the one constraint as given below:

$$\begin{array}{ll} \text{Minimize} & \sigma_K^2 = F(\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_n) \\ \text{subject to} & \sum_{i=1}^N \lambda_i = 1 \end{array} \quad (5.8)$$

*Note that the character λ is used for the weights in this section, even though the same character is used to represent the Lagrange multipliers in Chapter 4.

This constrained optimization problem can be readily solved by the method of Lagrange multiplier.

Forming the Lagrangean function $L(\lambda_i's, u)$, one obtains Equation 5.9 below:

$$L(\lambda_1, \lambda_2, \dots, \lambda_n, u) = \sigma_v^2 - 2 \sum_{i=1}^N \lambda_i \sigma_{vx_i} + \sum_{i=1}^N \sum_{j=1}^N \lambda_i \lambda_j \sigma_{X_i X_j} - 2u \left(\sum_{i=1}^N \lambda_i - 1 \right) \quad (5.9)$$

In the above equation, it should be noted that "2u" instead of "u" was used for the Lagrange multiplier, and that Equation 5.9 is a quadratic equation having $N + 1$ variables, i.e., $\lambda_1, \dots, \lambda_N$, plus one additional variable "u", i.e., the Lagrange multiplier. Using the classical calculus approach to determine the stationary point of Equation 5.9, we differentiate with respect to λ_i 's and u and obtain a set of simultaneous equations having $N + 1$ equations.

For the convenience of notation and understanding, v is replaced by o and X's are dropped keeping only the subscripts in Equation 5.9. The resulting equation is given by Equation 5.10 below:

$$\begin{aligned} L(\lambda's, u) &= \sigma_v^2 - 2 \sum_{i=1}^N \lambda_i \sigma_{vx_i} + \sum_{i=1}^N \sum_{j=1}^N \lambda_i \lambda_j \sigma_{X_i X_j} - 2u \left(\sum_{i=1}^N \lambda_i - 1 \right) = 0 \\ &= \sigma_o^2 - 2 \sum_{i=1}^N \lambda_i \sigma_{oi} + \sum_{i=1}^N \sum_{j=1}^N \lambda_i \lambda_j \sigma_{ij} - 2u \left(\sum_{i=1}^N \lambda_i - 1 \right) = 0 \end{aligned} \quad (5.10)$$

Expanding Equation 5.10 gives the following:

$$\begin{aligned}
 L(\lambda's, u) = & \sigma_0^2 - 2\lambda_1 \sigma_{01} - 2\lambda_2 \sigma_{02} - 2\lambda_3 \sigma_{03} - \dots - 2\lambda_N \sigma_{0N} \\
 & + \lambda_1^2 \sigma_{11} + \lambda_2^2 \sigma_{22} + \lambda_3^2 \sigma_{33} + \dots + \lambda_N^2 \sigma_{NN} \\
 & + 2\lambda_1 \lambda_2 \sigma_{12} + 2\lambda_1 \lambda_3 \sigma_{13} + 2\lambda_1 \lambda_4 \sigma_{14} + \dots + 2\lambda_1 \lambda_N \sigma_{1N} \\
 & \quad + 2\lambda_2 \lambda_3 \sigma_{23} + 2\lambda_2 \lambda_4 \sigma_{24} + \dots + 2\lambda_2 \lambda_N \sigma_{2N} \\
 & \quad \quad + 2\lambda_3 \lambda_4 \sigma_{34} + \dots + 2\lambda_3 \lambda_N \sigma_{3N} \\
 & \quad \quad \quad + 2\lambda_4 \lambda_N \sigma_{4N} \\
 & \quad \quad \quad \cdot \\
 & \quad \quad \quad \cdot \\
 & \quad \quad \quad \cdot \\
 & \quad \quad \quad + 2\lambda_{N-1} \lambda_N \sigma_{N-1,N} \\
 & - 2u(\lambda_1 + \lambda_2 + \lambda_3 + \dots + \lambda_N - 1)
 \end{aligned} \tag{5.11}$$

Next, partial differentiation of Equation 5.11 is performed for each variable and the resulting quantity is set equal to zero to obtain the stationary point. For example:

$$\begin{aligned}
 \frac{\partial L}{\partial \lambda_1} = & -2\sigma_{01} + 2\lambda_1 \sigma_{11} + 2\lambda_2 \sigma_{12} + 2\lambda_3 \sigma_{13} + \dots + 2\lambda_N \sigma_{1N} - 2u = 0 \\
 \text{or} \quad & -\sigma_{01} + \lambda_1 \sigma_{11} + \sum_{j=2}^N \lambda_j \sigma_{1j} - u = 0
 \end{aligned} \tag{5.12}$$

$$\text{or} \quad \sum_{j=1}^N \lambda_j \sigma_{1j} - u = \sigma_{01}$$

Similarly,

$$\frac{\partial L}{\partial \lambda_2} = -2\sigma_{02} + 2\lambda_2\sigma_{22} + 2\lambda_1\sigma_{12} + 2\lambda_3\sigma_{23} + 2\lambda_4\sigma_{24} + \dots + 2\lambda_N\sigma_{2N} - 2u = 0$$

or

$$\sum_{j=1}^N \lambda_j \sigma_{2j} - u = \sigma_{02} \quad (5.13)$$

$$\frac{\partial L}{\partial \lambda_3} = -2\sigma_{03} + 2\lambda_3\sigma_{33} + 2\lambda_1\sigma_{13} + 2\lambda_2\sigma_{23} + 2\lambda_4\sigma_{34} + 2\lambda_N\sigma_{3N} - 2u = 0$$

or

$$\sum_{j=1}^N \lambda_j \sigma_{3j} - u = \sigma_{03} \quad (5.14)$$

The general form of Equations 5.12, 5.13, and 5.14 can be given in Equation 5.15

$$\frac{\partial L}{\partial X_i} = 0 \implies \sum_{j=1}^N \lambda_j \sigma_{ij} - u = \sigma_{0i} \quad (5.15)$$

Finally, differentiating Equation 5.11 with respect to u , we obtain the original constraint equation.

$$\sum_{j=1}^N \lambda_j = 1 \quad (5.16)$$

The resulting set of $N + 1$ simultaneous equations are:

$$\sum_{j=1}^N \lambda_j \sigma_{ij} - u = \sigma_{0i} \quad \text{For } i = 1, \dots, N. \quad (5.17)$$

$$\sum_{i=1}^N \lambda_i = 1$$

In matrix notation, Equation 5.17 can be expressed as $AX = B$ where A , X , and B are as given below:

$$A = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \dots & \sigma_{1n} & -1 \\ \sigma_{21} & \sigma_{22} & \dots & \sigma_{2n} & -1 \\ \vdots & \vdots & & \vdots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \dots & \sigma_{nn} & -1 \\ 1 & 1 & \dots & 1 & 0 \end{bmatrix} \quad X = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_n \\ u \end{bmatrix} \quad B = \begin{bmatrix} \sigma_{01} \\ \sigma_{02} \\ \vdots \\ \sigma_{0n} \\ 1 \end{bmatrix} \quad (5.18)$$

The desired solution is obtained by pre-multiplying the Equation 5.18 by A^{-1} , or $X = A^{-1}B$. Having obtained the solution which provides the optimal weights λ_i 's and the Lagrange multiplier u , the kriging variance σ_K^2 for the block can be computed from Equation 5.7 which was given earlier and is also repeated below.

$$\begin{aligned} \sigma_K^2 &= \sigma_V^2 - 2 \sum_{i=1}^N \lambda_i \sigma_{VX_i} + \sum_{i=1}^N \sum_{j=1}^N \lambda_i \lambda_j \sigma_{X_i X_j} \\ &= \sigma_0^2 - 2 \sum_{i=1}^N \lambda_i \sigma_{0i} + \sum_{i=1}^N \sum_{j=1}^N \lambda_i \lambda_j \sigma_{ij} \end{aligned} \quad (5.7)$$

(After replacing v by o and dropping X 's from Equation 5.7)

In practice, however, the above equation is somewhat modified in order to utilize the already computed value, i.e., u . The desired modification is accomplished as follows:

Consider Equation 5.17 which resulted when the Lagrangean function was differentiated with respect to each variable.

$$\sum_{j=1}^N \lambda_j \sigma_{ij} - u = \sigma_{oi} \quad \text{For } i = 1, 2, \dots, N \quad (5.17)$$

If we multiply λ_i to both sides of the i -th equation and sum up the resulting quantities, we obtain Equation 5.19 below:

$$\sum_{j=1}^N \sum_{i=1}^N \lambda_j \lambda_i \sigma_{ij} - u \sum_{i=1}^N \lambda_i = \sum_{i=1}^N \sigma_{oi} \lambda_i \quad (5.19)$$

However, the original constraint requires that $\sum_{i=1}^N \lambda_i = 1$. Therefore, one obtains the following from Equation 5.19:

$$\sum_{j=1}^N \sum_{i=1}^N \lambda_j \lambda_i \sigma_{ij} = \sum_{i=1}^N \sigma_{oi} \lambda_i + u \quad (5.20)$$

Substituting Equation 5.20 into Equation 5.7, one obtains Equation 5.21 which is the one actually utilized in kriging variance determination.

$$\begin{aligned} \sigma_K^2 &= \sigma_0^2 - 2 \sum_{i=1}^N \lambda_i \sigma_{oi} + \sum_{i=1}^N \lambda_i \sigma_{oi} + u \\ &= \sigma_0^2 - \sum_{i=1}^N \lambda_i \sigma_{oi} + u \end{aligned} \quad (5.21)$$

5.5 EXAMPLES OF KRIGING

Two illustrative example problems, one for point kriging and the other for block kriging, are provided here. The point kriging example demonstrates the fact that kriging in fact minimizes the estimation variance.

Point Kriging Example

This is an identical example problem given in Reference 1 (Kim and Knudsen).

Figure 5.2 shows two different sampling grids each with 3 holes. The problem will be to estimate the grade at point X_0 for each grid. Assume that the theoretical variogram model is a transitive variogram described by Equation 5.22.

$$\begin{aligned}\gamma(h) &= 0.01h && \text{for } h \leq 400 \text{ ft.} \\ \gamma(h) &= 4.0 && \text{for } h > 400 \text{ ft.}\end{aligned}\tag{5.22}$$

The following steps show the calculations for the first grid.

Step 1. Calculate the covariance between each of the sample points.

Remember that $K(h) = K(0) - \gamma(h)$.

$$\sigma_{(1,1)} = \sigma_{(2,2)} = \sigma_{(3,3)} = K(0) = 4.0$$

$$\sigma_{(1,2)} = \sigma_{(2,1)} = \sigma_{(1,3)} = \sigma_{(3,1)} = K(200) = 4.0 - .01(200) = 2.0$$

$$\sigma_{(2,3)} = \sigma_{(3,2)} = K(20) = 4.0 - .01(20) = 3.8$$

Figure 5.2

HYPOTHETICAL SAMPLING GRIDS TO ILLUSTRATE KRIGING

Step 2. The covariance between each sample point and the point X_0 is computed.

$$\sigma_{(1,X_0)} = \sigma_{(2,X_0)} = \sigma_{(3,X_0)} = K(100) = 4.0 - .01(100) = 3.$$

Step 3. The matrices of Equation 5.18 can now be filled in.

$$\begin{bmatrix} 4.0 & 2.0 & 2.0 & -1 \\ 2.0 & 4.0 & 3.8 & -1 \\ 2.0 & 3.8 & 4.0 & -1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ u \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 1 \end{bmatrix}$$

Step 4. Solving this system of equations gives the following weights and Lagrange multiplier.

$$\lambda_1 = 0.487$$

$$\lambda_2 = 0.256$$

$$\lambda_3 = 0.256$$

$$u = -0.026$$

Step 5. The kriging variance calculated from Equation 5.21 is 0.974.

$$\sigma_K^2 = \sigma_0^2 + u - \sum_{i=1}^3 \lambda_i \sigma_{oi}$$

$$= 4.0 - .026 - 0.487(3) + 0.256(3) + 0.256(3)$$

$$= 0.974$$

For the second grid Steps 3 through 5 are shown below.

Step 3. The matrices for the second grid are:

$$\begin{bmatrix} 4.0 & 2.27 & 2.27 & -1 \\ 2.27 & 4.0 & 2.27 & -1 \\ 2.27 & 2.27 & 4.0 & -1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ u \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 1 \end{bmatrix}$$

Step 4. Solving the set of equations gives the following results:

$$\lambda_1 = \lambda_2 = \lambda_3 = 0.333$$

and

$$u = -0.847$$

Step 5. The kriging variance is 0.13

$$\sigma_K^2 = 4.0 - 0.847 - 3.0 = 0.13$$

As can be seen in Step 4 above, each sample has the same weight for the second grid. This solution is obvious in retrospect by just looking at the grid. It also seems reasonable to assume that the kriging variance for the second grid should be less than for the first grid. This fact is borne out in Step 5.

The above example clearly shows that kriging was able to distinguish between the two sampling grids and to adjust the weights to give minimum kriging variance. The sample points 2 and 3 in the first grid were given the same weight but the weight was less than the weight given to sample 1. This is the "transfer of influence" phenomena and is due to the points being close together in relation to the other sampling point.

Block Kriging Example

A more common estimation problem is to estimate the grade of a block by the nearby drill holes. Figure 5.3 shows a block with 10 surrounding drill holes. For this example, the problem is to estimate the thickness of ore in the block from the 10 surrounding drill holes. The theoretical variogram used for this example is described by Equation 5.23.

$$\begin{aligned}\gamma(h) &= 30.0 + 44 \left[1.5\left(\frac{h}{a}\right) - .5\left(\frac{h}{a}\right)^3 \right] && \text{for } h \leq a \\ \gamma(h) &= 74.0 && \text{for } h > a\end{aligned}\tag{5.23}$$

The same sequence of steps is followed as in the previous example.

Step 1. Calculate the covariance between each of the sample points.

$$\begin{aligned}\sigma(i,i) &= K(o) = 74 && \text{for } i = 1,10 \\ \sigma(1,2) &= \sigma(2,1) = K(o) - \gamma(61') = 32.78 \\ \sigma(1,3) &= \sigma(3,1) = K(o) - \gamma(75') = 29.93 \\ &\text{etc.}\end{aligned}$$

Step 2. The covariance between the sample point and the block is computed. The block is represented by 16 points spread over the block. Figure 5.4 shows sample point 5 and the 16 grid points within the block.

$$\begin{aligned}\sigma_{(5v)} &= K(o) - \frac{1}{16} \sum \gamma(\text{pt5} - Y_i) \quad i = 1,16 \\ \sigma_{(5v)} &= K(o) - \frac{1}{16} [\gamma(\text{pt5} - Y_1) - \gamma(\text{pt} - Y_2) - \dots] \\ &= K(o) - \frac{1}{16} [\gamma(60') - \gamma(80') \dots] \\ \sigma_{(5v)} &= 74. - 46.31 = 27.69\end{aligned}$$

Scale 1 inch to 100 ft.

Figure 5.3

BLOCK KRIGING EXAMPLE

Figure 5.4

SAMPLE POINT 5 AND BLOCK(v)

Step 3. The matrices of Equation 5.18 can be filled in as shown in Figure 5.5.

Step 4. The solution to the system gives the following weights:

$$\begin{array}{ll} \lambda_1 = .18 & \lambda_6 = .06 \\ \lambda_2 = .14 & \lambda_7 = .07 \\ \lambda_3 = .16 & \lambda_8 = .08 \\ \lambda_4 = .14 & \lambda_9 = .05 \\ \lambda_5 = .07 & \lambda_{10} = .06 \\ & u = -.779 \end{array}$$

Step 5. The Kriging variance is 7.20

$$\begin{aligned} \sigma_K^2 &= 36.51 + (-.779) \\ &\quad - [.18(35.77) + .14(32.46) + .16(32.33) + .14(30.45) + .07(27.69) \\ &\quad + .06(25.41) + .07(25.36) + .08(24.63) + .05(23.76) + .06(21.23)] \\ &= 36.51 - .779 - 30.08 = 7.20 \end{aligned}$$

This example illustrates the numerous calculations necessary for kriging. The number of computations for each block is related to the number of drill holes included and the number of grid points used in approximating the block. In the example above only 10 holes were used in the kriging, but of those the nearest 4 accounted for 63% of the weight.

$$\begin{bmatrix}
 74.0 & 32.7 & 29.9 & 26.8 & 33.0 & 30.7 & 28.4 & 24.4 & 26.7 & 20.1 & -1.00 \\
 32.7 & 74.0 & 24.9 & 30.7 & 23.4 & 25.4 & 18.3 & 16.3 & 30.5 & 30.4 & -1.00 \\
 29.9 & 24.9 & 74.0 & 30.5 & 23.1 & 18.2 & 25.3 & 31.3 & 15.4 & 15.3 & -1.00 \\
 26.8 & 30.7 & 30.5 & 74.0 & 17.2 & 16.0 & 15.9 & 19.0 & 18.4 & 25.1 & -1.00 \\
 33.0 & 23.4 & 23.1 & 17.2 & 74.0 & 34.6 & 34.2 & 23.8 & 23.3 & 12.5 & -1.00 \\
 30.7 & 25.4 & 18.2 & 16.0 & 34.6 & 74.0 & 25.3 & 16.4 & 30.5 & 15.7 & -1.00 \\
 28.4 & 18.3 & 25.3 & 15.9 & 34.2 & 25.3 & 74.0 & 31.2 & 15.6 & 8.3 & -1.00 \\
 24.4 & 16.3 & 31.3 & 19.0 & 23.8 & 16.4 & 31.2 & 74.0 & 10.3 & 7.4 & -1.00 \\
 26.7 & 30.5 & 15.4 & 18.4 & 23.3 & 30.5 & 15.6 & 10.3 & 74.0 & 25.3 & -1.00 \\
 20.1 & 30.4 & 15.3 & 25.1 & 12.5 & 15.7 & 8.3 & 7.4 & 25.3 & 74.0 & -1.00 \\
 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 0.00
 \end{bmatrix}
 \cdot
 \begin{bmatrix}
 \lambda_1 \\
 \lambda_2 \\
 \lambda_3 \\
 \lambda_4 \\
 \lambda_5 \\
 \lambda_6 \\
 \lambda_7 \\
 \lambda_8 \\
 \lambda_9 \\
 \lambda_{10} \\
 u
 \end{bmatrix}
 =
 \begin{bmatrix}
 35.77 \\
 32.46 \\
 32.33 \\
 30.45 \\
 27.69 \\
 25.41 \\
 25.36 \\
 24.63 \\
 23.76 \\
 21.23 \\
 1.00
 \end{bmatrix}$$

Figure 5.5

MATRICES OF EQUATION 5.18 FOR KRIGING

REFERENCES

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APPENDIX A

VARIOGRAM COMPUTATION PROGRAM "UGAMM"

USER'S GUIDE

SECTION I: PROGRAM IDENTIFICATION

- 1.1 PROGRAM TITLE: Variogram Calculation
- 1.2 PROGRAM CODE NAME: UGAMM
- 1.3 PROGRAM WRITER: H. Peter Knudsen
- PROGRAM DOCUMENTATION: Young C. Kim
- 1.4 ORGANIZATION: Department of Mining and Geological Engineering
University of Arizona
Tucson, Arizona
- 1.5 DATE: October 15, 1976
- 1.6 UPDATES: None. Version 1.
- 1.7 SOURCE LANGUAGE: CDC 6400 Computer Systems - FORTRAN Extended -
Version IV.
- 1.8 ABSTRACT:

This program is specifically tailored for use by the U. S. ERDA personnel to compute variograms directly from 'SCOR' tapes. However, it is a general purpose variogram computation program having many user options. Some such options are: 1) choice of input data device, 2) automatic scaling of the variogram plot on the line printer, 3) punch out option for the variogram display program 'VARPLOT', and 4) drill hole data selection option which allows inclusion of only those data points within the specified boundary, during the variogram computation.

Program UGAMM is capable of processing a single data group or multiple groups of data, all during the same run. It is currently dimensioned to handle up to 1000 total data points within a single data group.

SECTION II: ENGINEERING DOCUMENTATION

2.1 NARRATIVE DESCRIPTION:

Program UGAMM has been specifically tailored for use by the U. S. ERDA personnel to compute variograms directly from 'SCOR' tapes. However, it is a general purpose variogram calculation program written to accept a variety of data taken either on a regular or an irregular grid.

Variograms are calculated in any specified direction within a plane, using the data that have coordinates and assay values. The plane need not be a horizontal plane, although the horizontal variograms are the rule rather than an exception for most tabular deposits such as roll-front type uranium deposit. If a vertical variogram is desired from a set of drill hole assays, it can be obtained by treating depth as the east coordinate and by letting the north coordinate as zero (i.e., the same direction)

Program UGAMM is capable of processing a single data group or multiple groups of data, all during the same run. The program can handle up to 1000 data points within a single data group. Also, the data need not be ordered within each group.

Being a general purpose program, the program possesses many user options. These options are:

1. Choice of input data device, i.e., the 'SCOR' tape or the card input through the system card reader.
2. Logarithmic transformation of data.
3. Computation of the average variogram from all the data groups, if more than one set of data are processed during the run.
4. Option to punch out the variogram results on cards in a format that is directly compatible to the input data format of the Calcomp plot display program of a variogram "VARPLOT" (See Appendix B of this report.)

5. Automatic scaling of the variogram plot on the line printer, for each variogram that is generated.
6. Drill hole data selection option which allows inclusion of only those data points that lie within a specified boundary, during a variogram computation.

For each variogram that is computed, the program produces two types of output as shown in Figures 1 and 2. Figure 1 is a display of the results in numeric form whereas Figure 2 is a display in graphic form. For the latter type of display, the user must prespecify as to whether or not one desires a plot of a) GAMMA(H) or b) MOMENT CENTER. In general, GAMMA(H) should be specified if the data lie on a regular grid and if the class interval specified is equal to the grid interval. However, MOMENT CENTER is more frequently specified in practice due to the fact that most drill hole data are on an irregular grid.

Up to 20 class intervals along a given direction are allowed in variogram computation. In addition, the specified class interval is also utilized to limit the distance between data points within which the variogram is to be computed. For example, if the distance between two assay data exceed 20 times the class interval distance, variogram computation for these points is by-passed.

Since a variogram is a vector function, i.e., it has a direction as well as the value, the direction along which the computation is to be performed must be specified by the user. In specifying the desired direction, one must also specify the allowable spread in degrees from the direction within which to accept the assay data. This allowable spread, i.e., a tolerance is required simply because most drill holes seldom lie on a straight line. If the spread is 90 degrees, then each and every hole is

VARIOGRAM

URANIUM PROJECT
GRADE

DATA USED IN CALCULATIONS

DIRECTION = 0. WINDOW 90.
CLASS SIZE = 100.
MAX DISTANCE = 2000.
LOGARITHMS - NO. RELATIVE VARIOGRAM - NO.
COORDINATE SELECTION- NORTH(71800. 72800.) EAST(09700. 11100.)

MEAN =
VARIANCE = .397E-02
STD DEVIATION = .630E-01
NO.OF SAMPLES = 146

DISTANCE	NO. OF SAMPLES	DRIFT	GAMMA (H)	MOMENT CENTER	AVERAGE DISTANCE
0	100	126.	.216E-02	.228E-02	87.8
100	200	527.	.342E-02	.344E-02	144.3
200	300	918.	.345E-02	.347E-02	245.4
300	400	1042.	.355E-02	.355E-02	343.8
400	500	1233.	.405E-02	.403E-02	444.8
500	600	1267.	.418E-02	.419E-02	545.5
600	700	1175.	.439E-02	.433E-02	645.2
700	800	1157.	.491E-02	.492E-02	745.0
800	900	899.	.473E-02	.473E-02	845.1
900	1000	784.	.397E-02	.398E-02	943.5
1000	1100	519.	.306E-02	.306E-02	1043.9
1100	1200	402.	.333E-02	.334E-02	1140.8
1200	1300	295.	.337E-02	.336E-02	1240.5
1300	1400	152.	.319E-02	.318E-02	1341.0
1400	1500	39.	.270E-02	.269E-02	1437.7
1500	1600	8.	.135E-02	.134E-02	1533.3

Figure 1

SAMPLE OUTPUT OF COMPUTATION RESULTS

GAMMA

II

Figure 2

PRINTER PLOT OF VARIOGRAM RESULTS

included thus resulting in an average variogram in all directions. Program UGAMM is capable of producing the average variogram from several variograms that are being computed using several different data sets. An example of such situation is to compute a variogram from each bench of an open pit mine, and then obtain the average variogram from all benches.

If desired, a relative variogram can also be obtained using program UGAMM. In a relative variogram, the obtained value for a given distance h is divided by the square of the mean of samples that were used for computation.

2.2 METHOD OF SOLUTION:

The theoretical basis of the variogram definition as well as the physical meaning of a variogram are not discussed here. Instead the reader is referred to the main text of this report (See Chapters 2 and 3). The subject matter being discussed in this section deals mainly with the practical aspects of computing variograms from a variety of data taken either on a regular or an irregular grid.

Theoretically, a variogram is defined as Equation 1 below:

$$\gamma(h) = \frac{1}{2V} \int_V [f(x) - f(x+h)]^2 dx \quad (1)$$

A variogram, $\gamma(h)$, is a vector function in three-dimensional space and it is a function of the distance h between a point x and another point $x+h$. The function, $f(x)$, simply defines the value of interest such as the grade at a point x .

When a variogram is to be computed using a discrete number of points such as drill hole assays, the integral sign in Equation 1 must be replaced by a summation sign as shown in Equation 2.

$$\gamma(h) = \frac{1}{2N} \sum_{j=1}^N [f(x_j) - f(x_j + h)]^2 \quad (2)$$

In Equation 2, it is assumed that there are N pairs of samples, each pair being separated by distance h. In addition, all these samples are assumed to lie on a straight line, along which the variogram computation is being performed.

In practice, however, most drill holes neither lie on a straight line nor are spaced at a uniform grid. Consequently, one must not only specify the direction for the variogram computation, but also the allowable spread in degrees from the direction within which to accept the assay data. Similarly, the distance h in Equation 2 which corresponds to a uniform grid spacing must be modified to allow an interval instead of a point. The modified version of Equation 2 which computes the variogram using a distance that falls within some interval is given by Equation 3:

$$\gamma(h) = \frac{\sum_{j=1}^N (h_j) x [f(x) - f(x + h_j)]^2}{2 \sum_{j=1}^N (h_j)} \quad (3)$$

The basic unit used for the interval (h) in Equation 3 is the class size

which is also specified by the user. For example, if the class size is 100 ft., any pair of data points whose distance is between 0 to 100 ft. are included in the computation. In using Equation 3, the actual distance h_j for the j -th pair of data points is used instead of the midpoint of the class interval such as 50 ft. in the above example. Hence, the obtained $\gamma(h)$ value is the weighted average value rather than the arithmetic average of Equation 2.

After computing $\gamma(h)$ value for the first class size, i.e., 100 ft., the next class size of 200 ft. (2×100) is used to obtain pairs of data points whose distances fall between 100 ft. to 200 ft. The $\gamma(h)$ value is then computed using those pairs that have been obtained earlier. The whole process is repeated for a total of twenty (20) class intervals, i.e., 2000 ft. in the above example. Hence, the specified class interval distance effectively limits the maximum distance for which the variogram is to be computed by program UGAMM.

Program UGAMM calculates $\gamma(h)$ values using both Equations (2) and (3) above. Values of $\gamma(h)$ calculated by Equation 2 are listed under the heading of GAMMA(H) in Figure 1, whereas those obtained by Equation 3 are listed under MOMENT CENTER. As mentioned earlier, GAMMA(H) contains the arithmetic average whereas MOMENT CENTER contains the weighted average.

The values of $\gamma(h)$ calculated by Equation 3 can be compared with those calculated by Equation 2 to determine if the class size chosen was good. Generally the class size is chosen to be the average grid size. If the differences between the GAMMA(H) and MOMENT CENTER values are large, the class size should be adjusted slightly. In cases where the data is

from a truly regular grid, the values will be the same.

Program UGAMM also calculates the average difference between samples separated by distance h. Listed under the heading DRIFT, this value is useful in determining if drift, i.e., a general increase or decrease in grade with distance for the direction specified, is present.

The last column in Figure 1 gives the average distance of all the pairs of data points whose distances fell within the given class interval. If desired, the values of $\gamma(h)$ calculated by Equations 2 or 3 can be divided by the mean squared to get the values in relative terms.

For each variogram being computed, program UGAMM produces a two page output as shown in Figures 1 and 2. Figure 2 is simply a graphic display of a part of the results given in Figure 1. For example, the program user must specify in advance as to which column (i.e., GAMMA(H) or MOMENT CENTER) is to be plotted on the graph, since both graphs can not be printed simultaneously. In Figure 2, the distance h for each $\gamma(h)$ value is the average distance and not the midpoint of each class interval.

The following four (4) formulas, in essence, summarizes the method of solution.

$$\text{Gamma(h)} \quad \gamma(h) = \frac{\text{Cum. Difference Squared}}{2 \times (\text{No. of Samples})}$$

$$\text{Moment Center} \quad \gamma(h) = \frac{\text{Cum. (Distance x Diff. Squared)}}{2 \times (\text{Cum. Distance})}$$

$$\text{Average Distance} = \frac{\text{Cum. Distance}}{\text{No. of Samples}}$$

$$\text{Drift} = \frac{\text{Cum. Difference}}{\text{No. of Samples}}$$

2.3 PROGRAM CAPABILITIES:

A maximum of 1000 data cards may be included in each data set and any number of data sets included per job. If desired, the data set may be selected from a larger data set by limiting the data to points that fall within a user specified rectangular block. A variogram is calculated for each data set, and if desired, an average variogram for all the data sets may be calculated. The variogram values may be divided by the mean squared to transform their magnitude to percents of the mean squared. The sample values may be transformed to logarithms.

2.4 DATA INPUTS:

Figure 3 below describes the data inputs whereas Figure 4 shows a 80-80 listing of sample input data.

2.5 PRINTED OUTPUT:

Two pages of printed output are produced for each variogram. Page one is a listing of the sample mean, sample variance, variogram direction, and a tabulation of the variogram results (see Figure 1). Page two is a plot of either $\text{GAMMA}(H)$ or the MOMENT CENTER versus distance.

2.6 PUNCHED CARD OUTPUT:

For each variogram calculated, if desired, the following cards are punched out. These cards can be used as input for the variogram display program "VARPLOT" (See Appendix B of this report.)

COL.	FORMAT	NAME	DESCRIPTION
***	*****	****	*****
CARD ONE. HEADER CARD FOR RUN IDENTIFICATION.			
1-80	8A10	HEAD	ALPHANUMERIC DESCRIPTION.
CARD TWO. VARIOGRAM OPTIONS SPECIFICATION CARD.			
1	I1	INPT	INPUT DATA DEVICE. 1='SCOR' TAPE. 0= CARD READER USING VARIABLE FORMAT CARD.
2	I1		NOT USED.
3	I1	LOG	OPTION FOR LOG TRANSFORMATION OF DATA. 1=YES, 0=NO, (NORMALLY=0)
4	I1	ITAL	AVERAGE VARIOGRAM OPTION. 1=YES, 0=NO. THIS IS THE AVERAGE OF ALL THE VARIOGRAMS FROM THE SEVERAL DATA GROUPS. (NORMALLY=1)
5	I1	IPC	PUNCH OUT OPTION FOR PROGRAM 'VARPLOT'. 1= YES, 0=NO.
6	I1	LL	VARIOGRAM DISPLAY OPTION. 1=MOMENT CENTER, 0=GAMMA(H). (NORMALLY=1)
7	I1	ISCAL	AUTOMATIC SCALING OF THE GRAPH. 1=YES, 0=NO. (NORMALLY=1)
8	I1	NREL	RELATIVE VARIOGRAM OPTION. 1=YES, 0=NO.
9	I1	ICOMP	ASSAY DATA READ IN OPTION. 1='SCOR' TAPE, 0=SYSTEM CARD READER.
11-20	F10.0	CLAS	CLASS INTERVAL TO BE USED TO GROUP DISTANCES. IT IS ALSO USED TO LIMIT THE DISTANCE BETWEEN HOLES FOR WHICH THE VARIOGRAM IS TO BE COMPUTED, I.E., 20 X CLAS = MAX. DISTANCE (H).
21-30	F10.0	ANG	ANGLE IN DEGREES ALONG WHICH THE VARIOGRAM IS TO BE CALCULATED. 0. = EAST-WEST 45. = NE-SW 90. = NORTH-SOUTH -45. = NW-SE
31-40	F10.0	SPR	ALLOWABLE ANGLE DEVIATION IN DEGREES.
41-50	F10.0	TMAX	MAXIMUM VARIOGRAM VALUE ON Y-AXIS OF THE GRAPH WHEN AUTOMATIC SCALING OPTION IS NOT USED. IF ISCAL = 1, THIS VALUE IS IGNORED.
CARD THREE. INPUT DATA SPECIFICATION CARD.			
1	I1	LD	MUST HAVE A VALUE OF EITHER 1, 2, OR 3. IF 'INPT' VALUE IS 1 INDICATING THAT THE INPUT IS FROM THE 'SCOR' TAPE, VARIOUS LD VALUES REFER TO THE FOLLOWING: 1 = GRADE, 2 = THICKNESS, 3 = GT PRODUCT.
2	I1	NO	IF CARD INPUT, LD SPECIFIES THE LOCATION OF ASSAY VALUE (I.E., IS IT THE FIRST, SECOND OR THIRD ON THE DATA COLUMN).
3	I1	IE	NOT USED IF 'INPT' IS 1. OTHERWISE, LOCATION OF NORTHING. (1, 2, OR 3)
4	I1	ISEL	NOT USED IF 'INPT' IS 1. OTHERWISE, LOCATION OF EASTING. (1, 2, OR 3)
11-20	F10.0	YMIN	DATA SELECTION OPTION. 1=YES, 0=NO. IF YES, THE APPROPRIATE BOUNDARIES WITHIN WHICH TO SEARCH THE ASSAY DATA MUST BE SPECIFIED ON THE SUBSEQUENT FOUR (4) COLUMNS.
21-30	F10.0	YMAX	MINIMUM NORTH COORDINATE IF ISEL = 1.
31-40	F10.0	XMIN	NOT USED OTHERWISE.
41-50	F10.0	YMAX	MAXIMUM NORTH COORDINATE IF ISEL = 1.
			MINIMUM EAST COORDINATE IF ISEL = 1.
			MAXIMUM EAST COORDINATE IF ISEL = 1.
CARD FOUR. VARIABLE FORMAT CARD FOR DATA READ IN. INCLUDE ONLY IF 'INPT' IS EQUAL TO ZERO.			
1-80	8A10	IFRMT	FORMAT SPECIFICATION. MUST BE ENCLOSED BY ().
CARD FIVE. TITLE OF THE VARIOGRAM TO BE OUTPUT FOR EACH DATA GROUP			
1-80	8A10	TITLE	ALPHA NUMERIC DESCRIPTION.
CARD SIX. DATA CARDS FOR COORDINATES AND ASSAY VALUES.			
END OF DATA FOR EACH SET OF DATA GROUP IS A BLANK CARD OR AN EOF CARD.			
EACH VARIOGRAM TO BE CALCULATED MUST CONSIST OF A TITLE CARD (CARD FIVE), DATA CARDS, AND A BLANK OR AN EOF CARD.			

Figure 3
DESCRIPTION OF INPUT DATA

URANIUM PROJECT		0.	90	0.00	
000111100	100.	72800	09700	11100	
3211	0				
(10X, 2F10.0, 15X, F10.0)					
GRADE VARIOGRAM					
025390	11089	711165029.	0.076	2.000	0.151
025425	11413	711205069.	0.071	5.500	0.391
030390	11069	711685036.	0.070	2.000	0.140
030415	11307	711725059.	0.062	2.000	0.125
030420	11358	711675059.	0.078	12.500	0.973
033420	11360	711855055.	0.106	17.000	1.801
035345	10614	712305016.	0.090	10.000	0.904
035365	10807	712285036.	0.060	3.500	0.210
035375	10919	712235025.	0.064	6.000	0.382
035385	11018	712215027.	0.072	15.000	1.081
035415	11312	712125056.	0.201	20.000	4.013
036411	11270	712205045.	0.101	20.500	2.079
040330	10459	712655019.	0.082	4.000	0.328
040335	10508	712805018.	0.113	12.000	1.361
040340	10559	712635021.	0.096	16.000	1.541
040360	10757	712625030.	0.127	23.500	2.992
040370	10855	712615034.	0.117	26.000	3.034
040371	10870	712615032.	0.185	30.000	5.549
040380	10968	712615047.	0.063	10.000	0.634
040400	11165	712565068.	0.064	2.000	0.127
040408	11249	712535056.	0.109	12.500	1.366
040410	11264	712525054.	0.100	9.500	0.954
040420	11365	712645067.	0.088	10.000	0.876
045330	10461	713305014.	0.110	12.500	1.380
045335	10512	713235020.	0.247	12.500	3.088

Figure 4

SAMPLE LISTING OF INPUT DATA

A. Title Card.

<u>COLUMN</u>	<u>FORMAT</u>	<u>REMARKS</u>
2-80	7A10,A9	Title of Variogram

B. Parameter Card.

<u>COLUMN</u>	<u>FORMAT</u>	<u>REMARKS</u>
1-10	E10.3	Direction of variogram.
11-20	E10.3	Allowable angle deviation.
21-30	E10.3	Sample mean.
31-40	E10.3	Sample variance.
41-50	E10.3	Distance interval.
51-60	A3	Log transform of data (yes or no).
61-70	I10	Number of samples.

C. Variogram Cards.

<u>COLUMN</u>	<u>FORMAT</u>	<u>REMARKS</u>
1-10	E10.3	Number of samples.
11-20	E10.3	DRIFT
21-30	E10.3	GAMMA(H)
31-40	E10.3	MOMENT CENTER.
41-50	E10.3	Average distance.

SECTION III: SYSTEM DOCUMENTATION

3.1 COMPUTER EQUIPMENT:

Program UGAMM was written for a CDC-6400 computer. A card reader, a card punch, and a line printer are necessary to run the program.

3.2 SOURCE PROGRAM:

The source listing of Program UGAMM is given at the end of the Appendix.

3.3 VARIABLES AND SUBROUTINES:

The major variables are defined in the program listing at the end of this Appendix. The program consists of the following subroutines:

1. UGAMM: Main program for input and variogram calculations.
2. VAROUT: Output subroutine.
3. RDN: Data input for variable format option.
4. RSCOR: Data input for SCOR tapes.
5. System subroutines - Subroutines for logarithm, sine, cosine, and square root are required by the program and are provided by the system.

3.4 STORAGE REQUIREMENTS:

As presently structured program UGAMM requires 50K storage on the CDC 6400.

3.5 MAINTENANCE AND UPDATES:

None to date.

SECTION IV: OPERATING DOCUMENTATION

4.1 OPERATING INSTRUCTIONS:

The program is operated under the SCOPE operating system.

4.2 OPERATING MESSAGES:

No special operating messages are produced.

4.3 CONTROL CARDS:

The program may be executed with standard SCOPE 3.4 control cards. As run on the University of Arizona CDC 6400, the makeup of the input deck is as follows:

```
Job Card
FTN.
LGO.
7/8/9
<SOURCE PROGRAM>
7/8/9
<INPUT DATA>
6/7/8/9
```

4.4 ERROR RECOVERY:

The program will terminate if errors are found in the control card parameters. The program must be restarted on error.

4.5 RUN TIME:

Run time depends on the number of variograms plotted and on the number of sample points in the data. A typical run for 1 variogram with about 200 data points is 5 seconds.

UGAMM PROGRAM LISTING

```

010 PROGRAM UGAMM (INPUT, OUTPUT, PUNCH, TAPE1=INPUT, TAPE5=OUTPUT, TAPE6=
020 1 PUNCH, TAPE4)
030 DIMENSION TOT (20, 5), TOTVAR (20, 5)
040 COMMON VAR (20, 5), HEAD (8), LOG, CLAS, DLIM, ANG, SMEAN, VARI, STD, N
050 COMMON IPC, L, SPR, ISCAL, TMAX, NREL
060 COMMON /IO/ IOUT, IPT, IPCH, ICOMP, ITAPE, TITLE (8), MAXN
070 COMMON /DAT/ DATA (1000, 4), IFRMT (8), LD, NO, IE, IA
080 COMMON /DAT2/ ISEL, YMIN, YMAX, XMIN, XMAX
090 DATA MAXN, IPT, IOUT, ITAPE, IPCH / 1000., 1, 5, 4, 6 /
100 DATA TOT, TSUM, TSUM2 / 102 * 0.0 /
110 C*****
120 C THIS PROGRAM WAS WRITTEN BY HARVEY P. KNUDSEN AND DOCUMENTED
130 C BY YOUNG C. KIM.
140 C DEPARTMENT OF MINING & GEOLOGICAL ENGINEERING
150 C UNIVERSITY OF ARIZONA, TUCSON, ARIZONA.
160 C OCTOBER, 1976.
170 C THIS PROGRAM IS SPECIALLY TAILORED FOR USE BY THE U.S. ERDA
180 C PERSONNEL TO COMPUTE VARIOGRAMS DIRECTLY FROM 'SCOR' TAPES.
190 C HOWEVER, IT IS A GENERAL PURPOSE VARIOGRAM COMPUTATION PROGRAM
200 C HAVING MANY USER OPTIONS. SOME SUCH OPTIONS ARE: 1) CHOICE OF
210 C INPUT DATA DEVICE, 2) AUTOMATIC SCALING OF THE VARIOGRAM PLOT ON
220 C THE LINE PRINTER, 3) PUNCH OUT OPTION FOR THE VARIOGRAM DISPLAY
230 C PROGRAM 'VARPLOT', & 4) DRILL HOLE DATA SELECTION OPTION WHICH
240 C ALLOWS INCLUSION OF ONLY THOSE DATA POINTS WITHIN THE SPECIFIED
250 C BOUNDARY, DURING THE VARIOGRAM COMPUTATION.
260 C
270 C PROGRAM UGAMM CALCULATES HORIZONTAL VARIOGRAMS IN ANY DIRECTION.
280 C THE PROGRAM CAN ALSO CALCULATE VERTICAL VARIOGRAMS USING DRILL
290 C HOLE ASSAYS, BY TREATING DEPTH AS THE EAST COORDINATE AND BY
300 C LETTING THE NORTH COORDINATE AS ZERO (I.E., THE SAME DIRECTION)
310 C EACH INPUT DATA RECORD MUST HAVE AN IDENTIFICATION, COORDINATE &
320 C ASSAY VALUES. HOWEVER, DATA NEED NOT BE ORDERED.
330 C
340 C THIS PROGRAM IS CAPABLE OF PROCESSING A SINGLE DATA GROUP
350 C OR MULTIPLE GROUPS OF DATA, ALL DURING THE SAME RUN.
360 C IT IS CURRENTLY DIMENSIONED TO HANDLE UP TO 1000 TOTAL DATA
370 C POINTS WITHIN A SINGLE DATA GROUP.
380 C UP TO 20 CLASS INTERVALS ALONG A GIVEN DIRECTION ARE ALLOWED
390 C IN VARIOGRAM COMPUTATION.
400 C
410 C THE FIRST THREE (3) CARDS ARE USED FOR RUN CONTROL PURPOSES, AND
420 C THESE CARDS ARE READ IN FROM THE SYSTEM CARD READER.
430 C THE REMAINING INPUT DATA CAN BE EITHER ON TAPE (USE UNIT 4)
440 C OR ON THE SYSTEM CARD READER.
450 C BE PROVIDED.
460 C*****
470 C
480 C**** DESCRIPTION OF INPUT DATA.
490 C COL. FORMAT NAME DESCRIPTION
500 C *** *****
510 C
520 C CARD ONE. HEADER CARD FOR RUN IDENTIFICATION.
530 C 1-80 8A10 HEAD ALPHANUMERIC DESCRIPTION.
540 C
550 C CARD TWO. VARIOGRAM OPTIONS SPECIFICATION CARD.
560 C 1 I1 INPT INPUT DATA DEVICE. 1='SCOR' TAPE. 0= CARD
570 C READER USING VARIABLE FORMAT CARD.
580 C 2 I1
590 C 3 I1 LOG OPTION FOR LOG TRANSFORMATION OF DATA. 1=YES,
600 C 0=NO, (NORMALLY=0)
610 C 4 I1 ITAL AVERAGE VARIOGRAM OPTION. 1=YES, 0=NO.
620 C THIS IS THE AVERAGE OF ALL THE VARIOGRAMS
630 C FROM THE SEVERAL DATA GROUPS. (NORMALLY=1)

```

00640	C	5	I1	IPC	PUNCH OUT OPTION FOR PROGRAM 'VARPLOT'.
00650					1= YES, 0=NO.
00660	C	6	I1	LL	VARIOGRAM DISPLAY OPTION. 1=MOMENT CENTER,
00670	C				0=GAMMA(H). (NORMALLY=0)
00680	C	7	I1	ISCAL	AUTOMATIC SCALING OF THE GRAPH. 1=YES, 0=NO.
00690	C				(NORMALLY=1)
00700	C	8	I1	NREL	RELATIVE VARIOGRAM OPTION. 1=YES, 0=NO.
00710	C	9	I1	ICOMP	ASSAY DATA READ IN OPTION. 1='SCOR' TAPE,
00720	C				0=SYSTEM CARD READER.
00730	C	11-20	F10.0	CLAS	CLASS INTERVAL TO BE USED TO GROUP DISTANCES.
00740	C				IT IS ALSO USED TO LIMIT THE DISTANCE BETWEEN
00750	C				HOLES FOR WHICH THE VARIOGRAM IS TO BE
00760	C				COMPUTED, I.E., 20 X CLAS = MAX. DISTANCE (H).
00770	C	21-30	F10.0	ANG	ANGLE IN DEGREES ALONG WHICH THE VARIOGRAM IS
00780	C				TO BE CALCULATED.
00790	C				0. = EAST-WEST 45. = NE-SW
00800	C				90. = NORTH-SOUTH -45. = NW-SE
00810	C	31-40	F10.0	SPR	ALLOWABLE ANGLE DEVIATION IN DEGREES.
00820	C	41-50	F10.0	TMAX	MAXIMUM VARIOGRAM VALUE ON Y-AXIS OF THE GRAPH
00830	C				WHEN AUTOMATIC SCALING OPTION IS NOT USED.
00840	C				IF ISCAL = 1, THIS VALUE IS IGNORED.
00850	C				
00860	C	CARD THREE.		INPUT DATA SPECIFICATION CARD.	
00870	C	1	I1	LD	MUST HAVE A VALUE OF EITHER 1,2, OR 3.
00880	C				IF 'INPT' VALUE IS 1 INDICATING THAT THE INPUT
00890	C				IS FROM THE 'SCOR' TAPE, VARIOUS LD VALUES
00900	C				REFER TO THE FOLLOWING:
00910	C				1 = GRADE, 2 = THICKNESS, 3 = GT PRODUCT.
00920	C				IF CARD INPUT, LD SPECIFIES THE LOCATION OF
00930	C				ASSAY VALUE (I.E., IS IT THE FIRST, SECOND OR
00940	C				THIRD ON THE DATA COLUMN).
00950	C	2	I1	NO	NOT USED IF 'INPT' IS 1. OTHERWISE, LOCATION
00960	C				OF NORTHING. (1,2,OR 3)
00970	C	3	I1	IE	NOT USED IF 'INPT' IS 1. OTHERWISE, LOCATION
00980	C				OF EASTING. (1,2,OR 3)
00990	C	4	I1	ISEL	DATA SELECTION OPTION. 1=YES, 0=NO.
01000	C				IF YES, THE APPROPRIATE BOUNDARIES WITHIN
01010	C				WHICH TO SEARCH THE ASSAY DATA MUST BE
01020	C				SPECIFIED ON THE SUBSEQUENT FOUR(4) COLUMNS.
01030	C	11-20	F10.0	YMIN	MINIMUM NORTH COORDINATE IF ISEL = 1.
01040	C				NOT USED OTHERWISE.
01050	C	21-30	F10.0	YMAX	MAXIMUM NORTH COORDINATE IF ISEL = 1.
01060	C	31-40	F10.0	XMIN	MINIMUM EAST COORDINATE IF ISEL = 1.
01070	C	41-50	F10.0	YMAX	MAXIMUM EAST COORDINATE IF ISEL = 1.
01080	C				
01090	C	CARD FOUR.		VARIABLE FORMAT CARD FOR DATA READ IN. INCLUDE ONLY IF	
01100	C				'INPT' IS EQUAL TO ZERO.
01110	C	1-80	8A10	IFRMT	FORMAT SPECIFICATION. MUST BE ENCLOSED BY ().
01120	C				
01130	C	CARD FIVE.		TITLE OF THE VARIOGRAM TO BE OUTPUT FOR EACH DATA GROUP	
01140	C	1-80	8A10	TITLE	ALPHA NUMERIC DESCRIPTION.
01150	C				
01160	C	CARD SIX.		DATA CARDS FOR COORDINATES AND ASSAY VALUES.	
01170	C				END OF DATA FOR EACH SET OF DATA GROUP IS
01180	C				A BLANK CARD OR AN EOF CARD.
01190	C				EACH VARIOGRAM TO BE CALCULATED MUST CONSIST OF A TITLE CARD
01200	C				(CARD FIVE), DATA CARDS, AND A BLANK OR AN EOF CARD.
01210	C				
01220	C	*****			*****
01230	C	MAJOR VARIABLES USED IN PROGRAM UGAMM			
01240	C	DATA		ARRAY CONTAINING SAMPLES AND COORDINATES	
01250	C	Z		ARRAY USED FOR TEMPORARY STORAGE OF INPUT DATA	
01260	C	X		DIFFERENCE IN EAST COORDINATES BETWEEN TWO SAMPLES	

```

01270 C      Y      DIFFERENCE IN WEST COORDINATES BETWEEN TWO SAMPLES
01280 C      Q      DIFFERENCE IN SAMPLE VALUES
01290 C      DIS     DISTANCE BETWEEN TWO SAMPLES
01300 C      QSQR    Q SQUARED
01310 C      DISQ    DIS TIMES Q SQUARED
01320 C      TOT(I,1) TOTVAR(I,1) CUMULATIVE DISTANCE FOR I-TH CLASS
01330 C      INTERVAL.
01340 C      TOT(I,2) TOTVAR(I,2) CUMULATIVE DIFFERENCE
01350 C      TOT(I,3) TOTVAR(I,3) CUMULATIVE QSQR
01360 C      TOT(I,4) TOTVAR(I,4) CUMULATIVE DISTANCE TIMES DIFFERENCE SQ.
01370 C      TOT(I,5) TOTVAR(I,5) CUMULATIVE NUMBER OF SAMPLES
01380 C      MAXN     MAXIMUM NO. OF DATA POINTS CURRENTLY ALLOWED.
01390 C      IPT      UNIT DEVICE ON WHICH THE RUN CONTROL DATA ARE STORED
01400 C      ITAPE    UNIT DEVICE ON WHICH ASSAY DATA IS STORED.
01410 C      IOUT     UNIT DEVICE FOR PRINTED OUTPUT.
01420 C      IPCH    UNIT DEVICE FOR PUNCHED OUTPUT OF THE VARIOGRAM
01430 C      IEXIT    CODE USED TO TERMINATE THE RUN.
01440 C      1 = MEANS DATA WAS FOUND. HENCE PROCESS IT.
01450 C      2 = MEANS NORMAL END OF THE RUN. NO MORE DATA GROUP
01460 C      WAS FOUND.
01470 C      3 = MEANS ABRUPT END OF THE RUN DUE TO INPUT DATA
01480 C      ERROR.
01490 C*****
01500 C****READ IN CONTROL CARD ONE
01510 C      READ(IPT,1) HEAD
01520 C      10      FORMAT(3A10)
01530 C      IF(EOF(IPT).NE.C) GO TO 200
01540 C****READ IN CONTROL CARD TWO
01550 C      READ(IPT,20) INPT,LOG,ITAL,IPC,LL,ISCAL,NREL,
01560 C      11COMP,CLAS,ANG,SPR,TMAX
01570 C      20      FORMAT(I1,1X,7F11,1X,4F10.0)
01580 C      IF(EOF(IPT).NE.C) GO TO 200
01590 C****SIMPLE LOGIC CHECKS
01600 C      L=LL+3
01610 C      IF(LL.GT.3) L = 4
01620 C      IF(1COMP.EQ.0) ITAPE=IPT
01630 C      DLIM = 2)*CLAS
01640 C      DANG = ANG*.017453292519943
01650 C      CST = COS(DANG)
01660 C      SIT = SIN(DANG)
01670 C      DSPR = SPR*.017453292519943
01680 C      CSPR = COS(DSPR)
01690 C****READ IN CARD THREE FOR INPUT OPTION (INPT) ZERO
01700 C      READ(IPT,22) LD,NO,IE,ISEL,YMIN,YMAX,XMIN,XMAX
01710 C      22      FORMAT(4I1,6X,4F10.0)
01720 C      IF(EOF(IPT).NE.C) GO TO 200
01730 C      IF(INPT.EQ.1) GO TO 23
01740 C      READ(IPT,71) IFRMT
01750 C      71      FORMAT(3A10)
01760 C      IF(EOF(IPT).NE.0) GO TO 200
01770 C      23      IF(INPT.EQ.1.AND.LD.LE.3) GO TO 75
01780 C      IF((LD.LE.3).AND.(NO.LE.3).AND.(IE.LE.3)) GO TO 75
01790 C      WRITE(IOUT,30)
01800 C      30      FORMAT(" SPECIFIED ORDER OF DATA IS IN ERROR")
01810 C****STOP THIS RUN
01820 C      GO TO 200
01830 C****READ IN ASSAY DATA FROM APPROPRIATE SUBROUTINES BASED ON INPT VALUE
01840 C      75      IF(INPT.EQ.2) CALL RDN(N,IEXIT)
01850 C      IF(INPT.EQ.1) CALL RSCOR(N,IEXIT)
01860 C      GO TO (35,185,201) IEXIT
01870 C      85      IF(LOG.NE.1) GO TO 91
01880 C      DO 90 I=1,N
01890 C      DATA(I,1) = ALOG(DATA(I,1))

```



```

01900 90 CONTINUE
01910 91 NI = N-1
01920 DO 130 I=1,NI
01930 Q1 = DATA(I,1)
01940 Y1 = DATA(I,2)
01950 X1 = DATA(I,3)
01960 II = I+1
01970 DO 130 J = II,N
01980 Q2 = DATA(J,1)
01990 Y2 = DATA(J,2)
02000 X2 = DATA(J,3)
02010 X = X1-X2
02020 Y = Y1-Y2
02030 Q = Q1-Q2
02040 DIS = SORT(Y*Y + X*X) + .0001
02050 IF(DIS.GT.DLIM.OR.DIS.LT.0.0001) GO TO 130
02060 C*****COMPARE DIRECTION WITH ACCEPTABLE DIRECTIONS.
02070 DOT=(X*CST + Y*SIT)/DIS
02080 ADOT=ABS(DOT)
02090 IF(ADOT.LT.CSPR) GO TO 130
02100 K = DIS/CLAS + 1
02110 IF(K.GT.20) GO TO 130
02120 C*****KEEP TRACK OF DATA POINTS USED IN THE CALCULATION OF THE VARIOGRAM
02130 DATA(I,4)=1
02140 DATA(J,4)=1
02150 DIS = DIS - .0001
02160 Q=SIGN(Q,DOT)
02170 QSQR = Q*Q
02180 DISQ = DIS*QSQR
02190 TOT(K,1) = DIS + TOT(K,1)
02200 TOT(K,2) = Q + TOT(K,2)
02210 TOT(K,3) = QSQR + TOT(K,3)
02220 TOT(K,4) = DISQ + TOT(K,4)
02230 TOT(K,5) = TOT(K,5) + 1.0
02240 130 CONTINUE
02250 C*****CALCULATE MEAN AND STANDARD DEVIATION OF SAMPLES.
02260 SUM = 0.0
02270 SUM2 = 0.0
02280 DO 100 I=1,N
02290 IF(DATA(I,4).NE.1) GO TO 100
02300 SUM = SUM + DATA(I,1)
02310 SUM2 = SUM2 + DATA(I,1)**2
02320 100 CONTINUE
02330 VARI = (FLOAT(N)*SUM2-SUM*SUM)/(FLOAT(N)*FLOAT(N-1))
02340 SMEAN = SUM/FLOAT(N)
02350 SMU2=SMEAN*SMEAN
02360 STD = SORT(VARI)
02370 DO 150 I = 1,20
02380 AN = TOT(I,5)
02390 IF(AN.EQ.0.0) GO TO 150
02400 VAR(I,1) = TOT(I,1)/AN
02410 VAR(I,2) = TOT(I,2)/AN
02420 IF(NREL.EQ.0) GO TO 148
02430 TOT(I,3)=TOT(I,3)/SMU2
02440 TOT(I,4)=TOT(I,4)/SMU2
02450 148 VAR(I,3) = TOT(I,3)/(2.0*AN)
02460 IF(TOT(I,4).EQ.0) GO TO 149
02470 VAR(I,4) = TOT(I,4)/(TOT(I,1)*2.0)
02480 149 VAR(I,5) = TOT(I,5)
02490 150 CONTINUE
02500 C***** PRINT OUT RESULTS OF VARIOGRAM
02510 CALL VAROUT
02520 C**** COMBINE DATA IF AVERAGE VARIOGRAM IS TO BE CALCULATED.

```

```

C2530      IF (ITAL.EQ.0) GO TO 170
C2540      DO 160 I = 1,20
C2550      DO 160 J = 1,5
C2560      TOTVAR(I,J) = TOTVAR(I,J) + TOT(I,J)
C2570      CONTINUE
C2580      160  TSUM = SUM + TSUM
C2590      TSUM2 = SUM2 + TSUM2
C2600      NN = NN + N
C2610      C*****ZERO TOT AND VAR ARRAYS
C2620      170  DO 130 I = 1,20
C2630      DO 180 J = 1,5
C2640      TOT(I,J) = 0.0
C2650      VAR(I,J) = 0.0
C2660      180  CONTINUE
C2670      GO TO 75
C2680      185  IF (ITAL.EQ.0) GO TO 194
C2690      C*****CALCULATE AVERAGE VARIOGRAM
C2700      VARI = (FLOAT(NN)*TSUM2-TSUM*TSUM)/(FLOAT(NN)*FLOAT(NN-1))
C2710      SMEAN = TSUM/FLOAT(NN)
C2720      STD = SQRT(VARI)
C2730      N = NN
C2740      DO 190 I = 1,20
C2750      AN = TOTVAR(I,5)
C2760      IF (AN.EQ.0.0) GO TO 190
C2770      VAR(I,1) = TOTVAR(I,1)/AN
C2780      VAR(I,2) = TOTVAR(I,2)/AN
C2790      VAR(I,3) = TOTVAR(I,3)/(AN*2.0)
C2800      IF (TOTVAR(I,4).EQ.0) GO TO 189
C2810      VAR(I,4) = TOTVAR(I,4)/(TOTVAR(I,1)*2.0)
C2820      189  VAR(I,5) = AN
C2830      190  CONTINUE
C2840      C*****PRINT OUT RESULTS OF AVERAGE VARIOGRAM
C2850      TITLE(1) = "AVERAGE"
C2860      TITLE(2) = "VARIOGRAM"
C2870      CALL VAROUT
C2880      194  WRITE(IOUT,195)
C2890      195  FORMAT(1H1,20X," NORMAL END OF JOB")
C2900      GO TO 201
C2910      200  WRITE(IOUT,191)
C2920      191  FORMAT(" END OF FILE ENCOUNTERED. ERROR IN JOB SETUP.")
C2930      201  STOP
C2940      END
C2950      SUBROUTINE VAROUT
C2960      C*****
C2970      C SUBROUTINE VAROUT PRINTS OUT THE RESULTS OF THE VARIOGRAM CALCULATION
C2980      C*****
C2990      DIMENSION A(100),C(11),D(50),E(13)
C3000      COMMON VAR(20,5),HEAD(8),LOG,CLAS,DLIM,ANG,SMEAN,VARI,STD,NN
C3010      COMMON IPC,L,SPR,ISCAL,TMAX,NREL
C3020      COMMON /IO/IOUT,IPT,IPCH,ICOMP,ITAPE,TITLE(8),MAXN
C3030      COMMON /DAT2/ISEL,YMIN,YMAX,XMIN,XMAX
C3040      DIMENSION ILOG(2)
C3050      DATA ILOG/2HNO,3HYES/
C3060      DATA D/20*1H,1HG,1HA,1HM,1HM,1HA,1H,1H,1H*,1HH,1H*,20*1H /
C3070      DATA E/1HM,1HO,1HM,1HE,1HN,1HT,1H,1HC,1HE,1HN,1HT,1HE,1HR/
C3080      IF (L.EQ.3) GO TO 8
C3090      DO 7 I = 1,13
C3100      D(I+18) = E(I)
C3110      7  CONTINUE
C3120      8  C(1) = 0.
C3130      DO 85 I = 2,11
C3140      C(I) = CLAS*2.0 + C(I-1)
C3150      85  CONTINUE

```

```

03160 C*****PRINT OUT PAGE ONE OUTPUT
03170 WRITE(IOUT,10)
03180 10 FORMAT(1H1,60X,"VARIOGRAM")
03190 WRITE(IOUT,20) TITLE
03200 20 FORMAT(1H0,25X,8A10)
03210 NL=LOG+1
03220 NR=NREL+1
03230 WRITE(IOUT,20) HEAD
03240 WRITE(IOUT,25)
03250 25 FORMAT(78X,"DATA USED IN CALCULATIONS")
03260 WRITE(IOUT,30) ANG,SPR,SMEAN,CLAS,VARI,DLIM,STD,ILOG(NL)
03270 1,ILOG(NR),NN
03280 30 FORMAT("DIRECTION =" ,F5.0,3X"WINDOW",F5.0,45X"MEAN",
03290 110X,"="E10.3,"/" CLASS SIZE ="F5.0,59X"VARIANCE ="E10.3
03300 2/" MAX DISTANCE =" ,F10.0,54X"STD DEVIATION =" ,E10.3,
03310 3/" LOGARITHMS -" ,A5,5X,"RELATIVE VARIOGRAM -" ,A5,30X,
03320 4"NO. OF SAMPLES =" ,I10)
03330 IF(ISEL.NE.0) WRITE(IOUT,110) YMIN,YMAX,XMIN,XMAX
03340 110 FORMAT(" COORDINATE SELECTION- NORTH(" ,2F10.0," ) "
03350 13X,"EAST(" ,2F10.0," ) ")
03360 C*****PUNCH OUTPUT CARDS OR STORE ON DEVICE UNIT NO. 6.
03370 IF(IPC.EQ.0) GO TO 39
03380 WRITE(IPCH,21) TITLE
03390 21 FORMAT(1X,7A10,A9)
03400 WRITE(IPCH,23) ANG,SPR,SMEAN,VARI,CLAS,ILOG(NL),NN
03410 23 FORMAT(5E10.3,A4,6X,I10)
03420 22 FORMAT(6E10.3)
03430 39 WRITE(IOUT,40)
03440 40 FORMAT(1H0,10X,"DISTANCE",13X,"NO. OF SAMPLES",15X,"DRIFT"
03450 1,13X,"GAMMA (H) ",8X,"MOMENT CENTER",4X" AVERAGE DISTANCE")
03460 DO 50 I = 1,20
03470 LOW = (I-1)*CLAS
03480 LUP = I*CLAS
03490 IF(VAR(I,5).GT.0.) WRITE(IOUT,45) LOW,LUP,VAR(I,5),VAR(I,2),
03500 1VAR(I,3),VAR(I,4),VAR(I,1)
03510 IF(IPC.EQ.1.AND.VAR(I,5).GT.0) WRITE(IPCH,22) VAR(I,5),VAR(I,2),V
03520 1AR(I,3),VAR(I,4),VAR(I,1)
03530 45 FORMAT(1H ,4X,I6," ---",I6,10X,F8.0,5X,3(11X,E10.3),10X,F10.1)
03540 50 CONTINUE
03550 WRITE(IOUT,10)
03560 C*****PRINT OUT PAGE TWO OUTPUT
03570 WRITE(IOUT,20) TITLE
03580 WRITE(IOUT,20) HEAD
03590 WRITE(IOUT,100)
03600 100 FORMAT(1H0)
03610 C IF AUTOMATIC SCALING IS NOT DESIRED SKIP TO 56
03620 IF(ISCAL.EQ.0.AND.TMAX.NE.0.0) GO TO 56
03630 C*****FIND MAX VALUE
03640 TMAX = VAR(1,L)
03650 DO 55 I=2,20
03660 IF(VAR(I,L).GT.TMAX) TMAX = VAR(I,L)
03670 55 CONTINUE
03680 56 UNIT = CLAS/5.
03690 DIV = TMAX/50.
03700 TEMP = TMAX
03710 DO 75 K=1,50
03720 TOP = TEMP
03730 BOT = TEMP - DIV
03740 DO 65 I = 1,20
03750 IF(VAR(I,5).EQ.0.) GO TO 65
03760 IF((VAR(I,L).GT.TOP).OR.VAR(I,L).LE.BOT) GO TO 65
03770 J = VAR(I,1)/UNIT + 1
03780 A(J) = 1HX

```

```

03790      IF (VAR(I,5).LT.30) A(J) = "#"
03800      CONTINUE
03810      C      ARRAY "A" CONTAINS BLANKS AS WELL AS DATA POINT MARKERS.
03820      WRITE(IOUT,60) D(K),TEMP,A
03830      60      FORMAT(1H,2X,A1,2X,E10.3," *",100A1)
03840      TEMP = BOT
03850      DO 70 I=1,100
03860      A(I) = " "
03870      70      CONTINUE
03880      75      CONTINUE
03890      WRITE(IOUT,80)
03900      80      FORMAT(1H,17X,10("$*****$*****"),"$")
03910      WRITE(IOUT,90)C
03920      90      FORMAT(1H,13X,11(F6.0,4X))
03930      RETURN
03940      END
03950      SUBROUTINE RDN(N,IEXIT)
03960      C*****
03970      C      SUBROUTINE RDN READS INPUT DATA WITH USER SPECIFIED FORMAT.
03980      C*****
03990      COMMON /DAT/DATA(1000,4),IFRMT(8),LD,NO,IE,IA
04000      COMMON /DAT2/ISEL,YMIN,YMAX,XMIN,XMAX
04010      COMMON /IO/IOUT,IPT,IPCH,ICOMP,ITAPE,TITLE(8),MAXN
04020      DIMENSION Z(3)
04030      N=0
04040      IEXIT = 2
04050      READ(ITAPE,10) TITLE
04060      10      FORMAT(8A10)
04070      IF(EOP(ITAPE).NE.0) RETURN
04080      80      READ(ITAPE,IFRMT) Z(1),Z(2),Z(3)
04090      C**** BLANK CARD CAN BE USED TO SIGNAL END OF DATA
04100      IF(EOP(ITAPE).NE.0) GO TO 85
04110      IF(Z(1).EQ.0..AND.Z(2).EQ.0..AND.Z(3).EQ.0.) GO TO 85
04120      IF(ISEL.EQ.0) GO TO 81
04130      IF(Z(NO).LT.YMIN.OR.Z(NO).GT.YMAX) GO TO 80
04140      IF(Z(IE).LT.XMIN.OR.Z(IE).GT.XMAX) GO TO 80
04150      81      N = N+1
04160      IF(N.GT.MAXN) GO TO 83
04170      DATA(N,1) = Z(LD)
04180      DATA(N,2) = Z(NO)
04190      DATA(N,3) = Z(IE)
04200      DATA(N,4) = 0.
04210      GO TO 80
04220      85      IEXIT = 1
04230      IF(N.GT.1) RETURN
04240      83      WRITE(IOUT,210) N
04250      210      FORMAT(" INPUT DATA ERROR",3X,I5," DATA POINTS PRESENT")
04260      IEXIT = 3
04270      RETURN
04280      END
04290      SUBROUTINE RSCOR(N,IEXIT)
04300      C*****
04310      C      SUBROUTINE RSCOR INPUT DATA ACCORDING TO SCOR CARD FORMAT.
04320      C*****
04330      COMMON /DAT/DATA(1000,4),IFRMT(8),LD,NO,IE,IA
04340      COMMON /DAT2/ISEL,YMIN,YMAX,XMIN,XMAX
04350      COMMON /IO/IOUT,IPT,IPCH,ICOMP,ITAPE,TITLE(8),MAXN
04360      DIMENSION Z(5)
04370      N=0
04380      IEXIT = 2
04390      READ(ITAPE,10) TITLE
04400      10      FORMAT(8A10)
04410      IF(EOP(ITAPE).NE.0) RETURN

```

```

04420 80 READ(ITAPE,20)Z
04430 20 FORMAT(20X,F5.0,F7.0,F8.0,19X,2F8.0)
04440 IF(EOP(ITAPE).NE.0)GO TO 85
04450 C*****BLANK CARD CAN BE USED TO SIGNAL END OF DATA
04460 IF(Z(1).EQ.0..AND.Z(2).EQ.0..AND.Z(3).EQ.0.) GO TO 85
04470 IF(ISEL.EQ.0) GO TO 81
04480 IF(Z(4).LT.YMIN.OR.Z(4).GT.YMAX) GO TO 80
04490 IF(Z(5).LT.XMIN.OR.Z(5).GT.XMAX) GO TO 80
04500 81 IF(Z(LD).EQ.0.)GO TO 80
04510 N = N+1
04520 IF(N.GT.MAXN) GO TO 83
04530 DATA(N,1) = Z(LD)
04540 DATA(N,2) = Z(4)
04550 DATA(N,3) = Z(5)
04560 DATA(N,4) = 0.
04570 GO TO 80
04580 85 IEXIT = 1
04590 IF(N.GT.1) RETURN
04600 83 WRITE(IOUT,210) N
04610 210 FORMAT(" INPUT DATA ERROR",3X,I5," DATA POINTS PRESENT")
04620 IEXIT = 3
04630 RETURN
04640 END

```

APPENDIX B

VARIOGRAM DISPLAY PROGRAM "VARPLOT".

USER'S GUIDE

SECTION I: PROGRAM IDENTIFICATION

- 1.1 PROGRAM TITLE: Calcomp Plotting Program of Experimental Variograms
- 1.2 PROGRAM CODE NAME: VARPLOT
- 1.3 PROGRAM WRITER: Harvey P. Knudsen
Young C. Kim
- PROGRAM DOCUMENTATION: Young C. Kim
- 1.4 ORGANIZATION: Department of Mining and Geological Engineering
University of Arizona
Tucson, Arizona
- 1.5 DATE: November 10, 1976
- 1.6 UPDATES: One. Version: II
- 1.7 SOURCE LANGUAGE: CDC 6400 Computer Systems - FORTRAN Extended -
Version IV.
- 1.8 ABSTRACT:

Program VARPLOT is a Calcomp plotter program for plotting the experimental variogram values. To use this program, the user must have an access to a Calcomp plotter as well as the standard Calcomp subroutines.

The program is currently dimensioned to plot a maximum of five different variograms on one graph. The program limits the total number of plots during one run to fifteen (15). If desired, a spherical variogram model can be superimposed on the same graph or drawn as a new graph.

SECTION II: ENGINEERING DOCUMENTATION

2.1 NARRATIVE DESCRIPTION:

Program VARPLOT is a Calcomp plotter program which plots the experimental variogram values. To use this program, the user must have an access to a Calcomp plotter as well as the standard Calcomp subroutines. Five such subroutines are;

1. AXIS Subroutine
2. SYMBOL Subroutine
3. NUMBER Subroutine
4. PLOT Subroutine
5. LINE Subroutine.

Prior to executing this program, a variogram computation program such as the Program GAMMA (see Appendix A of this report) should be executed to generate the above variogram values. These values, then, become the necessary input data to VARPLOT program.

Since this is a Calcomp plotter program, run control parameters specifically for the plotter must be specified ahead of the main input data. Examples of such parameters are the size of a graph, size and color of the pen, and type of symbols to use. These parameters are read into the program only once during the program run.

The program is currently dimensioned to plot a maximum of five different variograms on the same graph using different symbols. The program allows a total of fifteen different plots during one run. If desired, a spherical variogram model can be superimposed on the same graph or drawn on a new graph. If the model is to be drawn on a new graph, it must be the first plot for that run.

2.2 METHOD OF SOLUTION:

The method of solution is straightforward, if one is familiar with the Calcomp subroutines.* Use of these subroutines is not described in this user's guide, however.

The computed experimental variogram values which have been obtained from another program such as GAMMA, are plotted on a graph whose dimensions are specified by the user. The recommended dimensions are 8 inches along Y-axis and 10 inches along X-axis. The variogram values are plotted along Y-direction whereas the distance values are plotted along X-direction.

In addition to the variogram values, other pertinent information regarding the variogram results is also given on the bottom of the graph as shown in Figure 1. This information consists of 1) title of the plot, 2) direction window, and class size used, and 3) mean, variance and sample size.

If more than one variogram is to be plotted on the same graph, the above information regarding the second and the subsequent plots are not given, however. Similarly, if a spherical variogram model is either superimposed on the same graph or drawn on a new graph, neither the title of the model nor the model parameters are given on the plot.

In the program, several indices are used to control the type of plotting operation being performed. For example, the index "NEW" is used to designate if the plot is 1) a new one, 2) to be plotted on the same graph, or 3) for a variogram model. The "NSY" index is used as a counter for multiple plots on one graph, whereas the "NPLOT" index is used as a counter for the total number of plots during the same run. The variable "ICK" is a switch used for drawing the axes of the graph for a new plot.

* _____, Programming Calcomp Pen Plotters, California Computer Products, Inc., 305 Muller Ave., Anaheim, California 92083, September 1969.

1590.0 - 1500.0 BENCHES. NW-SE DIRECTION. 30 DEG WINDOW.
 DIRECTION = 45. WINDOW = 30. MEAN = 0.582
 CLASS SIZE = 25.00 VARIANCE = 0.1810
 LOGARITHMS - NO. OF SAMPLES = 893

Figure 1
A SAMPLE OUTPUT OF A VARIOGRAM PLOT

2.3 PROGRAM CAPABILITIES:

Program VARPLOT limits the total number of plots during one run to fifteen (15). This limit can be easily modified by the user by changing the "NPMAX" value given in the DATA statement. Similarly, the program is currently dimensioned to allow a total of five (5) multiple plots on the same graph. If more than five multiple plots are called for by the user, a message is printed and the current plot is ignored. The program proceeds to the next plot until a new plot is encountered.

As it is currently dimensioned, there can be a maximum of twenty (20) data points for each variogram. If there are more than 20 data points, the remaining points are ignored.

The program performs other checks on the possible inconsistencies of the input data. If data points fall outside the limits of the graph, these points are reset to coincide with the graph boundaries. If the index "NEW" is not specified in the first column of the title card, then the program assumes that the plot is a new plot. If the spherical variogram model parameters are improperly specified, plotting the model is omitted after printing a message.

2.4 DATA INPUTS:

1. CARD TYPE ONE. RUN PARAMETER CARD (NEEDED ONLY ONCE).

<u>COLUMN</u>	<u>FORMAT</u>	<u>VARIABLE NAME</u>	<u>DESCRIPTION</u>
1-5	F5.0	PEN	PLOT PEN DESIRED (NORMALLY = 0.3).
6-10	I5	INK	COLOR OF INK DESIRED (0 = BLACK).
11-20	F10.0	TOP	VARIOGRAM VALUE AT TOP OF GRAPH.
21-30	F10.0	BOT	VARIOGRAM VALUE AT BOTTOM OF GRAPH.
31-40	F10.0	EAST	DISTANCE VALUE AT EAST EDGE OF GRAPH.
41-50	F10.0	WEST	DISTANCE VALUE AT WEST EDGE OF GRAPH.
51-60	F10.0	XLEN	LENGTH OF X-AXIS IN INCHES (NORMALLY = 10).
61-70	F10.0	YLEN	LENGTH OF Y-AXIS IN INCHES (NORMALLY = 8).

2. CARD TYPE TWO. PLOT CONTROL AND TITLE CARD (REQUIRED FOR EACH PLOT).

<u>COLUMN</u>	<u>FORMAT</u>	<u>VARIABLE NAME</u>	<u>DESCRIPTION</u>
1	I1	NEW	CODE USED TO DISTINGUISH THE TYPE OF THE CURRENT PLOT. IF NEW = 1, NEW PLOT. THEREFORE, DRAW THE AXES ALSO. IF NEW = 2, PLOT ON SAME GRAPH. IF NEW = 3, PLOT VARIOGRAM MODEL.
2-80	7A10,A9	TITLE	DESCRIPTION OF THE PLOT TO BE WRITTEN OUT ON THE GRAPH. USED ONLY WHEN NEW = 1.

3. CARD TYPE THREE. VARIOGRAM INFORMATION CARD (NEEDED IF NEW = 1 OR 2).

<u>COLUMN</u>	<u>FORMAT</u>	<u>VARIABLE NAME</u>	<u>DESCRIPTION</u>
1-10	E10.3	ANG	DIRECTION IN WHICH THE VARIOGRAM WAS COMPUTED.
11-20	E10.3	WIND	THE SPREAD OR THE WINDOW ALLOWED IN COMPUTATION.
21-30	E10.3	SMEAN	SAMPLE MEAN OF ALL DATA POINTS.
31-40	E10.3	VARI	SAMPLE VARIANCE OF ALL DATA POINTS.
41-50	E10.3	CLASS	CLASS SIZE USED TO GROUP THE DATA POINTS.
51-53	A3	DLOG	LOGARITHMIC TRANSFORMATION OF DATA. YES OR NO.
61-70	F10.0	SN	TOTAL NUMBER OF DATA POINTS.

4. CARD TYPE FOUR. DATA CARDS (ONE CARD FOR EACH DATA POINT). NEEDED IF
NEW = 1 OR NEW = 2.

<u>COLUMN</u>	<u>FORMAT</u>	<u>VARIABLE NAME</u>	<u>DESCRIPTION</u>
31-40	E10.3	V	ARRAY USED FOR VARIOGRAM VALUES.
41-50	E10.3	DIS	ARRAY USED FOR THE CORRESPONDING DISTANCES.

5. CARD TYPE FIVE. END OF DATA CARDS (NEEDED IF NEW = 1 OR 2). A 7/8/9 CARD
IS USED TO INDICATE END OF DATA.

NOTE: CARD TYPES 2, 3 AND 4 CAN BE AUTOMATICALLY GENERATED BY EXECUTING
THE PROGRAM GAMMA, WHICH COMPUTES THE EXPERIMENTAL VARIOGRAM VALUES. THEY ARE
NEEDED IF THE INDEX VALUE OF NEW IS EITHER 1 OR 2. IF THE VALUE OF NEW IS
EQUAL TO 3, A SPHERICAL VARIOGRAM MODEL PARAMETERS MUST BE SPECIFIED USING THE
FORMAT GIVEN BELOW, RIGHT AFTER THE CARD TYPE TWO.

6. CARD TYPE SIX. MODEL PARAMETER CARD (NEEDED IF NEW = 3).

<u>COLUMN</u>	<u>FORMAT</u>	<u>VARIABLE NAME</u>	<u>DESCRIPTION</u>
1-10	F10.0	CO	NUGGET VALUE OF A SPHERICAL VARIOGRAM MODEL.
11-20	F10.0	C	"C" VALUE.
21-30	F10.0	RANG	THE RANGE OF VARIOGRAM.

NO END OF DATA CARD IS REQUIRED AFTER TYPE SIX CARD.

A 80-80 listing of sample input data is shown in Figure 2.

2.5 PROGRAM OPTIONS:

None.

2.6 OUTPUT:

There are basically three types of output:

1. The plot of variogram computation results.
2. The plot of the user specified spherical variogram model, if desired.
3. Error or limitation messages, if encountered.

Figure 3 shows a sample output of multiple plots including the user specified variogram model which is superimposed.

.3	.32	0	700	0	10	8
1590.0	1500.0	BENCHES.	ALL DIRECTIONS.			
0.000E+00	0.900E+02	0.582E+00	0.181E+00	0.350E+02	NO	893
0.510E+02	0.114E+00	0.107E+00	0.119E+00	0.242E+02		
0.281E+03	0.443E-01	0.958E-01	0.982E-01	0.565E+02		
0.117E+04	0.540E-01	0.132E+00	0.132E+00	0.892E+02		
0.119E+04	0.661E-01	0.162E+00	0.165E+00	0.122E+03		
0.151E+04	0.742E-01	0.154E+00	0.155E+00	0.159E+03		
0.187E+04	0.834E-01	0.162E+00	0.162E+00	0.193E+03		
0.204E+04	0.101E+00	0.169E+00	0.168E+00	0.228E+03		
0.224E+04	0.140E+00	0.184E+00	0.184E+00	0.263E+03		
0.220E+04	0.102E+00	0.173E+00	0.173E+00	0.297E+03		
0.252E+04	0.164E+00	0.195E+00	0.195E+00	0.333E+03		
0.247E+04	0.140E+00	0.174E+00	0.174E+00	0.366E+03		
0.261E+04	0.142E+00	0.194E+00	0.194E+00	0.403E+03		
0.251E+04	0.232E+00	0.215E+00	0.216E+00	0.437E+03		
0.254E+04	0.157E+00	0.187E+00	0.187E+00	0.472E+03		
0.247E+04	0.195E+00	0.178E+00	0.178E+00	0.507E+03		
0.264E+04	0.188E+00	0.199E+00	0.199E+00	0.543E+03		
0.228E+04	0.199E+00	0.192E+00	0.193E+00	0.577E+03		
0.227E+04	0.211E+00	0.178E+00	0.178E+00	0.613E+03		
0.226E+04	0.200E+00	0.194E+00	0.194E+00	0.646E+03		
0.195E+04	0.191E+00	0.184E+00	0.184E+00	0.683E+03		
7/8/9						
3	TEST PLOT OF VARFLOT					
0.003	0.17	200.				
2	1590.0	1500.0	BENCHES.	N-S DIRECTION, 30 DEG WINDOW.		
0.900E+02	0.300E+02	0.582E+00	0.181E+00	0.350E+02	NO	893
0.380E+02	0.132E-02	0.853E-01	0.783E-01	0.533E+02		
0.221E+03	0.160E-01	0.154E+00	0.155E+00	0.887E+02		
0.174E+03	0.580E-01	0.119E+00	0.124E+00	0.122E+03		
0.297E+03	0.774E-01	0.177E+00	0.176E+00	0.160E+03		
0.344E+03	0.260E-01	0.140E+00	0.140E+00	0.191E+03		
0.328E+03	0.768E-02	0.157E+00	0.157E+00	0.230E+03		
0.425E+03	0.682E-02	0.182E+00	0.180E+00	0.261E+03		
0.372E+03	0.483E-02	0.220E+00	0.222E+00	0.297E+03		
0.474E+03	0.133E-02	0.175E+00	0.176E+00	0.333E+03		
0.451E+03	0.977E-01	0.171E+00	0.171E+00	0.366E+03		
0.466E+03	0.542E-01	0.235E+00	0.236E+00	0.402E+03		
0.494E+03	0.581E-01	0.196E+00	0.197E+00	0.437E+03		
0.439E+03	0.470E-01	0.201E+00	0.201E+00	0.473E+03		
0.455E+03	0.138E-01	0.215E+00	0.215E+00	0.506E+03		
0.531E+03	0.115E-01	0.183E+00	0.183E+00	0.543E+03		
0.453E+03	0.539E-01	0.180E+00	0.180E+00	0.578E+03		
0.427E+03	0.141E-01	0.149E+00	0.148E+00	0.615E+03		
7/8/9						

Figure 2

A 80-80 LISTING OF SAMPLE INPUT DATA
(See Figure 3 for Plots)

1590.0 - 1500.0 BENCHES. ALL DIRECTIONS.
 DIRECTION = 0. WINDOW = 90. MEAN = 0.582
 CLASS SIZE = 35.00 VARIANCE = 0.1810
 LOGARITHMS - NO. OF SAMPLES = 893

Figure 3

A SAMPLE OUTPUT OF MULTIPLE PLOTS

SECTION III: SYSTEM DOCUMENTATION

3.1 COMPUTER EQUIPMENT:

The VARPLOT program was run on a CDC 6400 computer using 50K-core central memory.

3.2 PERIPHERAL EQUIPMENT:

The following peripheral equipment was used in the execution of the VARPLOT program:

CDC 405 card reader
CDC 512 line printer
Calcomp Plotter (11" or 30")

3.3 SOURCE PROGRAM:

A source program listing is given at the end of this Appendix.

3.4 VARIABLES AND SUBROUTINES:

A listing of all major variables is given in the program listing.

Program VARPLOT utilizes the following Calcomp subroutines:

1. AXIS: Draws the axes of the graph using the user specified scaling factor.
2. SYMBOL: Draws the user specified symbols.
3. NUMBER: Plots the number in FORTRAN F-type format.
4. PLOT: Generates the actual plotter command.
5. LINE: Produces a line plot of the pairs of data values in two arrays (X and Y).
6. INITIAL: See comments below.

In addition to the above five standard Calcomp subroutines, the program utilizes one (1) nonstandard University of Arizona subroutine called "INITIAL". This subroutine is called at the beginning only once for the purpose of

initializing the plotter subroutines. As such, this INITIAL subroutine performs the task of Calcomp "PLOTS" subroutine.

3.5 DATA STRUCTURES:

No files are created or read by this program.

3.6 STORAGE REQUIREMENTS:

As presently structured, program VARPLOT requires approximately 50K storage on the CDC 6400.

3.7 MAINTENANCE AND UPDATES:

This is the second version of the variogram plotting program. The first version was written by Harvey P. Knudsen in July 1976.

SECTION IV: OPERATING DOCUMENTATION

4.1 OPERATING INSTRUCTIONS:

Program VARPLOT is operated under the CDC 6400, SCOPE 3.4, operating system.

4.2 OPERATING MESSAGES:

Error messages produced by the program are self-explanatory. In addition, normal system messages are produced.

4.3 CONTROL CARDS:

The program may be executed with standard SCOPE 3.4 control cards. As run on the University of Arizona CDC 6400, the makeup of the input deck is as follows:

```
Job Card
FTN.
LGO.
7/8/9
<SOURCE PROGRAM>
7/8/9
<INPUT DATA>
6/7/8/9
```

4.4 ERROR RECOVERY:

If certain errors occur, the program will make assumptions and continue execution of the program. A warning message will be printed. If a fatal error occurs in execution, the program must be restarted.

4.5 RUN TIME:

Run time depends on number of plots. Typical run time is about 1 second per plot. A typical plotter time is 3-5 minutes per plot on the 11" plotter.

VARPLOT PROGRAM LISTING

```

010  PROGRAM VARPLOT (INPUT, OUTPUT, PLOT, TAPE1=INPUT, TAPE3=OUTPUT, TAPE2=
020  1=PLOT)
030  DIMENSION DIS (22), V (22), TITLE (8), NSYMB (5)
00040  DATA INPT, IPR, IUNIT, NPLT, NSY, ICK/1, 3, 7, 0, 0, 0/
00050  DATA NPMAX, NSMAX, NDMAX/15, 5, 20/
00060  DATA YSN, PAPER/0.0, 0.0/
00070  DATA NSYMB/3, 2, 1, 0, 5/
00080  C*****
00090  C THIS PROGRAM WAS WRITTEN BY HARVEY P. KNUDSEN.
00100  C DEPARTMENT OF MINING & GEOLOGICAL ENGINEERING
00110  C UNIVERSITY OF ARIZONA, TUCSON, ARIZONA.
00120  C JULY, 1976.
00130  C THE ORIGINAL PROGRAM WAS MODIFIED BY YOUNG C. KIM
00140  C ON NOVEMBER 10, 1976.
00150  C
00160  C PROGRAM VARPLOT IS A CALCOMP PLOTTER PROGRAM FOR PLOTTING THE
00170  C EXPERIMENTAL VARIOGRAM VALUES. IT IS CURRENTLY DIMENSIONED
00180  C TO ACCEPT A TOTAL OF 20 DATA POINTS FOR A VARIOGRAM.
00190  C PROGRAM IS CAPABLE OF PLOTTING A MAXIMUM OF FIVE DIFFERENT
00200  C VARIOGRAMS ON ONE GRAPH, AND OF PLOTTING A TOTAL OF FIFTEEN (15)
00210  C DIFFERENT PLOTS DURING ONE RUN. IF DESIRED, A SPHERICAL VARIO-
00220  C GRAM MODEL CAN BE SUPERIMPOSED ON THE SAME GRAPH OR DRAWN ON
00230  C A NEW GRAPH. HOWEVER, NEITHER THE TITLE NOR THE MODEL PARAMETE
00240  C ARE SHOWN ON THE GRAPH.
00250  C
00260  C *****
00270  C DEFINITION OF VARIABLES.
00280  C PEN = SIZE OF INK PEN DESIRED (RECOMMENDED SIZE = 0.3)
00290  C INK = COLOR OF INK (RECOMMENDED CODE = 0 AT U. OF A.)
00300  C TOP = VARIOGRAM VALUE AT TOP OF GRAPH
00310  C BOT = VARIOGRAM VALUE AT BOTTOM OF GRAPH
00320  C EAST = DISTANCE VALUE AT EAST EDGE OF GRAPH
00330  C WEST = DISTANCE VALUE AT WEST EDGE OF GRAPH
00340  C XLEN = LENGTH OF X AXIS IN INCHES. (RECOMMENDED LENGTH = 10)
00350  C YLEN = LENGTH OF Y AXIS IN INCHES. (RECOMMENDED LENGTH = 8)
00360  C NEW = CODE USED TO DISTINGUISH THE TYPE OF THE CURRENT PLO
00370  C IF NEW = 1, NEW PLOT. THEREFORE, DRAW THE AXES ALSO.
00380  C IF NEW = 2, PLOT ON SAME GRAPH.
00390  C IF NEW = 3, PLOT VARIOGRAM MODEL
00400  C TITLE = TITLE OF GRAPH.
00410  C ICK = CODE USED TO CHECK IF THE AXES HAVE BEEN DRAWN EARLI
00420  C NSY = INDEX USED TO SPECIFY THE TYPE OF SYMBOLS TO BE USED
00430  C IT IS ALSO USED TO CHECK THE TOTAL NUMBER OF PLOTS O
00440  C THE SAME GRAPH.
00450  C NSYMB = ARRAY FOR STORING THE INDEX FOR SYMBOL CHARACTERS.
00460  C NPLT = COUNTER FOR THE TOTAL NUMBER OF GRAPHS BEING PLOTTED
00470  C XMOVE = THE DISTANCE TO MOVE THE ORIGIN ALONG X-AXIS FOR NEW
00480  C PLOT.
00490  C NPMAX = MAX. NUMBER OF TOTAL PLOTS ALLOWED DURING ONE RUN.
00500  C NSMAX = MAX. NUMBER OF PLOTS ALLOWED WITHIN ONE GRAPH.
00510  C NDMAX = MAX. NUMBER OF DATA POINTS CURRENTLY DIMENSIONED.
00520  C ANG = DIRECTION IN WHICH THE VARIOGRAM WAS COMPUTED.
00530  C WIND = THE SPREAD OR THE WINDOW ALLOWED IN COMPUTATION
00540  C SMEAN = SAMPLE MEAN OF ALL DATA POINTS
00550  C VARI = SAMPLE VARIANCE OF ALL DATA POINTS.
00560  C CLASS = CLASS SIZE USED TO GROUP THE DATA POINTS.
00570  C DLOG = LOGARITHMIC TRANSFORMATION OF DATA. YES OR NO.
00580  C SN = TOTAL NUMBER OF DATA POINTS.
00590  C V = ARRAY USED FOR VARIOGRAM VALUES.
00600  C DIS = ARRAY CONTAINING THE CORRESPONDING DISTANCES.
00610  C *****
00620  C ***READ IN PLOTTER CONTROL CARD***
00630  C READ (INPT, 5) PEN, INK, TOP, BOT, EAST, WEST, XLEN, YLEN

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00640 5      FORMAT(F5.0,I5,6F10.0)
00650      CALL INITIAL(YBN,IUNIT,PEN,INK,PAPER)
00660      XMOVE = XLEN + 3.0
00670      CALL PLOT(0.0,2.0,-3)
00680 C***READ IN TITLE OF GRAPH
00690 6      READ (INPT,7) NEW,TITLE
00700      IF(NEW.EQ.0.OR.NEW.GT.3) NEW = 1
00710      IF(EOF(INPT).NE.0) GO TO 99
00720 7      FORMAT(I1,7A10,A9)
00730 C      IF THIS IS A NEW GRAPH, MOVE THE ORIGIN BY XMOVE DISTANCE IN
00740 C      INCHES ALONG X-DIRECTION.
00750      IF(NEW.LE.1.AND.NPLOT.GT.0) CALL PLOT(XMOVE,0.0,-3)
00760      IF(NEW.LE.1.AND.NPLOT.GT.0) NSY=0
00770      IF(NEW.GT.1) CALL PLOT(0.0,0.0,-3)
00780      IF(NEW.EQ.3) GO TO 31
00790 C***READ IN VARIOGRAM OUTPUT (FROM GAMMA PROGRAM)
00800 C      7/8/9 CARD IS USED TO INDICATE END OF DATA
00810      READ (INPT,21) ANG,WIND,SMEAN,VARI,CLASS,DLOG,SN
00820 21     FORMAT(5E10.3,A3,7X,F10.0)
00830      I = 1
00840 17     READ (INPT,20) V(I),DIS(I)
00850      IF(V(I).GT.TOP) V(I)=TOP
00860      IF(DIS(I).GT.EAST) DIS(I)=EAST
00870 20     FORMAT(30X,E10.3,E10.3)
00880      IF(EOF(INPT).NE.0) GO TO 30
00890 C****IF THERE ARE MORE DATA POINTS THAN DIMENSIONED, READ AND IGNORE
00900      IF(I.EQ.NDMAX) GO TO 17
00910      I = I+1
00920      GO TO 17
00930 30     I = I-1
00940 C****IF THERE ARE NSMAX NO. OF PLOTS ON THIS GRAPH, IGNORE THIS PLOT
00950 C      AND GO TO THE NEXT GRAPH. ALSO RESET ICK VALUE TO ZERO.
00960 31     IF(NSY.LT.NSMAX) GO TO 33
00970      WRITE(IPR,240)
00980 240    FORMAT("//20X," ATTEMPTS ARE BEING MADE TO PLOT MORE THAN 'NSMAX
00990      1 NO. OF PLOTS ON ONE GRAPH"/20X," IGNORE THIS PLOT AND GO TO TH
01000      2NEXT GRAPH"//)
01010      ICK = 0
01020      GO TO 6
01030 33     NSY = NSY + 1
01040      IF (NEW.EQ.3.AND.ICK.GT.0) GO TO 40
01050      ICK = 1
01060      IF(NEW.GT.1) GO TO 32
01070      CALL SYMBOL(0.,-0.75,.12,TITLE,0.,79)
01080      CALL SYMBOL(0.,-1.0,.12,12HDIRECTION = ,0.,12)
01090      CALL NUMBER(999.,-1.0,.12,ANG,0.,0)
01100      CALL SYMBOL(2.0,-1.0,.12,9HWINDOW = ,0.,9)
01110      CALL NUMBER(999.0,-1.0,.12,WIND,0.0,0)
01120      CALL SYMBOL(4.,-1.0,.12,7HMEAN = ,0.,7)
01130      CALL NUMBER(999.,-1.0,.12,SMEAN,0.,3)
01140      CALL SYMBOL(0.,-1.25,.12,13HCLASS SIZE = ,0.,13)
01150      CALL NUMBER(999.,-1.25,.12,CLASS,0.,2)
01160      CALL SYMBOL(4.,-1.25,.12,11HVARIANCE = ,0.,11)
01170      CALL NUMBER(999.,-1.25,.12,VARI,0.,4)
01180      CALL SYMBOL(0.,-1.50,.12,13HLOGARITHMS = ,0.,13)
01190      CALL SYMBOL(999.,-1.50,.12,DLOG,0.,10)
01200      CALL SYMBOL(4.,-1.50,.12,17HNO. OF SAMPLES = ,0.,17)
01210      CALL NUMBER(999.,-1.50,.12,SN,0.,-1)
01220 C**** COMPUTE HORIZONTAL AND VERTICAL SCALES FOR AXES.
01230 32     DIFY = (TOP - BOT)/YLEN
01240      V(I+1) = BOT
01250      V(I+2) = DIFY
01260      DIFX = (EAST - WEST)/XLEN

```

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01270      DIS(I+1) = WEST
01280      DIS(I+2) = DIFX
01290      IF (NEW.EQ.2) GO TO 34
01300      CALL AXIS(0.,0.0,5HGAMMA,5,YLEN,90., V(I+1),V(I+2),10.)
01310      CALL AXIS(0.5,0.0,4HFEET,-4,XLEN,0.,DIS(I+1),DIS(I+2),10)
01320      CALL PLOT(XLEN,0.0,3)
01330      CALL PLOT(XLEN,YLEN,2)
01340      CALL PLOT(0.,YLEN,2)
01350 C**** DRAW LINES AND SYMBOLS OF VARIOGRAM VALUES.
01360 IF (NEW.EQ.3) GO TO 40
01370 34 CALL LINE(DIS,V,I,1,1,NSYMB(NSY))
01380 NPLT = NPLT+1
01390 IF (NPLT.EQ.NPMAX) GO TO 210
01400 GO TO 6
01410 C***READ IN VARIOGRAM MODEL. ASSUME A SPHERICAL VARIOGRAM MODEL.
01420 C CO = NUGGET VALUE
01430 C C = C VALUE
01440 C RANG = RANGE OF VARIOGRAM
01450 C****IF THERE IS AN ERROR IN INPUT DATA, SKIP PLOTTING THIS VARIOGRAM.
01460 40 READ(INPT,45)CO,C,RANG
01470 IF (EOF(INPT).NE.0) GO TO 99
01480 45 FORMAT(3F10.0)
01490 SILL = CO + C
01500 IF (RANG. LT.0.0. OR.CO.LT.0.0) GO TO 230
01510 IF (SILL.GT.TOP.OR.RANG.GT.EAST) GO TO 230
01520 NINC=1)*XLEN
01530 X = 0.0
01540 Y = (CO-BOT)/DIFX
01550 CALL PLOT(X,Y,3)
01560 DO 50 J = 1,NINC
01570 X = X+0.1
01580 H = X*DIFX
01590 IF (H.LT.RANG) GO TO 47
01600 GH = CO + C
01610 GOTO 48
01620 47 GH = CO + C*(1.5*H/RANG -.5*(H/RANG)**3)
01630 48 Y = (GH-BOT)/DIFX
01640 CALL PLOT(X,Y,2)
01650 50 CONTINUE
01660 NPLT = NPLT +1
01670 IF (NPLT.EQ.NPMAX) GO TO 210
01680 GO TO 6
01690 230 WRITE(IPR,231)
01700 231 FORMAT(//20X," ERROR IN VARIOGRAM MODEL SPECIFICATION. IGNORE P
01710 10TTING THE MODEL"//)
01720 GO TO 6
01730 210 WRITE(IPR,211)
01740 211 FORMAT(//20X," TOTAL NO. OF GRAPHS EXCEEDS THE MAXIMUM ALLOWED
01750 1 HENCE, PLOTTING, IS BEING TERMINATED."//)
01760 GO TO 212
01770 220 WRITE(IPR,221)
01780 221 FORMAT(//20X," IMPROPER VALUE OF 'NEW' VARIABLE -- TERMINATE P
01790 1TTING"//)
01800 GO TO 212
01810 99 WRITE(IPR,200)
01820 200 FORMAT(1H1,"END OF FILE ENCOUNTERED. ASSUME NORMAL END OF JOB.
01830 212 CALL PLOT(0.0,0.,999)
01840 90 STOP
01850 END

```

APPENDIX C

KRIGING PROGRAM "UKRIG" USER'S GUIDE

SECTION I: PROGRAM IDENTIFICATION

- 1.1 PROGRAM TITLE: Kriging Program for Bedded Deposit
- 1.2 PROGRAM CODE NAME: UKRIG
- 1.3 PROGRAM WRITER: Harvey P. Knudsen
- PROGRAM DOCUMENTATION: Young C. Kim
- 1.4 ORGANIZATION: Department of Mining and Geological Engineering
University of Arizona
Tucson, Arizona
- 1.5 DATE: November 3, 1976
- 1.6 UPDATES: None. Version: 1.
- 1.7 SOURCE LANGUAGE: CDC 6400 Computer Systems - FORTRAN Extended -
Version IV.
- 1.8 ABSTRACT:

This is a kriging program designed for bedded tabular deposits, such as roll-front type uranium deposits. Both the thickness and grade-thickness product are estimated by kriging, using user specified number of surrounding drill hole assay values. The variogram model used for kriging is assumed to be a spherical variogram.

Printed output from the program consists of 1) parameter values used for the run, and 2) the kriged block values and associated kriging variances. The program also generates a data file containing the kriging results similar to those that are printed.

SECTION II: ENGINEERING DOCUMENTATION

2.1 NARRATIVE DESCRIPTION:

Program UKRIG is a kriging program written in FORTRAN IV. It is designed especially for bedded tabular deposits such as roll-front type uranium deposits. For each user specified block, the thickness of ore and the grade thickness product are estimated by kriging using a specified number of drill holes near the block center.

The variogram model used for kriging is assumed to be a spherical variogram. It is also assumed that any anisotropy present is of the simple geometric type. Hence only the range (a), nugget effect (C_0), C value (c), horizontal anisotropy factor (AFH), vertical anisotropy factor (AFV) and the anisotropy angle (ANG) are read into the program.

The horizontal anisotropy factor is defined as the maximum range of influence divided by the minimum range of influence in the horizontal plane. The anisotropy angle (ANG) is measured in degrees from E-W direction to the major axis (i.e., maximum range of influence). The angle is positive if it is measured in counterclock direction. It is negative, otherwise. The vertical anisotropy factor is the maximum range of influence in the horizontal plane divided by the vertical range of influence.

If three-dimensional instead of two-dimensional kriging is desired, it can be accomplished by specifying a nonzero value for the vertical anisotropy factor. If this factor is zero, then two-dimensional kriging is performed.

The program is modularized so that the basic mechanics of the kriging computations are performed in subroutine KRIG, which is readily adaptable

to any other driving (or main) program.

Printed output from the program consists of 1) parameter values used for the run, and 2) the kriged block values and associated kriging variances. The program also generates a data file containing the kriging results similar to those that are printed.

If debug option switch is on, additional information is printed during the program run. Similarly, the entire input data assays are printed if list option switch is on.

2.2 METHOD OF SOLUTION:

The theoretical basis of kriging procedure is not discussed here. Instead the reader is referred to other suitable references or to Chapter 4 of this report.

Since kriging is an extension procedure which provides an unbiased estimate having the least extension error, the objective of kriging is to find the appropriate weights to use in extending the assays of the surrounding holes to the block. Mathematically, the extension error is defined as follows:

$$\sigma_E^2 = \sigma_K^2 = \sigma_V^2 - 2 \sum_{i=1}^N \lambda_i \sigma_{VX_i} + \sum_{i=1}^N \sum_{j=1}^N \lambda_i \lambda_j \sigma_{X_i X_j} \quad (1)$$

Equation (1) above is a quadratic equation having N variables, i.e., $\lambda_1, \lambda_2, \dots, \lambda_n$. Here, the value N is equal to the number of data points to be used in kriging and λ_i 's are the variables of interest, i.e., the kriging weights.

The first term σ_E^2 in Equation (1) is the block variance* within the deposit. Naturally, there is a unique block variance for a given size block in a given deposit. Consequently, one needs to compute the block variance only once during kriging operation.

The second term in Equation (1) is the covariance between the current block and the i -th data point. This covariance must be computed for all N data points for the current block. The process must be repeated for all blocks to be computed.

Similarly, the third term in Equation (1) represents the covariance between two data points, i.e., x_i and x_j , and must be computed for all N data points and for all blocks.

In the program, a maximum number of data points to be used is specified by the user, and a search is made within a prespecified radius of search from the center of the current block. Consequently, the actual number, i.e., N can vary between block to block depending on whether or not enough data points have been found during this search. If less than two (2) data points are found for the current block, then the block is not kriged. Instead, the program proceeds to the next block.

In attempting to minimize the extension error given in Equation (1), there is however a constraint regarding the kriging weights, λ_i 's. The constraint is that the sum of these weights adds up to 1, as given in Equation (2) below.

$$\sum_{i=1}^N \lambda_i = 1 \quad (2)$$

*Assume one divides the entire deposit into blocks of a given size. Then the block variance is the variance of these block grades.

Consequently, the kriging problem now becomes the constrained optimization problem of N variables. This constrained optimization problem is solved using the method of Lagrange multiplier, as discussed in Chapter 4.3 of this report.

Next, the actual mechanics of computational steps used in the program is described briefly. After reading in the necessary run parameters such as the area definition, variogram values, anisotropy factors, the program first computes the block variance (σ_v^2) for the specified block size. This computation is accomplished in SUBROUTINE BLOCK and it is performed only once during the run.

Afterwards, kriging of each block commences row by row, starting from the N-W corner of the area. To krig a block, the user specified number of holes located nearest to the current block is first determined, by searching within some maximum search radius.

The obtained holes are then used to krig the block by calling SUBROUTINE KRIG, which in turn calls other appropriate subroutines for computing the covariance between a hole and the block (σ_{vx_i}) as well as covariance between two samples ($\sigma_{x_i x_j}$).

SUBROUTINE COVAR determines the covariance between a hole and the block, whereas the covariance between two samples is directly obtained by the difference in the variogram values from SUBROUTINE SUBGAM.

Finally, a set of simultaneous linear equations are solved for the kriging weights to be used for the current block in SUBROUTINE FINDT. The method of solution used here is the Gaussian Elimination Technique.

In computing the block variance as well as the covariance between a point and the block, a total of 16, equally spaced points within the block is

used to approximate the total area of integration.

The method of solution is summarized in Figure 1.

2.3 PROGRAM CAPABILITIES:

Program UKRIG is currently dimensioned to handle up to 500 assay data points. If there are more data points, only the first 500 data points are used for kriging. A message is printed out and the remaining data points are ignored.

These assay data points are read under the user specified format. The following information for each assay data is assumed to be read into the program during execution, under the variable names given below. A data point is assumed to be given as one record or one data card.

- a) Hole identification under HOLID(I).
- b) X, Y, and Z coordinates of data point location under XE(I), YN(I), ZL(I).
- c) Three values, such as grade, thickness and G-T, under GD(I,J), J = 1, 2, 3.

In kriging a block, a maximum of fifty (50) surrounding assay data points can be used. However, between 10 to 20 points are sufficient for all practical purposes.

The program assumes that the variogram model used for kriging is a spherical variogram. However, any other variogram model can be used by modifying one subroutine called SUBGAM. The same variable arrays used to store the coefficients of a spherical variogram model can be used to store other variogram model coefficients.

Since a block is kriged one at a time and the results are printed as well as stored on a data file, there is no limit as to the number of blocks that can be kriged at one run. The program krigs both thickness and grade-thickness.

Figure 1

METHOD OF SOLUTION SUMMARY

2.4 DATA INPUTS:

The following describes the data inputs:

INPUT CARDS

	<u>COLUMN</u>	<u>VARIABLE NAME</u>	<u>DESCRIPTION</u>
CARD ONE.	DEFINITION OF AREA TO BE KRIGED		
	1-5	NBC	STARTING COLUMN
	6-10	NEC	ENDING COLUMN
	11-15	NBR	STARTING ROW
	16-20	NER	ENDING ROW
	21-30	YMAX	MAXIMUM NORTHING (ROW 1)
	31-40	XMIN	MINIMUM EASTING (COL 1)
	41-50	WIDY	BLOCK WIDTH IN NORTH DIRECTION
	51-60	WIDX	BLOCK WIDTH IN EAST DIRECTION
CARD TWO.	HOLE ACCEPTANCE AND DEBUGGING PARAMETERS		
	1-5	RMAX	MAXIMUM DISTANCE FROM BLOCK FOR HOLE TO BE INCLUDED IN KRIGING
	6-10	NK	MAXIMUM NUMBER OF HOLES TO BE USED FOR EACH BLOCK. MUST BE LESS THAN OR EQUAL TO 50
	11	IDBUG	DEBUG (1=YES, 0=NO)
	12	ILIST	LIST DRILL HOLES (1=YES, 0=NO)
CARD THREE.	VARIOGRAM PARAMETERS FOR GRADE X THICKNESS.		
	1-10	CO	NUGGET VALUE
	11-20	C	C VALUE OF VARIOGRAM
	21-30	RANG	RANGE OF VARIOGRAM
	31-40	ANG	ANGLE OF ANISOTROPY IN DEGREES FROM X-AXIS TO THE MAJOR AXIS. THE ANGLE IS PLUS IF COUNTERCLOCKWISE, MINUS IF CLOCKWISE.
	41-50	AFH	HORIZONTAL ANISOTROPY FACTOR (MAJOR AXIS OVER MINOR AXIS)
	51-50	AFV	VERTICAL ANISOTROPY FACTOR (SET EQUAL TO 0 FOR BEDDED DEPOSITS, I.E. FOR 2-DIMENSIONAL KRIGING)
CARD FOUR.	VARIOGRAM PARAMETERS FOR THICKNESS. USE SAME FORMAT AS CARD THREE		

DATA CARDS

CARD ONE. VARIABLE FORMAT CARD TO READ IN SUBSEQUENT ASSAY DATA
CARD TWO & ON:

HOLID	HOLE IDENTIFICATION NAME
XE	X-COORDINATE OF THE DATA POINT. (E-W)
YN	Y-COORDINATE OF THE DATA POINT. (N-S)
XL	Z-COORDINATE OR ELEVATION OF DATA POINT.
GD	ARRAY CONTAINING THREE ASSAY VALUES.

THE ABOVE INFORMATION REGARDING A DATA POINT MUST BE GIVEN AS ONE RECORD OR ONE DATA CARD.

A sample 80-80 listing of input data is given in Figure 2.

2.5 PROGRAM OPTIONS:

There are three program options. Extreme care in the use of these options is strongly emphasized because of the voluminous printed output and potential mistake in 3-D kriging.

1. Debug Option - If the debug option switch (IDBUG) is on during the program execution, the following additional information is printed for the prespecified number of blocks (MAXPR).
 - a) augmented matrix value elements.
 - b) the holes used to krig each block.
2. List Option - If this switch (ILIST) is on, the entire drill hole assay data are listed prior to performing kriging computation.
3. Three (3) Dimensional Kriging - The program is capable of performing 3-D kriging, although most bedded deposits are more amenable to 2-D kriging. For kriging in 3-dimensions, the vertical anisotropy factor (AFV) must have some positive value. In addition, the value of B(3) array must be initialized to some datum plane (see Line 1460 in the source program listing given at the end of this user's guide).

2.6 OUTPUT:

Figure 3 shows a typical printed output. This output is obtained after setting both the debug and list option switches to zero, i.e., off position.

Figure 4 shows an 80-80 listing of the created data file which contains the kriged results of each block.

1	3	1	3	71800	10500	100	100
200	610						
	27		16	350	0	1	0
	30		44	350	0	1	0
(A5,5X,2F10.0,F5.0,10X3F10.0)							
025390		11089		711165029.		0.076	2.000
025425		11413		711205069.		0.071	5.500
030390		11069		711685036.		0.070	2.000
030415		11307		711725059.		0.062	2.000
030420		11358		711675059.		0.078	12.500
033420		11360		711855055.		0.106	17.000
035345		10614		712305016.		0.090	10.000
035365		10807		712285036.		0.060	3.500
035375		10919		712235025.		0.064	6.000
035385		11018		712215027.		0.072	15.000
035415		11312		712125056.		0.201	20.000
036411		11270		712205045.		0.101	20.500
040330		10459		712655019.		0.082	4.000
040335		10508		712805018.		0.113	12.000
040340		10559		712635021.		0.096	16.000
040360		10757		712625030.		0.127	23.500
040370		10855		712615034.		0.117	26.000
040371		10870		712615032.		0.185	30.000
040380		10968		712615047.		0.063	10.000
040400		11165		712565068.		0.064	2.000
040408		11249		712535056.		0.109	12.500
040410		11264		712525054.		0.100	9.500
040420		11365		712645067.		0.088	10.000

Figure 2

SAMPLE 80-80 LISTING OF INPUT DATA

DESCRIPTION OF AREA TO BE KRIGED
 NORTHING OF ROW 1 71800.
 EASTING OF COL 1 10500.
 KRIGED AREA IS BOUNDED BY
 COLUMNS 1 AND 3
 ROWS 1 AND 3

BLOCK DIMENSIONS ARE (100.X 100.)

MAX RANGE OF INFLUENCE OF HOLE 250.
 MAX NUMBER OF HOLES USED TO KRIG A BLOCK 6

VARIOGRAM PARAMETERS

CO 27.0000
 C 16.0000
 RANGE 350.
 ANGLE OF ROTATION 0.
 HOR. ANISOTROPY 1.
 VERT. ANISOTROPY 0.

VARIOGRAM PARAMETERS

CO 30.0000
 C 44.0000
 RANGE 350.
 ANGLE OF ROTATION 0.
 HOR. ANISOTROPY 1.
 VERT. ANISOTROPY 0.

BLOCK VARIANCES 14.28563 36.51985

OUTPUT OF KRIGED RESULTS

COLUMN	ROW	X-COORD.	Y-COORD.	ELEVATION	G-T	THICK	G-T VAR.	THICK VAR.
1	1	10650.	71750.	5018.	.11	13.50	7.9979	11.2191
2	1	10750.	71750.	5026.	.11	12.32	7.6536	10.9735
3	1	10850.	71750.	5017.	.14	18.50	7.4257	9.9048
1	2	10650.	71650.	5022.	.12	15.39	8.0389	11.4355
2	2	10750.	71650.	5018.	.10	13.10	7.9807	11.3158
3	2	10850.	71650.	5019.	.13	18.21	8.2087	11.5521
1	3	10650.	71550.	5027.	.12	11.04	8.1134	11.5269
2	3	10750.	71550.	5030.	.09	9.52	7.7809	10.7565
3	3	10850.	71550.	5024.	.08	10.90	7.7836	10.7082

Figure 3

10650.	71750.5018.	.1121E+00	.1350E+02	.8298E-02	.7998E+01	.1122E+02
10750.	71750.5026.	.1078E+00	.1232E+02	.8748E-02	.7854E+01	.1097E+02
10850.	71750.5017.	.1377E+00	.1850E+02	.7445E-02	.7426E+01	.9905E+01
10650.	71650.5022.	.1162E+00	.1539E+02	.7553E-02	.8039E+01	.1144E+02
10750.	71650.5018.	.1040E+00	.1310E+02	.7941E-02	.7981E+01	.1132E+02
10850.	71650.5019.	.1301E+00	.1821E+02	.7141E-02	.8209E+01	.1159E+02
10650.	71550.5027.	.1195E+00	.1104E+02	.1082E-01	.8113E+01	.1153E+02
10750.	71550.5030.	.9341E-01	.9523E+01	.9809E-02	.7781E+01	.1076E+02
10850.	71550.5024.	.8169E-01	.1090E+02	.7496E-02	.7784E+01	.1071E+02

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Figure 4

A 80-80 LISTING OF CREATED DATA FILE

SECTION III: SYSTEM DOCUMENTATION

3.1 COMPUTER EQUIPMENT:

The UKRIG program was run on a CDC 6400 computer using 60K-core central memory.

3.2 PERIPHERAL EQUIPMENT:

The following peripheral equipment was used in the execution of the UKRIG program:

CDC 405 card reader
CDC 512 line printer

3.3 SOURCE PROGRAM:

A source program listing is given at the end of this user's guide.

3.4 VARIABLES AND SUBROUTINES:

A listing of all major variables is given in Figure 5.

Program UKRIG consists of the following subroutines:

1. UKRIG: Main program which reads input data, controls the kriging operation, and outputs the results.
2. KRIG: Performs the basic mechanics of kriging computations by sequentially calling other supporting subroutines.
3. BLOCK: Computes the block variance for a given size block in the deposit.
4. COVAR: Computes the covariance between a hole and a block.
5. SUBGAM: Computes the variogram value for a given distance.
6. FINDT: Solves a set of simultaneous linear equations using the Gaussian elimination technique.
7. DSORT: Sorts the selected drill hole assays into increasing order of distance.

<u>VARIABLE NAME</u>	<u>DESCRIPTION</u>
A	= A SYMMETRIC MATRIX.
AFH	= ANISOTROPY FACTOR (HORIZONTAL)
AFV	= ANISOTROPY FACTOR (VERTICAL)
ANG	= ANGLE OF ANISOTROPY
B	= TEMPORARY ARRAY FOR STORING THE DISTANCE COMPONENTS
C	= C VALUE OF VARIOGRAM
CO	= NUGGET VALUE
GA	= ARRAY WHICH CONTAINS THE COVARIANCE BETWEEN THE BLOCK = THE CONSTANT TERMS CONSISTING OF THE COVARIANCES BETWEEN THE BLOCK AND SURROUNDING DATA POINTS
GD	= ARRAY CONTAINING THREE ASSAY VALUES
GH	= VARIOGRAM VALUE. OBTAINED FROM SUBROUTINE SUBGAM.
GMOY	= AVERAGE VALUE OF VARIOGRAM IN THE BLOCK.
HOLID	= HOLE IDENTIFICATION NAME.
IDBUG	= DEBUG (1 = YES, 0 = NO).
II	= ACTUAL NO. OF DATA POINTS USED TO KRIG THE CURRENT BLOCK.
ILIST	= LIST DRILL HOLES (1 = YES, 0 = NO).
M	= 1 MEANS G-T PRODUCT.
M	= 2 MEANS THICKNESS ONLY.
MAXH	= MAX. NO. OF DATA POINTS CURRENTLY DIMENSIONED.
MAXN	= MAX. NO. OF SURROUNDING POINTS TO BE USED FOR KRIGING
NBC	= STARTING COLUMN.
NBR	= STARTING ROW.
NEC	= ENDING COLUMN.
NER	= ENDING ROW.
NK	= MAXIMUM NUMBER OF HOLES TO BE USED FOR EACH BLOCK.
R	= ARRAY CONTAINING THE KRIGING WEIGHTS. = SOLUTION IN TERMS OF KRIGING WEIGHTS.
RANG	= RANGE OF VARIOGRAM.
RMAX	= MAXIMUM DISTANCE FROM BLOCK FOR HOLE TO BE INCLUDED IN KRIGING.
WIDX	= BLOCK WIDTH IN EAST DIRECTION
WIDY	= BLOCK WIDTH IN NORTH DIRECTION.
XE	= X-COORDINATE OF THE DATA POINT. (E-W).
XMIN	= MINIMUM EASTING (COL 1).
YMAX	= MAXIMUM NORTHING (ROW 1).
YN	= Y-COORDINATE OF THE DATA POINT. (N-S).
ZL	= Z-COORDINATE OR ELEVATION OF DATA POINT.

Figure 5

LISTING OF ALL MAJOR VARIABLES

3.5 DATA STRUCTURES:

A data file containing the kriged results is generated on system logical unit device #5. Structure of this data file is shown in Figure 4. Each record contains X and Y coordinates of the block, the elevation of the first data point, computed grade-thickness, thickness and grade of the block, and kriging variances.

3.6 STORAGE REQUIREMENTS:

As presently structured, program UKRIG requires approximately 60K storage on the CDC 6400.

3.7 MAINTENANCE AND UPDATES:

None to date.

SECTION IV: OPERATING DOCUMENTATION

4.1 OPERATING INSTRUCTIONS:

Program UKRIG is operated under the CDC 6400, SCOPE 3.4, operating system.

4.2 OPERATING MESSAGES:

Error messages produced by the program are self-explanatory. In addition, normal system messages are produced.

4.3 CONTROL CARDS:

The program may be executed with standard SCOPE 3.4 control cards. As run on the University of Arizona CDC 6400, the makeup of the input deck is as follows:

```
Job Card
FTN.
LGO.
7/8/9
<SOURCE PROGRAM>
7/8/9
<INPUT DATA>
6/7/8/9
```

4.4 ERROR RECOVERY:

If certain errors occur, the program will make assumptions and continue execution of the program. A warning message will be printed. If a fatal error occurs in execution, the program must be restarted.

4.5 RUN TIME:

Run time depends on number of holes selected for kriging a block and the number of blocks to be kriged. Typical run time is about 20 to 50 seconds for kriging a medium size open-pit bench area.

UKRIG PROGRAM LISTING


```

00010 PROGRAM UKRIG (INPUT,OUTPUT,PUNCH,TAPE1=INPUT,TAPE3=OUTPUT,
00020 1TAPE5)
00030 COMMON /SYS/IPT,IOUT,IPR,IDBUG,IDPR,MAXPR
00040 COMMON /BLK/WX2,WY2,WX4,WY4,WX8,WY8
00050 COMMON /GAM/CO(2),C(2),RANG(2),CST(2),SIT(2),APH(2),APV(2)
00060 COMMON /KRG/IHOL(51),GA(51),R(51)
00070 COMMON /DRILL/HOLID(800),YN(800),XE(800),ZL(800),GD(800,3)
00080 DIMENSION B(3),IFRMT(20),DIST(51),BLKVAR(2),VARKG(2),ANG(2)
00090 DIMENSION GR(2)
00100 DATA MAXH,MAXK,MAXPR,IDPR/800,50,5,0/
00110 DATA IPT,IOUT,IPR/1,5,3/
00120 C*****
00130 C THIS PROGRAM WAS WRITTEN BY HARVEY P. KNUDSEN.
00140 C DEPARTMENT OF MINING & GEOLOGICAL ENGINEERING
00150 C UNIVERSITY OF ARIZONA, TUCSON, ARIZONA.
00160 C NOVEMBER, 1975.
00170 C PROGRAM UKRIG IS A KRIGING PROGRAM DESIGNED FOR BEDDED
00180 C TABULAR DEPOSITS. BOTH THE THICKNESS AND GRADE-THICKNESS
00190 C PRODUCT ARE KRIGED.
00200 C THIS PROGRAM ASSUMES THAT THE VARIOGRAM MODEL FOR KRIGING
00210 C IS A SPHERICAL VARIOGRAM.
00220 C PROGRAM UKRIG IS PRESENTLY DIMENSIONED TO ACCEPT UP TO
00230 C 500 ASSAY VALUES.
00240 C
00250 C DESCRIPTION OF INPUT CARDS.
00260 C CARD ONE. DEFINITION OF AREA TO BE KRIGED.
00270 C COL 1-5 NBC STARTING COLUMN
00280 C 6-10 NEC ENDING COLUMN
00290 C 11-15 NBR STARTING ROW
00300 C 16-20 NER ENDING ROW
00310 C 21-30 YMAX MAXIMUM NORTHING (ROW 1)
00320 C 31-40 XMIN MINIMUM EASTING (COL 1)
00330 C 41-50 WIDY BLOCK WIDTH IN NORTH DIRECTION
00340 C 51-60 WIDX BLOCK WIDTH IN EAST DIRECTION
00350 C CARD TWO. HOLE ACCEPTANCE AND DEBUGGING PARAMETERS
00360 C COL 1-5 RMAX MAXIMUM DISTANCE FROM BLOCK FOR
00370 C HOLE TO BE INCLUDED IN KRIGING
00380 C 6-10 NK MAXIMUM NUMBER OF HOLES TO BE
00390 C USED FOR EACH BLOCK. MUST BE LESS THAN
00400 C OR EQUAL TO 50.
00410 C 11 IDBUG DEBUG (1=YES, 0=NO)
00420 C 12 ILIST LIST DRILL HOLES (1=YES, 0=NO)
00430 C CARD THREE. VARIOGRAM PARAMETERS FOR GRADEXTHICKNESS.
00440 C COL 1-10 CO NUGGET VALUE
00450 C 11-20 C C VALUE OF VARIOGRAM
00460 C 21-30 RANG RANGE OF VARIOGRAM
00470 C 31-40 ANG ANGLE OF ANISOTROPY IN DEGREES FROM
00480 C X-AXIS TO THE MAJOR AXIS. THE ANGLE IS
00490 C PLUS IF COUNTERCLOCKWISE, MINUS IF CLOCK
00500 C -WISE.
00510 C 41-41 APH HORIZONTAL ANISOTROPY FACTOR (MAJOR AXIS
00520 C OVER MINOR AXIS)
00530 C 51-50 APV VERTICAL ANISOTROPY FACTOR
00540 C (SET EQUAL TO 0 FOR BEDDED DEPOSITS, I.E.
00550 C FOR 2-DIMENSIONAL KRIGING)
00560 C CARD FOUR. VARIOGRAM PARAMETERS FOR THICKNESS
00570 C USE SAME FORMAT AS CARD THREE
00580 C
00590 C DATA CARDS.
00600 C CARD ONE. VARIABLE FORMAT CARD TO READ IN SUBSEQUENT ASSAY DATA
00610 C CARD TWO & ON: HOLID HOLE IDENTIFICATION NAME
00620 C XE X-COORDINATE OF THE DATA POINT. (E-W)
00630 C

```

```

00640      C      YN      Y-COORDINATE OF THE DATA POINT. (N-S)
00650      C      ZL      Z-COORDINATE OR ELEVATION OF DATA POINT.
00660      C      GD      ARRAY CONTAINING THREE ASSAY VALUES.
00670      C      THE ABOVE INFORMATION REGARDING A DATA POINT MUST BE
00680      C      GIVEN AS ONE RECORD OR ONE DATA CARD.
00690      C
00700      C*****
00710      C      MAXH = MAX. NO. OF DATA POINTS CURRENTLY DIMENSIONED.
00720      C      MAXN = MAX. NO. OF SURROUNDING POINTS TO BE USED FOR KRIGING
00730      C      MAXPR = MAX. NO. OF DEBUG PRINTOUTS ALLOWED.
00740      C      IDPR = TOTAL DEBUG PRINTOUT COUNTER.
00750      C      B( ) = TEMPORARY ARRAY FOR STORING THE DISTANCE COMPONENTS.
00760      C      II = ACTUAL NO. OF DATA POINTS USED TO KRIG THE CURRENT BLOCK
00770      C      GA = ARRAY WHICH CONTAINS THE COVARIANCE BETWEEN THE BLOCK
00780      C      AND THE SURROUNDING DATA POINTS. OBTAINED DURING KRIGING
00790      C      R = ARRAY CONTAINING THE KRIGING WEIGHTS.
00800      C*****
00810      C      READ FIRST CARD.
00820      C      READ(IPT,1) NBC, NEC, NBR, NER, YMAX, XMIN, WIDY, WIDX
00830      10      FORMAT(4F15.4,F10.0)
00840      C      READ(IPT,12) RMAX, NK, IDBUG, ILIST
00850      12      FORMAT(F5.0, I5, 3I1)
00860      C      IF(NK.GT.MAXK) NK=MAXK
00870      C      WRITE(IPR,15) YMAX, XMIN, NBC, NEC, NBR, NER
00880      15      FORMAT(1H1, "DESCRIPTION OF AREA TO BE KRIGED"/, 5X
00890      1" NORTHING OF ROW 1", T25, F10.0, /, 5X, "EASTING OF COL 1"
00900      2 T25, F10.0 / 5X "KRIGED AREA IS BOUNDED BY"/
00910      35X "COLUMNS", T25, I5, " AND" I5, /, 5X, "ROWS", T25, I5,
00920      4" AND", I5)
00930      C      WRITE(IPR,16) WIDY, WIDX
00940      16      FORMAT(1H0, "BLOCK DIMENSIONS ARE (" , F5.0, "X", F5.0, ")")
00950      C      WRITE(IPR,17) RMAX, NK
00960      17      FORMAT(1H0, "MAX RANGE OF INFLUENCE OF HOLE "T45, F6.0, /,
00970      1" MAX NUMBER OF HOLES USED TO KRIG A BLOCK", T45, I6)
00980      C**** READ IN THIRD AND FOURTH CONTROL CARDS- VARIOGRAM PARAMETERS.
00990      C      DO 26 I=1, 2
01000      C      READ(IPT,2) CO(I), C(I), RANG(I), ANG(I), AFH(I), AFV(I)
01010      20      FORMAT(6F10.0)
01020      C      WRITE(IPR,25) CO(I), C(I), RANG(I), ANG(I), AFH(I), AFV(I)
01030      25      FORMAT(1H0, "VARIOGRAM PARAMETERS", /, 5X "CO", T29, F10.4, /,
01040      15X "C", T29, F10.4, /, 5X "RANGE", T25, F10.0, /, 5X "ANGLE OF ROTATION",
01050      2 T25, F10.0, /, 5X "HOR. ANISOTROPY ", T25, F10.0, /,
01060      35X "VERT. ANISOTROPY ", T25, F10.0)
01070      C      RA = ANG(I) * 0.17453292
01080      C      CST(I) = COS(RA)
01090      C      SIT(I) = SIN(RA)
01100      26      CONTINUE
01110      C      WX2 = WIDX/2.0
01120      C      WY2 = WIDY/2.0
01130      C      WX4 = WIDX/4.0
01140      C      WY4 = WIDY/4.0
01150      C      WX8 = WIDX/8.0
01160      C      WY8 = WIDY/8.0
01170      C**** READ IN VARIABLE FORMAT CARD FIRST
01180      C      READ(IPT,30) IFRMT
01190      30      FORMAT(2I4)
01200      C      I = 0
01210      C**** READ IN DRILL HOLE DATA
01220      40      I=I+1
01230      C      READ(IPT, IFRMT) HOLID(I), XE(I), YN(I), ZL(I), (GD(I,J), J=1,3)
01240      C      IF(EOP(IPT).GT.0) GO TO 50
01250      C      IF(I.LT.MAXH) GO TO 40
01260      C      WRITE(IPR,45)

```

```

270 45 FORMAT(//10X,"***THERE ARE MORE DATA POINTS THAN ALLOWED --- IGN
01280 13RE THE REMAINING DATA POINTS"/)
01290 50 NHOL= I - 1
01300 IF(ILIST.EQ.0)GO TO 55
01310 WRITE(IPR,51)
01320 51 FORMAT(1H0," HOLE"T16,"NORTH"T27,"EAST"T37,"ELEV"T47,"GRADE"
01330 1T57,"THICK"T70,"GT")
01340 DO 54 I=1,NHOL
01350 WRITE(IPR,52)HOLID(I),YN(I),XE(I),ZL(I),(GD(I,J),J=1,3)
01360 52 FORMAT(1H,A5,5X,3F10.0,3F10.2)
01370 54 CONTINUE
01380 C****CALCULATE THE VARIANCE OF A BLOCK IN THE DEPOSIT
01390 55 CALL BLOCK(BLKVAR(1),1)
01400 CALL BLOCK(BLKVAR(2),2)
01410 WRITE(IPR,56)BLKVAR
01420 56 FORMAT(1H0,"BLOCK VARIANCES",T25,F12.5,T40,F12.5)
01430 C****START KRIGING THE BLOCKS
01440 YM = YMAX +.5*WIDY
01450 XM = XMIN -.5*WIDX
01460 C*****NOTE** B(3) VALUE IS NOT USED IN 2-D KRIGING. HENCE,SET IT TO 0.
01470 B(3)=0.0
01480 WRITE(IPR,62)
01490 62 FORMAT(1H1,10X,"***OUTPUT OF KRIGED RESULTS***"//3X,"COLUMN Row
01500 1" T20,"X-COORD.",T30,"Y-COORD.",T40,"ELEVATION G-T",T60,
01510 2"THICK",T70,"G-T VAR.",T85,"THICK VAR."/)
01520 DO 80 J = NBC,NEC
01530 B(1) = XM + J*WIDX
01540 DO 80 K = NBR,NER
01550 B(2) = YM - K*WIDY
01560 C*****FIND NEAREST NK HOLES FOR KRIGING
01570 IDPR= IDPR + 1
01580 R2=RMAX**2
01590 II=0
01600 DO 60 L = 1,NHOL
01610 DY=B(2)-YN(L)
01620 DX=B(1)-XE(L)
01630 DZ=B(3)-ZL(L)
01640 DIS=(DX*CST(2)+DY*SIT(2))**2+(AFH(2)*(DY*CST(2)-DX*SIT(2))**2
01650 1+(AFV(2)*DZ)**2
01660 IF(DIS.GT.R2) GO TO 60
01670 IF(II.GE.NK.AND.DIS.GT.DIST(II))GO TO 60
01680 II = II+1
01690 DIST(II)=DIS
01700 IHOL(II) = L
01710 if(ii .eq. 1) go to 60
01720 C****ARRANGE HOLES IN INCREASING ORDER OF DISTANCE BY CALLING DSORT.
01730 CALL DSORT(DIST,IHOL,II)
01740 IF(II.GT.NK)II=NK
01750 60 CONTINUE
01760 IF(II.LE.2) GO TO 80
01770 C*****KRIG THE BLOCK IF THERE ARE MORE THAN 2 DATA POINTS.
01780 DO 76 M=1,2
01790 CALL KRIG(B,II,M,IFERR)
01800 IF(IFERR.EQ.0) GO TO 61
01810 WRITE(IPR,73)J,K
01820 73 FORMAT(" ERROR IN MATRIX SOLUTION BLOCK ",2I5)
01830 DO 900 III =1,II
01840 LL=IHOL(III)
01850 WRITE(IPR,75)HOLID(LL),YN(LL),XE(LL)
01860 900 CONTINUE
01870 GO TO 80
01880 C****CALCULATE THE KRIGED VALUE AND THE KRIGING VARIANCE
01890 61 AG = 0.

```

```

01900 GR(M)=3.
01910 IF(IDBUG.GE.1.AND.IDPR.LT.MAXPR)WRITE(IPR,65)
01920 65 FORMAT(1H0," HOLE" T11,"NORTH" T22,"EAST" T32,"ELEV" T42,"GRADE"
01930 1T52,"WEIGHT")
01940 DO 70 L=1,II
01950 LL=IHOL(L)
01960 AG=AG+GA(L)*R(L)
01970 GR(M)=GR(M)+GD(LL,M)*R(L)
01980 IF(IDBUG.GE.1.AND.IDPR.LT.MAXPR)WRITE(IPR,75)HOLID(LL),YN(LL),
01990 1XE(LL),ZL(LL),GD(LL,M),R(L)
02000 75 FORMAT(1H,A5,3F10.0,2F10.2)
02010 70 CONTINUE
02020 VARKG(M)=BLKVAR(M)-AG-R(II+1)
02030 76 CONTINUE
02040 C****NOTE***FOR 2-D KRIGING, THE ELEVATION OF ORE IN THE NEAREST HOLE
02050 C IS ASSIGNED TO THE BLOCK.
02060 KK=IHOL(1)
02070 ELEV=ZL(KK)
02080 WRITE(IPR,81) J,K,B(1),B(2),ELEV,GR,VARKG
02090 81 FORMAT(4X,2I5,3X,3F10.0,2F8.2,2F15.4)
02100 GRADE=GR(1)/GR(2)
02110 WRITE(IOUT,78)B(1),B(2),ELEV,GR,GRADE,VARKG
02120 78 FORMAT(2F10.0,2F5.0,5E11.4)
02130 80 CONTINUE
02140 STOP
02150 END
02160 SUBROUTINE KRIG(B,NS,M,IFERR)
02170 C*****
02180 C THIS SUBROUTINE PERFORMS THE BASIC MECHANICS OF KRIGING
02190 C BY SEQUENTIALLY CALLING OTHER SUPPORTING SUBROUTINES.
02200 C*****
02210 COMMON /SYS/IPT,IOUT,IPR,IDBUG,IDPR,MAXPR
02220 COMMON /KRG/IHOL(51),GA(51),R(51)
02230 COMMON /DRILL/HOLID(800),YN(800),XE(800),ZL(800),GD(800,3)
02240 COMMON /GAM/CO(2),C(2),RANG(2),CST(2),SIT(2),AFH(2),AFV(2)
02250 DIMENSION P1(3),P2(3),B(3),A(51,52)
02260 C***CALCULATE COVARIANCES AND SET-UP MATRIX FOR SIMULTANEOUS EQUATION:
02270 C A = A SYMETRIC MATRIX
02280 C R = SOLUTION IN TERMS OF KRIGING WEIGHTS.
02290 C GA = THE CONSTANT TERMS CONSISTING OF THE COVARIANCES BETWEEN
02300 C THE BLOCK AND SURROUNDING DATA POINTS.
02310 C GH = VARIOGRAM VALUE. OBTAINED FROM SUBROUTINE SUBGAM.
02320 C GMOY = AVERAGE VALUE OF VARIOGRAM IN THE BLOCK.
02330 DO 15 K=1,NS
02340 KK=IHOL(K)
02350 P1(1)=XE(KK)
02360 P1(2)=YN(KK)
02370 P1(3)=ZL(KK)
02380 K1=K+1
02390 C****COMPUTE COVARIANCE BETWEEN THE CURRENT HOLE AND OTHER HOLES.
02400 IF(K1.GT.NS)GO TO 11
02410 DO 10 L=K1,NS
02420 LL=IHOL(L)
02430 P2(1)=XE(LL)
02440 P2(2)=YN(LL)
02450 P2(3)=ZL(LL)
02460 CALL SUBGAM(P1,P2,GH,M)
02470 A(K,L)=CO(M)+C(M)-GH
02480 A(L,K)=CO(M)+C(M)-GH
02490 10 CONTINUE
02500 C****COMPUTE COVARIANCE BETWEEN THE CURRENT HOLE AND THE BLOCK.
02510 11 A(K,K)=CO(M)+C(M)
02520 CALL COVAR(B,P1,GMOY,M)

```

```

02530      NS1=NS+1
02540      NS2=NS+2
02550      A(K,NS2)=CO(M)+C(M)-GMOY
02560      GA(K) = A(K,NS2)
02570      A(K,NS1) = 1.0
02580      A(NS1,K) = 1.0
02590      15 CONTINUE
02600      A(NS1,NS2) = 1.0
02610      A(NS1,NS1) = 0.0
02620      C***SOLVE SIMULTANEOUS EQUATIONS
02630      IF (IDBUG. NE. 1. OR. IDPR. GE. MAXPR) GO TO 20
02640      WRITE(IPR,90) (GA(IX),IX=1,NS2)
02650      WRITE(IPR,18)
02660      18 FORMAT(1H0,"AUGMENTED MATRIX")
02670      DO 19 IX = 1,NS1
02680      WRITE(IPR,90) (A(IX,IY),IY=1,NS2)
02690      19 CONTINUE
02700      90 FORMAT(15F7.2)
02710      20 CALL FINDT(A,R,NS1,IFERR)
02720      C***R CONTAINS THE SOLUTION
02730      RETURN
02740      END
02750      SUBROUTINE BLOCK(BLKVAR,M)
02760      C*****
02770      C THIS SUBROUTINE COMPUTES THE BLOCK VARIANCE IN THE DEPOSIT
02780      C FOR A GIVEN SIZE BLOCK, USING 16 EQUALLY SPACED DATA POINT
02790      C APPROXIMATION FOR THE BLOCK AREA INTEGRATION.
02800      C*****
02810      COMMON /BLK/WX2,WY2,WX4,WY4,WX8,WY8
02820      COMMON /GAM/CO(2),C(2),RANG(2),CST(2),SIT(2),AFH(2),AFV(2)
02830      DIMENSION P1(3),P2(3)
02840      GMOY=.
02850      P1(1)=WX2
02860      P1(2)=WY2
02870      P1(3)=0.0
02880      P2(3)=0.0
02890      DO 10 I = 1,4
02900      P2(1) = WX2 + (I-3)*WX4 + WX8
02910      DO 10 J = 1,4
02920      P2(2) = WY2 + (J-3)*WY4 + WY8
02930      CALL COVAR(P1,P2,GH,M)
02940      GMOY = GMOY + GH
02950      10 CONTINUE
02960      BLKVAR = CO(M) + C(M) -GMOY/16.
02970      RETURN
02980      END
02990      SUBROUTINE COVAR(P1,P2,GH,M)
03000      C*****
03010      C THIS ROUTINE COMPUTES THE COVARIANCE BETWEEN A HOLE AND A BLOCK
03020      C USING 16 POINT APPROXIMATION.
03030      C*****
03040      COMMON /GAM/CO(2),C(2),RANG(2),CST(2),SIT(2),AFH(2),AFV(2)
03050      COMMON /BLK/WX2,WY2,WX4,WY4,WX8,WY8
03060      DIMENSION D(3),P1(3),P2(3)
03070      GMOY = 0.
03080      D(3) = P1(3)
03090      D(1) = P1(1) - 2.*WX4 - WX8
03100      DO 10 I = 1,4
03110      D(1) = D(1) + WX4
03120      D(2) = P1(2) - 2.*WY4 - WY8
03130      DO 10 J = 1,4
03140      D(2) = D(2) + WY4
03150      CALL SUBGAM(D,P2,GH,M)

```

```

03160      GMOY = GMOY+GH
03170      CONTINUE
03180 10      GH = GMOY/16
03190      RETURN
03200      END
03210      SUBROUTINE SUBGAM(P1,P2,GH,M)
03220 C*****
03230 C      THIS ROUTINE COMPUTES THE VARIOGRAM VALUE FOR A GIVEN DISTANCE
03240 C      USING A SPHERICAL VARIOGRAM MODEL. CONSEQUENTLY, THIS ROUTINE
03250 C      MUST BE MODIFIED, IF SOME OTHER TYPE MODEL IS TO BE USED.
03260 C*****
03270      COMMON /GAM/CO(2),C(2),RANG(2),CST(2),SIT(2),AFH(2),AFV(2)
03280      DIMENSION P1(3),P2(3)
03290 C      M = 1 MEANS G-T PRODUCT.
03300 C      M = 2 MEANS THICKNESS ONLY.
03310      XD=P1(1)-P2(1)
03320      YD=P1(2)-P2(2)
03330      ZD=P1(3)-P2(3)
03340      D=SQRT((XD*CST(M)+YD*SIT(M))**2+AFH(M)*AFH(M)*(YD*CST(M)-
03350 1XD*SIT(M))**2+AFV(M)*AFV(M)*ZD*ZD)
03360      IF(D.GT.RANG(M)) GO TO 20
03370      GH = CO(M)+C(M)*(1.5*D/RANG(M)-.5*(D/RANG(M))**3)
03380      IF(D.EQ.0.) GH=0.0
03390      RETURN
03400 20      GH = CO(M) +C(M)
03410      RETURN
03420      END
03430      SUBROUTINE FINDT(A,X,N,IFERR)
03440 C*****
03450 C      THIS ROUTINE SOLVES A SET OF SIMULTANEOUS LINEAR EQUATIONS
03460 C      USING THE GAUSSIAN ELIMINATION TECHNIQUE.
03470 C      THE FACT THAT THE DIAGONAL ELEMENT IS ALWAYS THE LARGEST
03480 C      ONE IN KRIGING IS TAKEN INTO ACCOUNT IN SOLUTION.
03490 C*****
03500      DIMENSION A(51,52),X(51)
03510      IFERR=0
03520      MP = N +1
03530      DO 10 I = 1,N
03540      IP = I +1
03550 C---PERFORM ELIMINATION. I IS PIVOT ROW A(I,I) IS PIVOT ELEMENT.
03560      DO 10 J = 1,N
03570      IF(I.EQ.J) GO TO 10
03580      IF(A(I,I).EQ.0) GO TO 30
03590      F = (-A(J,I))/A(I,I)
03600      DO 9 K = IP,MP
03610 9      A(J,K) = A(J,K) + F*A(I,K)
03620      CONTINUE
03630 10      DO 20 I = 1,N
03640 20      X(I) = A(I,N+1)/A(I,I)
03650      RETURN
03660 30      IFERR=1
03670      END
03680      SUBROUTINE DSORT(DS,IHOL,N)
03690 C*****
03700 C      THIS IS A SORTING ROUTINE IN INCREASING ORDER.
03710 C*****
03720      DIMENSION DS(1),IHOL(1)
03730      DNEW=DS(N)
03740      NEW=IHOL(N)
03750      N1=N-1
03760      DO 20 I=1,N1
03770      K=I
03780      IF(DNEW.LT.DS(I)) GO TO 30

```

03790	20	CONTINUE
03800		RETURN
03810	30	JK=0
03820		DO 40 I=K,N1
03830		J=N1-JK
03840		JK=JK + 1
03850		DS(J+1) = DS(J)
03860		IHOL(J+1) =IHOL(J)
03870	40	CONTINUE
03880		DS(K) =DNEW
03890		IHOL(K) =NEW
03900	50	RETURN
03910		END

APPENDIX D

MAP CREATION PROGRAM "MAP"

USER'S GUIDE

SECTION I: PROGRAM IDENTIFICATION

- 1.1 PROGRAM TITLE: Program To Create A Map Of Block Values
- 1.2 PROGRAM CODE NAME: MAP
- 1.3 PROGRAM WRITER: Harvey P. Knudsen
- 1.4 ORGANIZATION: Department of Mining and Geological Engineering
University of Arizona
Tucson, Arizona
- 1.5 DATE: November 20, 1976
- 1.6 UPDATES: None.
- 1.7 SOURCE LANGUAGE: CDC 6400 Computer Systems - FORTRAN Extended -
Version IV.
- 1.8 ABSTRACT:

This program creates a plan map of block values on the line printer. The block values can be the estimated grades by kriging or by other extension methods. Or, they can be any other attributes such as thickness, G-T, etc.

The size of a block on the printed map is 1/2 inch square, although the actual block need not be a square block. If the true size of the block is 100 ft. by 100 ft., the printed map will have a scale of 1 inch to 200 ft.

This program must be executed with a line printer capable of printing eight (8) lines per inch, in order to obtain 1/2 inch square blocks. Up to three integer values, each value not exceeding four (4) characters, may be printed within each block. Consequently, the block values must be properly scaled by the user, using the user specified scaling parameters.

Program MAP is currently dimensioned to handle a maximum of 100 blocks along the Y-direction (N-S) and an unlimited number of blocks along the X-direction.

SECTION II: ENGINEERING DOCUMENTATION

2.1 NARRATIVE DESCRIPTION:

Program MAP is a FORTRAN IV program written to create a plan map of block values on the line printer. This program is designed to display grades assigned to individual blocks by kriging or other interpolation methods. This program can also be used for any other attributes of the block such as thickness, grade-thickness, etc.

The size of a block on the printed map is 1/2 inch square if the line printer prints eight (8) lines per inch. Up to three integer values, each value not exceeding four (4) characters, may be printed within each block. Appropriate scaling factors must be specified to convert the actual block values to the values printed on the map.

If the line printer prints six (6) lines per inch instead of eight (8) and if the user desires for the printed block size of 1/2 inch square, the program requires a slight modification. In this instance, only two instead of three values can be printed within each block.

The blocks may be square or rectangular but if the blocks are square the resulting printer map will have no distortion. For the square blocks, the scale of the map is 1 inch to two times the true block dimension. Thus if the blocks are 100 feet by 100 feet the map will have a scale of 1 inch to 200 feet.

The program is presently dimensioned to output a map up to 100 blocks in the Y (North-South) dimension and an unlimited number of blocks in the X-direction. The horizontal direction on the line printer is the X-direction. The map is printed in strips of 20 blocks in the X-direction and up

to 100 blocks in the Y-direction. If the map is wider than 20 blocks the strips will have to be taped together to form the entire map.

The block values to be printed are read in one card per block, using the user specified format. The items to read in from each card are; 1) north coordinate, 2) east coordinate, and 3) up to three values of block attributes. Because of the manner in which the map is printed, the block values must be ordered in increasing east coordinates prior to being read into the computer.

2.2 METHOD OF SOLUTION:

The method of solution is simple. The area to be mapped is defined by the user. The coordinates of the Northwest corner of the first block in row one and column one, and the beginning and ending rows and columns bounding the area are specified by the user. Next, the block dimensions in both X and Y-directions are specified, regardless whether the block is a square or a rectangular block. Finally, the scaling parameters for the maximum of three (3) values to be printed as well as the format for reading in these values and their coordinates are specified by the user.

Figure 1 shows a sample printer map of block values created by the program MAP. As it can be seen from Figure 1, each block is 1/2 inch square if the printer prints eight lines per inch.

It should be noted in Figure 1 that the coordinates printed on the printer map are coordinates of the edges of the blocks, not the centers. The values printed within each block, however, are assumed to be either those at the center of the block or the average values for that block. In other words, the coordinates of the block associated with the input data

8500													9500
73700	+	+	+	+	+	+	+	+	+	+	+	+	+
*	*	40*	115*	186*	201*	*	*	*	*	*	*	*	*
*	*	4*	6*	9*	10*	*	*	*	*	*	*	*	*
*	*	9*	17*	20*	19*	*	*	*	*	*	*	*	*
+	+	+	+	+	+	+	+	+	+	+	+	+	+
*	*	45*	106*	189*	190*	134*	88*	73*	94*	131*	208*	*	*
*	*	5*	7*	9*	8*	8*	7*	8*	9*	10*	11*	*	*
*	*	9*	14*	21*	21*	15*	11*	8*	9*	12*	17*	*	*
+	+	+	+	+	+	+	+	+	+	+	+	+	+
*	*	74*	147*	201*	175*	87*	60*	84*	105*	147*	175*	*	*
*	*	6*	9*	11*	9*	7*	6*	8*	10*	11*	10*	*	*
*	*	11*	14*	17*	18*	12*	8*	9*	10*	12*	16*	*	*
+	+	+	+	+	+	+	+	+	+	+	+	+	+
*	*	71*	125*	202*	139*	70*	40*	63*	86*	129*	115*	*	*
*	*	7*	10*	13*	9*	5*	4*	6*	9*	11*	10*	*	*
*	*	9*	12*	15*	13*	12*	8*	9*	9*	11*	11*	*	*
+	+	+	+	+	+	+	+	+	+	+	+	+	+
*	*	61*	73*	160*	105*	37*	25*	23*	39*	62*	105*	*	*
*	*	7*	6*	10*	8*	4*	3*	3*	5*	7*	9*	*	*
*	*	8*	10*	15*	12*	8*	7*	6*	7*	8*	11*	*	*
+	+	+	+	+	+	+	+	+	+	+	+	+	+
*	*	25*	41*	76*	85*	37*	29*	23*	17*	27*	110*	*	*
*	*	3*	4*	8*	8*	4*	3*	2*	2*	3*	6*	*	*
*	*	6*	8*	8*	10*	8*	8*	8*	6*	6*	15*	*	*
+	+	+	+	+	+	+	+	+	+	+	+	+	+
*	*	41*	37*	64*	82*	34*	28*	24*	23*	31*	96*	*	*
*	*	3*	4*	7*	8*	4*	3*	2*	3*	3*	5*	*	*
*	*	12*	7*	8*	9*	7*	8*	8*	7*	8*	16*	*	*
+	+	+	+	+	+	+	+	+	+	+	+	+	+
*	*	59*	25*	25*	28*	21*	21*	17*	24*	28*	70*	*	*
*	*	3*	3*	4*	4*	3*	2*	2*	2*	3*	4*	*	*
*	*	18*	6*	5*	6*	7*	8*	7*	8*	7*	14*	*	*
+	+	+	+	+	+	+	+	+	+	+	+	+	+
*	*	19*	23*	20*	17*	17*	15*	11*	19*	27*	58*	*	*
*	*	2*	2*	2*	2*	2*	1*	2*	2*	2*	5*	*	*
*	*	9*	11*	7*	6*	7*	7*	5*	8*	9*	11*	*	*
+	+	+	+	+	+	+	+	+	+	+	+	+	+
*	*	24*	25*	52*	40*	22*	23*	24*	54*	77*	55*	*	*
*	*	2*	2*	3*	3*	2*	2*	2*	2*	4*	4*	*	*
*	*	10*	9*	14*	10*	7*	9*	9*	19*	19*	11*	*	*
72700	+	+	+	+	+	+	+	+	+	+	+	+	+
*	*	26*	26*	73*	58*	21*	32*	35*	47*	68*	51*	*	*
*	*	2*	2*	4*	4*	2*	2*	3*	3*	3*	4*	*	*
*	*	9*	8*	14*	12*	7*	11*	11*	15*	17*	12*	*	*
+	+	+	+	+	+	+	+	+	+	+	+	+	+
8500													9500

Figure 1

SAMPLE PRINTER MAP OF BLOCK VALUES

should be as close to the midpoint of the block, if not exactly at the center of the block.

When reading in the block values, these values must be presorted by increasing eastings. This is necessary because the map is printed in strips of 20 blocks in the X-direction. Thus for a strip the blocks are read in until a block is encountered that lies on the next strip. The strip is printed and the program starts to read in blocks for the new strip. This is continued until the map is completed.

Up to three values may be printed in each block if the line printer prints eight (8) lines per inch. If the line printer only prints six (6) lines per inch the program will have to be modified and only two values could then be printed within the block. The scaling parameters for the block values should be chosen carefully so that the value can be printed within 4 spaces. For example, the value 13.45 could be scaled by multiplying by 100. The printed value would then be 1345.

Up to 100 blocks may be printed in the Y-direction. If this is insufficient the program can be modified by increasing the dimensions of the MAP and the variable MAXR to the required size.

2.3 PROGRAM CAPABILITIES:

Program MAP is presently dimensioned to print a map 100 blocks in the Y-direction and an unlimited number of blocks in the X-direction. Up to three integer values, each not exceeding 4 characters can be printed within each block, using eight lines per inch line printer. The same number of values can be printed for each block using six lines per inch line printer. However, the block dimension will no longer be 1/2 inch square. If a 1/2

inch square block is desired using six lines per inch printer, only two values can be printed after the necessary program modification.

All scaling parameters are user specified. The block values are read in under user specified format. In addition to the northing and easting the program assumes that three additional values will be read in for each block.

2.4 DATA INPUTS:

All data is read in from cards. The following describes the data inputs:

CARD ONE.

<u>COLUMN</u>	<u>FORMAT</u>	<u>VARIABLE NAME</u>	<u>DESCRIPTION</u>
1-10	F10.0	YMAX	NORTH COORD. OF UPPER EDGE ROW ONE.
11-20	F10.0	XMIN	EAST COORDINATE OF COL ONE.
21-25	F5.0	WIDY	BLOCK SIZE IN Y DIMENSION.
26-30	F5.0	WIDX	BLOCK SIZE IN X DIMENSION.
31-35	I5	NBC	BEGINNING COLUMN NUMBER OF AREA TO BE MAPPED.
35-40	I5	NEC	ENDING COLUMN.
41-45	I5	NBR	BEGINNING ROW.
46-50	I5	NER	ENDING ROW.

CARD TWO.

<u>COLUMN</u>	<u>FORMAT</u>	<u>VARIABLE NAME</u>	<u>DESCRIPTION</u>
1-10	F10.0	SCALE(1)	SCALING PARAMETER FOR VARIABLE 1. THE SCALING PARAMETERS SHOULD BE CHOSEN TO MAKE THE VARIABLE AN INTERGER.
11-20	F10.0	SCALE(2)	SCALING PARAMETER FOR VARIABLE 2.
21-30	F10.0	SCALE(3)	SCALING PARAMETER FOR VARIABLE 3.

CARD THREE.

<u>COLUMN</u>	<u>FORMAT</u>	<u>VARIABLE NAME</u>	<u>DESCRIPTION</u>
1-80	8A10	IFRMT	VARIABLE FORMAT FOR INPUT DATA. THE FOLLOWING INFORMATION MUST BE READ IN FOR EACH BLOCK (ONE CARD PER BLOCK) IN THE FOLLOWING ORDER: NORTH COORDINATE EAST COORDINATE VARIABLE 1 (THIS WILL BE PLOTTED ON LINE 1 OF THE PRINTED BLOCK.) VARIABLE 2 VARIABLE 3

A sample listing of input data is given in Figure 2.

2.5 PROGRAM OPTIONS:

Other than the ability to specify the map size and various scaling parameters as described in Section 2.4 there are no options.

2.6 OUTPUT:

Figure 1 shows a typical printed map. There is no other output from this program.

75000	7000	100	100	16	26	14	24
100.	1.0	100.					
(2F10.0, 5X, 3E11.4)							
08650.	73650.	4959.	.4099E+00	.4491E+01	.9126E-01	.2702E+01	.4464E+02
08650.	73550.	4959.	.4576E+00	.5031E+01	.9097E-01	.9429E+00	.1817E+02
08650.	73450.	4946.	.7482E+00	.6309E+01	.1186E+00	.3579E+00	.8492E+01
08650.	73350.	4941.	.7149E+00	.7360E+01	.9713E-01	.3549E+00	.8173E+01
08650.	73250.	4914.	.6163E+00	.7238E+01	.8515E-01	.2990E+00	.7210E+01
08650.	73150.	4943.	.2591E+00	.3703E+01	.6997E-01	.3535E+00	.8424E+01
08650.	73050.	5010.	.4103E+00	.3251E+01	.1262E+00	.1038E+01	.1921E+02
08650.	72950.	5010.	.5960E+00	.3259E+01	.1829E+00	.2434E+01	.4087E+02
08650.	72850.	5056.	.1927E+00	.2000E+01	.9634E-01	.2359E+01	.3977E+02
08650.	72750.	4996.	.2433E+00	.2291E+01	.1062E+00	.1649E+01	.2808E+02
08650.	72650.	5011.	.2622E+00	.2774E+01	.9445E-01	.8804E+00	.1663E+02
08650.	72550.	5011.	.2974E+00	.3922E+01	.7584E-01	.4873E+00	.1063E+02
08650.	72450.	5035.	.3676E+00	.4877E+01	.7539E-01	.4750E+00	.9948E+01
08750.	73650.	4954.	.1156E+01	.6576E+01	.1758E+00	.2715E+01	.4483E+02
08750.	73550.	4954.	.1068E+01	.7233E+01	.1476E+00	.9443E+00	.1797E+02
08750.	73450.	4943.	.1471E+01	.9865E+01	.1492E+00	.3833E+00	.8812E+01
08750.	73350.	4942.	.1253E+01	.1005E+02	.1247E+00	.3277E+00	.7703E+01
08750.	73250.	4913.	.7346E+00	.6767E+01	.1086E+00	.3323E+00	.7722E+01
08750.	73150.	5058.	.4132E+00	.4932E+01	.8379E-01	.3507E+00	.8110E+01
08750.	73050.	5008.	.3772E+00	.4829E+01	.7812E-01	.9084E+00	.1704E+02
08750.	72950.	5008.	.2527E+00	.3635E+01	.6952E-01	.1821E+01	.2983E+02
08750.	72850.	5056.	.2349E+00	.2080E+01	.1129E+00	.2078E+01	.3551E+02
08750.	72750.	5056.	.2500E+00	.2698E+01	.9267E-01	.1136E+01	.2088E+02
08750.	72650.	5058.	.2603E+00	.2968E+01	.8769E-01	.6568E+00	.1310E+02
08750.	72550.	5016.	.2632E+00	.3815E+01	.6899E-01	.4716E+00	.9866E+01
08750.	72450.	4996.	.5725E+00	.6520E+01	.8780E-01	.3607E+00	.8426E+01
08850.	73650.	4968.	.1868E+01	.9207E+01	.2029E+00	.2772E+01	.4565E+02
08850.	73550.	4968.	.1896E+01	.9026E+01	.2101E+00	.9741E+00	.1869E+02
08850.	73450.	4968.	.2018E+01	.1151E+02	.1753E+00	.6469E+00	.1265E+02
08850.	73350.	4940.	.2027E+01	.1348E+02	.1504E+00	.4889E+00	.1033E+02
08850.	73250.	4933.	.1604E+01	.1068E+02	.1502E+00	.2991E+00	.7240E+01

Figure 2

A 80-80 LISTING OF SAMPLE INPUT DATA

SECTION III: SYSTEM DOCUMENTATION

3.1 COMPUTER EQUIPMENT:

Program MAP was run on a CDC 6400 computer.

3.2 SOURCE PROGRAM:

A source program listing is given at the end of this user's guide.

3.3 VARIABLES:

A listing of major variables is given in the program listing.

3.4 STORAGE REQUIREMENTS:

Program MAP requires 50K storage on the CDC 6400.

3.5 MAINTENANCE AND UPDATES:

None.

SECTION IV: OPERATING DOCUMENTATION

4.1 OPERATING INSTRUCTIONS:

Program MAP is operated under the CDC SCOPE 3.4 operating system.

4.2 OPERATING MESSAGES:

No special operating messages are provided by the program.

4.3 CONTROL CARDS:

The program is executed with standard SCOPE 3.4 control cards.

4.4 ERROR RECOVERY:

There are no error checks in the program. If the map specified is too large the size is adjusted to the maximum allowable size with no error message being printed.

4.5 RUN TIME:

Run time depends on the number of blocks to be mapped. For instance, to block 100 blocks takes about 1 second.

MAP PROGRAM LISTING

```

10 PROGRAM MAPBLOC(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
20 DIMENSION IXCORD(3),IYCORD(100),MAP(20,3,100),IVAR(3)
30 DIMENSION YCRD(100),SCALE(3),VAR(3),IFRMT(8)
40 DATA IPT,IOUT,BLNK,BLNK2/5,6,5H----+,5H /
00050 DATA YCRD/100*10H /
00060 C*****
00070 C PROGRAM MAP CREATES A PLAN MAP OF BLOCK VALUES ON
00080 C THE LINE PRINTER. THE BLOCKS ARE ONE HALF INCH SQUARE
00090 C AS SHOWN BELOW. UP TO THREE VALUES MAY BE PRINTED
00100 C WITHIN EACH BLOCK. THE VALUES MAY BE UP TO FOUR
00110 C CHARACTERS IN LENGTH. ALL SCALING PARAMETERS ARE USER
00120 C SPECIFIED.
00130 C NOTE,IF THE TRUE SIZE OF THE BLOCK IS 100'X100'
00140 C THEN THE PRINTED MAP WILL HAVE A SCALE OF
00150 C 1INCH TO 200 FT.
00160 C PROGRAM MAP IS CURRENTLY DIMENSIONED TO OUTPUT A MAP
00170 C UP TO 100 BLOCKS IN LENGTH IN THE Y DIMENSION AND
00180 C ANY LENGTH IN THE X DIMENSION. THE MAP IS PRINTED
00190 C WITH 20 BLOCKS PER PAGE IN THE X DIMENSION.
00200 C EXAMPLE OF OUTPUT IS GIVEN BELOW
00210 C
00220 C      71800 +---+---+---+---+
00230 C            * 11* 10* 13*
00240 C            * 13* 12* 18*
00250 C            * 0* 0* 0*
00260 C            +---+---+---+---+
00270 C            * 11* 10* 13*
00280 C            * 15* 13* 18*
00290 C            * 0* 0* 0*
00300 C            +---+---+---+---+
00310 C      12500
00320 C*****
00330 C CONTROL CARDS
00340 C CARD ONE.
00350 C COL 1 -10 F10.0 YMAX NORTH COORD. OF UPPER EDGE ROW ONE.
00360 C      11-20 F10.0 XMIN EAST COORDINATE OF COL ONE.
00370 C      21-25 F5.0 WIDY BLOCK SIZE IN Y DIMENSION
00380 C      26-30 F5.0 WIDX BLOCK SIZE IN X DIMENSION
00390 C      31-35 I5 NBC BEGINNING COLUMN NUMBER OF AREA
00400 C TO BE MAPPED.
00410 C      35-40 I5 NEC ENDING COLUMN
00420 C      41-45 I5 NBR BEGINNING ROW
00430 C      46-50 I5 NER ENDING ROW.
00440 C CARD TWO.
00450 C      1 -10 F10.0 SCALE(1) SCALING PARAMETER FOR VARIABLE 1.
00460 C THE SCALING PARAMETERS SHOULD BE
00470 C CHOSEN TO MAKE THE VARIABLE AN
00480 C INTERGER.
00490 C      11-20 F10.0 SCALE(2) SCALING PARAMETER FOR VARIABLE 2.
00500 C      21-30 F10.0 SCALE(3) SCALING PARAMETER FOR VARIABLE 3.
00510 C CONTROL CARD THREE.
00520 C COL 1-80 8A10 IFRMT VARIABLE FORMAT FOR INPUT DATA.
00530 C THE FOLLOWING INFORMATION MUST BE
00540 C READ IN FOR EACH BLOCK (ONE CARD
00550 C PER BLOCK) IN THE FOLLOWING ORDER.
00560 C NORTH COORDINATE
00570 C EAST COORDINATE
00580 C VARIABLE 1 (THIS WILL BE PLOTTED
00590 C ON LINE 1 OF THE PRINTED BLOCK.
00600 C VARIABLE 2
00610 C VARIABLE 3
00620 C*****
00630 C*****READ IN CONTROL CARD ONE.

```

```

00640      READ(IPT,20) YMAX,XMIN,WIDY,WIDX,NBC,NEC,NBR,NER
00650      20      FORMAT(2F10.0,2F5.0,4I5)
00660      C*****READ IN CONTROL CARD TWO.
00670      READ(IPT,25) SCALE
00680      25      FORMAT(3F10.0)
00690      C*****READ IN CONTROL CARD THREE.
00700      READ(IPT,26) IPRMT
00710      26      FORMAT(8A10)
00720      C*****SET UP MAP PAGING INDEXES
00730      ITCOL=NEC-NBC+1
00740      JROW=NER-NBR+1
00750      IF(JROW.GT.100) JROW=100
00760      NPAGES = (ITCOL + 19) / 20
00770      XNEW = XMIN + (NBC-1) * WIDX
00780      YNEW = YMAX - (NBR-1) * WIDY
00790      N = JROW / 10 + 1
00800      C*****CALCULATE COORDINATES FOR NORTHING TIC MARKS ON MAP.
00810      DO 35 I=1,N
00820      II = I*10
00830      IYCORD(II) = YNEW - I*10*WIDY
00840      35      ENCODE(10,38,YCRD(II)) IYCORD(II)
00850      38      FORMAT(I10)
00860      C*****START PRINTING THE MAP, PAGE BY PAGE.
00870      DO 200 MPAGE=1,NPAGES
00880      ICOL = 20
00890      IF(ICOL*MPAGE.GT.ITCOL) ICOL=ITCOL-(MPAGE-1)*20
00900      NC=ICOL/10 + 1
00910      WRITE(IOUT,110)
00920      110      FORMAT(1H1)
00930      C*****CALCULATE COORDINATES FOR EASTING TIC MARKS ON MAP.
00940      DO 40 II=1,NC
00950      IXCORD(II) = XNEW + (II-1)*10*WIDX
00960      C*****PRINT OUT UPPER BORDER OF MAP.
00970      WRITE(IOUT,45) (IXCORD(II),II=1,NC)
00980      45      FORMAT(T7,I8,T57,I8,T107,I8)
00990      IYNEW=YNEW
01000      ENCODE(10,38,IY) IYNEW
01010      WRITE(IOUT,50) IY, (BLNK,I=1,ICOL)
01020      50      FORMAT(A10,"+",20A5)
01030      C*****READ IN BLOCK DATA- QUIT WHEN A BLOCK FROM THE NEXT PAGE IS FOUND
01040      55      READ(IPT,IPRMT) XE,YN,VAR
01050      IF(EOF(IPT).NE.0) GO TO 99
01060      C*****CALCULATE POSITION OF THIS BLOCK IN MAP
01070      JPOS=(YNEW - WIDY/2. - YN+0.0001)/WIDY + 1
01080      IPOS=(XE-WIDX/2.0 - XNEW+0.0001)/WIDX + 1
01090      IF(IPOS.GT.20) GO TO 100
01100      C*****FILL IN THE VALUES TO BE PRINTED IN THIS BLOCK
01110      DO 62 II=1,3
01120      IVAR(II)=VAR(II)*SCALE(II)
01130      ENCODE(4,65,MAP(IPOS,II,JPOS)) IVAR(II)
01140      62      CONTINUE
01150      65      FORMAT(I4)
01160      GO TO 55
01170      C*****PRINT OUT PAGE
01180      99      IEOF=1
01190      DO 140 I=1,JROW
01200      DO 130 K=1,3
01210      130      WRITE(IOUT,120) (MAP(J,K,I),J=1,ICOL)
01220      120      FORMAT(1H,9X"***",20(A4,"**"))
01230      WRITE(IOUT,50) YCRD(I), (BLNK,L=1,ICOL)
01240      140      CONTINUE
01250      WRITE(IOUT,45) (IXCORD(I),I=1,NC)
01260      C*****PAGE OUTPUT COMPLETE GO ON TO NEXT PAGE OR EXIT IF EOF HAS

```

```

10 C****BEEN ENCOUNTERED.
30 IF (IEOF.EQ.1) GO TO 201
1300 DO 160 I=1,100
1310 DO 160 J=1,3
1320 DO 160 K=1,20
160 MAP(K,J,I)=BLNK2
1330 JPOS=(YNEW-WIDY/2.-YN+0.0001)/WIDY +1
1340 IPOS=(XE-WIDX/2.-XNEW+0.0001)/WIDX +1
1350 DO 162 II=1,3
1360 IVAR(II)=VAR(II)*SCALE(II)
1370 ENCODE(4,65,MAP(IPOS,II,JPOS)) IVAR(II)
162 CONTINUE
1390 XNEW = XNEW + 20.*WIDX
1400 CONTINUE
200 WRITE(IOUT,205)
1410 FORMAT(" LAST PAGE PRINTED. NORMAL END OF JOB.")
205 GO TO 203
1430 WRITE(IOUT,202)
201 FORMAT(" END OF FILE ENCOUNTERED NORMAL END OF JOB")
1440 202
1450 203
1460 STOP
1470 END

```

APPENDIX E

ESTIMATION VARIANCE PROGRAM "ESTVAR"

USER'S GUIDE

SECTION I: PROGRAM IDENTIFICATION

- 1.1 PROGRAM TITLE: Estimation Variance Calculation
- 1.2 PROGRAM CODE NAME: ESTVAR
- 1.3 PROGRAM WRITER: H. P. Knudsen
- PROGRAM DOCUMENTATION: H. P. Knudsen
- 1.4 ORGANIZATION: Department of Mining and Geological Engineering
University of Arizona
Tucson, Arizona
- 1.5 DATE: November 20, 1976
- 1.6 UPDATES: None. Version: 1.
- 1.7 SOURCE LANGUAGE: CDC 6400 Computer Systems - FORTRAN Extended -
Version IV.
- 1.8 ABSTRACT:

Program ESTVAR calculates the estimation variance (estimation error) of assigning the average grade of a set of samples to a block. The block can be a rectangular 3-D block or an irregular 2-D block. Up to 15 nodes can be used to describe the irregular block and up to 500-data points may be included. The spherical variogram is assumed.

SECTION II: ENGINEERING DOCUMENTATION

2.1 NARRATIVE DOCUMENTATION:

Program ESTVAR calculates the variance of the error (estimation variance) made by assigning the grade of a sample or samples to a block. This program can be used to solve many interesting problems concerning the estimation variance of various sampling grids. It can be used to calculate the estimation variance for a given set of samples. It can be used to test which of several sampling grids will give the smallest probable error. Or it can be used to calculate the reduction in the probable error made by adding one or more new samples, even before the samples are taken. If the unknown true grade of a block V is $Z(V)$ and the known grade of a block v is $Z(v)$ then the variance of the error of assigning the grade $Z(v)$ to $Z(V)$ is given by

$$\text{VAR}[Z(V)-Z(v)] = \sigma_E^2 = \frac{2}{V} \int_V dx \int_V \gamma(x-y) dy - \frac{1}{V^2} \int_V dx \int_V \gamma(x-y) dy - \frac{1}{V^2} \int_V dx \int_V \gamma(x-y) dy \quad (1)$$

where $\gamma(x-y)$ is the value of the variogram for the distance between points x and y . Thus, once the variogram is known, this equation can be used to calculate the magnitude of the estimation errors made during an ore reserve analysis.

In most cases instead of assigning to $Z(V)$ the grade $Z(v)$ of block v , the average grade of a set of samples is assigned to $Z(V)$. In this common case the estimation variance is given by

$$\sigma_E^2 = \frac{2}{NV} \sum_{i=1}^N \int_V \gamma(x_i - x) dx - \frac{1}{V^2} \int_V dx \int_V \gamma(x-y) dy - \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N \gamma(x_i - x_j) \quad (2)$$

where N is the number of samples.

The block V can be of two types, a regular block like a cube, or an irregular shaped block such as a polygon. The regular block is described by specifying its length, width and depth and its spatial position in the orebody, i.e., its northing, easting and elevation. The irregular block is described by specifying the coordinates of a set of nodes that outline the block. Up to 15 nodes may be used to define the block.

Up to 500 samples may be included in each run. The spatial location of each sample must be read in.

The spherical variogram model is assumed in the program. However subroutine SUBGAM may be modified if other models are desired. It is also assumed that any anisotropy in the variogram is of the simple geometric type.

2.2 METHOD OF SOLUTION:

For a given set of samples and a given block the estimation variance is calculated according to Equation 2. The only difficulty involved in the calculations is the evaluation of the integrals. This is accomplished by using a discrete summation to approximate the integral. The volume over which the integration is performed is approximated by a set of regular grid points within the volume. For example, the square shown below could be approximated by the 16 points in the grid shown.

Number of points in X-Direction = $IX=4$

Number of points in Y-Direction = $IY=4$

Total number of points = $(IX \cdot IY) = 16$

The quality of the approximation is dependent on the number of grid

```

01      .10      10      10      01      140      50.0      50.1      0
(3P)    50.      0.00      50.0      50.0      50.1      1.
      12.5      25.0      25.0      1.0
      12.5      37.5      1.0
      37.5      12.5      1.0
      37.5      37.5      1.0

```

Figure 1
EXAMPLE INPUT DATA

VARIOGRAM PARAMETERS

```

CO      .1000E+00
C      .1500E+00
RANGE      140.
ANGLE OF ROTATION      0.
HOR. ANISOTROPY      1.
VERT. ANISOTROPY      0.

```

DEFINITION OF BLOCK

```

COORDINATES OF NW CORNER { 50.X 50.X 1.0 0. 50. )
BLOCK DIMENSIONS

```

ESTIMATION VARIANCE = 3.20652E-01

VARIANCE OF A POINT WITHIN THE BLOCK = 0.14087E+00

DRILL HOLE LOCATIONS

NORTH	EAST	ELEV.	NORTH	EAST	ELEV.
25.0	25.0	1.0	12.5	12.5	1.0
37.5	12.5	1.0	37.5	37.5	1.0
12.5	37.5	1.0			

Figure 2
EXAMPLE OUTPUT

points used. The total number of grid points is given by $IX \cdot IY \cdot IZ$. The program is presently dimensioned to allow a maximum of 100 grid points. The number of grid points to use in a regular block is specified by the number of grid points in the X, Y and Z directions.

For irregular shaped blocks the grid is specified in the same manner as the regular block except that the vertical dimension is ignored, i.e., the block is considered to be two dimensional. This assumption is for simplicity in the program, because each of the grid points must be tested to see if it lies within the irregular shaped block.

This assumption is not overly restrictive because in many cases such as in open pit mines the sample length is the same as the block height, thus allowing the block to be considered two dimensional.

2.3 PROGRAM CAPABILITIES:

A maximum of 500 samples may be included in each data set. Either a regular block or irregular shaped block may be input. A maximum of 15 nodes may be used to describe the irregular block.

2.4 DATA INPUT:

Data is input from punched cards. The order of the control cards and data cards is given below. The data cards are read in according to a user specified format. Figure 1 is a listing of sample input data.

1. CARD ONE. PROGRAM RUN PARAMETERS.

<u>COLUMN</u>	<u>VARIABLE NAME</u>	<u>DESCRIPTION</u>
1	IBLK	TYPE OF BLOCK 0 = RECTANGULAR, 1 = IRREGULAR.
2	IPRT	PRINT OUT HOLES IF = 1
11-15	IY	NUMBER OF POINTS IN Y DIRECTION.
16-20	IX	NUMBER OF POINTS IN X DIRECTION.
21-25	IZ	NUMBER OF POINTS IN Z DIRECTION.

2. CARD TWO. VARIOGRAM PARAMETERS.

<u>COLUMN</u>	<u>VARIABLE NAME</u>	<u>DESCRIPTION</u>
1-10	CO	NUGGET VALUE
11-20	C	C VALUE OF VARIOGRAM
21-30	RANG	RANGE OF VARIOGRAM
31-40	ANG	ANGLE OF ANISTROPY IN DEGREES FROM X-AXIS TO THE MAJOR AXIS.. THE ANGLE IS PLUS IF COUNTERCLOCK- WISE, MINUS IF CLOCKWISE.
41-41	AFH	HORIZONTAL ANISOTROPY FACTOR(MAJOR AXIS OVER MINOR AXIS)
51-50	AFV	VERTICAL ANISOTROPY FACTOR.

3. CARD THREE. FOR RECTANGULAR BLOCK.

<u>COLUMN</u>	<u>VARIABLE NAME</u>	<u>DESCRIPTION</u>
1-10	YMAX	NORTH COORDINATE OF UPPER LEFT CORNER.
11-20	XMIN	EAST COORDINATE OF UPPER LEFT CORNER.
21-30	ZMAX	ELEVATION OF UPPER EDGE OF BLOCK.
31-40	WIDY	BLOCK WIDTH IN NORTH DIRECTION.
41-50	WIDX	BLOCK WIDTH IN EAST DIRECTION.
51-60	WIDZ	BLOCK DEPTH.

4. CARD THREE. FOR IRREGULAR BLOCK.

<u>COLUMN</u>	<u>VARIABLE NAME</u>	<u>DESCRIPTION</u>
1-10	YNO(I)	NORTH COORDINATE OF FIRST NODE.
11-20	XEA(I)	EAST COORDINATE OF FIRST NODE.
		UP TO 15 NODES MAY BE USED TO DEFINE THE BLOCK. ONE NODE PER CARD. A 999.0 IN EACH COORDINATE POSITION INDICATES THE LAST NODE HAS BEEN READ IN.

5. CARD FOUR. VARIABLE FORMAT CARD FOR ASSAY DATA.

<u>COLUMN</u>	<u>VARIABLE NAME</u>	<u>DESCRIPTION</u>
1-80	IFRMT	VARIABLE FORMAT. ONLY THE NORTHING, EASTING, AND ELEVATION ARE READ IN FOR EACH DRILL HOLE.

DRILL HOLE DATA CARDS. THE DATA CARDS FOLLOW CARD THREE AND ARE
FOLLOWED BY A 7/8/9 CARD.

2.5 OUTPUT:

Output from program ESTVAR consists of a listing of the input parameters, the calculated estimated variance and a listing of the sample points.

Figure 2 is an example of the output.

SECTION III: SYSTEM DOCUMENTATION

3.1 COMPUTER EQUIPMENT:

Program ESTVAR was run on a CDC-6400 computer.

3.2 SOURCE PROGRAM:

A listing of the source listing is given at the end of this Appendix.

3.3 VARIABLES AND SUBROUTINES:

A listing of major variables is given in the program listing.

Subroutines included in program ESTVAR are:

SUBGAM - calculates the value of the variogram between two points.

PTLOC - determines whether a point is within a closed boundary.

Subroutine PTLOC was written by John K. Hall of the geological survey of Israel. Documentation and explanation of this subroutine were published in Mathematical Geology, Volume 7, November 1, 1975.

3.4 STORAGE REQUIREMENTS:

Program ESTVAR requires 46K storage on the CDC 6400.

3.5 MAINTENANCE AND UPDATES:

None

SECTION IV: OPERATING DOCUMENTATION

4.1 OPERATING INSTRUCTIONS:

Program ESTVAR is operated under the CDC 6400 SCOPE 4.3 operating system.

4.2 OPERATING MESSAGES:

No operating messages are generated by program ESTVAR.

4.3 CONTROL CARDS:

The program is executed with standard SCOPE 3.4 control cards.

4.4 ERROR RECOVERY:

The program will terminate if certain errors are encountered. Self explanatory error messages will be printed.

4.5 RUN TIME:

The run time is dependent on the number of grid points used and the number of sample points. The example run shown in Figure 2 required 1 second on the CDC-6400.

ESTVAR PROGRAM LISTING

```

00500 PROGRAM ESTVAR(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
00600 COMMON /GAM/CO,C,RANG,CST,SIT,AFH,AFV
00700 DIMENSION YN(500),XE(500),ZL(500),IFRMT(8),P(3,100),P1(3)
00800 DIMENSION P2(3),YNO(16),XEA(16)
00900 DATA IPT,IOUT,MAXH,MAXN,MAXPT/5,6,500,15,1000/
01000 C*****
01100 C PROGRAM ESTVAR CALCULATES THE ESTIMATION VARIANCE OF
01200 C A BLOCK FROM A SET OF SAMPLES.
01300 C CONTROL CARDS
01400 C CARD ONE. PROGRAM RUN PARAMETERS.
01500 C COL 1 IBLK TYPE OF BLOCK
01600 C 2 IPRT PRINT OUT HOLES IF = 1
01700 C 11-15 IY NUMBER OF GRID POINTS IN Y DIRECTION.
01800 C 16-20 IX NUMBER OF GRID POINTS IN Y DIRECTION
01900 C 21-25 IZ NUMBER OF GRID POINTS IN Z "
02000 C CARD TWO. VARIOGRAM PARAMETERS.
02100 C COL 1-10 CO NUGGET VALUE
02200 C 11-20 C C VALUE OF VARIOGRAM
02300 C 21-30 RANG RANGE OF VARIOGRAM
02400 C 31-40 ANG ANGLE OF ANISTROPY IN DEGREES FROM
02500 C X-AXIS TO THE MAJOR AXIS. THE
02600 C ANGLE IS PLUS IF COUNTERCLOCKWISE,
02700 C MINUS IF CLOCKWISE.
02800 C 41-41 AFH HORIZONTAL ANISOTROPY FACTOR (MAJOR
02900 C AXIS OVER MINOR AXIS)
03000 C 51-50 AFV VERTICAL ANISOTROPY FACTOR.
03100 C CARD THREE. FOR RECTANGULAR BLOCK
03200 C 1-10 YMAX NORTH COORDINATE OF UPPER LEFT CORNER.
03300 C 11-20 XMIN EAST COORDINATE OF UPPER LEFT CORNER.
03400 C 21-30 ZMAX ELEVATION OF UPPER EDGE OF BLOCK
03500 C 31-40 WIDY BLOCK WIDTH IN NORTH DIRECTION
03600 C 41-50 WIDX BLOCK WIDTH IN EAST DIRECTION
03700 C 51-60 WIDZ BLOCK DEPTH.
03800 C CARD THREE. FOR IRREGULAR BLOCK.
03900 C 1-10 YNO(I) NORTH COORDINATE OF ITH NODE.
04000 C 11-20 XEA(I) EAST COORDINATE OF ITH NODE.
04100 C UP TO 15 NODES MAY BE USED TO
04200 C DEFINE THE BLOCK. THE NODES MUST
04300 C BE IN CLOCKWISE ORDER. ONE NODE PER
04400 C CARD. A 999.0 IN EACH COORDINATE
04500 C POSITION INDICATES THE LAST NODE
04600 C HAS BEEN READ IN.
04700 C CARD FOUR. VARIABLE FORMAT CARD FOR ASSAY DATA.
04800 C 1-80 IFRMT VARIABLE FORMAT.
04900 C ONLY THE NORTHING, EASTING, AND
05000 C ELEVATION ARE READ IN FOR EACH
05100 C DRILL HOLE.
05200 C DRILL HOLE DATA CARDS. THE DATA CARDS FOLLOW CARD THREE
05300 C AND ARE FOLLOWED BY A 7/8/9 CARD.
05400 C *****
05500 C MAJOR VARIABLES USED IN PROGRAM ESTVAR.
05600 C YN ARRAY CONTAINING NORTH COORDINATE OF SAMPLE POINTS
05700 C XE ARRAY CONTAINING EAST COORD. OF SAMPLE POINT.
05800 C ZL ARRAY CONTAINING ELEVATION OF SAMPLE POINT.
05900 C P ARRAY CONTAINING COORDINATES OF GRID POINTS.
06000 C P1 COORDINATES OF SAMPLE POINT.
06100 C P2 COORDINATES OF SAMPLE POINT.
06200 C GH VALUE OF VARIGRAM FOR A GIVEN DISTANCE.
06300 C

```

```

C      BLKVAR  VARIANCE OF A POINT IN THE BLOCK.
C      COSAM   AVERAGE COVARIANCE BETWEEN SAMPLE POINTS.
C      COBLK   AVERAGE COVARIANCE BETWEEN SAMPLES AND BLOCK.
C      EST     ESTIMATION VARIANCE.
C*****
C      READ IN CONTROL CARD ONE.
C      READ(IPT,10)IBLK,IPRT,IY,IX,IZ
C      FORMAT(2I1,8X,3I5)
C      READ IN CONTROL CARD TWO
C      READ(IPT,20)CO,C,RANG,ANG,AFH,AFV
C      FORMAT(6F10.0,3I5)
C      WRITE(IOUT,25)CO,C,RANG,ANG,AFH,AFV
C      FORMAT(" VARIOGRAM PARAMETERS",/5X"CO",T25,E10.4,/,
15X"C",T25,E10.4,/,5X"RANGE",T25,F10.0,/,5X"ANGLE OF ROTATION",
2T25,F10.0,/,5X"HOR. ANISOTROPY ",T25,F10.0,/,
35X"VERT. ANISOTROPY ",T25,F10.0)
C      IF(IBLK.EQ.1) GO TO 32
C*****READ IN CONTROL CARD THREE.
C      READ(IPT,20)YMAX,XMIN,ZMAX,WIDY,WIDX,WIDZ
C      WRITE(IOUT,30)YMAX,XMIN,ZMAX,WIDY,WIDX,WIDZ
C      FORMAT(1H0,"DEFINITION OF BLOCK",/5X"COORDINATES OF NW CORNER",
30 12X"("2(F10.0,""),F10.0,"")"
2/5X"BLOCK DIMENSIONS",9X"(",2(F5.0,"X"),F5.0,"")")
C*****CALCULATE POINTS WITHIN THE RECTANGULAR BLOCK
C      DX =WIDX/IX
C      DY =WIDY/IY
C      DZ =WIDZ/IZ
C      M=0
C      DO 60 I=1,IX
C      DO 60 J=1,IY
C      DO 60 K=1,IZ
C      M=M+1
C      IF(M.GT.MAXPT)GO TO 220
C      P(1,M) = XMIN +DX/2 + (I-1)*DX
C      P(2,M) = YMAX -DY/2 - (J-1)*DY
C      P(3,M) = ZMAX -DZ/2 - (K-1)*DZ
60  CONTINUE
C      GO TO 37
C*****READ IN CONTROL CARD THREE. IRREGULAR BLOCK.
32  I=1
C      YIN=99999999.
C      YMAX=0.000
C      XMIN=99999999.
C      XAX=0.0
33  READ(IPT,20)YNO(I),XEA(I)
C      IF(YNO(I).EQ.999.0.AND.XEA(I).EQ.999.0) GO TO 39
C      IF(I.GT.MAXN) GO TO 200
C      IF(YNO(I).GT.YMAX) YMAX=YNO(I)
C      IF(YNO(I).LT.YIN) YIN=YNO(I)
C      IF(XEA(I).GT.XAX) XAX=XEA(I)
C      IF(XEA(I).LT.XMIN) XMIN=XEA(I)
C      I=I+1
C      GO TO 33
39  NODES = I-1
C      WRITE(IOUT,36)
C      WRITE(IOUT,38) (YNO(I),XEA(I),I=1,NODES)
36  FORMAT(1H0,"DEFINITION OF BLOCK -NODES")
38  FORMAT(1H,"(",F10.0,"",F10.0,"")")
C      WIDY =YMAX-YIN
C      WIDX =XAX-XMIN
C*****CALCULATE POINTS WITHIN THE IRREGULAR BLOCK
C      DX =WIDX/IX
C      DY =WIDY/IY

```

```

12700      M=0
12800      DO 41 I=1,IX
12900      DO 41 J=1,IY
13000      X= XMIN +DX/2 + (I-1)*DX
13100      Y= YMAX -DY/2 - (J-1)*DY
13200      CALL PTLOC(NODES,XEA,YNO,X,Y,LOC)
13300      IF(LOC.LE.0) GO TO 41
13400      M=M+1
13500      IF(M.GT.100)GO TO 37
13600      P(1,M)=X
13700      P(2,M)=Y
13800      41 CONTINUE
13900      37 READ(IPT,35) IFRMT
14000      35 FORMAT(8A10)
14100      ANGR =ANG*0.01745329252
14200      CST= COS(ANGR)
14300      SIT=SIN(ANGR)
14400      C*****READ IN SAMPLE DATA
14500      I=1
14600      40 READ(IPT,IFRMT) YN(I),XE(I),ZL(I)
14700      IF(I.GT.MAXH) GO TO 205
14800      IF(EOP(IPT).NE.0) GO TO 50
14900      I=I+1
15000      GO TO 40
15100      50 NHOL = I-1
15200      C*****CALCULATE THE VARIANCE OF A BLOCK
15300      M1=M-1
15400      GMOY =0.
15500      DO 70 I=1,M1
15600      II=I+1
15700      DO 70 J=II,M
15800      CALL SUBGAM(P(1,I),P(1,J),GH)
15900      GMOY = GMOY +GH
16000      70 CONTINUE
16100      BLKVAR = 2.*GMOY/(M*M) + CO/M
16200      C*****CALCULATE COVARIANCES
16300      COSAM =0.
16400      COBLK =0.
16500      DO 100 I=1,NHOL
16600      II=I+1
16700      P1(1)=XE(I)
16800      P1(2)=YN(I)
16900      P1(3)=ZL(I)
17000      IF(II.GT.NHOL) GO TO 91
17100      DO 90 J =II,NHOL
17200      P2(1)=XE(J)
17300      P2(2)=YN(J)
17400      P2(3)=ZL(J)
17500      CALL SUBGAM(P1,P2,GH)
17600      COSAM =COSAM +GH
17700      90 CONTINUE
17800      91 DO 95 K=1,M
17900      CALL SUBGAM(P1,P(1,K),GH)
18000      COBLK =COBLK +GH
18100      95 CONTINUE
18200      100 COSAM = COSAM/(.5*NHOL*NHOL) +CO/NHOL
18300      COBLK = COBLK/(NHOL*M)
18400      C*****ESTIMATION VARIANCE IS
18500      EST = 2.*COBLK - BLKVAR - COSAM
18600      WRITE(IOUT,110) EST,BLKVAR
18700      110 FORMAT(1H0,"ESTIMATION VARIANCE =",E12.5,/, " VARIANCE OF
18800      2 A POINT WITHIN THE BLOCK =",E12.5)
18900      WRITE(IOUT,120)

```

```

10      WRITE(IOUT,125) (YN(I),XE(I),ZL(I),I=1,NHOL)
120      FORMAT(1H0,"DRILL HOLE LOCATIONS",/,
13X,3(7X"NORTH"6X"EAST"5X"ELEV.",3X))
125      FORMAT(3(5X,3F10.1))
130      GO TO 130
200      WRITE(IOUT,140)
140      FORMAT(" TOO MANY NODES- JOB TERMINATED")
130      GO TO 130
220      WRITE(IOUT,225)
225      FORMAT(" TOO MANY POINTS IN THE NUMERICAL APPROXIMATION")
130      GO TO 130
205      WRITE(IOUT,150)
150      FORMAT(" TOO MANY SAMPLE POINTS")
130      STOP
      END
      SUBROUTINE SUBGAM(P1,P2,GH)
C*****
C      SUBROUTINE SUBGAM CALCULATES THE VALUE OF THE VARIOGRAM
C      BETWEEN THE POINTS P1 AND P2.
C*****
      COMMON /GAM/CO,C,RANG,CST,SIT,APH,AFV
      DIMENSION P1(3),P2(3)
      XD=P1(1)-P2(1)
      YD=P1(2)-P2(2)
      ZD=P1(3)-P2(3)
      D=SQRT((XD*CST+YD*SIT)**2+APH*APH*(YD*CST-
1XD*SIT)**2+AFV*AFV*ZD*ZD)
      IF(D.GT.RANG) GO TO 20
      GH = CO+C*(1.5*D/RANG-.5*(D/RANG)**3)
      RETURN
20      GH = CO +C
      RETURN
      END
      SUBROUTINE PTLOC(NP,X,Y,XP,YP,LOC)
C*****
C      SUBROUTINE PTLOC DETERMINES WHETHER A POINT IS WITHIN A
C      CLOSED BOUNDARY.
C      IF LOC =-1 THE POINT IS OUTSIDE.
C      0 THE POINT LIES ON THE BOUNDARY.
C      1 THE POINT LIES WITHIN THE BOUNDARY.
C      2 THE POINT LIES ON THE NODE X(I1),Y(I2).
C      PROGRAM PTLOC WAS WRITTEN BY JOHN K. HALL OF THE
C      GEOLOGICAL SURVEY OF ISRAEL. DOCUMENTATION AND
C      EXPLANATION OF THIS ALGORITHM WAS PUBLISHED IN
C      MATHEMATICAL GEOLOGY, VOL 7, NOV.1,1975.
C*****
      DIMENSION X(1),Y(1)
      DATA C180/0.999999998/
      I1 = 0
      I2 = 0
      ISR = 0
      IEC = 0
      TSC = 0.0
      SC = 0.0
      TCC = 1.0
      CC = 1.0
      IP = NP
      X1 = X(NP)-XP
      Y1 = Y(NP)-YP
      R1 = SQRT(X1*X1 + Y1*Y1)
      IF(R1.EQ.0.0) GO TO 65
      DO 45 I = 1,NP
      X2 = X(I) -XP

```

```

25300 Y2 = Y(I) - YP
25400 R2 = SORT(X2*X2 + Y2*Y2)
25500 VPL = R1*R2
25600 IF(R2.EQ.0.0) GO TO 65
25700 ST = (X1*Y2 - X2*Y1)/VPL
25800 CT = (X1*X2 + Y1*Y2)/VPL
25900 IF((CT + C180).LE.0.0) GO TO 55
26000 IF((I-NP).LT.0) GO TO 25
26100 IF((IEC/2)*2 - IEC) 60,50,50
26200 25 TSC = SC*CT + CC*ST
26300 TCC = CC*CT - SC*ST
26400 IF((TSC*SC).GT.0) GO TO 40
26500 IF((TSC*ST).GE.0) GO TO 40
26600 IEC = IEC + 1
26700 40 SC = TSC
26800 CC = TCC
26900 X1 = X2
27000 Y1 = Y2
27100 R1 = R2
27200 IP = I
27300 45 CONTINUE
27400 50 LOC = -1
27500 RETURN
27600 55 LOC = 0
27700 I1 = IP
27800 I2 = I
27900 RETURN
28000 60 ISR = 1
28100 LOC = 1
28200 IF(SC.LT.0.0) ISR = -1
28300 RETURN
28400 65 LOC = 2
28500 I1 = I
28600 I2 = I
28700 IF(R1.NE.0.0) RETURN
28800 I1 = IP
28900 I2 = IP
29000 RETURN
29100 END

```