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Analytical Relations Between Elastic-Plastic Fracture Criteria

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OAK RIDGE NATIONAL LABORATORY

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ANALYTICAL RELATIONS BETWEEN ELASTIC-PLASTIC
FRACTURE CRITERIA

J. G. Merkle

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FOREWORD

The work reported herein was performed at the Oak Ridge National Laboratory, under Budget Activity No. YA 01 19 03 2, Project Title, "Fracture and Irradiation Effects." This work is sponsored by the U.S. Nuclear Regulatory Commission, Office of Standards Development, and is directed at ORNL by J. D. Whitman. The NRC cognizant engineer for this work is P. H. Randall of the Structures and Components Standards Branch.

ANALYTICAL RELATIONS BETWEEN ELASTIC-PLASTIC FRACTURE CRITERIA

J. G. Merkle

ABSTRACT

The equation of the normalized COD design curve recently proposed in the UK as a basis for determining allowable crack sizes is derived from the Equivalent Energy approximation for the J Integral. It is also shown that another approximation for the J Integral recently proposed by Westinghouse is mathematically equivalent to the normalized COD approach.

INTRODUCTION

There are now in existence several proposed criteria for fracture in the elastic-plastic range, each of which has a certain amount of experimental evidence to support its validity. Prominent among these criteria are the Crack Opening Displacement, the J Integral and the K_{Icd} parameter of the Equivalent Energy method. Progress has been made in developing practical experimental and analytical procedures for applying each of these criteria separately. Nevertheless, future progress in the development of codes and standards could be facilitated by a demonstration that these criteria have a partially common analytical basis and should therefore approximately agree with each other. In the U.S., efforts are being made to standardize the procedures for obtaining fracture toughness values from nonlinear load-displacement test records, using either the J Integral or the Equivalent Energy method of analysis. In the United Kingdom, a proposal has already been made to establish allowable defect sizes in structures by means of a Crack Opening Displacement analysis,¹ despite the fact that problems of definition and measurement still exist with respect to determining critical COD values. The purpose of this discussion is to present a derivation of the proposed UK relationship between allowable defect sizes and critical COD values, using both the Equivalent Energy method and the J Integral as bases for the analysis.

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THE UK PROPOSED COD DESIGN CURVE

In developing a basis for an inelastic fracture analysis method, one approach is to assume that a parameter defined on the basis of linear elastic fracture mechanics still has essentially the same meaning in the plastic range. Burdekin and Dawes¹ proposed the graphical relationship between allowable effective defect size, $\bar{a}_{max}$, and critical COD, δ_c , shown in Fig. 1, by relating φ , the normalized COD, to ϵ/ϵ_Y , the ratio of the nominal strain to the uniaxial yield strain. The definition of φ is based on linear elastic fracture mechanics, beginning with the general expression for the crack tip stress intensity factor, which is

$$K_I = C\sigma \sqrt{\pi a} . \quad (1)$$

The effective defect size, $\bar{a}$, is defined by

$$\bar{a} = C^2 a , \quad (2)$$

so that, combining Eqs. (1) and (2) gives

$$K_I = \sigma \sqrt{\pi \bar{a}} \quad (3)$$

For plane stress, the plastic zone radius, r_Y , is estimated by the formula

$$r_Y = \frac{1}{2\pi} \left(\frac{K_I}{\sigma_Y} \right)^2 . \quad (4)$$

In addition, the elastic strain energy release rate, ϕ_I , is related to K_I by

$$K_I^2 = E\phi_I , \quad (5)$$

and ϕ_I is related to δ by²

$$\phi_I = c_Y \delta . \quad (6)$$

Substituting Eq. (6) into Eq. (5), and then Eq. (5) into Eq. (4) gives

$$r_Y = \frac{\delta}{2\pi\epsilon_Y}, \quad (7)$$

where ϵ_Y is the uniaxial yield strain. Dividing both sides of Eq. (7) by $\bar{a}$ gives the definition of φ , the normalized COD, which is

$$\varphi = \frac{r_Y}{\bar{a}} = \frac{\delta}{2\pi\epsilon_Y\bar{a}}. \quad (8)$$

The normalized COD design curve shown in Fig. 1 is roughly parabolic in the elastic range, and becomes a straight line in the plastic range. The linear shape of the curve in the plastic range is semi-empirical, as discussed in Ref. 1 and 2. The calculations for the two labeled points in the elastic range in Fig. 1 were done by two different methods. The calculation for design conditions was done by a conventional linear elastic fracture mechanics formula. The calculation for proof test conditions was done with the equation for the strip yield model in fracture mechanics.^{2,3} For design conditions, the relationship used is

$$K_I = 1.2 \sigma_Y \sqrt{\pi \bar{a}}, \quad (9)$$

where the factor 1.2 provides an adjustment for both plastic zone size and free surface effects. For $\sigma_Y = 215$, which represents the B.S. 1515 and ASME Code Section III primary membrane stress limits, with respect to yield stress, for normal service conditions, Eq. (9) gives

$$\bar{a} = 0.5 \left(\frac{K_I}{\sigma_Y} \right)^2. \quad (10)$$

Combining Eqs. (5) and (6) gives

$$K_I^2 = E\sigma_Y\delta, \quad (11)$$

from which it follows that

$$\left(\frac{K_I}{\sigma_Y}\right)^2 = \frac{\delta}{\epsilon_Y} . \quad (12)$$

Thus, substituting Eq. (12) into Eq. (10) gives, for the case of $\sigma/\sigma_Y = 2/3$,

$$\bar{a} = 0.5 \left(\frac{\delta}{\epsilon_Y}\right) , \quad (13)$$

which agrees with Fig. 1. Rearranging Eq. (8) gives

$$\varphi = \frac{1}{2\pi} \frac{\left(\frac{\delta}{\epsilon_Y}\right)}{\bar{a}} , \quad (14)$$

and substituting Eq. (13) into Eq. (14) gives, for the case of $\sigma/\sigma_Y = 2/3$,

$$\varphi = 0.32 , \quad (15)$$

which also agrees with Fig. 1.

The working equation of the strip yield model¹⁻³ is

$$\delta = \frac{8\sigma_Y \bar{a}}{\pi E} \ln \sec \left(\frac{\pi \sigma}{2 \sigma_Y} \right) , \quad (16)$$

which rearranged gives

$$\bar{a} = \frac{\left(\frac{\delta}{\epsilon_Y}\right)}{\frac{8}{\pi} \ln \sec \left(\frac{\pi \sigma}{2 \sigma_Y} \right)} . \quad (17)$$

Reference 1 assumes that under proof test conditions, the pressure is 1.3 times the design pressure, and that consequently $\sigma/\sigma_Y = 0.87$. For this value of σ/σ_Y , Eq. (17) gives

$$\bar{\epsilon} = 0.25 \left(\frac{\epsilon}{\epsilon_Y} \right), \quad (18)$$

which agrees with Fig. 1. Substituting Eq. (18) into Eq. (14) gives

$$\psi = 0.64, \quad (19)$$

which also agrees with Fig. 1.

Because two different equations were used in Ref. 1 for the calculations pertaining to the labeled points on the elastic portion of the curve in Fig. 1, the equation of this part of the curve is uncertain. In addition, it appears that the slopes of the two portions of the curve in Fig. 1 do not quite match at their point of intersection. Instead, the elastic range curve appears to lie slightly below the downward projection of the plastic range curve as the intersection point of the two parts of the curve is approached. From the information given in Fig. 1, it can be determined that the equation of the linear portion of the curve in Fig. 1 is

$$\psi = \left(\frac{\epsilon}{\epsilon_Y} \right) - 0.25. \quad (20)$$

If Eq. (20) is extrapolated back into the elastic range, the value of ψ for design conditions, for which $\sigma/\sigma_Y = 2/3$, is 0.42, which exceeds the elastically calculated value of 0.32. Thus the two parts of the curve shown in Fig. 1 probably do not meet at a point of tangency.

ANALYTICAL REPRESENTATION OF THE NORMALIZED COD DESIGN CURVE

The anomalies in the shape of the normalized COD design curve mentioned above have recently been remedied by Dawes,⁴ who suggested that the curve should be represented by the equation

$$\varphi = (\epsilon/\epsilon_Y)^2, \quad \text{for } \epsilon/\epsilon_Y \leq 0.5, \quad (21)$$

and by Eq. (20), for $\epsilon/\epsilon_Y \geq 0.5$. Equation (21) defines the parabola that joins the straight line defined by Eq. (20) at a point of tangency, at which $\epsilon/\epsilon_Y = 0.5$.

Equation (21) was derived by Dawes⁵ by writing the first term in the series expression for the quantity on the right hand side of Eq. (16), as discussed in Ref. 2, and then multiplying the result by a safety factor of 2.0. While the analytical basis of Eq. (21) is thus clear, it would still be desirable to find a single analytical model, even though it might be approximate, that would produce the entire normalized COD design curve, and therefore lead to the derivation of both Eqs. (20) and (21). As it turns out, such an analytical model does exist.

It has been established⁶ that the Equivalent Energy Method⁷ provides an approximate estimate of the J Integral, by means of the equation

$$J = 2\pi C^2 aU \left(\frac{E'}{E} \right), \quad (22)$$

where U is the nominal work per unit volume at a reference point near the location of the flaw, E' is the initial slope of the nominal stress versus nominal strain curve at the same point, and the other terms in Eq. (22) are as previously defined. If uniaxial tension is assumed, then

$$\frac{E'}{E} = 1. \quad (23)$$

Therefore, substituting Eqs. (2) and (23) into Eq. (22) gives

$$J = 2\pi a^2 U. \quad (24)$$

For nonlinear conditions, J is considered to be a generalization of ϕ_I , so that

$$J = \sigma_Y \delta . \quad (25)$$

Combining Eqs. (24) and (25) gives

$$\delta = \frac{2\pi a U}{\sigma_Y} , \quad (26)$$

and substituting Eq. (26) into Eq. (8) gives

$$\varphi = \frac{U}{\sigma_Y \epsilon_Y} . \quad (27)$$

Since

$$\sigma_Y = E \epsilon_Y \quad (28)$$

Eq. (27) can also be written in the form

$$\varphi = \frac{U}{E \epsilon_Y^2} . \quad (29)$$

Thus the expression for φ depends on the expression for U , which depends on the shape of the stress-strain curve. The shape of the stress-strain curve for an elastic-ideally plastic material is shown by the solid curve in Fig. 2. If U is expressed as a function of strain, then a factor of safety can be applied to the flaw size by overestimating the value of U for a given value of strain. This can be done by applying the safety factor to the elastic modulus, without changing the yield stress, as indicated by the dashed curve in Fig. 2. The effect is to establish a reduced yield strain which is the actual yield strain divided by the factor of safety. Below the reduced yield strain,

$$U = \frac{n}{2} E \epsilon^2 , \quad (30)$$

ε

where n is the factor of safety on flaw size below the reduced yield strain. Substituting Eq. (30) into Eq. (29) gives

$$\varphi = \frac{n}{2} \left(\frac{\varepsilon}{\varepsilon_Y} \right)^2, \quad \varepsilon \leq \frac{\varepsilon_Y}{n}. \quad (31)$$

Above the reduced yield strain

$$U = \sigma_Y \varepsilon - \frac{\sigma_Y \varepsilon_Y}{2n}, \quad (32)$$

and substituting Eq. (32) into Eq. (27) gives

$$\varphi = \left(\frac{\varepsilon}{\varepsilon_Y} \right) - \frac{1}{2n}, \quad \varepsilon \geq \frac{\varepsilon_Y}{n}. \quad (33)$$

For a value of $n = 2$, Eqs. (31) and (33) give

$$\varphi = \left(\frac{\varepsilon}{\varepsilon_Y} \right)^2, \quad \varepsilon \leq 0.5 \varepsilon_Y, \quad (21)$$

and

$$\varphi = \left(\frac{\varepsilon}{\varepsilon_Y} \right) - 0.25, \quad \varepsilon \geq 0.5 \varepsilon_Y, \quad (20)$$

which are the same as the equations for the normalized COD design curve proposed by Dawes.⁴ From Eq. (27) it follows that

$$\frac{d\varphi}{d\left(\frac{\varepsilon}{\varepsilon_Y}\right)} = \frac{\sigma}{\sigma_Y}. \quad (34)$$

Consequently, Eqs. (31) and (33) represent a parabola and a straight line that meet at a single point of tangency, and there are no anomalies in the shape of the normalized COD curve proposed by Daves.⁴

APPLICATIONS

For a safety analysis, it is usually necessary to estimate an allowable crack size for a given strain level, and an allowable strain level for a given crack size. Suppose that the following information is given:

$$K_{Ic} = 200 \text{ ksi } \sqrt{\text{in.}}$$

$$\sigma_Y = 65 \text{ ksi}$$

$$\sigma = 36 \text{ ksi}$$

$$\bar{a}_f = 1.0 \text{ in.}$$

where $\bar{a}_f$ is the estimated final effective flaw size. Substituting Eq. (12) into Eq. (8) gives

$$\varphi = \frac{\left(\frac{K_{Ic}}{\sigma_Y}\right)^2}{2\pi\bar{a}_{\max}}, \quad (35)$$

which rearranged gives

$$\bar{a}_{\max} = \frac{\left(\frac{K_{Ic}}{\sigma_Y}\right)^2}{2\pi\varphi}. \quad (36)$$

Since $\epsilon/\epsilon_Y = \sigma/\sigma_Y = 36/65 = 0.554$, φ is determined from Eq. (20) to be 0.304. Then, using Eq. (36), the value of $\bar{a}_{\max}$ is 4.95 in. Thus the final effective flaw size is smaller than the allowable effective flaw size by a factor of 4.95. For the allowable strain corresponding to the

estimated final effective flaw size, the value of φ , from Eq. (35), is 1.50. Then, from Eq. (20), the value of ϵ/ϵ_Y is 1.75, indicating that yielding should precede fracture. In this case, the allowable strain exceeds the applied strain by a factor of 3.16.

DISCUSSION

Another approach to this problem, that produced a mathematically identical result, was the analysis of Begley, Landes and Wilson,⁸ based on the J Integral. The estimating equations for the J Integral given in Ref. 8, when written in terms of the effective flaw size, are

$$J = \epsilon^2 E \bar{\alpha} , \quad \epsilon \leq \epsilon_Y , \quad (37)$$

and

$$J = E \bar{\alpha} \epsilon_Y (2\epsilon - \epsilon_Y) , \quad \epsilon \geq \epsilon_Y . \quad (38)$$

Using Eqs. (25) and (8), Eqs. (37) and (38) reduce to

$$\varphi = \frac{1}{2} \left(\frac{\epsilon}{\epsilon_Y} \right)^2 , \quad \epsilon \leq \epsilon_Y , \quad (39)$$

and

$$\varphi = \left(\frac{\epsilon}{\epsilon_Y} \right) - 0.5 , \quad \epsilon \geq \epsilon_Y \quad (40)$$

which are identical to the equations that result from using $n = 1$ in Eqs. (31) and (33). Thus the result of the analysis developed in Ref. 8 is simply a normalized COD curve with a factor of safety of 1.0. Unfortunately, there is no mention of this fact in Ref. 8.

The fact that an approximate analytical relation has been established between the J Integral, the Equivalent Energy Method and the Normalized COD approach implies that these methods may share a partially common basis

and also that they may be subject to some similar limitations. The close relation between the J Integral and the value of δ is further substantiated by the elastic-plastic finite element calculations performed by Sumpter and Turner,⁹ for flaws emanating from a circular hole in a flat plate. These calculations indicate that J/c_Y and δ do follow very similar trend curves with σ/σ_Y , at least for the geometry investigated. However, Egan¹⁰ has also shown experimentally that different flaw geometries may actually follow different ϕ vs σ/σ_Y trend curves, the most noticeable example studied being the trend curves for surface cracked and center through cracked tensile specimens. The Equivalent Energy Method may be subject to the same kind of limitation.

CONCLUSION

It has been shown that an equation proposed recently in the UK for estimating an allowable flaw size as a function of the critical crack opening displacement can be derived from the equation that gives the Equivalent Energy approximation for the J Integral. It has also been shown that a method proposed more recently by Westinghouse for estimating the J Integral reduces mathematically to the same approach.

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NOMENCLATURE

a	Defect size, generally half-length, in.
$\bar{a}$	Effective defect size, in.
$\bar{a}_f$	Final effective flaw size, in.
$\bar{a}_{\max}$	Allowable effective defect size, in.
C	Shape factor, dimensionless
E	Modulus of elasticity, ksi
E'	Initial tangent modulus of the curve of nominal stress versus nominal strain at the location of a flaw, ksi
ϕ_I	Elastic strain energy release rate for plane strain conditions, in.-kips/in. ²
J	The J Integral, in.-kips/in. ²
K_I	The elastic crack tip stress intensity factor, ksi·in. ^{1/2}
K_{Ic}	Critical value of the elastic crack tip stress intensity factor for plane strain conditions, ksi·in. ^{1/2}
n	Safety factor on flaw size below the reduced yield strain, dimensionless
r_Y	Plastic zone radius, in.
U	Nominal strain energy density, ksi
δ	Crack opening displacement, in.
δ_c	Critical value of the crack opening displacement, in.
ϵ	Strain, in./in.
ϵ_Y	Yield strain, in./in.
σ	Stress, ksi
σ_Y	Yield stress, ksi
φ	Normalized crack opening displacement, dimensionless

Fig. 1. Normalized COD design curve proposed in Ref. 1. The letters A.W. stand for "as welded," and the letters S.R. stand for "stress relieved."

Fig. 2. Elastic-ideally plastic stress-strain curves used to derive the equations of the modified COD design curve.