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by

B. F. Gibson

A. T. Hess

los alamos
scientific laboratory

of the University of California

LOS ALAMOS, NEW MEXICO 87545

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ELASTIC AND CHARGE-EXCHANGE SCATTERING OF
PIONS FROM ^3He AND ^3H

by

B. F. Gibson and A. T. Hens

ABSTRACT

We have examined 1) the elastic scattering of pions from the isodoublet ^3He and ^3H and 2) the single charge-exchange reaction $^3\text{H}(\pi^+, \pi^0)^3\text{He}$ using a formalism which incorporates the π -N multiple scattering to all orders. Emphasis is placed on numerical results which illustrate those features of the differential cross sections that are expected to be of interest to the experimentalist. Realistic nuclear densities corresponding to the form factors of elastic electron scattering were used. Charge-exchange cross sections are presented in terms of angular distributions for both the π^0 and the recoil nucleus. In elastic scattering, Coulomb-nuclear interference effects are significant at incident pion kinetic energies of less than 100 MeV; form factor effects are apparent at large momentum transfer. Comparison of data and theory for $\pi^+ - ^3\text{He}$ with that for $\pi^- - ^3\text{He}$ (or the conjugate $\pi^+ - ^3\text{H}$) will provide a test of the convergence of the fixed scatterer, multiple-scattering formalism utilized in this report.

I. INTRODUCTION

Both the elastic and the charge-exchange scattering of pions from ^3He and ^3H are of current interest, theoretically and experimentally.¹⁻⁵ The elastic scattering cross sections provide information about several aspects of the scattering process. First, the small-angle region reflects the Coulomb-nuclear interference mechanism. Second, the large-angle region is sensitive to the nuclear form factor. Finally, a comparison of $\pi^\pm$ scattering from a given target, or $\pi^+(\pi^-)$ scattering from mirror targets, will provide information about the convergence of the multiple-scattering formulation of the problem: a comparison between cross sections in which two of the target constituents and the projectile are found in the dominant $T = 3/2$ state and in which only one of the target constituents and the projectile are in a pure $T = 3/2$ state.

The charge-exchange reaction $^3\text{He}(\pi^-, \pi^0)^3\text{H}$,

or the isospin reflected $^3\text{H}(\pi^+, \pi^0)^3\text{He}$, is an isobaric analog transition. Unlike previous pion charge-exchange experiments which were only possible by activation techniques,⁶ this reaction permits one to determine angular distributions by directly observing the recoiling residual nucleus. The additional information contained in the angular distribution should help clarify any questions relating to the charge-exchange analog transition beyond the point of whether the total cross section exhibits a minimum near the (3,3) resonance.

Previous calculations for both the elastic-scattering and charge-exchange reactions were carried out using the Glauber multiple-scattering formalism.⁷ While this may be a reasonable approach for small-angle elastic pion-nucleus scattering, the extension to charge-exchange reactions is questionable because of the lack of strong forward peaking in the basic pion-nucleon charge-exchange amplitude,

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an assumption inherent in the Glauber approximation.

In the calculations reported here, we use a formalism free from the small-angle, forward-peaked Glauber approximations as well as the low-density approximation of first-order optical model calculations. This formalism is described briefly in Sec. II. Numerical results for elastic scattering and the single charge-exchange reaction are presented and discussed in Secs. III and IV, respectively. Our conclusions are summarized in Sec. V.

II. FORMALISM

A complete description of the multiple-scattering theory which we have used can be found in Ref. 8. We review the relevant results here. The pion-nucleon t -matrix is assumed to have the form⁹⁻¹² in the laboratory frame

$$\begin{aligned} \langle \vec{q}' | t(\omega) | \vec{q} \rangle = & \lambda_0(\omega) v_0(q) v_0(q') \\ & + \lambda_1(\omega) \vec{q} \cdot \vec{q}' v_1(q) v_1(q'), \end{aligned} \quad (1)$$

where $\omega = (\vec{k}^2 + \mu^2)^{1/2}$ is the pion energy in the π -N center of mass frame and μ is the pion mass. The factor

$$\lambda_\ell(\omega) = \frac{2\ell + 1}{2ik} \frac{\exp(2i\delta_\ell(\omega)) - 1}{k^{2\ell}} \quad (2a)$$

where k is the pion laboratory momentum, ensures that the t -matrix has the right on-shell dependence.¹³ The functions

$$v_\ell(q) = \frac{k^2 + \alpha_\ell^2}{q^2 + \alpha_\ell^2} \quad (2b)$$

describe the off-shell extension of the t -matrix and go to unity on-shell ($q = k$) as they should. The parameters α_ℓ have been determined previously from fits to π -deuteron absorption to be approximately:¹⁴ $\alpha_0 = 500$ MeV/c and $\alpha_1 = 100$ MeV/c. This t -matrix is then inserted into the self-consistent set of multiple-scattering equations of the form

$$\begin{aligned} G_i(\vec{k}, \vec{q}) = & t_i(\vec{k}, \vec{q}) e^{i\vec{k} \cdot \vec{r}_i} \\ & + \sum_{j \neq i} \int \frac{d^3 p}{(2\pi)^3} t_i(\vec{p}, \vec{q}) \\ & \times G_0(k, p) e^{i\vec{p} \cdot \vec{r}_j} G_j(\vec{k}, \vec{p}). \end{aligned} \quad (3)$$

Here t_i is the free pion-nucleon scattering amplitude, G_0 is the pion propagator, and G_i is the amplitude which describes the multiple scattering to all orders, the last scattering taking place on the i^{th} nucleon, and it depends on all A nucleons. The total pion-nucleus amplitude is related to the G_i by

$$F(\vec{k}, \vec{q}) = \sum_{i=1}^A e^{-i\vec{q} \cdot \vec{r}_i} G_i(\vec{k}, \vec{q}). \quad (4)$$

This total pion-nucleus amplitude for elastic scattering must then be averaged over the nuclear wave functions, the $3A$ dimensional integral being done by Monte Carlo techniques. By applying partial wave expansions to the quantities in Eq. (3), using the separable t -matrices of Eq. (1), and applying matrix inversion techniques, one can solve the resulting system of coupled equations exactly to obtain the G_i called for in Eq. (4). Thus, one calculates multiple scattering to all orders in the fixed-nucleon approximation.

For elastic scattering of an incident π^+ , we can write Eq. (3) in the shorthand form

$$G_1^+ = t_1^{++} + t_1^{++} \sum_{j \neq 1} G_j^+. \quad (5a)$$

By allowing for charge exchange, we can construct a set of coupled equations, similar in form to Eq. (5), but containing amplitudes for charge exchange as well as elastic scattering.¹⁵

$$\begin{aligned} G_1^+ &= t_1^{++} + t_1^{++} \sum_{j \neq 1} G_j^+ + t_1^{+0} \sum_{j \neq 1} G_j^0 \\ G_1^0 &= t_1^{0+} + t_1^{0+} \sum_{j \neq 1} G_j^+ + t_1^{00} \sum_{j \neq 1} G_j^0. \end{aligned} \quad (5b)$$

Matrix techniques can then be used to solve for the total charge-exchange amplitudes. G_1^0 , as well as

lastic-scattering amplitudes, G_1^+ .

We have examined a method for accounting for the fact that the successive nucleons which are struck are not at rest but have some initial momentum as well as a final recoil momentum. The $\vec{q} \cdot \vec{q}$ factor in Eq. (1) is replaced by the Galilean invariant form

$$\vec{q}_f \cdot \vec{q}_f = (\vec{q} - \frac{M}{M} \vec{P}_i) \cdot (\vec{q} - \frac{M}{M} \vec{P}_f), \quad (6)$$

where M is the nucleon mass and $\vec{P}_i$ and $\vec{P}_f$ are the initial and final nucleon momenta, respectively. Since each nucleon is struck successively in the various terms in the expansion of Eq. (3), we can use this effective angle transformation in the entire multiple-scattering solution. If we assume that, after a given pion-nucleon scattering, the nucleus recoils with momentum $\vec{Q}$, it is easy to show that $\vec{P}_i = -\frac{A-1}{A} \vec{Q}$ and $\vec{P}_f = \frac{\vec{Q}}{A}$. This implies that in the incident channel a particular component of Fermi motion is selected. Using these values for $\vec{P}_i$ and $\vec{P}_f$ in Eq. (6) and keeping terms of order (v/M) we have

$$\vec{q}_f \cdot \vec{q}_f = \vec{q} \cdot \vec{q} - \frac{M}{A} \frac{Q^2}{2}. \quad (7a)$$

This angle transformation therefore modifies the s- and p-wave pion-nucleon amplitudes; in particular, a sizeable effective s-wave amplitude is induced from the basic π -N p-wave amplitude. (A complete relativistic treatment of this correction introduces in addition an effective d-wave component.) A similar modification to the spin-flip amplitude corresponding to Eq. (1) leads to

$$\vec{\sigma} \cdot (\vec{q}_f \times \vec{q}_f) = \left(1 + \frac{M}{A}\right) \vec{\sigma} \cdot (\vec{q} \times \vec{q}). \quad (7b)$$

III. ELASTIC SCATTERING

Below we present our results for the elastic differential cross sections for incident pion kinetic energies ranging from 25 MeV through 200 MeV. Charge form factors from electron scattering^{16,17} were converted to body form factors¹⁸ which were used to represent the mass distributions of the ground states (see the Appendix). Spin-flip of the

unpaired nucleon in the scattering process was included; this accounts for some 10-20% of the elastic differential cross section in the region of the minimum. Coulomb-nuclear interference effects were included.¹⁹⁻²² In Figs. 1-9 the solid curve represents the $\pi^+ - {}^3\text{He}$ cross section and the dot-dashed curve represents $\pi^- - {}^3\text{He}$. The long dashed curve represents the $\pi^+ - {}^3\text{H}$ cross section, and for 100 and 180 MeV the short dashed curve represents $\pi^- - {}^3\text{H}$. The angle transformation of Eq. (7) is included in all of the curves shown. The variation in the parameters a_0 and a_1 allowed by the π -d capture study reported in Ref. 14 restricts the variations in the calculated cross sections to 11% within the Monte Carlo noise.

We note several trends in the results. First, for low energies (less than about 100 MeV) the differential cross sections for $\pi^- - {}^3\text{He}$ and $\pi^+ - {}^3\text{H}$ exhibit Coulomb-nuclear interference effects at angles out to about 60° . Differences that arise beyond this point are generally due to differences in the form factors, although at very low energies such as 25 MeV, form factor effects are not really visible at any angle. As the incident pion kinetic energy increases, the Coulomb-nuclear interference region moves to smaller angles, where measurement becomes more difficult. For π^- scattering the attractive Coulomb interaction provides constructive interference, so that there is only a single minimum in the angular distribution resulting from the p-wave nature of the dominant π -N scattering amplitude. For π^+ scattering there is destructive interference leading to a second minimum in the angular distribution; for 25 MeV it is evident that the two minima have coalesced.

Second, we note the difference between the $\pi^+ - {}^3\text{He}$ and $\pi^- - {}^3\text{He}$ cross sections at all energies. In the $\pi^+ - {}^3\text{He}$ scattering the projectile and two of the target nucleons are in a pure $T = 3/2$ state; in the $\pi^- - {}^3\text{H}$ scattering the projectile and only one of the target nucleons are in a pure $T = 3/2$ state. Thus at energies above 75 MeV, where the p-wave scattering dominates, one expects the π^+ cross section to be larger than the π^- cross section due to the size of the (3,3) scattering amplitude relative to the other p-wave amplitudes. In fact, the relative sizes of the s-wave scattering amplitudes and the mixing of the s- and p-wave

Fig. 1. Elastic scattering of pions from ${}^3\text{He}$ and ${}^3\text{H}$ at a pion laboratory kinetic energy of 25 MeV. The solid curve depicts $\pi^+ - {}^3\text{He}$ scattering; the dashed curve depicts $\pi^+ - {}^3\text{H}$ scattering; the dot-dashed curve depicts $\pi^- - {}^3\text{He}$ scattering.

Fig. 2. Elastic scattering of pions from ${}^3\text{He}$ and ${}^3\text{H}$ at a pion laboratory kinetic energy of 50 MeV. The solid curve depicts $\pi^+ - {}^3\text{He}$ scattering; the dashed curve depicts $\pi^+ - {}^3\text{H}$ scattering; the dot-dashed curve depicts $\pi^- - {}^3\text{He}$ scattering.

Fig. 3. Elastic scattering of pions from ${}^3\text{He}$ and ${}^3\text{H}$ at a pion laboratory kinetic energy of 75 MeV. The solid curve depicts $\pi^+ - {}^3\text{He}$ scattering; the dashed curve depicts $\pi^+ - {}^3\text{H}$ scattering; the dot-dashed curve depicts $\pi^- - {}^3\text{He}$ scattering.

Fig. 4. Elastic scattering of pions from ${}^3\text{He}$ and ${}^3\text{H}$.

Fig. 5. Elastic scattering of pions from ${}^3\text{He}$ and ${}^3\text{H}$.

Fig. 6. Elastic scattering of pions from ^3He and ^3H .

Fig. 8. Elastic scattering of pions from ^3He and ^3H .

Fig. 7. Elastic scattering of pions from ^3He and ^3H .

Fig. 9. Elastic scattering of pions from ^3He and ^3H .

amplitudes due to the angle transformation ensures that the π^+ cross section is larger beyond the Coulomb-nuclear interference region at the lower pion energies also. (The opposite relation for the $\pi^+ - {}^3\text{H}$ and $\pi^- - {}^3\text{H}$ cross sections holds.)

Third, for the single strong-scatterer cases $\pi^- - {}^3\text{He}$ and $\pi^+ - {}^3\text{H}$, where there is only one $T = 3/2$ π -N amplitude, the minimum near 80° at 180 MeV moves only slightly to larger angles as the pion kinetic energy is lowered. In contrast, that minimum for $\pi^+ - {}^3\text{He}$ (and $\pi^- - {}^3\text{H}$), where there are two $T = 3/2$ π -N amplitudes, moves noticeably to smaller angles as the pion kinetic energy is decreased. Both of these effects result from the mixing of the s- and p-wave amplitudes by the angle transformation of Eq. (7). This same mixing fixes the angle of that minimum in $\pi^+ - {}^3\text{He}$ scattering as a function of pion kinetic energy,²³ but for the $\pi^- - {}^3\text{He}$ ($\pi^+ - {}^3\text{H}$) scattering the single $T = 3/2$ amplitude out of three is insufficient to hold the minimum fixed and it drifts out to larger angles as the pion kinetic energy is decreased, and for $\pi^+ - {}^3\text{He}$ ($\pi^- - {}^3\text{H}$) scattering the two $T = 3/2$ amplitudes out of three is more than sufficient to move the minimum into smaller angles as the pion kinetic energy is decreased. As was pointed out above, this minimum moves far enough in the $\pi^+ - {}^3\text{He}$ case to coalesce with the Coulomb-nuclear interference minimum at 25 MeV.

Because it becomes difficult to separate the Coulomb effects from the differences in the form factors, we have plotted in Figs. 10 and 11 for pion kinetic energies of 100 and 180 MeV the elastic scattering from ${}^3\text{He}$ and ${}^3\text{H}$ assuming that both of the trinucleons have the same ${}^3\text{He}$ density. Thus one can easily differentiate the region where the Coulomb-nuclear interference effects are important and the region where one $T = 3/2$ scattering differs markedly from two $T = 3/2$ scatterings. Clearly, by comparison of these figures with the previous ones, it is possible to ascertain the significant form factor effects at high momentum transfer.

Finally, we note that the success of multiple-scattering calculations (which utilize the fixed-scatterer approximation) in π -d elastic scattering has been attributed by some to the single-scattering dominance of the process;²⁴ i.e., there is only one

$T = 3/2$ π -N amplitude in π -d scattering. If single-scattering dominance is required for a good representation of the scattering process by multiple-scattering calculations, then the $\pi^- - {}^3\text{He}$ and $\pi^+ - {}^3\text{H}$ cross sections presented here should be good predictions, while the $\pi^+ - {}^3\text{He}$ and $\pi^- - {}^3\text{H}$ cross sections should be poor predictions, and multiple-scattering calculations are good only where the impulse approximation is approximately valid. Because we have included the motion of the struck nucleon in the calculation, which leads to the angle transformation of Eq. (7), we do not expect the multiple-scattering formalism to fail. However, the $\pi^+ - {}^3\text{He}$ experiments should test this.

IV. CHARGE EXCHANGE

In Figs. 12-20 we present ${}^3\text{H}(\pi^+, \pi^0){}^3\text{He}$ differential cross-section results for incident pion kinetic energies ranging from 25 through 200 MeV. In these figures we have plotted averages of the cross sections obtained for both the ${}^3\text{H}$ and ${}^3\text{He}$ densities used in the elastic scattering. For other than extreme forward angles, the ${}^3\text{He}(\pi^-, \pi^0){}^3\text{H}$ cross sections differ little from the curves shown. Both spin-flip transitions and the angle transformation in Eq. (7) are included. The spin-flip contribution is significant, filling in the diffractive minimum substantially.⁵

For the benefit of the experimentalist, both the angular distribution of the outgoing π^0 and that for the recoiling residual nucleus are given. The following observations are of experimental interest: 1) Below 100 MeV the angular distribution of the recoiling nucleus is forward peaked. 2) Above 150 MeV that angular distribution peaks strongly near 90° . 3) As the incident pion kinetic energy increases, the energy of the peak in the angular distribution of the recoiling nucleus decreases. 4) For pion energies at or near the (3,3) resonance, there may be difficulty in extracting the recoil nucleus from the target near the peak of the angular distribution.

In Fig. 21 we present the total charge-exchange cross section as a function of energy. This is an analog transition. In heavier nuclei the charge exchange to the analog state exhibits a minimum near the (3,3) resonance⁶ due to the absorption generated

Fig. 10. Elastic scattering of pions assuming identical densities.

Fig. 12. Angular distribution for π^0 and ${}^3\text{He}$ in ${}^3\text{H}(\pi^+, \pi^0){}^3\text{He}$.

Fig. 11. Elastic scattering of pions assuming identical densities.

Fig. 13. Angular distribution for π^0 and ${}^3\text{He}$ in ${}^3\text{H}(\pi^+, \pi^0){}^3\text{He}$.

Fig. 14. Angular distribution for π^0 and ${}^3\text{He}$ in ${}^3\text{H}(\pi^+, \pi^0){}^3\text{He}$.

Fig. 16. Angular distribution for π^0 and ${}^3\text{He}$ in ${}^3\text{H}(\pi^+, \pi^0){}^3\text{He}$.

Fig. 15. Angular distribution for π^0 and ${}^3\text{He}$ in ${}^3\text{H}(\pi^+, \pi^0){}^3\text{He}$.

Fig. 17. Angular distribution for π^0 and ${}^3\text{He}$ in ${}^3\text{H}(\pi^+, \pi^0){}^3\text{He}$.

Fig. 18. Angular distribution for π^0 and ${}^3\text{He}$ in ${}^3\text{H}(\pi^+, \pi^0){}^3\text{He}$.

Fig. 20. Angular distribution for π^0 and ${}^3\text{He}$ in ${}^3\text{H}(\pi^+, \pi^0){}^3\text{He}$.

Fig. 19. Angular distribution for π^0 and ${}^3\text{He}$ in ${}^3\text{H}(\pi^+, \pi^0){}^3\text{He}$.

Fig. 21. Total cross sections for the single charge reaction ${}^3\text{H}(\pi^+, \pi^0){}^3\text{He}$. The dashed curve depicts the non-spin-flip cross section; the dot-dashed curve depicts the spin-flip cross section. The solid curve is the total cross section, the sum of the spin-flip and non-spin-flip components.

by the strong multiple scattering. In contrast, for the trinucleons we do not expect a deep minimum because of the few nucleons present, and this minimum is actually filled in by the spin-flip contribution as shown. We expect to see a cross section having essentially no structure and a maximum value of about 7 mb in the region of the (3,3) resonance.

V. CONCLUSIONS

We have studied the elastic scattering of pions from ^3He and ^3H as well as the isobaric analog reaction $^3\text{H}(\pi^+, \pi^0)^3\text{He}$ using a multiple-scattering formalism in which we utilized realistic pion-nucleon amplitudes which incorporate off-shell effects by means of a separable approximation. For energies of less than 100 MeV, the small-angle elastic-scattering differential cross section is sensitive to Coulomb-nuclear interference effects. Form factor differences are apparent at larger momentum transfer. A comparison of theory with experiment for $\pi^\pm - ^3\text{He}$ should provide a significant test of the convergence properties of the multiple-scattering formalism used. For the charge-exchange reaction no minimum was found for the total cross section; the spin-flip contribution is about 30% at some energies. The π^0 angular distribution

becomes more forward peaked as the energy of the incident pion increases, making it more difficult to extract the recoiling residual nucleus from the target. In addition to studying the structure of the trinucleons, the reaction should be useful in normalizing π^0 detectors.

We conclude with two remarks: 1) The calculations reported here do not include the π -N d-wave, either real or induced when relativistic transformations are used in Eq. (6). Consequently, the cross sections above 130 MeV, while illustrative of the qualitative features to be expected, are quantitatively inadequate at angles beyond the minimum near 90° . 2) The calculations reported here do not include absorption of the type (π, NN) , which is apparently important at low energies. Consequently, the cross sections below 50 MeV may not be quantitatively correct. The inclusion of these two effects must await our ability to incorporate them into the present computer codes.

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APPENDIX NUCLEAR DENSITIES

It is the single-particle density for each nucleon that is required in the multiple-scattering calculations described in this report. These densities are available from elastic electron scattering as the body form factors:

$$F_B(q^2) = F_{ch}(q^2)/F_p(q^2) \quad (A1)$$

Here $F_B(q^2)$ is the body form factor (as a function of the square of the momentum transfer), $F_{ch}(q^2)$ is the charge form factor measured in elastic electron scattering experiments,^{16,17} and $F_p(q^2)$ is the measured proton form factor. The proton form factor can be adequately represented as¹⁸

$$F_p(q^2) = \frac{0.58}{1 + q^2/8.19} + \frac{1.25}{1 + q^2/15.7} - \frac{0.80}{1 + q^2/26.7} - 0.03 \quad (A2)$$

The body form factors that result from the procedure described in Eq. (A1) can be fitted by the sum of exponentials:

$$F_B(q^2) = e^{-a^2 q^2} - b^2 q^2 e^{-c^2 q^2} \quad (A3)$$

For the case of ^3He we obtain parameter values of $a = 0.613$ fm, $b = 0.346$ fm, and $c = 0.846$ fm. For ^3H we obtain parameter values of $a = 0.584$ fm, $b = 0.180$ fm, and $c = 0.738$ fm.

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