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FINITE LARMOR RADIUS STABILIZED Z-PINCHES*

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ABSTRACT

The possibility of Finite-Larmor-Radius stabilization of a straight, diffuse, Z-pinch is considered using the FLR corrected MHD fluid equations. Stabilization of all MHD modes is indicated if a_i/a is sufficiently large. If a_i/a is too large however, current-driven instabilities are found. Comparison with several experiments suggests that a "stability window" may exist. Scaling to reactor conditions is considered and, because of the possibility of operating at very high plasma density, sufficient $n\tau$ for a pulsed D-D system of small size and energy content is suggested.

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I. Introduction

Finite-Larmor-radius stabilization may provide a possible explanation for the long lived toroidal Z-pinch experiment of Es'kov, et al⁽²⁾ and several other Z-pinches which are apparently hydromagnetically stable. In this report general scaling arguments for FLR stabilized Z-pinches are given, microstability is discussed along with FLR stabilization of all MHD modes using fluid equations with the ion pressure tensor with FLR terms. Finally, high density reactor possibilities are considered.

II. Scaling Arguments

FLR effects are important if the number of ion gyroradii in the pinch, $S = a/a_i$ is not too large. From the Bennett relation, for equal ion (Deuterium) and electron temperatures,

$$S = \left[\frac{na^2}{5 \times 10^{14}} \right]^{1/2}, \quad (\text{cm}^{-3} \text{ cm}^2)^{1/2}.$$

Note that the FLR parameter S is independent of the plasma temperature and depends only on na^2 , a fact which was apparently not appreciated by early workers who felt high temperature was necessary to achieve FLR stabilization⁽³⁾. (The condition $\lambda_{ii} > a_i$ is readily satisfied and is given by $I > 1.9Sn^{1/8}$.)

The energy content in plasma per unit length depends only on S and the temperature and is given by

$$U_p' = 3eT\pi a^2 n = 0.75S^2 T_{\text{keV}} \quad \text{joules/cm.}$$

Two types of Z-pinchs of interest can be distinguished, a pulsed linear pinch with end losses, and a flow-thru pinch. For the first type a length $L = \alpha v_{\text{sound}} \tau$ is required where $\alpha = 0.1$ with solid end conductors⁽⁴⁾ and $\alpha \approx 1$ for open ends. The total plasma energy is approximately,

$$U_p = LU_p' = \frac{2.2 \times 10^7 \alpha S^2 (n\tau) T^{3/2} (\text{keV})}{n} \quad \text{joules}$$

The nuclear energy QU_p is plotted vs n in Fig. 1 for typical parameters.

The second type, a flow-thru pinch, may be sustained for a number of axial sound transits thereby increasing the total energy yield each pulse. The nuclear power of a flow-through Z-pinch is roughly.

$$P = QU_p' \alpha v_{\text{sound}}$$

or

$$P = 22.5 QS^2 T_{\text{keV}}^{3/2} \alpha \quad \text{M. watts}$$

for $T_e = T_i$.

III. Stability

Providing sufficiently small S to achieve FLR stabilization of MHD modes places rather strong restrictions on microinstability transport-except at high plasma density. This is illustrated in Fig. 2 which shows the ratio of the confinement time τ to $\tau_{\text{classical}}$, τ_{sound} , and τ_{Bohm} as a function of S for fixed $n\tau$. Roughly speaking, if a reasonable $n\tau$ value is required, τ/τ_{Bohm} is large unless the plasma density is high $n \geq 10^{20} \text{ cm}^{-3}$.

Current-driven instabilities are likely sources of anomalous resistivity, rapid heating, and the concomitant blow-up of the pinch. This conclusion is suggested by noting that, although S decreases with decreasing na^2 as $S \sim \sqrt{na^2}$, the ratio of the electron drift velocity to the ion thermal velocity increases with decreasing n as

$$\frac{v_{\text{De}}}{v_{\text{ti}}} \approx \left[\frac{2.5 \times 10^{15}}{na^2} \right]^{1/2} = \frac{2.2}{S}.$$

In arriving at this equation the Bennett relation was used and the current was assumed to be uniform.

Licwer and Krall⁽⁵⁾ discuss the lower hybrid drift instabilities which appear when $v_{\text{De}} \geq v_{\text{ti}}$. From their analysis, the resistivity η is,

$$\eta \epsilon_0 = \eta_{\text{classical}}$$

$$\text{if } v_{\text{De}} < v_{\text{ti}} \text{ and, } \eta \epsilon_0 \approx \frac{.012}{\omega_{\text{pi}}} \frac{v_{\text{De}}}{c} \left(\frac{M}{m} \right)^{1/2} e^{-v_{\text{ti}}/v_{\text{De}}}$$

if $v_{\text{De}} > v_{\text{ti}}$. The most important aspect of enhanced resistivity is that resistive heating rapidly blows the whole pinch apart or leads to superthermal, low density skin layers which expand also. Balancing ohmic heating, Bremsstrahlung, and expansion work gives

$$I^2 \frac{\eta}{\pi a^2} \approx P_B + 2neT \frac{d}{dt} (\pi a^2).$$

For $\eta = \eta_{\text{classical}}$, the Bennett equilibrium occurs at the Pease-Braginski current $I \approx 10^6$ amperes for which $\frac{d}{dt} (a^2) = 0$. If the resistivity is anomalous then $\eta \gg \eta_{\text{classical}}$, P_B is small, and

$$\frac{I^2 \eta}{\pi a^2} \approx 2neT \frac{d}{dt} (\pi a^2)$$

Using the Bennett relation gives the rate of pinch expansion

$$\frac{d}{dt} (a^2) = 9 \times 10^8 \eta_{\text{ohm-cm}} \text{ cm}^2 - \text{sec}^{-1}.$$

Using the anomalous resistivity expression given earlier,

$$\frac{da}{dt} = \frac{9.9 \times 10^{15}}{na^2} T_{\text{ev}}^{1/2} \exp(-S/2.2) \text{ cm-}\mu\text{s}^{-1}$$

or

$$\frac{da}{dt} = \frac{19.8}{S^2} T_{\text{ev}}^{1/2} \exp(-S/2.2) \text{ cm} - \mu\text{s}^{-1}$$

for $S > 2.2$. Blow-up of the pinch by anomalous heating is very rapid as the above equation shows and indicates the necessity of not letting S become too small.

If $S > 2.2$, but initially skin current is present, enhanced resistivity may be expected to cause a rapid relaxation of the pinch pressure profile. A possible relaxed profile may be close to that described by Bennett⁽⁶⁾ for which

$$P = p_0 / (1 + \frac{r^2}{a^2})^2$$

and for which $v_{De}/v_{ti} = \text{constant}$, $T = \text{constant}$. Following Reference (5), turbulent resistivity effects have been added to the Magpie Code and the above questions are being calculated numerically.

IV. Finite-Larmor-Radius Stabilization of MHD Modes.

The linearized fluid equation with no plasma flow,

$$\rho_0 \frac{\partial^2 \vec{v}}{\partial t^2} = \bar{\nabla}[(\vec{v} \cdot \bar{\nabla})p_0 + \gamma p_0 \bar{\nabla} \cdot \vec{v}] + \bar{J}_0 \times [\bar{\nabla} \times (\vec{v} \times \bar{B}_0)] - \bar{B}_0 \times \bar{\nabla} \times [\bar{\nabla} \times (\vec{v} \times \bar{B}_0)] \quad (1)$$

can be written

$$\rho_0 \frac{\partial^2 \vec{v}}{\partial t^2} = \bar{\nabla} R + \bar{M}.$$

For the z-pinch equilibrium $J_0 = J_0(r)\hat{z}$, $\bar{B}_0 = B_0(r)\hat{\theta}$, $\rho_0 = \rho_0(r)$, the pressure term R is,

$$R = v_r p_0' + \gamma p_0 \bar{\nabla} \cdot \vec{v} \quad (2)$$

and the magnetic-force term is,

$$\begin{aligned} \bar{M} = \hat{r} & \left[2J_0 B_0 (\partial_r v_r + \partial_z v_z) + \partial_r (J_0 B_0 v_r) + \frac{B_0^2}{r^2} (v_r + \partial_\theta^2 v_r) \right. \\ & \left. + B_0^2 (\partial_r^2 v_r - \frac{\partial_r v_r}{r} + \partial_r \partial_z v_z) \right] \\ & + \hat{\theta} \left[\frac{\partial_\theta}{r} (J_0 B_0 v_r) \right] \\ & + \hat{z} \left[\partial_z (J_0 B_0 v_r) + B_0^2 \left(\frac{\partial_\theta^2}{r} v_z + \partial_z^2 v_z + \partial_r \partial_z v_r - \frac{\partial_z v_r}{r} \right) \right]. \end{aligned} \quad (3)$$

Taking $v \sim \exp i\omega t$, Eq. 1 becomes

$$\begin{aligned} -\rho_0 \omega^2 \vec{v} = \hat{r} & \left[\partial_r (\gamma p_0 \bar{\nabla} \cdot \vec{v}) + 2J_0 B_0 (\partial_r v_r + \partial_z v_z) \right. \\ & \left. + \frac{B_0^2}{r^2} (v_r + \partial_\theta^2 v_r) + B_0^2 (\partial_r^2 v_r - \frac{\partial_r v_r}{r} + \partial_r \partial_z v_z) \right] \\ & + \hat{\theta} \left[\frac{\partial_\theta}{r} (\gamma p_0 \bar{\nabla} \cdot \vec{v}) \right] \\ & + \hat{z} \left[\partial_z (\gamma p_0 \bar{\nabla} \cdot \vec{v}) + B_0^2 \left(\frac{\partial_\theta^2}{r^2} v_z + \partial_r \partial_z v_r - \frac{\partial_z v_r}{r} + \partial_z^2 v_z \right) \right] \end{aligned} \quad (4)$$

since $J_0 B_0 = -p_0'$ for equilibrium. Equation 4 admits translation $\bar{v} = v(r)\hat{z}$, rotation $\bar{v} = v(r)\hat{\theta}$, and solid-body displacement $\bar{v} = v_0\hat{x} = (\hat{r}v_0 + \hat{\theta}iv_0)e^{i\theta}$ for $\omega^2 = 0$ consistent with zero restoring force for these motions. The latter motion forms a departure point for long wavelength, $m = 1$ kink modes as will be discussed later.

Fourier analyzing $v \sim \exp i(m\theta + kz)$ gives from Eq. 4,

$$\begin{aligned} -\rho_0\omega^2\bar{v} = \hat{r} \left[\frac{B_0^2}{r^2} (1 - m^2) - 2p_0'(v_r' + ikv_z) + (\gamma p_0 \bar{v} \cdot \bar{v})' + B_0^2 \left(v_r'' - \frac{v_r'}{r} + ikv_z \right) \right] \\ + \hat{\theta} \left[\frac{im}{r} \gamma p_0 \bar{v} \cdot \bar{v} \right] \\ + \hat{z} \left[ik\gamma p_0 \bar{v} \cdot \bar{v} + B_0^2 \left(-\left(\frac{m^2}{r^2} + k^2 \right) v_z + ik \left(v_r' - \frac{v_r}{r} \right) \right) \right]. \quad (5) \end{aligned}$$

The $\hat{\theta}$ component of Eq. (5) gives,

$$\left(1 - \frac{m^2 v_s^2}{r^2 \omega^2} \right) \bar{v} \cdot \bar{v} = v_r' + \frac{v_r}{r} + ikv_z \quad (6)$$

where $v_s^2 = \gamma p_0 / \rho_0$. The $\hat{r}$ and $\hat{z}$ components of Eq. (5) can be combined with Eq. (6) to obtain a differential equation for v_r ,

$$\begin{aligned} \left(x_A + \frac{2}{\beta\gamma} \right) v_r'' + \left[\frac{4W}{r\beta\gamma} + x_A U + \left(\frac{1}{x_s} + \frac{2}{\beta\gamma} \right) \left(W \left(\frac{x_A}{W} \right)' + \frac{x_A}{r} - \frac{4k^2 k_s^{-2}}{r\beta\gamma} \right) \right] v_r' \\ - \left[W \left(k_s^2 + \frac{2K}{r} + \frac{2}{\beta\gamma r^2} \right) + \left(-\frac{x_A}{r} + \frac{4k^2 k_s^{-2}}{r\beta\gamma} \right) U \right] v_r = 0 \quad (7) \end{aligned}$$

where $m = 1$, it is assumed $v_s = \text{constant}$ and,

$$k_s = \omega/V_s$$

$$X_A = 1 - \frac{1}{\beta \gamma k_s^2 r^2}$$

$$X_s = 1 - \frac{1}{k_s^2 r^2}$$

$$W = \frac{2k_s^2 k_s^{-2}}{\beta \gamma} - X_A + k_s^2 k_s^{-2} \left[\frac{1}{X_s} - \frac{1}{\beta \gamma} \right]$$

$$\beta = \frac{2p}{B^2}$$

$$U = \left(2 + \frac{\gamma}{X_s} \right) K - \frac{X_s'}{X_s^2}$$

$$K = \frac{p_0}{\gamma p_0}$$

Since Eq. 7 is too complicated to study in general, consider nearly solid body displacements with long axial wavelengths such that $kr \ll 1$. Clearly $kr \ll 1$ holds only in the center regions of a diffuse pinch extending to $r = \infty$. The component equations of Eq. 5, ($m = 1$) are approximately,

$$-\rho_0 \omega^2 v_r = -2p_0' i k v_z \quad (8)$$

$$-\rho_0 \omega^2 v_z = B_0^2 \left(-\left(\frac{1}{r^2} + k^2 \right) v_z - \frac{i k v_r}{r} \right) \quad (9)$$

where higher order terms in k_r , v_r'' , v_r' , v_z' , and $\bar{\nabla} \cdot \bar{v}$, have been dropped. Neglecting kr terms in Eq. (9) and combining with Eq. (8) gives a growing root $\omega = -i\gamma$, where

$$\gamma = k V_s \sqrt{-Kr} \quad (10)$$

or,

$$\gamma = k V_A \sqrt{-2 \left(\frac{r B_0'}{B_0} + 1 \right)}.$$

Note that kink growth tends to occur only where B_0 falls off less rapidly than r^{-1} , a well-known result obtained from the energy principle.

Next add the off-diagonal, FLR viscosity force term $\bar{\lambda}$ of Roberts and Taylor⁽⁷⁾ to Eq. (5). Strictly speaking $\bar{\lambda}$ is only correct for low β , weakly unstable, small gyroradius systems so that the results obtained pertain only to the outer, low β pinch region where $\gamma \rightarrow 0$; that is, where $rB'_0/B_0 \approx -1$ or where $k \rightarrow 0$. The linearized force term components perpendicular to $\bar{B}_0 = B_0 \hat{z}$ are,

$$\begin{aligned}\lambda_x &= \partial_x [\rho_0(\partial_x v_y + \partial_y v_x)] - \partial_y [\rho_0(\partial_x v_x - \partial_y v_y)] \\ \lambda_y &= -\partial_y [\rho_0(\partial_x v_y + \partial_y v_x)] - \partial_x [\rho_0(\partial_x v_x - \partial_y v_y)]\end{aligned}\quad (11)$$

where the force is $F = v\lambda$ where $v = a_i^2 \Omega_i / 4$. Choosing a local cylindrical coordinate system such that $\hat{x} \rightarrow \hat{r}$, $\hat{y} \rightarrow \hat{z}$, $\hat{z} \rightarrow \hat{\theta}$, corresponding to large radius the components of $\bar{\lambda}$ are

$$\begin{aligned}\lambda_r &= \partial_r [\rho_0(\partial_r v_z + \partial_z v_r)] - \partial_z [\rho_0(\partial_r v_r - \partial_z v_z)] \\ \lambda_z &= -\partial_z [\rho_0(\partial_r v_z + \partial_z v_r)] - \partial_r [\rho_0(\partial_r v_r - \partial_z v_z)]\end{aligned}\quad (12)$$

Adding $i\omega \bar{F}$ to Eq. (5) and following the same procedure used to obtain Eq. (10) we note that only the term $\partial_r [\rho_0 \partial_z v_r]$ enters significantly and obtain the dispersion relation,

$$\omega^2 - kv(\gamma K + \frac{1}{r}) + \gamma^2 = 0. \quad (13)$$

The roots of Eq. (13) are,

$$\omega = \frac{kv(1 + \gamma Kr)}{2r} \pm \sqrt{\left[\frac{kv(1 + \gamma Kr)}{2r}\right]^2 - \gamma^2} \quad (14)$$

or, from Eq. 10, stability occurs when,

$$\frac{kv(1 + \gamma kr)}{2r} > \gamma \quad (15)$$

The local stability condition can be written,

$$\frac{a_i}{r} \geq \frac{8\sqrt{-2Kr}}{1 + \gamma Kr} \quad (16)$$

for the outer regions. Extending Eq. (16) towards the center of the pinch where $\beta > 1$ and $B_0 \sim r$ stability is "suggested" when

$$\frac{a_i}{r} \geq 8/\sqrt{\beta} .$$

The above results are of course only heuristic but they suggest that the kink instability couples to FLR stabilization forces even in the nearly incompressible, long wave length limit. The important result suggested is that stability may be achieved for $a_i/r_p \approx 1$ for all MHD modes of a diffuse Z-pinch. (A similar FLR argument can be given for the sausage modes however marginal stability does exist in this case even in the MHD limit.) Also note that since only local stability conditions are considered, the above arguments carry over the pinches with strong skin current.

To the extent that the preceding FLR arguments are correct, the usual stabilization picture applies as illustrated in Fig. 3. The equilibrium configuration has a net diamagnetic drift velocity along Z which is in the direction of $\bar{k}$ both for the sausage and kink modes. Flute-like perturbations which are purely growing in the MHD limit tend to propagate when FLR effects are included. This comes about because a phase shift between $\hat{n}$ and $\hat{\theta}$ occurs since ions move less than the local

$\hat{E}/B$ drift velocity. For a plane layer driven unstable by gravity the dispersion relation is,

$$\omega = \frac{\omega^*}{2} \pm \sqrt{\frac{\omega^*{}^2}{2} - \gamma^2}.$$

Stability occurs when $\frac{\omega^*}{2} = kV_{DM}/2 \gtrsim \gamma$ which gives results similar to the previous treatment.

V. Experimental Evidence

The foregoing discussion indicates that FLR stabilization would enter at low values of S , or low na^2 where

$$S = \left[\frac{\bar{Z}(1 + \frac{\bar{Z}T_e}{T_i})}{2(M_i/M_D)} \right]^{1/2} \left[\frac{n_e a^2}{5 \times 10^{14}} \right]^{1/2}$$

for a plasma with ions of mass M_i and average charge $\bar{Z}$. Small S suggests that pinches with low $\bar{Z}$, large M_i , and, most importantly, low $n_e a^2$ should give possible evidence for FLR Stabilization. As mentioned earlier the experiment of Es'kov et al falls in the small S category and it appears to be MHD stable.

Early work on snow-plow pinches sheds little light on small S . Since $n_e a^2 = 5 \times 10^{14} \text{ cm}^{-3} - \text{cm}^2$ corresponds to $p_0 a_0^2 = 7 \mu D_2 - \text{cm}^2$, small diameter pinches studies at low pressure (with small $\frac{\bar{Z}}{M_i}$) would be relevant. Most snow-plow pinch experiments^(8,9,10) were conducted in 10 cm or so diameter vessels with $p \geq 10 \mu D_2$. (At $p \leq 10 - 20 \mu D_2$ breakdown could not be achieved or was unreliable.) Typically D_2 pinches had $pa^2 \approx 10^3 \mu\text{-cm}^2$ leading to $S \geq 10$ and sausage and kink modes were seen. To the author's knowledge only one experiment⁽¹¹⁾ was conducted well into the low S region in X_e with $a = 0.6 \text{ cm}$ and $p_{X_e} = 2 \mu$ or $pa^2 = 0.72 \mu\text{-cm}^2$. This experiment was of very short duration $\tau \approx 50 \text{ nsec}$, it had low current $I \approx 60 \text{ KA}$, it was inertia dominated, and very little diagnostic information is available.

The plasma focus is more stable than MHD theory suggests and FLR stabilization has been invoked as an explanation⁽¹²⁾. Although $na^2 \gg 5 \times 10^{14}$ for the focus, it is felt that the effective dimension might be the current skin-depth rather than the plasma radius.

Recently some z-pinches formed in different ways have shown considerable stability. "Exploded wire" plasmas have been studied at $I \leq 1.5$ MA, $\tau \approx 50$ ns using the Gamble II supply⁽¹³⁾. Low Z polyethylene wires were studied for $d > 10\mu\text{m}$. Isotropic neutron emission and good agreement with a 1-D code was obtained for $d \geq 50\mu\text{m}$. Although sausage-like perturbations were observed, no strong necking-off occurred and kink motions were never seen⁽¹⁴⁾ over a pinch duration of up to 100 sound transits across the radius. At $d = 50\mu\text{m}$, $n_{\text{atoms}} a^2 = 10^{16}$ corresponding to $S \approx 8$ ($\bar{Z} \approx 3$, $M_i \approx 6M_D$). Below $d = 50\mu\text{m}$ the discharge becomes anomalously resistive, the neutrons are anisotropic, and very energetic particles appear. This could be the result of the appearance of the lower hybrid or Buneman current driven instabilities.

Microscopic pinches have been observed to occur in several experiments. In the Lee experiment⁽¹⁵⁾ Laser-induced breakdown of a vacuum gap from Fe anode vapor produced small x-ray emitting regions attributed to pinches with $a \approx 15\mu\text{m}$. It is estimated that the electron temperature and ion temperature are about 10 keV the electron density is $n_e \approx 10^{21}$, and the lifetime is 1-10 nsec. This lifetime is roughly 100 radial sound transits suggesting MHD stability. Taking $\bar{Z} \approx 10$, gives approximately the value $S = 3$. Similar microscopic pinches recently reported by Morozov⁽¹⁶⁾ are observed to last up to several microseconds.

VI. Reactor Possibilities

Two features of FLR stabilized pinches stand out for reactor considerations; small energy content, and the possibility of achieving high ion temperature. The first consideration suggests high pulse repetition rate systems in the 1-10 MJ energy release range or, possibly, a pulsed flow-through pinch.^(17,18) (Since only $n > 10^{19} \text{ cm}^{-3}$ is considered it appears likely any system would be pulsed.). High ion temperature and the short length made possible by $n > 10^{19} \text{ cm}^{-3}$ suggest that large $n\tau$ at high T_i may be achieved leading to possibly a DD reactor system.

Thusfar scaling of pinches with $S = a/a_i$, and n has been considered. For the purposes of this section it is more useful to fix S at possible interesting values and determine the scaling requirements as a function of $n\tau$ or $Q = U_{\text{nuclear}}/U_{\text{in}}$. For a straight pinch at high density and with no refueling during the pulse, it appears difficult to reduce the magnetic energy U_M to less than about $10 U_{\text{plasma}} \approx U_{\text{in}}$. Numerical calculations in 1-dimension suggest that Q 's of 2 and possibly as high as 5 might be obtained for DT. Low Q values of 2-5 appear to require direct electrical conversion of the pinch magnetic energy into capacitor charge⁽¹⁹⁾ however, in contrast to earlier studies where multimegavolts and tens of megamperes were required to overcome MHD instability, with FLR stabilization the electrical requirements are considerably reduced.

Figures 4-6 show approximately the $(n\tau)$ dependence of length L , total plasma energy U_p , and the pulse voltage V required to establish the

pinch. In all the figures $T = 100$ keV was chosen. Since, from the Bennett relation,

$$I_{\text{amps}}^2 = 10^6 S^2 T_{\text{ev}}$$

for $T = 100$ keV, the current required is,

$$I \approx 10^6 \frac{S}{3} \text{ amps.}$$

Note that the voltage - current requirements are quite modest.

Roughly speaking, from the figures, a D-D, pulsed pinch with $n\tau \approx 2 \times 10^{16}$ ($Q = U_{\text{nuclear}}/U_{\text{plasma}} \approx 10$) would operate in the parameter range $n = 10^{20} - 10^{21}$, $L \approx 10^3 - 10^4$ cm, $U_p \approx 10^5 - 10^6$ joules ($U_{\text{nuclear}} \approx U_M \approx 10 U_p \approx 10^6 - 10^7$ joules), $V \approx 10^3$ volts, and $I \approx 1$ MA ($S = 3$). The large $n\tau = 2 \times 10^{16}$ requirements would require essentially classical confinement $n\tau_{\text{classical}} \approx 2.5 \times 10^{12} S^2 T_{\text{keV}}^{3/2}$ ($\beta \approx 1$) and a substantial degree of electrostatic confinement of the ions. The requirements of a D-T system are reduced with lower $n\tau \approx 10^{15}$ or, alternatively, larger $n\tau$ would possibly remove the requirement of direct electrical recovery of the pinch magnetic energy.

Roughly speaking the magnetic energy directly associated with plasma confinement in the pinch is $1-2 U_p$ while the remainder $U_M \approx 5-10 U_p$ is stored in the parasitic inductance arising from the separation of the pinch from the walls. In a flow-through pinch, about $2 U_p$ energy must be fed into the pinch each flow-through time τ . If the pinch can be sustained for $5-10\tau$ the effective $Q = \frac{U_{\text{nuclear}}}{U_{\text{in}}}$ is increased by factors of 5 or so over the straight pinch with no refueling making direct recovery of the magnetic energy less crucial.

Morozov⁽¹⁷⁾ has suggested one possible flow-through Z-pinch configuration which appears to be most readily adaptable to low density ($B < 500$ kG) operation. At higher density, discussed here, it may be possible to configure the pinch driving electrodes to achieve ablative refueling.

VII. Summary

If the calculation of FLR stabilization of all MHD modes of a Z-Pinch is correct, the requirements for a high density Z-pinch for fusion are greatly relaxed. If, furthermore, near-classical confinement is possible, a straight pinch of modest length ($10^3 - 10^4$ cm) could provide sufficient $n\tau$ to make possible a D-D fusion system.

Useful electrical power production by a simple pulsed pinch without refueling would probably require direct electrical recovery of the pinch magnetic energy for D-D but possibly not for D-T. If a flow-through pinch mode can be made to work, direct recovery may be unnecessary for high efficiency.

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Figure 1

Nuclear Energy Released vs n for DT and DD Plasma

Figure 2

Scaling of τ/τ_{sound} , τ/τ_{Bohm} , $\tau/\tau_{\text{classical}}$ vs S

Figure 3

Schematic of Sausage and kink Modes.

Figure 4

Pinch Length vs $(n\tau)$

Figure 5

Pinch Plasma Energy vs $(n\tau)$

Figure 6

Pulse Voltage required to supply $U_m = 10U_p$ in $\tau_{\text{rise}} = 0.1\tau$.

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