MASTER

COOLANT MIXING IN LMFBR ROD BUNDLES AND
OUTLET PLENUM MIXING TRANSIENTS

Progress Report

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Reports Issued Under This Contract

A. Quarterly Progress Reports

COO-2245-1 Period June 1, 1972 - November 30, 1972
COO-2245-2 Period December 1, 1972 - February 28, 1973
COO-2245-3 Period March 1, 1973 - May 31, 1973
COO-2245-6 Period June 1, 1973 - August 31, 1973
COO-2245-7 Period September 1, 1973 - November 30, 1973
COO-2245-8 Period December 1, 1973 - February 28, 1974
COO-2245-10 Period March 1, 1974 - May 31, 1974
COO-2245-13 Period June 1, 1974 - August 31, 1974
COO-2245-14 Period September 1, 1974 - November 31, 1974
COO-2245-15 Period December 1, 1974 - February 28, 1975
COO-2245-23 Period March 1, 1975 - May 31, 1975
COO-2245-25 Period June 1, 1975 - August 31, 1975
COO-2245-26 Period September 1, 1975 - November 30, 1975
COO-2245-28 Period December 1, 1975 - February 29, 1976
COO-2245-30 Period March 1, 1976 - May 31, 1976

Reports Issued Under This Contract

B. Topical Reports

E. Khan and N. Todreas, "A Review of Recent Analytical and Experimental Studies Applicable to LMFBR Fuel and Blanket Assembly Design," COO-2245-4TR, MIT, Sept. 1973.

E. Khan, W. Rohsenow, A. Sonin and N. Todreas, "A Simplified Approach for Predicting Temperature Distribution in Wire Wrapped Assemblies," COO-2245-5TR, MIT, September 1973.

T. Eaton and N. Todreas, "Instrumentation Methods for Interchannel Coolant Mixing Studies in Wire-Wrap Spaced Nuclear Fuel Assemblies," COO-2245-9TR, MIT, June 1974.

Y. B. Chen, K. Ip, N. E. Todreas, "Velocity Measurements in Edge Subchannels of Wire Wrapped LMFBR Fuel Assemblies," COO-2245-11TR, MIT, September 1974.

E. Khan, N. Todreas, W. Rohsenow, A. A. Sonin, "Analysis of Mixing Data Relevant to Wire-Wrapped Fuel Assembly Thermal-Hydraulic Design," COO-2245-12TR, MIT, Sept. 1974.

E. Khan, W. Rohsenow, A. Sonin, N. Todreas, "A Porous Body Model for Predicting Temperature Distributions in Wire Wrapped Fuel and Blanket Assemblies of a LMFBR," COO-2245-16TR, MIT, March 1975.

E. Khan, W. M. Rohsenow, A. Sonin, N. Todreas, "Input Parameters to the ENERGY Code (To be used with the ENERGY Code Manual) COO-2245-17TR, MIT, May 1975.

E. Khan, W. Rohsenow, A. Sonin, N. Todreas, "Manual for ENERGY Codes I, II, III," COO-2245-18TR, MIT, May 1975.

P. Carajilescov and N. Todreas, "Experimental and Analytical Study of Axial Turbulent Flows in an Interior Subchannel of a Bare Rod Bundle," COO-2245-19TR.

B. Chen and N. Todreas, "Prediction of Coolant Temperature Field in a Breeder Reactor Including Interassembly Heat Transfer," COO-2245-20TR, MIT, May 1975.

F. Carre and N. Todreas, "Development of Input Data to Energy Code for Analysis of Reactor Fuel Bundles," COO-2245-21TR, MIT, May, 1975.

Reports Issued Under This Contract

B. Topical Reports, Continued

H. Ninokata and N. E. Todreas, "Turbulent Momentum Exchange Coefficients for Reactor Fuel Bundle Analysis," COO-2245-22TR, MIT, June 1975.

R. Anoba and N. Todreas, "Coolant Mixing in LMFBR Rod Bundles and Outlet Plenum Mixing Transients," COO-2245-24TR, MIT, August 1975.

B. Bosy, "Fabrication Details for Wire Wrapped Fuel Assembly Components," COO-2245-27TR, MIT, November 1975.

Ralph G. Bennett and Michael W. Golay, "Interferometric Investigation of Turbulently Fluctuating Temperature in an LMFBR Outlet Plenum Geometry," COO-2245-29TR, MIT, June 1976.

Reports Issued under this Contract

C. Papers and Summaries

Yi Bin Chen, Ka-Lam Ip, Neil E. Todreas, "Velocity Measurements in Edge Channels of Wire-Wrapped LMFBR Fuel Assemblies," American Nuclear Society Transactions Vol. 19, 1974, pp. 323-324.

P. Carajilescov, N. Todreas, "Experimental and Analytical Study of Axial Turbulent Flows in an Interior Subchannel of a Bare Rod Bundle," J. of Heat Transfer, Vol. 98, No. 2, May 1976, pp. 262-268 (Included as Appendix to Quarterly Progress Report, COO-2245-15).

E. Khan, W. Rohsenow, A. Sonin, N. Todreas, "A Porous Body Model for Predicting Temperature Distribution in Wire-Wrapped Fuel Rod Assemblies," Nuclear Engineering and Design, 35 (1975) 1-12.

E. Khan, W. Rohsenow, A. Sonin, N. Todreas, "A Porous Body Model for Predicting Temperature Distribution in Wire-Wrapped Rod Assemblies Operating in Combined Forced and Free Convection," Nuclear Engineering and Design, 35 (1975) 199-211.

Ralph G. Bennett and Michael W. Golay, "Development of an Optical Method for Measurement of Temperature Fluctuation in Turbulent Flows," American Nuclear Society Transactions, Vol. 22, 1975, p. 581.

B. Chen and N. Todreas, "Prediction of the Coolant Temperature Field in a Breeder Reactor Including Interassembly Heat Transfer," Nuclear Engineering and Design 35, (1975) 423-440 (Included as Appendix to Quarterly Progress Report, COO-2245-23).

R. Bennett and M.W. Golay, "Interferometric Investigation of Turbulently Fluctuating Temperature in an LMFBR Outlet Plenum Geometry," Accepted for the ASME Winter Annual Meeting, Dec., 1976, (Included as Appendix in Quarterly Progress Report, COO-2245-30).

Commencing with report COO-2245-30, a new task, TASK IV, which has been added to the contract, was reported. This TASK IV and TASK IID differ in that TASK IV is initially concentrated on thermal analyses using slug and laminar velocity profiles while TASK IID is concentrated on hydrodynamic analyses of turbulent velocity fields.

COOLANT MIXING IN LMFBR ROD BUNDLES AND
OUTLET PLENUM MIXING TRANSIENTS

Contract AT(11-1)-2245

Quarterly Progress Report

June 1, 1976 - August 31, 1976

The work of this contract has been divided into the following Tasks:

TASK I: BUNDLE GEOMETRY (WRAPPED AND BARE RODS)

TASK IA: Assessment of Available Data

TASK IB: Experimental Bundle Water Mixing Investigation

TASK IC: Experimental Bundle Peripheral Velocity Measurements (Laser Anemometer)

TASK ID: Analytic Model Development - Bundles

TASK II: SUBCHANNEL GEOMETRY (BARE RODS)

TASK IIA: Assessment of Available Data

TASK IIB: Experimental Subchannel Water Mixing Investigation

TASK IIC: Experimental Subchannel Local Parameter Measurements (Laser Anemometer)

TASK IID: Analytic Model Development - Subchannels

TASK III: LMFBR OUTLET PLENUM FLOW MIXING

TASK IIIA: Analytical and Experimental Investigation of Velocity and Temperature Fields

TASK IV: THEORETICAL DETERMINATION OF LOCAL TEMPERATURE FIELDS IN LMFBR FUEL ROD BUNDLES

TASK I: BUNDLE GEOMETRY (WRAPPED AND BARE RODS)

TASK IA: Assessment of Available Data and Codes (Kerry Basehore)

This quarter an attempt was made to match the flow data from the Argonne National Lab experimental 61 pin test bundle XX07 with SUPERENERGY by both MIT and ANL. The bundle could be evaluated for the isolated assembly case with poor results due to the exclusion of interassembly heat transfer effects. The more significant multiassembly mock-up failed due to problems with the SUPERENERGY code. Briefly stated, the problems are as follows:

1. The interassembly subchannel geometric linkage in the code will handle only two cases, i.e., like bundles next to each other or 217 pin bundles next to 91 pin bundles. The ANL XX07 is a 61 pin bundle surrounded by EBR-II 91 pin fuel assemblies. The coding will be changed in the future to relax this constraint.
2. Problems with numerical stability developed because of the small interassembly gap (0.027 inches) between the EBR-II fuel assemblies. This problem is explored in more detail in TASK ID of this report.

Also during this quarter a comparison was made between the Westinghouse adiabatic COTEC code and SUPERENERGY (isolated wall boundary condition) and between the multiassembly versions

TRITON and SUPERENERGY (interassembly heat transfer boundary condition) for a seven bundle cluster in the CRBR blanket region. The COTEC and TRITON output used in this comparison is taken from a report given by M. Carelli at the June LMFBR Rod Bundle T&H Working Group meeting. The blanket bundle operating conditions input into SUPERENERGY were as specified by WARD. The method of comparison used was as follows:

1. By using SUPERENERGY with the isolated wall (adiabatic) boundary condition, and varying the ϵ^* and C_1 empirical input parameters, the COTEC temperature profile for blanket assembly A2 along three radial skew was matched. ϵ^* represents a nondimensionalized enhanced eddy diffusivity for heat and C_1 the ratio of the circumferential swirl velocity to the average bundle velocity. The best match between COTEC and SUPERENERGY is shown in Fig. 1, at the axial height corresponding to the top of the heated length and in Fig. 2 at the bundle exit.
2. Using ϵ^* and C_1 best fit determined above the interassembly heat transfer boundary condition with the seven radial blanket bundle cluster, a run was performed with SUPERENERGY in an attempt to match TRITON. Figures 3 and 4 are examples of a typical radial temperature profile at two axial

levels corresponding to the top of the heated length and the bundle exit respectively. As can be seen when Fig. 3 and 4 are compared with their appropriate adiabatic counterparts in Fig. 1 and Fig. 2, interassembly heat transfer effects are quite visible in the bundle centers.

From this study an interesting comparison results. This is the close similarity of the TRITON and SUPERENERGY temperature profiles with the exception that SUPERENERGY seems to be a constant 10 degrees lower than TRITON. This is not a result of an energy imbalance in SUPERENERGY (see Table I) or of an approximation error due to the average set of properties that SUPERENERGY uses. Therefore the difference may lie in some problem not yet found in SUPERENERGY or for that matter TRITON.

This work does emphasize the importance of getting the proper values of C_1 and ϵ^* for blanket bundles. This task is now being experimentally undertaken with 61 pin salt tests.

TASK IB.1: 61-Pin Fuel Bundle Experiments (Alan S. Hanson)

No work was performed this quarter.

TASK IB.2: Experimental Bundle Water Mixing Investigation
Repeat of Laminar and Transition Flow - 61 Pin Fuel Mixing
Experiments (Stuart Glazer)

During the current quarter, salt mixing experiments were temporarily suspended pending an investigation by C. Chiu into flow redistribution caused by the flow separator mounted at the exit plane of the bundle. The study concluded that the flow separator would cause excessive flow redistribution unless its length was reduced to approximately 5 inches, and the twisted wires, inserted in each subchannel to promote mixing, were removed. As a result, the flow separator for the 61 pin fuel bundle has been dismounted from the bundle, its wires have been removed, and is currently awaiting machining in the machine shop.

While the bundle was disassembled, it was discovered that 2 of the 3 injection rods built several years ago by Alan Hanson were unusable due to warpage. It was therefore decided to replace all 3 injection rods with one similar in design to those used by Carl Oosterman. The new injection rods will not have the axial slots throughout the salt injection length characteristic of the old rods. The injection rods have been designed and fabrication is nearly completed.

Trial mixing experiments were attempted during the quarter for the purpose of checkout and debugging of test equipment. As a result of removing the twisted wires from the flow separator, the pressure drop through the bundle was

reduced to the point where the small positive displacement pump can produce enough head to inject salt solution directly into the bundle. Previously, without the small precision pump, it had been necessary to fill a large stainless steel tank with the saline solution, and inject the contents of the tank into the bundle by pressurizing the tank with compressed air. Bypassing the tank and the laboratory compressed air system will simplify the experimental hardware as well as reduce corrosion and air bubble problems in the piping.

One of the students hired to assist in laboratory work, Brad Basler, had done complete trial runs of the mixing experiment. He also has compiled a set of notes describing the experimental procedure including recently adopted changes from that of Alan Hanson, resulting from changes in piping, injection techniques, and probe construction.

During the current quarter, work continued on the automated data acquisition system, which will eventually replace the relatively slow electronics designed and built for Alan Hanson to record the outputs from the salt conductivity probes. The new system is expected to greatly simplify and speed up the data acquisition and recording functions during the mixing tests. Modifications to the Interdata computer enabling it to receive multiplexed signals from up to 512 individual probes, has been nearly completed. Hardware completion and checkout is tentatively scheduled within the first few weeks of the new quarter.

The need was established for a software package to run on the Interdata Computer during the course of the mixing experiments, to facilitate the data collection. This software package is required to accept certain information about each individual set of probe data collected, differentiate between mixing data and calibration data, permit real time examination of the data, store the data in specified formats on low cost disk units, and be structured to permit the running of salt mixing experiments in the most rapid, convenient, and flexible formats possible. An undergraduate student, Ms. Pei-ti Tung, was hired to assist in the writing of this program. The program has been written on a similarly configured Interdata Computer, and is nearly completed. Final checkout of the operating system is planned as soon as the experimental hardware is completed.

The need has also been recognized for a data reduction program. Similar programs, written by Alan Hanson for use in the MIT Information Processing Center with data collected on paper tape, are not adaptable for use with the salt data as collected on floppy disks by the automated data acquisition system. Detailed plans and flow charts have been developed for the data reduction programs, and programming will be accomplished by Ms. Tung during the next quarter. The data reduction program will be designed to run on the Interdata Computer in the Joint Mechanical - Civil Engineering computer facility, which has extensive interactive plotting capability.

Pending the completion of salt mixing tests on the 61 pin blanket bundle by C. Chiu, complete mixing tests will be rerun on the 61 pin fuel bundle, to verify and extend the range of applicability of the data taken by Alan Hanson in the laminar and transition flow regimes. It is anticipated that the automated data acquisition system will be available for use at that time.

TASK IB.3: 217 Pin Mixing Experiments (Stuart Glazer)

During the current quarter, fabrication of the 217 pin bundle flow separator was temporarily suspended due to the urgent priority of the required hardware for the 61 pin fuel and blanket bundles. The 217 pin flow separator is expected to be completed during the new quarter. Fabrication of some of the mounting fixtures for the 217 pin bundle was continued. Additional salt conductivity probes for use in the mixing tests were fabricated.

TASK IB.4 61 Pin Blanket Bundle Experiment (Chong Chiu)

In this quarter, experimental results on the flow split have been obtained at the following Re numbers

- (1) 1700 (Pitot tube measurement)
- (2) 3200 (isokinetic)
- (3) 4343 (isokinetic)
- (4) 5452 (isokinetic)
- (5) 8160 (isokinetic)
- (6) 9650 (Pitot tube measurement)

Moreover, two data reduction programs have been developed to reduce the data stated above. The results of this experiment will be issued as a topical report in the near future.

The flow split in terms of V_2/V_1 , where V_2 and V_1 are the averaged velocity for the edge and interior channels respectively, is higher than that predicted by Novenstern's geometric correlation. At this time, this discrepancy is believed due to the different friction factors for these two types of channels.

TASK IC: Experimental Bundle Peripheral Velocity Measurement (Laser Anemometer)

No work was performed this quarter since the equipment is being used for TASK III experiments.

TASK ID: Analytic Model Development - Bundles

TASK ID.1: Steady State SUPERENERGY (Kerry Basehore)

During this quarter an attempt was made to develop an expression to predict stability limits. This was indicated when a SUPERENERGY evaluation of an EBR-II type fuel assembly failed due to an instability (see TASK IA: Assessment of Available Data). The stability expressions were developed quite simply by using a nodal energy balance on the various subchannel types (see Fig. 7) and requiring that the temperature at subchannel 0 remain positive. The following expressions for a minimum axial step size result:

1. Interior subchannels (Fig. 5)

$$\Delta Z_A \leq \frac{A_1 \rho C_p V_1}{(P-D)} \left[\frac{\eta_1 \eta_A}{2\eta_A (k + \rho C_p \epsilon_{H_1}) \eta_1 (k + \rho C_p \epsilon_{H_A})} \right] \quad (1)$$

where the subscript

A = 1 if node 3 is interior subchannel

A = 2 if node 3 is an edge subchannel

and

A_1 = interior subchannel flow area

V_1 = interior subchannel average fluid axial velocity

η_1 = interior to interior subchannel nodal distance

η_2 = interior to edge subchannel nodal distance

ϵ_{H_1} = effective enhanced eddy diffusivity between interior subchannels

ϵ_{H_2} = effective enhanced eddy diffusivity between interior and edge subchannels

P = pin pitch

D = pin diameter

ρ = coolant density

C_p = coolant specific heat

k = coolant conductivity

2. Edge subchannel (Fig. 5)

$$\Delta Z_B \leq \frac{\rho C_p A_2 V_2}{\left[\frac{(P-D)}{\eta_2} (k + \rho C_p \epsilon_{H_2}) + \frac{g}{\eta_A} (k + \rho C_p \epsilon_{H_A}) + \frac{g}{\eta_A} (k + \rho C_p \epsilon_{H_B}) + \frac{P}{R_2} + \rho C_p g V_\theta \right]} \quad (2)$$

where the subscript

A = 4 if node 2 is a corner subchannel

A = 3 if node 2 is an edge subchannel

B = 4 if node 3 is a corner subchannel

B = 3 if node 3 is an edge subchannel

and

A_2 = edge subchannel flow area

V_2 = edge subchannel average fluid axial velocity

η_3 = edge to edge subchannel nodal distance

η_4 = edge to corner subchannel nodal distance

ϵ_{H_3} = effective enhanced eddy diffusivity between edge subchannels

ϵ_{H_4} = effective enhanced eddy diffusivity between edge and corner subchannels

g = width of the gap between the duct wall and neighboring fuel pin

V_θ = azimuthal velocity in the edge and corner subchannels

R_2 = $1/h_2 + S_1/2 kw$, inverse equivalent heat transfer coefficient for the edge subchannel

h_2 = heat transfer coefficient between the edge subchannel and the inside duct wall

S_1 = duct wall thickness

kw = conductivity of the duct wall

3. Corner subchannel (Fig. 5)

$$\Delta Z_5 \leq \frac{\rho C_p V_3 A_3}{\left[\frac{2g}{\eta_4} (k + \rho C_p \epsilon_{H_4}) + \frac{X_1}{R_3} + \rho C_p V_{\theta} g \right]} \quad (3)$$

where

A_3 = corner subchannel flow area

V_3 = corner subchannel average fluid axial velocity

R_3 = $1/h_3 + S_1/1.732$ kw; inverse equivalent heat transfer coefficient for the corner subchannel

h_3 = heat transfer coefficient between the corner subchannel and the inside duct wall

X_1 = length of the inside duct wall corresponding to the corner subchannel

4. Duct wall

Not important in stability considerations since, the temperature obtained for the duct is a simple resistance averaging of the neighboring bundle and gap subchannels.

5. Gap edge subchannel (Fig. 6)

$$\Delta Z_c \leq \frac{\rho C_p V_6 A_6}{\left[\frac{P}{R_6} + \frac{2kP}{S_2} + \frac{S_2 k}{\eta_A} + \frac{S_2 k}{\eta_B} \right]} \quad (4)$$

where the subscript

A = 3 if node 2 is a gap edge subchannel

A = 6 if node 2 is a corner edge subchannel

B = 3 if node 3 is a gap edge subchannel

B = 6 if node 3 is a gap corner subchannel

$C = 6$ if node 2 is a gap edge subchannel

$C = 7$ if node 2 is a gap corner subchannel

and

A_6 = gap edge subchannel flow area

V_6 = gap edge subchannel average fluid axial velocity

$R_6 = 1/h_6 + S_1/2 \text{ kw}$; inverse equivalent heat transfer coefficient for the gap edge subchannel

h_6 = heat transfer coefficient between the gap edge subchannel and the outside duct wall

S_2 = width of the interassembly gap

η_6 = gap edge to gap edge subchannel nodal distance

η_7 = gap edge to gap corner subchannel nodal distance

6. Gap corner subchannel (Fig. 6)

$$\Delta Z_8 \leq \frac{\rho C_p V_7 A_7}{\left[\frac{X_2}{R_7} + \frac{2X_3 k}{S_2} + \frac{2S_2 k}{\eta_6} \right]} \quad (5)$$

where

A_7 = gap corner subchannel flow area

V_7 = gap corner subchannel average fluid axial velocity

X_2 = length of the outside duct wall corresponding to the gap corner subchannel

X_3 = length of the gap centerline corresponding to gap corner subchannel

$R_7 = 1/h_7 + S_1/1.732 \text{ kw}$; inverse equivalent heat transfer coefficient for the gap corner subchannel

h_7 = heat transfer coefficient between the gap corner subchannel and the outside duct wall

The total number of axial steps necessary for a given bundle then is simply

$$N = \frac{L}{\Delta Z_{\min}}$$

where

L = bundle axial height

$\Delta Z_{\min}$ = minimum axial step size from the above relations

For some assemblies the above limits put severe economic restrictions on the applicability of SUPERENERGY use for bundles with low gap flow rates or small gap widths, since Eqs. (4) and (5) appear generally to be the most limiting. For example, to use SUPERENERGY to analyze an EBR-II type fuel assembly (91 pins, interassembly gap $S_2 = 0.027"$) with a gap flow rate of 50 lbm/hr would require about 9800 axial steps. To get around this restriction it was decided to develop a conduction solution across the interassembly gap. This enables more economic solutions with only a small loss of accuracy due to the inherent stagnant gap flow assumption necessary. This solution technique and a more detailed discussion of stability will be explored in a topical report to be released next quarter.

TASK ID.2 Analytical Model Development - Transient SUPERENERGY
(Stuart Glazer).

During the current quarter, much effort was devoted to the development of an analytical method of calculation of transient heat flux from fuel rods. An accurate fuel model

of this type is necessary, as the thermal capacitance of the fuel rods significantly affect coolant temperatures during flow, power, and inlet coolant temperature transient events.

The requirements for a fuel model to be used in the transient version of SUPERENERGY are very fast running time, minimum storage requirements, and ease of interface with the coolant temperature model. The determination of the temperature distribution in fuel elements is not strictly required by SUPERENERGY in the calculation of coolant temperatures, since SUPERENERGY models the heat flux from the fuel elements as internal heat generation within the coolant subchannels. Therefore it is only necessary to know the heat flux from the rods as a function of time.

One commonly used technique to determine heat fluxes from fuel elements is a finite difference scheme which requires multiple radial modalization of the rod. Typically, for a high accuracy solution, 10 or more nodes may be required in the fuel rod. Consequently, running times are long, and storage requirements are large. An alternative model tested during this quarter involved the idealization of an entire fuel rod over an axial slice as a single lumped mass with a uniform temperature. The mass of the element is adjusted by means of an empirical factor. This factor is chosen for fuel pins of particular geometries so as to make the total instantaneous heat flux from the element match

the actual instantaneous heat flux as determined by finite difference codes, using many nodes in the fuel and clad to achieve high accuracy.

Several individual analyses were performed to compare the solution yielded by this effective thermal capacitance model with the "exact" solution. Since the spectrum of transients for this Clinch River Breeder Reactor Plant includes power and inlet coolant temperature variations with time, analyses were performed assuming exponential variation of rod power production and coolant temperature with time for a single region lumped model. A second two region lumped model, was formulated by representing the clad as one lumped region, and modeling the fuel as a second lumped region. An additional analysis permitting exponential variation in bundle flow rate, and hence the heat transfer coefficient from the clad surface to the coolant, was also initiated.

A typical fuel pin for the CRBRP reactor is shown schematically in Fig. 8. By suitably weighting the mass of the clad and the mass of the fuel, a single effective mass m is determined for the fuel pin shown in Fig. 8. Similarly, a weighted mean specific heat is also found for the entire rod. An energy balance written for an element of a fuel rod yields:

$$\rho_m C_p \frac{dT}{dt} = q''' V - h A_s (T - T_c) \quad (1)$$

where

m = wieghted mean mass

C_p = weighted mean specific heat

Γ = empirical factor

q''' = instantaneous volumetric power production
in the fuel rod

V = volume of element

h = heat transfer coefficient from the clad to
the coolant

A_s = clad surface area

D_p = clad OD

D_f = fuel OD

T = instantaneous element lumped temperature

T_c = instantaneous coolant temperature

t = time

and the initial condition is

$$T(o) = T_c(o) = \frac{q'''(o)D_p}{4h} \quad (2)$$

For the two region lumped model, the energy balance yields:

Region I (Fuel)

$$m_1 C_{p1} \Gamma_1 \frac{dT_1}{dt} = q''' V_1 - h_g A_g (T_1 - T_2) \quad (3)$$

Region II (Clad)

$$m_2 C_{p2} \Gamma_2 \frac{dT_2}{dt} = h_g A_g (T_1 - T_2) - h A_s (T_2 - T_c) \quad (4)$$

and the initial conditions are:

$$T_1(o) = T_c(o) + q'''(o)V_1\left(\frac{1}{hA_s} + \frac{1}{h_g A_g}\right) \quad (5)$$

$$T_2(o) = T_c(o) + \frac{q'''(o)V_1}{hA_s} \quad (6)$$

where subscripts 1 and 2 refer to the fuel and clad, respectively, and

h_g = effective heat transfer coefficient between the fuel OD and the clad ID

A_g = surface area of the clad fuel interface

The instantaneous power generation and coolant temperature forcing function were specified to be of exponential form. As shown in Figs. 9 and 10, the power generation and coolant temperature were defined as:

$$q'''(t) = q_o'''(1 - e^{-t/\tau_1}) \quad (7)$$

and

$$T_c(t) = T_{co}''' + T_{c\infty}(1 - e^{-t/\tau_2}) \quad (8)$$

where τ_1 and τ_2 are the time constants of the exponential forcing functions.

q_o''' = initial power generation

q_{∞}''' = total change in power generation during the transient

T_{co} = initial coolant temperature

$T_{c\infty}$ = total change in coolant temperature during the transient

The solution for instantaneous heat flow from the rod from Eq. (1) subject to the initial condition (2) and the forcing functions (7) and (8) is:

$$\begin{aligned}
 q''(t) = h \left\{ \left[-T_{c\infty} - \frac{q_{\infty}''' D}{4h} - \frac{\pi D^2 L q_{\infty}''' \tau_1}{4mC_p \Gamma - 4h\pi D L \tau_1} - \frac{h\pi D L T_{c\infty} \tau_2}{mC_p \Gamma - h\pi D L \tau_2} \right] e^{-\frac{h}{mC_p \Gamma} t} \right. \\
 + \frac{(q_0''' + q_{\infty}''') D}{4h} + \left[\frac{\pi D^2 L q_{\infty}''' \tau_1}{4mC_p \Gamma - 4h\pi D L \tau_1} \right] e^{-t/\tau_1} \\
 \left. + \left[\frac{h\pi D L T_{c\infty} \tau_2}{mC_p \Gamma - h\pi D L \tau_2} \right] e^{-t/\tau_2} + T_{c\infty} e^{-t/\tau_2} \right\} \quad (9)
 \end{aligned}$$

The solution for instantaneous heat flux from the rod from Eqs. (3) and (4), subject to initial conditions (5) and (6) and the forcing functions (7) and (8) is:

$$\begin{aligned}
 q''(t) = h_s \left\{ \frac{1}{(S_2 - S_1)} \left\{ \frac{(H\tau_1)(1 - S_2\tau_1)}{1 - A\tau_1 + B\tau_1^2} + \frac{(F\tau_2)(1 - S_2\tau_2)}{1 - A\tau_2 + B\tau_2^2} \right. \right. \\
 \left. \left. + S_2 \left(\frac{q_0''' V_1}{hA_s} + T_{co} - \frac{E+G}{B} \right) \right\} \cdot e^{-S_1 t} \right.
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{1}{(S_2 - S_1)} \left\{ \frac{(H\tau_1)(S_1\tau_1 - 1)}{1 - A\tau_1 + B\tau_1^2} + \frac{(F\tau_2)(S_1\tau_2 - 1)}{1 - A\tau_2 + B\tau_2^2} \right. \\
 & \left. - S_1 \left[\frac{q_o''' V_1}{hA_s} + T_{co} - \frac{E+G}{B} \right] \right\} \cdot e^{-S_2 t} \\
 & + \left(\frac{E+G}{B} \right) - (T_{co} + T_{c\infty}) + \left(\frac{H\tau_1^2}{1 - A\tau_1 + B\tau_1^2} \right) e^{-t/\tau_1} \\
 & + \left\{ \left(\frac{F\tau_2^2}{1 - A\tau_2 + B\tau_2^2} + T_{c\infty} \right) \cdot e^{-t/\tau_2} \right\}
 \end{aligned}$$

where

$$A = \frac{\left[h\pi D_p L + h_g \pi D_f L \left(1 + \frac{m_2 C_{p2} \Gamma_2}{m_1 C_{p1} \Gamma_1} \right) \right]}{m_2 C_{p2} \Gamma_2}$$

$$B = \frac{(h_g \pi D_f L)(h \pi D_p L)}{(m_1 C_{p1} \Gamma_1)(m_2 C_{p2} \Gamma_2)}$$

$$C = \frac{(h_g \pi D_f L)(\pi D_f^2 L)}{4(m_1 C_{p1} \Gamma_1)(m_2 C_{p2} \Gamma_2)}$$

$$E = B(T_{co} + T_{c\infty})$$

$$F = T_{c\infty} \left(-B + \frac{h\pi D_p L}{m_2 C_{p_2} \Gamma_2} \frac{1}{\tau_2} \right)$$

$$G = C(q''_0 + q''_\infty)$$

$$H = -Cq''_\infty$$

L = length of axial rod segment

For the purpose of solution comparison against each other and against the "exact" solution as obtained by running code COBRA III-C, dimensions and properties corresponding to typical CRBRP fuel and blanket rods were used as inputs. Initially, the lumped models were tested with only exponential changes in rod power, with time constants of 0, 1 and 10 seconds.

Results of this study are shown in Figs. 11 to 22. As can be seen, the 2 region lumped model does not yield significant accuracy improvements over the single region lumped model. Additionally, while the single region lumped model yields reasonable results for the case of the fuel pin, deviations from the finite difference solution are considered unacceptably large for the more massive blanket bundle pin.

As a result of this analysis, it was decided that neither the single no. 2 region lumped models were feasible for use in SUPERENERGY. It has been tentatively decided to use an alternative technique to determine

transient heat flux from the fuel rods, one which can additionally provide the temperature profile in the rods. This technique, employing the use of cubic Hermite polynomials in a collocation scheme, has been investigated by T. Chawla at ANL, and has been found to provide the temperature profile, and hence the heat flux, with extremely high accuracy, while requiring relatively little storage and computation time. During the current quarter, the collocation program written by Dr. Chawla, as well as information regarding its use and necessary modifications, were obtained from ANL. Plans call for the incorporation of the collocation techniques into the transient version of SUPERENERGY which is now being written. Completion of the code is expected during the next quarter.

FIGURE 1

COOLANT TEMPERATURE GRADIENT
TOP OF BUNDLE A2

$$\epsilon^* = 0.07$$

$$C_1 = 0.15$$

FIGURE 2

FIGURE 3

FIGURE 4

INTERIOR SUBCHANNELS

EDGE SUBCHANNELS

CORNER SUBCHANNELS

FIGURE 5

GAP EDGE SUBCHANNELS

GAP CORNER SUBCHANNELS

FIGURE 6

SUBCHANNELS IN AN ASSEMBLY

1. Interior subchannel	5. Duct corner subchannel
2. Edge subchannel	6. Gap edge subchannel
3. Corner subchannel	7. Gap corner subchannel
4. Duct edge subchannel	

Figure 1

Figure 8 Schematic of Fuel Rod

Figure 9. Power Generation vs Time

Figure 10. Temperature vs Time

DIMENSIONLESS HEAT FLUX, $\frac{Q''(T) - Q''(0)}{Q''(\infty) - Q''(0)}$

EXACT (DISTRIBUTED) ANALYSIS V.S.
SINGLE REGION LUMPED MODEL ANALYSIS
FOR POWER TRANSIENT ONLY, STEP DROP
OF POWER TO .03% INITIAL POWER, FOR FUEL PIN

$D_{\text{fuel}} = .200''$
 $D_{\text{clad}} = .230''$
 $h = 20000 \frac{\text{BTU}}{\text{hr} \cdot \text{ft}^2 \cdot ^\circ\text{F}}$
 $K_{\text{fuel}} = 1.5 \frac{\text{BTU}}{\text{hr} \cdot \text{ft} \cdot ^\circ\text{F}}$
 $K_{\text{clad}} = 12.4 \frac{\text{BTU}}{\text{hr} \cdot \text{ft} \cdot ^\circ\text{F}}$
 $Bi = \frac{hD}{K_{\text{mean}}} = \frac{(20000 \times .01917')}{4.159}$
 $Bi = 92.2$

Figure 11

Figure 12

Figure 13

Figure 14

Figure 15

LUMPED MODEL ANALYSIS FOR POWER TRANSIENT
ONLY, TIME CONSTANT OF EXPONENTIAL DROP =
10.0 SECONDS, FOR FUEL PIN

Figure 16

$C_{p_{fuel}} = .059 \frac{\text{BTU}}{\text{lbm} \cdot ^\circ\text{F}}$
 $D_{fuel} = .485''$
 $D = .520''$
 $h = 20000 \frac{\text{BTU}}{\text{hr} \cdot \text{ft}^2 \cdot ^\circ\text{F}}$
 $K_{fuel} = 1.5 \frac{\text{BTU}}{\text{hr} \cdot \text{ft} \cdot ^\circ\text{F}}$
 $K_{clad} = 12.4 \frac{\text{BTU}}{\text{hr} \cdot \text{ft} \cdot ^\circ\text{F}}$
 $Bi = \frac{hD}{k_{mean}} = \frac{(20000)(.0433)}{2.721}$
 $Bi = 318.3$

Figure 17

Figure 18

DIMENSIONLESS HEAT FLUX, $\frac{Q''(T) - Q''(0)}{Q''(\infty) - Q''(0)}$

Figure 19

Figure 20

EXACT (DISTRIBUTED) ANALYSIS V.S. TWO REGION
LUMPED MODEL ANALYSIS FOR POWER TRANSIENT
ONLY, TIME CONSTANT OF EXPONENTIAL POWER
DROP = 1.0 SEC, FOR BLANKET BUNDLE PIN

Figure 21

Figure 22

TABLE I
ASSESSMENT OF ENERGY BALANCE IN SUPERENERGY

Assembly	Q _{TOTAL IN} (kw)	ΔH _{assemb} (kw)	ΔH _{gap} (kw)
A2	747.6	673.1	45.2
B3	430.4	343.5	42.8
B2	710.4	668.3	46.3
B1	1185.0	1210.5	51.1
A1	1204.2	1206.9	50.3
E1	544.8	535.4	39.6
C3	427.3	345.0	43.3
TOTAL	5249.7	4982.7	318.6

$$\text{Error in energy balance} = \frac{5249.7 - (4982.7 + 318.6)}{5249.7} \times 100$$

$$= 0.983\%$$

TASK II: SUBCHANNEL GEOMETRY (BARE RODS)

TASK IIA: Assessment of Available Data

No new data was published which could be assessed this quarter.

TASK IIB: Experimental Subchannel Water Mixing Investigation
(Joseph M. Kelly)

Mixing tests begun in June were completed this quarter. Point concentration measurements in the gap region were initiated this quarter. Reduction of the data was initiated. This work at the P/D of about 1.12 will be completed and reported next quarter.

TASK IIC: Experimental Subchannel Local Parameter
Measurments (Wall Shear Stress Measurement
and Laser Doppler Anemometry)

TASCK IIC.1: Wall Shear Stress Measurements (Mohammed Fakory)

A literature survey was presented which concluded that the best available subchannel test section for doing this experiment is the subchannel test section which is being used by Task IIB.1. This test section hopefully will be available by early November. In order to measure total pressure point by point in peripheral direction around the rad, some design changes are necessary on this test section. Initial design for changing the subchannel test section to required form is done and as soon as the experiments of Task IIB.1 on the test section are finished, modifications of the test section to the desired form will be started.

TASK IIC.2 Laser Doppler Anemometry Measurements (John Bartzis)

In the present work the two-channel MARK II LDA arrangement [1] is used which is also used by Y.B. Chen in Task III.

The optical parameters are shown in Table I. As we can see from this Table we are going to use a 120 mm lens instead of 300 mm used by Chen. The reason is to decrease our spatial resolution error.

1.1 Calculation of the Parameters

The parameters of interest are

$$\overline{v_z}, \overline{v_r}, \overline{v_\theta}, \overline{v_z'^2}, \overline{v_r'^2}, \overline{v_\theta'^2}, \overline{v_r'v_z'}, \overline{v_\theta'v_z'}$$

The parameters that are convenient to be measured are

$$\overline{v_z}, \overline{v_r}, \overline{v_\theta}, \overline{v_z'^2}, \overline{v_x'^2}, \overline{v_y'^2}, \overline{v_x'v_z'}, \overline{v_x'v_z'}$$

The relations between these two groups of parameters are simple and are given in Table II. Essentially similar relations hold for the errors of the quantities of interest as functions of the errors of the quantities to be measured. Thus the error analysis will be presented in terms of the quantities to be measured.

1.2 Error Analysis

1.2.1 Turbulent Quantities vs Measured Quantities

The values to be obtained from the experiments for the normal turbulent stresses will not be the real ones since measured values include a series of Broadening effects.

The most important of these are:

1. Doppler Broadening (σ_D^2)
2. Electronics Broadening (σ_E^2)

$$\overline{v_{Turb}^{'2}} \approx (\overline{v^{'2}})_{Meas} - (\overline{v^{'2}})_D - (\overline{v^{'2}})_E \quad (1.1a)$$

We can transform these symbols to more familiar error notation

$$\sigma \equiv (\overline{v^{'2}})^{1/2}$$

$$\sigma^2 \equiv (\overline{v^{'2}})$$

existing thus

$$(\overline{v^{'2}})_{Turb} \approx (\overline{v^{'2}})_{Meas} - \sigma_D^2 - \sigma_E^2 \quad (1.1b)$$

For the turbulent shear stresses $\overline{v_i^{'2}v_j^{'2}}$ the broadening effects are insignificant since $v_i^{'2}$ and $v_j^{'2}$ signals are not the same and their correlation within the beam intersection region is rather insignificant.

1.2.1.1 Doppler Broadening

The causes for the Doppler broadening can be classified as follows:

1) Laser Light Linewidth.

It is expected to be very small and so it does not create any problem

2) Scattering Particle Brownian Motion.

Such an effect is considered to be important to very low velocity flows. Thus in our case it is not taken into consideration.

3) Finite Size of the Intersection Region.

In this category fall the main causes of Doppler Broadening. They can be distinguished as

a. Finite Transit Broadening

b. Velocity Gradient Broadening.

1.2.1.1.1 Finite Transit Time Broadening

The finite Transit Broadening is due to the fact that a particular particle scatters light only for a finite time — the time required to cross the intersection volume. Assuming that the aperture of photomultiplier is such that all the scattered light in the intersection region is heterodyned, the frequency broadening due to transit time effect is expected to be given by a relation [2]

$$(\overline{v'^2})^{1/2} = \text{CONST} \frac{U}{\ell} \quad (1.2)$$

where U is the mean particle velocity and ℓ a length scale representing the finite size of the probe volume and Dv_D is the rms value of Doppler frequency.

It is evident that ℓ will depend mainly on

- a. Beam waist at the intersection region (probe volume)
- b. Velocity vector orientation
- c. Angle of intersection of the two beams.

Thus a precise knowledge of $v_D'^2$ is rather difficult to obtain.

In the first order it is reasonable to take ℓ as the waist diameter $2b_0$ of the beam at the intersection region [3],

i.e.,

$$(\overline{v'^2})^{1/2} \approx \frac{U}{\pi\sqrt{2}(2b_0)} \quad (1.3)$$

or

$$\sigma_D^2 = C^2 U^2 / 2\pi^2 (2b_0)^2 = 8.92 \times 10^{-5} U^2 \quad (1.4)$$

Preliminary calculations show that

$$\frac{\sigma_D^2}{(\overline{v^2})_{\text{Turb}}} \sim 2\% \quad (1.5)$$

1.2.1.1.2 Velocity Gradient Broadening

The Velocity Gradient Broadening is due to the fact that the velocity is not constant within the intersection volume. Thus the velocity gradient produces a frequency spread which is not symmetric. Thus besides its effect on accuracy of the rms velocity, it affects the accuracy of the mean value as well. An exact analytical expression for velocity gradient broadening is very difficult to obtain. Approximate expressions have been obtained by many authors [4], [5], [2]. Melling [3] considers Gaussian intensity distribution within the elliptic intersection region and velocity gradient only along one of the principal axis of the elliptic control volume whose length is $2\sigma_1$ (see Fig. 1).

Then

$$U_{\text{Meas}} - U \approx \frac{\sigma_1^2}{2} \left(\frac{\partial^2 u}{\partial y^2} \right)_0 + \dots \quad (1.6)$$

$$\sigma_U^2 = \sigma_1^2 \left(\frac{\partial u}{\partial y} \right)^2 + \dots$$

1.2.1.2 Electronics Broadening

The Electronics broadening comes mainly from the uncertainty in the frequency Tracker read out which is 1% f.s.d. or 0.1V.

Translated into velocity

$$U\left(\frac{m}{s}\right) = C\left(\frac{m/s}{MHz}\right) v_{max}(MHz) \quad (1.7)$$

where C is the calibration factor given in Table II.1 and v_{max} is the maximum frequency in MHz. We consider here a uniform distribution. Thus

$$\sigma_E^2 \frac{\Delta U^2}{3} = 1.67 \times 10^{-4} v_{max}^2$$

Preliminary calculations indicate that the introduced broadening is of order 2-3%.

1.2.2 Error Analysis in Measured Quantities

1.2.2.1 Mean Values (i.e., v_x, v_y, v_z)

We can distinguish two main types of error.

1. Time average error (ϵ_1^T)
2. Reading error (ϵ_1^R)

Reading error is of order 0.1% and therefore negligible.

Thus

$$\epsilon_1 \approx \epsilon_1^T$$

From Appendix I Eq. (I-3) we have

$$\epsilon_1 \approx \left[2(\overline{v^2})^{1/2} \frac{l}{T} \right]^{1/2} \quad (1.8)$$

where ℓ is the length scale and T is the integration time.

As we can see From Appendix II, Table AII.2, the preliminary error estimations show that a time $T=3s$ is enough to decrease the relative error below 1% for v_z whereas in order to decrease the relative error of v_x , v_y below 15% we need probably some days of integration time! The time average seems to be the main constraint in getting a sufficiently accurate value of the secondary flow.

1.2.3 Error in Reynolds Stresses

The stresses as in the case of the mean velocities include

1. Finite integration time error (ϵ_{ij}^T)
2. Reading error ϵ_{ij}^R

The reading error is of order 1%, i.e.,

$$\epsilon_{ij}^R \approx 0.01 \, v_z' v_j' \quad (1.9)$$

The time average error is given by Eq. (I.7)

Appendix I, i.e.,

$$\epsilon_{ij} \approx 2\sqrt{2} \, (\overline{v_i' v_j'})^{3/4} \left[\frac{\ell}{T} \right]^{1/2} \quad (1.10)$$

Preliminary calculations shown in Appendix II show that an average time of 30s is needed to decrease the error below 10%.

TASK IID: Analytical Model Development (John Bartzis)

During this period a comprehensive summary of the differential equations, boundary conditions and method of solution was prepared. After typing and editing, it will be included in the next quarterly progress report.

REFERENCES

1. "55L Laser Doppler Anemometer MARK II" DISA Instruction Manual.
2. Melling, A., "Investigation of Flow in Non Circular Ducts and other Configurations by Laser Doppler Anemometry," University of London, Ph.D. Thesis, 1975.
3. F. Durst, A. Melling and J.H. Whitelaw "Principles and Practice of Laser Doppler Anemometry," Academic Press 1976.
4. Kreid, O.K., "Laser Doppler Velocimeter Measurement in Nonuniform Flow: Error Estimates," Applied Optics 13, No. 8, 1974.
5. Edwards, R.V., Angus, J.C., French, M.J. Dunning, J.W., "Spectral Analysis of the Singal from the Laser Doppler Flowmeter: Time Independent Systems," Journal of Applied Physics 42, No. 2, p. 837-850, 1971.
6. H. Tennekes and J.L. Lumley, "A First Course in Turbulence," The MIT Press 1972.

$$2\sigma_1 = 2b_o$$

$$2\sigma_2 = 2 \frac{2b_o}{\sin \beta} \sin \frac{\beta}{2}$$

$$2\sigma_3 = 2 \frac{2b_o}{\sin \beta} \cos \frac{\beta}{2}$$

Fig. 1 Beam Intersection

TABLE I
LDA OPTICAL PARAMETERS

<u>Parameter</u>	<u>Symbol</u>	<u>Value</u>	<u>Units</u>
1. Refractive Indices			
Air	n_o	1.00	
Water	n_w	1.33	
Plexiglas	n_p	1.50	
2. Laser			
Wavelength	λ	0.5145	μ
Beam Diameter	D	1.5	mm
3. Optical Unit			
Focal Length	f	120	mm
Beam Separation	h	40	mm
Half-Angle between Beams			
in air	$\theta/2$	664	dey
in water	$\theta/2$	499	dey
4. Beam Intersection (Fig. 1)			
Diameter of the Beam at Waist	$2b_o$	0.053	mm
Width	$2\sigma_1$	0.053	mm
Height	$2\sigma_2$	0.053	mm
Length	$2\sigma_3$	0.610	mm
5. Calibration Factor $c = \frac{\lambda}{2n \sin \frac{\beta}{2}}$	C	2.224	m/s _{MHZ}

TABLE II
PARAMETER AND ERROR RELATIONS

Quantity	Calculation	Error (e) (e = $ \phi_{\text{Meas}} - \phi $)
$\frac{V_z}{V}$	$\frac{V_z}{V} \cos \theta + \frac{V_y}{V} \sin \theta$	$e_z = e_x \cos \theta + e_y \sin \theta$
$\frac{V_r}{V}$	$\frac{V_x}{V} \cos \theta + \frac{V_y}{V} \sin \theta$	$e_r = e_x \cos \theta + e_y \sin \theta$
$\frac{V_\theta}{V}$	$-\frac{V_x}{V} \sin \theta + \frac{V_y}{V} \cos \theta$	$e_\theta = e_x \sin \theta + e_y \cos \theta$
$\frac{V_z}{V^2}$	$\frac{V_z}{V^2}$	e_{zz}
$\frac{V_r}{V^2}$	$\frac{V_x}{V^2} \cos \theta + \frac{V_y}{V^2} \sin \theta$	$e_{rr} = e_x \cos \theta + e_y \sin \theta$
$\frac{V_\theta}{V^2}$	$\frac{V_x}{V^2} \sin \theta - \frac{V_y}{V^2} \cos \theta$	$e_{\theta\theta} = e_x \sin \theta + e_y \cos \theta$
κ	$\frac{V_x^2}{V^2} + \frac{V_y^2}{V^2} + \frac{V_z^2}{V^2}$	$e_\kappa = 1/2(e_{xx} + e_{yy} + e_{zz})$
$\frac{V_x V_z}{V^2}$	$\frac{V_x V_z}{V^2} \cos \theta + \frac{V_y V_z}{V^2} \sin \theta$	$e_{xz} = e_x \cos \theta + e_y \sin \theta$
$\frac{V_x V_\theta}{V^2}$	$-\frac{V_x V_z}{V^2} \sin \theta + \frac{V_y V_z}{V^2} \cos \theta$	$e_{z\theta} = e_x \sin \theta + e_y \cos \theta$

APPENDIX I

INTEGRATION TIME ERROR

Due to finite integration time T an error is introduced given by the relation

$$\epsilon^2 = 2\sigma^2 \frac{F}{T} \quad (I.1)$$

where

σ is the standard deviation

F is the time scale defined as [6]

$$F = \int_0^\infty \frac{v(\tau) v'(t+\tau)}{\overline{v'^2}} d\tau$$

where v is the quantity under consideration it is evident that in order to estimate ϵ knowledge of σ and F is necessary.

Below we will try to estimate the above quantities and consequently ϵ .

I.1 Mean Velocities

For a time scale we will assume that the main effect comes from turbulence and the eddies are decorrelating in one overtuning time, i.e.,

$$F \sim \frac{l}{u}$$

where l/u are the length and velocity scales respectively.

let us take

$$u \approx (\overline{v'^2})^{1/2} \quad (I.2)$$

Then

$$\epsilon \approx \left[2(\overline{v_i'^2})^{1/2} \frac{l}{T} \right]^{1/2} \quad (I.3)$$

I.2 Stresses

The time scales corresponding to stresses are expected to be smaller than those corresponding to the mean velocity. This is based on the fact that when squaring a wave we move towards higher frequencies, i.e., lower time scales

$$\overline{F(v_i'v_j')} \leq \overline{F(v_i')} \quad (I.4)$$

Thus we can safely approximate

$$\overline{F(v_i'v_j')} \approx \overline{F(v_i')} \quad (I.5)$$

For the standard deviation of $v_i'v_j'$ we will assume that

$$\sigma_{ij}^2 \approx \overline{2v_i'v_j'} \quad (I.6)$$

The factor 2 is taken from Gaussian distribution. Taking

$$F \approx \frac{l}{(\overline{v'v'})^{1/2}}$$

Equation (I.1) becomes

$$\epsilon_{ij} \approx 2\sqrt{2} (\overline{v_i'v_j'})^{3/4} \left(\frac{l}{T}\right)^{1/2} \quad (I.7)$$

APPENDIX II

PRELIMINARY ERROR ESTIMATIONS

Let us take the following data

$$u_b = 1.5 \text{ m/s} \quad (\text{Re} = 30,000)$$

$$v_z = 1 \text{ m/s}$$

$$v_x = v_y = 10^{-3} \text{ m/s}$$

$$u^* = 0.15 \text{ Re}^{-1} \quad u_b = 0.08 \text{ m/s}$$

$$\hat{Y} = 0.0032 \text{ m}$$

$$\ell = 0.4 \quad \hat{Y} = 0.00126 \text{ m}$$

It is reasonable to take

$$\sigma_x \approx \sigma_y \approx \sigma_z = u^* = 0.08 \text{ m/s}$$

Let us take in addition that

$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = 2u^* = 0.16 \text{ m/s}$$

which is only true for a Gaussian distribution

II.1 Velocity Gradient Broadening

From Eq. (I.6) and

$$\left(\frac{\partial^2 v_z}{\partial y^2} \right) = \frac{u^*}{\kappa(r-R)^2}$$

and

$$\left(\frac{\partial u_z}{\partial y} \right)^2 = \frac{u^{*2}}{\kappa(r-R)^2}$$

we obtain

$$\frac{(v_z)_{\text{Meas}} - v_z}{v_z} = \frac{u^*/v_z}{20.4 \left(\frac{r-R}{b_o}\right)^2}$$

In Table AII.1 we see the values

$$\frac{(v_z)_{\text{Meas}} - v_z}{v_z} \quad \text{and} \quad \frac{\sigma_u^2}{u^{*2}}$$

as a function of

$$\frac{r-R}{b_o}$$

From Table AII.1 we can see that for $y^+ > 20$ the error in mean velocity is negligible where as for $\tau^+ > 50$ the error in $\overline{v_z'^2}$ becomes less than 1%.

II.2 Electronics Broadening

From Eq. (I.7) and $v_{\text{max}} = 1 \text{ MHz}$ we find $\sigma_E = 0.013 \text{ m/s}$

Thus from Eq. (I.1) neglecting velocity gradient

$$\left(\frac{\overline{v'^2}}{u^{*2}}\right)_{\text{Meas}} - \frac{\overline{u'^2}}{u_*^2} \text{ Turb} = \frac{\sigma_E^2}{2} = \frac{0.013^2}{0.08^2} = 0.026$$

II.3 Error Estimation

The errors have to be estimated from Eqs. (I.3) and (I.7). The error is strongly a function of the time average T . Relative errors of the quantities are given for $T = 3\text{s}, 10\text{s}, 30\text{s}$ and 100s , are given in Table AII.2.

TABLE AII.1
PRELIMINARY ESTIMATION OF VELOCITY

$\frac{r-R}{b_o}$	$\frac{(v_z)_{\text{Meas}} - v_z}{v_z} \%$	$\frac{\sigma_{uz}^2}{u^{*2}} \%$
1	10	625
10 ($y^+ \approx 20$)	1.0	6.25
20	NEGLIG	1.6
25 ($y^+ \approx 53$)	NEGLIG	1

TABLE AII.2
ERROR ESTIMATION

Error(%)	T = 3s	10s	30s	100s	1000s
$\frac{\epsilon_z}{v_z}$	0.86%	0.45	0.26	0.15	0
$\frac{\epsilon_x}{v_x}$	860	450	260	150	15
$\frac{\epsilon_y}{v_y}$	860	450	260	150	15
$\frac{\epsilon_{xx}}{u^2}$	20.5	11.2	6.5	3.6	
$\frac{\epsilon_{yy}}{u^2}$					
$\frac{\epsilon_{zz}}{u^2}$					
$\frac{\epsilon_{xz}}{u^2}$					
$\frac{\epsilon_{yz}}{u^2}$					

TASK III. LMFBR OUTLET PLENUM FLOW MIXING

III.1. Velocity Field Measurement and Prediction (Y. B. Chen)

In the analytical effort of this subtask, it was reported last quarter that VARR-II has been running successfully on the ERDA-owned CDC 7600 computer at Lawrence Berkeley Laboratory (LBL). However, during this quarter two major difficulties were encountered. Firstly, the code would not run successfully with derived (or no-slip) boundary conditions. Secondly, file transfer software between LBL and MIT was extremely cumbersome. Consequently, the computer results could not be transferred back reliably to MIT. Since considerable experience with VARR II was gained at LBL, it was decided to perform a code conversion such that the program could be run on local IBM 370 machine. This effort has been completed successfully. The major changes in the original code are the following:

- 1) The bit manipulation subroutine BITPIC has been eliminated, but the program has been adjusted for this change,
- 2) All of the original plotting subroutines have been removed, and a new velocity plotting subroutine for the local CALCOMP plotter has been added,
- 3) The problem with derived boundary condition has been resolved. It arose due to the improper index of the DO loop in the subroutine VM.
In the original Westinghouse version, this DO loop incorrectly enforces derived boundary

conditions for both fluid and non-fluid cells.

Several numerical experiments have also been performed in order to determine the best relaxation factor for Cartesian coordinates and the simulated FFTF geometry. It was found that the convergence rate improves as the relaxation factors increases from 1.7 to 1.95. But that the calculation does not converge at a value of 2.0. Thus a relaxation factor 1.95 has been selected as the optimal value.

In order to obtain the steady state field for velocity, temperature, and other turbulence quantities, several hundred iterations are usually necessary. It has been judged from several runs that 400 steps provide the best compromise between requirements for accuracy and limited computation time.

In the experimental part of this subtask, more velocity distribution data have been obtained. The velocity distribution maps are in Figures 1,2,3 for Run 1, and Figures 4,5, 6 for Run 2, along with TEACH and VARR-II predictions. The Reynolds numbers for Runs 1 and 2 are 20,000 and 33,000, respectively.

As can be seen from the figures that at 20,000 Reynolds number (Run 1), VARR-II does not give good predictions. To the contrary, the TEACH-code gives better results. At 33,000, Reynolds number, both codes give approximately similar results. But there still exist some differences between code predictions and experimental results.

It is expected that the code predictions would be more accurate at higher Reynolds numbers than at lower values. Thus, the data of run 2 are the more significant set. It is seen from a comparison of Figs. 4-7 that qualitative differences exist between both of the code predictions (VARR II and TEACH) and the experimental results.

It is seen that the center of the vortex flow in the chamber is predicted to occur at $Z=11.0$ mm, $R=8.2$ mm by TEACH (measured from the plenum floor, R measured from the plenum centerline); at $Z=8.6$ mm., $R=9.0$ mm, by VARR-II; while the experimental result shows the vortex center at $Z=11.7$ mm, $R=7.0$ mm. Thus, the vortex exhibits a lower circulation, and a position which is significantly higher and closer to the plenum center than either code predicts.

It is also seen that the velocity values along the plenum centerline and near the boundaries are somewhat greater in the code predictions than is observed experimentally. As is seen in Fig. 6 the momentum of the inlet jet decays spatially much more rapidly than is predicted, and the observed velocity gradient field is generally smaller than is predicted.

Some understanding for the causes of these disagreements can be obtained from an examination of Fig. 7. It is seen that the measured Reynolds stress is consistently greater than the predicted value by a factor which varies (spatially) as high as an order of magnitude. The same sort of disagreement is also seen in the comparison of measured and calculated turbulence kinetic energy values (Fig. 8). In both the VARR-II

and TEACH codes the Reynolds stress is calculated according to the equation

$$\rho \overline{u_i' u_j'} = \mu_t \left[\frac{\partial U_i}{\partial X_j} + \frac{\partial U_j}{\partial X_i} \right], \text{ where}$$

μ_t is the turbulent viscosity. The observed disagreements point to code predictions of too-low values of μ_t , or of viscous energy dissipation rate - depending upon the code. Such a cause would explain both too-high velocity and velocity gradient fields being predicted by the codes.

All of the code predictions discussed above are based on the turbulence constants suggested by the original authors of the codes. Their parameters are obtained from simple geometry flows (e.g., jet, pipe flow) and as has been indicated, the universality of the validity of these constants is questionable. Future efforts will be focussed upon adjusting the turbulence constants to match the experimental mean-flow and turbulence field results.

Since most turbulence models claim to give better results at high Reynolds numbers, we are also intended to go as high as possible in Reynolds number in future experiments. Due to the limitation of pipe diameter and pumping power of the test facility, the highest Reynolds number can be achieved at room temperature is only about 35,000. But the Reynolds number value can be increased to above 75,000 by increasing the water temperature through use of the heat exchanger in the loop and by using larger loop piping. This test modification is now undergoing and additional results will be reported next quarter.

III. 2 COMBINED TEMPERATURE AND VELOCITY MEASUREMENTS

(Vincent P. Manno)

An effort has been initiated to develop a measurement system which can detect both the temperature and velocity fields in a turbulent flow configuration similar to the 1/15 scale FFTF outer plenum geometry (See Fig. 9). The measurement system will utilize the interferometric temperature measurement system developed by R. Bennett and M. Golay¹ which provides a probeless detector which is also sensitive to the severe temperature fluctuations present in turbulent mixing. The velocity measurements will be performed using a Laser Doppler Anemometer (LDA). The LDA system also has the qualities of not interfering with the flow and high sensitivity to the rapid velocity fluctuations characteristic of turbulent flow. The interferometer and LDA will be aligned such that they will have the same measuring volume which will provide a simultaneous account of both velocity and temperature fluctuations at a particular position in the flow. Using the data acquired through these measurements, the correlating functional relationships between the temperature and velocity oscillations will be established for this steady state flow situation.

The effort is now being concentrated on the design of the experiment and equipment procurement. The flow circuit is basically the same used by Bennett and Golay which consists of a 1/2 hp air blower, heater and plexiglass test section

connected together with rubber hoses. The flow from the blower is split in half before arriving at the heater to allow one flow leg to remain at ambient temperature and the other flow leg to be heated. The ambient and heated flows are then directed into the test section where mixing of the coflowing hot and cold streams occurs. The optical components of the system are the following:

- a He-Ne laser
- a beamsplitter which divides the laser light into two perpendicular beams one of which drives the LDA system and the other of which leads into the interferometer,
- a Mach-Zehnder interferometer, and
- LDA transmitting optics and light detection components which consist of a photomultiplier and a photodiode.

A schematic diagram of the optical apparatus arrangement is presented in Fig. 10. All of the optical and associated electronic components are either on hand or on order from suppliers. New equipment ordered include mirrors, a beam-splitter and its mount and the LDA transmitting optics.

The work of the next quarter will center around completing the acquisition of the necessary components and assembling the integrated experimental system. Concurrent with these efforts will be development of a proper seeding scheme for the flow which is a basic requirement of an LDA system. (The seed is usually a suspended particle which follows the flow and provides light scattering centers which produce the Doppler frequency shift characteristic of the flow velocity which is measured by the LDA).

¹Bennett, R.G., and Golay, M.W., "Interferometric Investigation of Turbulently Fluctuating Temperature in a LMFBR Outlet Plenum Geometry," MIT Report COO 2245 29TR (May 1976).

III. 3 Enclosed in Appendix III.A is a copy of the paper R.G. Bennett and M.W. Golay, "Interferometric Investigation of Turbulently Fluctuating Temperature in an LMFBR Outlet Plenum Geometry," Paper No. 76-WA/HT-82, which will be presented at the 1976 Winter Annual Meeting of the American Society of Mechanical Engineers in New York in December, 1976. The work reported in the paper was performed under this subtask.

Figure 1: TEACH Prediction for Run 1

Figure 2: VARR-II Prediction for Run 1

Figure 3 : Experimental Results for Run 1

Figure 4 - TEACH Prediction for Run 2

Figure 5 : VARR-II Prediction for Run 2

Figure 6: Experimental Results for Run 2

Fig.9 Test cell dimensions.

FIGURE 10 Optical Configuration of Combined Velocity and Temperature Measurement System

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Appendix III.A

Interferometric Investigation of Turbulently Fluctuating Temperature in an LMFBR Outlet Plenum Geometry

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A novel of optical technique is developed for the measurement of turbulently fluctuating temperature in a transparent fluid flow. The turbulent sodium mixing in the ANL 1/15-scale FFTF outlet plenum is investigated with a scale model outlet mixing plenum, using flows of air. The electrical analog and the power spectral density of the fluctuating temperature is observed experimentally at seven stations in the flow.

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Interferometric Investigation of Turbulently Fluctuating Temperature in an LMFBR Outlet Plenum Geometry

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M. W. GOLAY

ABSTRACT

A novel optical technique is developed for the measurement of turbulently fluctuating temperature in a transparent fluid flow. The turbulent sodium mixing in the ANL 1/15-scale FFTF outlet plenum is investigated with a scale model outlet mixing plenum, using flows of air. The electrical analog and the power spectral density of the fluctuating temperature is observed experimentally at seven stations in the flow.

NOMENCLATURE

I	intensity of the light transmitted by the interferometer, mwatts/cm ²
I_o	the maximum interferometer throughput, expressed as an intensity, mwatts/cm ²
m	the order of interference, an odd integer
$n(s)$	index of refraction at the path length, s
n_{std}	index of refraction of the ambient air
n_{test}	index of refraction in the test cell
t	time, sec
$T(x,t)$	fluctuating temperature at point x , deg C
$v_i(x,t)$	fluctuating velocity in the x_i direction at point x , cm/sec
$\epsilon_{Hij}(x)$	eddy diffusivity of heat, a second rank tensor, cm ² /sec
λ	wavelength of the laser beam, 6328x10 ⁻⁸ cm
(...)	denotes averaged over time

INTRODUCTION

The conservative design of nuclear reactor structural components, guarding against thermal transients in the upper mixing plenum of liquid-metal-cooled fast-breeder reactors (LMFBR's), requires a predictive knowledge of the turbulent sodium mixing. Specifically, the eddy diffusivity of heat, ϵ_{Hij} :

$$\epsilon_{Hij}(x) \equiv \frac{[T(x,t)v_i(x,t)]}{[\partial T(x,t)/\partial x_j]} \quad (1)$$

describes the turbulent mixing that lessens thermal shock on the structural components. The eddy diffusivity of heat is often expediently defined in terms of conveniently known quantities (1) that fall short of describing the mixing accurately. Calculation of the eddy diffusivity of heat from Eq.(1) demands the mapping of the turbulently fluctuating temperature and velocity for any particular flow geometry.

This paper approaches the LMFBR turbulent sodium mixing problem with a transparent fluid scale-model;

measuring the fluctuating temperature by a novel optical method. The fluctuating velocity is not measured in this paper, but could be found with standard Laser-Doppler Anemometry techniques. This combination of optical systems could achieve the extremely fast time response needed to follow faithfully the turbulent fluctuations, while not intruding into the flow and disturbing its structure.

The recently developed (2,3) optical system for temperature measurements uses a Mach-Zehnder interferometer (4) of extremely short field (about 0.5 mm) in real time. Due to the short field, and other considerations, the instrument defines a temperature sensitive volume of 1 mm³. The detector system produces a nearly linear DC electrical analog of the fluctuating temperature over most of the 67 C (120 F) range of the instrument. The frequency response range is greater than 10 kHz for the inexpensive photodetector used in this work. The average temperature is not currently available from this optical technique. The average temperature is measured mechanically at a symmetrical, but remote, station in the flow.

The experiments reported seek to establish the capabilities of the temperature measurement technique, and to visualize the transport and mixing of heat in a 1/15-scale Fast Flux Test Facility (FFTF) outlet plenum model. The characterization of the turbulent temperature proceeds with photographic studies of the electrical temperature analog; Fourier analysis of the analog yields data regarding the power spectral density of the fluctuating temperature.

OPTICAL TECHNIQUE

The optical technique developed in this work requires that the flowing fluid be transparent. A flow of water is unsuitable in this application due to the large variation of the index of refraction of water with temperature, which would require that the thickness of the test cell be less than 100 microns. A flow of air is much more suitable, the exact thickness that is required is developed later in this section. Also, the Prandtl number for air is about one-tenth that of water at room temperature, which makes air closer to, though admittedly very far from, liquid sodium as a heat transfer medium.

The index of refraction of air varies, most importantly, with its temperature and pressure. The variation is quite small, being about 1x10⁻⁶ per deg C with temperature (5), and about 3x10⁻⁷ per millibar with pressure.¹ In this paper, the Mach-Zehnder

¹This is the value of the derivative of the Biot and Arago formula (5) with respect to pressure at STP.

interferometer is used as a differential refractometer (6) in which an extremely sensitive measurement is made of the difference of the index of refraction between a test fluid and a standard fluid. The design of the experiments imposes the responsibility for insuring that the temperature fluctuations always induce a much larger effect than the pressure fluctuations.

The Mach-Zehnder interferometer is shown schematically in Fig. 1. The incident monochromatic light ray divides in amplitude in the first beamsplitter; the rays reflect in their own mirrors, and recombine at the second beamsplitter. If the rays encounter

Fig. 1 Mach-Zehnder interferometer applied to temperature measurements. A, A'--mirrors; O, B--beamsplitters; D--photodiode detector system; V--variable aperture; S--standard cell with index of refraction n_{std} ; T--test cell with index of refraction n_{test} . The initial laser intensity I_0 is reduced to intensity I by the interferometer.

different conditions in the two cells through which they pass, respectively, their phases will generally differ, and they will interfere when recombined. All of the mirrors and beamsplitters are mutually parallel, and the two arms are of equal geometrical length. The optical path lengths experienced by the rays are governed by the refractive indices of the media through which the respective rays pass, as either $\int_{OA'B} n(s)ds$ or $\int_{OAB} n(s)ds$. With ideal optical components, total destructive interference of the two rays occurs when the condition (7)

$$\oint_{OABA'O} n(s)ds = \pm m \frac{\lambda}{2} \quad (2)$$

is satisfied, where odd integer values of m are the "order of interference." The transmitted intensity generally obeys the relation:

$$I = I_0 \cos^2 \left[\frac{\pi}{\lambda} \oint n(s)ds \right] \quad (3)$$

The argument of Eq.(3) must be kept within a range of $\pi/2$ as the index of refraction in the test cell fluctuates, so that the analog output intensity will correspond to a single value of the temperature. Further, the analog intensity is linear only in a broad range about the half-maximum of Eq.(3). To illustrate these points the interferometer characteristic (Eq.(3)) is sketched on the right hand side of Fig. 2. From these considerations, the maximum temperature range of the instrument should be specified to be somewhat larger than the expected range of the temperature fluctuations, so that the analog fluctuations will be linear; the maximum permissible thickness of the test cell is then governed by this maximum temperature range through Eq.(2). For this work, the temperature fluctuations ranged through approximately 45 Centigrade degrees, and the maximum range

Fig. 2 Visualization of parameters for interferometric temperature measurements. The interferometer characteristic curve on the right and the photodiode characteristic curve on the left share the interferometer throughput as a common axis.

of the instrument is 67 C above the ambient air temperature. The inside thickness of the test cell is 0.64 cm (0.025 in.), which corresponds to the first minimum condition ($m = 1$) of Eq.(2) for a uniform temperature distribution over the thickness of the test cell.

Schlieren effects, which can cause a misalignment of the interferometer is several ways, have been considered, and they have been found to produce less than 1% error in the temperature analog.

The analog output intensity of the interferometer is detected by a simple battery-powered photoconductive diode circuit. The characteristic curve of the photodiode is included in the left hand side of Fig. 2. In Fig. 2, a uniform test cell temperature of 33 C above ambient corresponds to a detector voltage of about 8 volts. The non-ideal fringe contrast is entirely suppressed with the oscilloscope preamplifier so that the display of the electrical analog is always a full-scale waveform of $\cos^2[(\pi/\lambda_0) \oint n(s)ds]$.

In the work being reported the electrical analog is displayed on a storage oscilloscope, and recorded via still photography. Additionally, the power spectral density of the analog is measured electronically, and recorded similarly with still photography.

EXPERIMENTS

The Argonne National Lab (ANL) 1/15-scale FFTF liquid sodium test loop is the standard geometry for the flow experiments. A cross-section of the cylindrical ANL test loop is represented by the thin (0.63 cm inside gap width) test assembly of Fig. 3. The average recirculating flow pattern in the test cell must be restricted to two dimensions in order to facilitate the temperature measurements. The test assembly is slightly larger than the ANL model in order to permit large Reynolds numbers, while decreasing the large flow rates. Even so, an entrance velocity of air of 119 m/sec (390 ft/sec) is needed to match the ANL inlet Reynolds numbers. The Reynolds numbers of the air flows investigated are about ten times smaller than those of the ANL model.

The experiments investigate the steady mixing of hot and cold coflowing inlet streams. The central 5 cm (2 in.) inlet admits hot air at about 56 C (100 F) above ambient. The two adjacent 2.5 cm (1 in.)

Fig. 3 Test cell dimensions. The test cell is constructed entirely of 0.64 cm (0.25 in.) laminated plexiglass. L--typical laser measurement station; T--typical thermocouple measurement station.

inlets admit ambient air. The difference in the average temperatures of the inlet flows is referred to as the inlet temperature upset. The entire flow is maintained by 1/4 HP blower, with about half of the flow being heated before entering the central inlet. Heated inlet Reynolds numbers of 10,000 and 20,000 are investigated. The inlet air velocities are matched with the flow heater turned off. When the central flow of air is heated, its velocity increases along with its specific volume, and shearing of the hot and cold inlet flows takes place. The effect of this flow slip on the average recirculating flow properties is not known accurately.

Due to the heating of the test cell walls, an unknown phase angle is opened between the arms of the interferometer. The rate of variation of the index of refraction of plexiglass with temperature is several orders of magnitude larger than that for air. This slow (but large) variation completely masks a measurement of the average temperature in the flow. Additionally, free convection of the air outside of the test cell introduces another slowly fluctuating phase angle, but this is greatly suppressed by wrapping the entire interferometer in thick fiberglass insulation, especially near the warm test cell. The turbulence of the flow within the cell produces a smaller, but much faster variation in the analog output intensity. This work seeks to examine only this rapidly fluctuating analog, and rejects the time-average of the analog as spurious information -- since it includes the wall heating, and uses a separate measurement of the average temperature to replace the mean. The symmetry of the test cell is exploited by placing a thin thermocouple in the flow at a station that is symmetric with respect to the laser station (see Fig. 3). A fine chromel-alumel thermocouple protrudes into the middle of the test cell through a 1.6 mm (1/16 in.) diameter hole in the front face of the cell. The thermocouple wires are 28-gauge, or 0.40 mm (0.0156 in.) in diameter.

Fig. 4 View of the seven interferometric temperature measurement stations in the test cell of Fig. 3. The average temperatures at the stations during rated flow conditions are listed in Table 1.

EXPERIMENTAL RESULTS

Temperature Analog Traces

Several illustrative traces of the temperature analog are presented here. Figure 4 presents a close-up view of the lower left hand portion of the test cell shown in Fig. 3, and it serves as a key to the seven laser measurement stations of the study. Table 1 includes the representative average temperatures at the seven stations under typical running conditions.

TABLE 1

Average Temperatures During Steady Flow Heating

Re = 9300

Ambient inlet at 65 °F

Heated inlet at 155 °F

Station	Average Temperature	
	°F	°C
A	117	47
A'	109	43
B	79	26
C	76	25
D	65	18
E	65	18
F	104	40

A typical noise trace (taken at station F) is presented in Fig. 5. The noise is independent of the particular station, and it shows a moderate frequency (several hundred Hz) ripple due to the motor-induced acoustical vibrations of the optical components of the interferometer. The noise generally represents a ± 3 C (5 F) error in the temperature signal. An equally strong source of low frequency noise in the data is due to the shaking of the building. However, due to the very short (10 millisecond) duration of the data trace, the shaking of the building (of the order of 10 Hz) cannot be seen. These errors in the temperature analog are certainly not fundamental, and can be reduced with more careful isolation of the interferometer. Note carefully that the ordinates of the temperature traces are nonlinear -- they are obtained from an inversion of Eq. 3, and they are

very nearly linear in the central 33 C range.

Fig. 5 Noise in the temperature analog; $Re = 8300$, and all of the inlets are at ambient temperature. The nonlinear ordinate is discussed in the text.

The two most vigorously fluctuating traces observed at $Re = 10,000$ are presented in Figs. 6 and 7.

Fig. 6 Temperature trace at station F (see Fig. 4); $Re = 8800$, and the heated inlet is at 47 C (85 F) above ambient. The non-linear ordinate is discussed in the text.

Fig. 7 Temperature trace at station B (see Fig. 4); $Re = 9300$, and the heated inlet is at 47 C (85 F) above ambient. The non-linear ordinate is discussed in the text.

The data of Fig. 6 are taken at station F, which is closest to the confluence of a hot and a cold inlet stream. The 40 C average temperature at station F (see Table 1) is very close to the mean temperature of the two streams, and the trace exhibits a very strong 300 Hz character. The trace is very suggestive of the initially uniform breakup of a shearing flow. A power spectral density trace of this signal is presented in the next section. The data of Fig. 7 are taken from station B, which is further along in the flow. The data trace has a much more random character, and is representative of the traces obtained at stations A and A', which differ only slightly from this data trace. The data trace taken at station C (not presented here) exhibits much less mixing. Note that station C is much farther along on the average streamlines of the recirculating flow of Fig. 3. The traces from stations D and E (also not presented here) show no measurable fluctuations above background noise. This is in agreement with the average temperature for stations D and E quoted in Table 1, which hover near the ambient inlet temperature during the experiments.

Power Spectral Densities

Several traces of the power spectral densities are also presented. Separate power spectral density (PSD) measurements are made of the temperature analog and of the background noise trace at each station. The background root-mean-square fluctuation at each frequency value is subtracted from the root-mean-square fluctuation data curve of the temperature analog in order to correct for the noise signal. The resultant PSD curves are normalized to unit area. Some error is introduced during the normalization because the full range of the PSD data is not determined experimentally. The composite error in the PSD curves is about 30%. Generally, the PSD technique suffers from short sampling times, but the qualitative behavior shown by the results is clear.

The PSD curves for stations F and B are presented in Figs. 8 and 9, respectively. The spectral

Fig. 8 Power spectral density curve at station F (see Fig. 4); $Re = 8800$ and the heated inlet is at 49 C (88 F) above ambient.

bandwidths of all of the various components in the temperature analog are shown in Fig. 10. It is found that the Fourier analysis is especially noise-free above several hundred Hertz. The PSD data curve at station F (Fig. 8) exhibits a strong 300 Hz component, and a harmonic of substantially less magnitude. A significant PSD signal persists well into the kHz range, even at this relatively low Reynolds number of 8800. The PSD data curve at station B

Fig. 9 Power spectral density curve at station B (see Fig. 4); $Re = 8800$, and the heated inlet is at $44^\circ C$ ($80^\circ F$) above ambient.

Fig 10 Frequency bandwidths of the component signals in the temperature analog. The three lower bands are all sources of noise in the analog. The solid portions of the two upper bands show the frequencies observed in the turbulent mixing experiments.

(Fig. 9) shows the reduced intensity of the 300 Hz component and the decay of the higher frequencies as the flow progresses. Note that both of the PSD curves do not resolve the low frequency behavior of the analog signal due to the high noise level, while the high frequency behavior is very well-represented -- this is quite contrary to the usual fast-response temperature measurement results (8,9).

Inlet Temperature Upset

Several PSD data curves at various inlet temperature upsets are shown in Fig. 11. The data of all of these curves are taken at station A'. The inlet temperature upset is found to have no measurable effect on the spectral distribution of the PSD, at least for stations similar to A'. That is, the turbulent mixing in these experiments is not measurably enhanced with the additional buoyant forces. Note that the higher Reynolds number ($Re = 19,000$) in this experiment causes the PSD to extend to considerably higher frequencies than the $Re = 8800$ results of Fig. 8.

The information foregone by normalizing the PSD

Fig. 11 Power spectral density curves at various inlet temperature upsets. The curves were found at station A'.

curves to unit area² has been regained in a different experiment. The spectrum analyzer is stopped at 1500 Hz, and is used to measure the value of the Fourier component at 1500 Hz for different values of the inlet temperature upset. This frequency is chosen because the measurement technique is most sensitive at this value, i.e., the highest signal to noise ratio is obtained here. The output data fluctuations are smoothed for several seconds in an integrating circuit, and the resulting data are plotted as a function of the applied temperature upset in Fig. 12. Two separate regressions are

Fig. 12 Variation of the RMS fluctuation (measured as ΔV) in the temperature signal at 1500 Hz versus the applied inlet temperature upset. The data is modeled by the two different regressions in the figure. The solid curve is the linear regression.

²The PSD curves have unit area by convention.

quoted in Fig. 12. They indicate that the RMS fluctuation (at 1500 Hz) increases very nearly linearly with the temperature upset. Note that the inlet temperature upset may affect the average flow pattern in the test cell.

CONCLUSIONS

The following conclusions have been derived in this work.

- 1 The temperature measurement method presented in this paper is an effective, and potentially, a very accurate way to implement an unintrusive turbulent temperature measurement.
- 2 The accuracy of the temperature analog, in its linear region, is about ± 3 C (5 F), with most of the noise originating from building vibrations and room noise affecting the interferometer. The accuracy may be improved in a straightforward manner, by seeking a quieter room, and by upgrading the quality of the optical bench. A special geometry for the incorporation of feedback to stabilize the system, and to refine the temperature analog has been considered (10).
- 3 The frequency response of the temperature analog is limited only by the choice of the photodetector. For the inexpensive detector in this work, the frequency response range is greater than 10 kHz.
- 4 The fluctuating temperature has a strong position dependence in the test cell. A large 300 Hz component in the power spectral density is generated in the inlet shearing layer, and it decays as the flow progresses along the average streamlines. This component is probably stimulated by a forced vibration of the inlet air columns -- this is suggested by considering the typical velocity and length-scales at the inlets.
- 5 The results demonstrate that the temperature upset in the test cell does not drive any measurable instabilities in the flow with buoyant forces. In effect, the temperature serves only to "color" the turbulent mixing that is driven almost solely by the shearing of the inlet flows.

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TASK IV: THEORETICAL DETERMINATION OF LOCAL TEMPERATURE
FIELDS IN LMFBR FUEL ROD BUNDLES

TASK IVA: Code Development for Solving the 2-D Multicell,
Multiregion Energy Equations (Man Kit Yeung)

Summary

During the last quarter, emphasis has been primarily put onto the iterative solution of coupling one corner cell to its neighboring internal cell. This problem has been satisfactorily solved for equal as well as unequal heat source densities in the fuel rods. Results of a small parametric study will be shown in terms of P/D , $2W/D$ and q_c''/q_I'' . Work has been already started on the coupled problem of a 4 cell case which depicts a 30° segment of a 19-pin bundle. Again, first results will be shown for that case, too.

No details will be given about the basic mathematical set up of the method at this time. These will follow in a topical report later on.

Work has been started to incorporate the wire and to account for fuel pin internal disturbances such as eccentric pellet position, cracked pellet and power tilting as well as heat source depression in the fuel.

1. Introduction

The objective of this study is to establish a two-

dimensional iterative method for a multicell thermal-hydraulic analysis of finite hexagonal bundles. The analysis employs an analytic method using a boundary least square technique for satisfying the various boundary and continuity conditions at the irregular cell boundaries which do not fit continuously the polar coordinate system.

This solution procedure is applied to the thermally fully developed slug flow, which serves, in this case, as an approximation to the real turbulent flow situation in Liquid Metal Heat Transfer.

The main objective of this study is to predict 2-D clad temperature fields without imposing the usually used but unrealistic adiabatic boundary condition on the cell boundaries. This automatically implies the extension of single cell methods used thus far to multicell calculations. This leads to a new parameter namely the ratio of heat source densities of neighboring rod rings which adds to the parameters pitch-to-diameter ratio and wall distance-to-diameter ratio. As a result mixing lengths and conduction form factors will be presented in terms of these three parameters. Based on the results of the sensitivity study concerning clad temperatures a criterion will be developed which will tell how many cells are necessary to cope with in the analysis under given geometric conditions

2

and heat source density distributions.

It is a well known fact that not only coolant region irregularities give rise to strong azimuthal clad temperature variations but also internal fuel pin distortions such as an eccentric pellet position and/or cracked pellet and two-dimensional power distributions will contribute to these variations. For the first time, this study will incorporate all these contributors in order to check what effect might contribute most and under what circumstances. Thus far, the internal disturbances have only dealt with by thermally decoupling the coolant region and using the concept of heat transfer coefficient. Only one exception has been reported in literature [1] where the combined effects of power tilt and coolant geometry were studied, with respect to their influence on thermoelastic stress and displacement fields.

In addition to all of these facts, there is one further aspect which should be noticed. When measuring clad temperature fields, thermocouples have to be imbedded into the clad material in order to get surface temperatures. The measured results are due to corrections which up till now mostly accounted only for radial conduction effects, whereas approximate correction factors for azimuthal conduction transport were simply taken from different coolant channel geometries, for instance from studies in

eccentric annuli. The results of this study can be used to determine realistic corrections for both directions because it solves a multiregion problem.

Finally, it should be noticed that some of the cell shapes bear resemblance to those used in the subchannel code THI3D. At a later date, the possibility of a combined lumped parameter and distributed parameter approach should be thoroughly checked.

2. Identification of Unit Cells in a Rod Bundle

In order to perform a multicell analysis for a rod bundle, it is essential to develop a scheme which can effectively identify any individual unit cell as a function of the cell number. The scheme consists of 2 parts; namely, cell numbering system and identification technique, which will be dealt with in the following section. This procedure has already been successfully applied for a hydrodynamic problem [2,3]. However, in case of the thermal problem the scheme has to be extended due to the variety of thermal boundary conditions.

A one-twelfth portion of a 127 pin bundle is shown in Fig. 1 with the artificial cell boundaries drawn as dashed lines. Throughout this report a double index ($m\ n$) system is used for the unit cells. The first index indicates the number of rows away from the center rod of the

bundle and the second one designates the distance between the cell center and the normal to the bundle wall in terms of number of pitches. This double index system also provides a fairly convenient way for numbering its neighboring cells. For illustration a typical hexagonal unit cell, along with the indices of its neighboring cells are shown in Fig. 2. However, the cells on the outermost row do not have the hexagonal or part of the hexagonal geometry; they also do not have neighbors on all sides. In this report the bundle wall (shown as solid line in Fig. 1) is assumed to be adiabatic.

In addition to the double index system, a "library" with all ten possible unit cells is developed. These unit cells are characterized by their geometries as well as their thermal boundary conditions. The boundary conditions (Dirichlet or Neumann type) for each unit cell will be prescribed during the course of multicell coupling calculation. The cell classification is summarized in Table 1.

The cell identification is accomplished by "teaching" the code the relationship between the double index and the cell type. The scheme is illustrated in Table 2. Thus by inputting the (m,n) index the code can automatically identify the cell type as well as the complete informations of the geometric parameters, (such as number of sides, normal distances, normal angles, etc.) and prescribed

boundary condition(s) of the specified cell being considered.

The geometry of an individual unit cell can be described by its number of sides, vertex angles (β_i), normal angles, (α_i), and normals to the sides (a_i). As shown in Fig. 3 the vertex angle is the angle made by the line connecting the center and the vertex and the X-axis; the normal is the distance from the center to the side and the normal angle is the angle between the normal and the X-axis. With these parameter the distance from the cell center to any point on any cell boundary is given by the relation:

$$\rho = a_i / \cos(\phi - \alpha_i)$$

$$\text{for } \beta_i \leq \alpha \leq \beta_i + 1 \quad i = 1, 2, \dots, I$$

Thus the location of any points on the cell boundary can be precisely described by these geometric parameters.

3. Solution Procedure

First the solutions for fuel, clad and coolant temperature fields were derived for all types of cells in a finite bundle under the assumption of adiabatic conditions at the cell boundaries. The solution of the coupled energy equations for one individual cell is sought in terms of one family of coefficients, Y_n . By imposing the adiabatic condition at the cell boundaries the set of Y_n

can be obtained by satisfying that boundary condition at discrete boundary points in a least square sense. Naturally, the separate adiabatic calculations of different cells result in discontinuity of temperatures at the imaginary matching boundary between two cells. The iterative method which has been developed removes this temperature difference in a systematic way by imposing heat flux continuity. The scheme will be explained for the example of a symmetry section of a 7-pin bundle as shown in Fig. 4 and goes as follows.

The coolant temperature field of the corner cell as resulting from the adiabatic calculation is given by

$$T_i^{(o)}(\rho, \phi) = A_i + \frac{q_i''' a^2}{2 k_c} \theta_i(\rho, \phi) \quad (1)$$

Similarly, the coolant temperature field for the small internal cell can be represented by

$$T_j^{(o)}(\rho, \phi) = A_j + \frac{q_j''' a^2}{2 k_c} \theta_j(\rho, \phi) \quad (2)$$

where θ_i and θ_j are the dimensionless temperatures for the i-th and j-th cell, respectively. Starting from these equations a procedure has been developed which results in an updated realistic coolant temperature field for the

corner cell given the known results of the initial adiabatic calculations for both cells. Following this updated temperature field is the calculation of an updated heat flux distribution at the common boundary which then is used as boundary condition for its neighboring cell. This in itself leads to a new temperature field for the neighboring cell. Starting the second loop of the iteration the temperature boundary condition will be applied again and so forth.

By means of this procedure it was found that a "satisfactory" solution already exists after the first two or three iterations, without achieving a very high degree of accuracy in the continuity condition of heat fluxes and temperatures at the matching boundaries between cells. However, in view of the fact that the designer will be mainly concerned about clad temperature variations we felt that we could sacrifice a certain amount in accuracy as long as we achieve sufficient convergence for the clad temperature fields. It is just this feature which makes the iterative procedure attractive compared to a direct overall solution of the multicell problem. The first results presented in the next chapter will show that it should be possible to establish a criterion which automatically stops further iterations done by the code.

4. Results

4.1 Two-Cell Calculations

A simple two-cell case with uniform power in both rods is used in this section as an example to demonstrate the applicability of this iterative procedure. The base case is followed by some parametric studies, such as the effects of wall to diameter ratio and power tilting on clad temperature behaviors.

Two sketches of azimuthal dependence of the dimensionless clad surface temperature for the corner and small internal cell are shown in Figs. 5 and 6 respectively. The flow regime is assumed to be fully-developed slug flow and the physical parameters are arbitrarily selected with $P/D = 1.2048$, $2W/D = 1.20$, $k_w = 0.221$ $k = 1.102$, $Bi = 1 \times 10^6$ and $Bi_s = 1 \times 10^6$. The calculation was done with twelve collocating points on each side of a unit cell. It has been observed that the clad temperature distributions of both cells converge to new solutions after approximately two iterations. The difference between the dimensionless temperature distributions of the second and the third iterations is less than 0.01, which, on an absolute scale for typical operating condition, is less than $1^\circ F$ or so. Thus the iterative method can be terminated at any step depending on the accuracy the designer desires. As could be expected, the higher degree of geometric irregularity of the corner cell

is reflected by its more pronounced azimuthal variation of the clad surface temperature. But before a physical explanation of the clad temperature behaviors are given, it would be helpful to look at the heat transport between cells more closely.

A sketch of the dimensionless heat flux distribution for the internal cell is shown in Fig. 7. It can be observed that the heat flux distribution varies quite a bit in the first few iterations and stabilizes out after approximately 4 iterations or so. However, despite the drastic variation of the heat flux distribution the clad temperature is relatively insensitive to the boundary condition. The positive q^* indicates that heat is transferred out from the small internal cell. This heat transport is expected because the coolant to fuel ratio of the corner cell is larger than that of the small internal cell in this particular example. (0.71 compared to 0.60 approximately) and for a fully developed flow the coolant temperature near the inner rod is relatively higher thus creates a net heat flow outward. As a result of the heat transfer out from the small internal cell the "effective" sink for the internal cell increase and this is illustrated by the decrease in its clad temperature variation. For the corner cell the opposite is true thus the clad temperature variation is even more pronounced. These results immediately

lead to the conclusion that assuming adiabatic condition on the imaginary cell boundaries does not necessarily yield conservative results - a surprising result, indeed. Naturally, this is dependent upon the parameters which are chosen.

Coupling the internal and corner cell results in an extra unknown coefficient, namely, the Y_o . Physically Y_o is the difference of normalized fuel centerline temperatures between neighboring cells and with some appropriate arrangement Y_o can infer the mean coolant temperature levels between cells. Defining an average coolant temperature in the i th cell and subtracting the result by the average coolant temperature of the j th cell we get after making the equation dimensionless by the average power.

$$\frac{\bar{T}_i - \bar{T}_j}{\bar{q}'' a^2 / 2kc} = 2Y_o + (q_1^+ \bar{\theta}_1 - q_j^+ \bar{\theta}_j) \quad (3)$$

This seemingly indicates that the mean coolant temperature level between cells can be calculated from the two-dimensional calculation for a fully developed slug flow. This quantity, coupled with the heat transferred across the common cell boundary calculated by evaluating the integral of the temperature gradient at the boundary, yields the artificial

length scale, l_{ij} similar to those being currently used in the subchannel codes.

4.1.1 Parametric Studies

In this section, the results of two parametric studies are shown to demonstrate how the wall-to-diameter ratio (W/D) and the power tilting in the bundle affect the clad temperature behavior.

In order to see the effect of the wall-to-diameter ratio three different 2W/D ratios, 1.05, 1.10 and 1.20 were used for calculations with other physical parameters kept constant. Fig. 8 is a plot for dimensionless heat flux distribution of the corner cell at the common boundary in conjunction with the internal cell. It can be seen that as wall distance decreases the direction of the heat flow is reversed. This is because the coolant area of the corner cell decreases with the wall distance and the decrease in heat sink leads heat to change its path to the internal cell. The mean coolant temperature differences between the two cells are listed in Table 3 as a function of 2W/D. The average effect of over cooling as a function of geometry can clearly be seen.

The artificial length scale, l_{ij} which has been widely used in many subchannel code can also be calculated from this method. By definition, l_{ij} is given by:

$$l_{ij} = \frac{(\bar{T}_i - \bar{T}_j)}{\frac{1}{S_{ij}} \int_{S_{ij}} q''_1 ds} = \frac{S_{ij} (\bar{T}_i - \bar{T}_j) / \frac{\bar{q}''_1 a^2}{2k_c}}{q_1^+ \int_{S_{ij}} q_1^* d\left(\frac{s}{b}\right)} \quad (4)$$

where S_{ij} is the common boundary between the i th and the j th cell. All these quantities can be calculated from the procedure mentioned earlier. The mean coolant temperature difference between cells, along with the artificial length scale l_{ij} are listed in Table 3 as a function of the $2W/D$ ratio. The distributions of the dimensionless clad temperature of the corner and the internal cells are shown in Figs. 9 and 10, respectively. It is clear that the clad temperature of the corner cell is extremely sensitive to the change of the wall distance. As seen from Fig. 9 a change of $2W/D$ from 1.20 to 1.05 (may result from a rod bowing) changes the temperature distribution drastically i.e., the difference between the maximum and the minimum is about 0.60 for the $2W/D = 1.20$ curve while the difference increases to about 1.50 for the $2W/D = 1.05$ case and reverses its sign. On the other hand, the internal cell remains quite insensitive regardless of the drastic change of boundary condition as seen from Fig. 10. Furthermore the maximum variation of clad temperature for the internal

cell is only about 0.03, which is almost 2 order magnitude less than of the corner cell. This is why corner cell clad temperature is of major concern in reactor design. Naturally, it should not be overlooked that the presented results are far too conservative because they assume thermally fully developed conditions.

4.1.2 Effect of Unequal Heat Source Densities in the Fuel Pins

Three two-cell calculations with different power densities in the two rods were calculated to roughly demonstrate the effect of tilting power across a bundle on the heat transfer in the individual cells. The dimensionless heat flux distribution of the corner cell at the common boundary is shown in Fig. 11 as a function of the power tilting ratio. The P/D and $2W/D$ ratios are held at 1.2048 and 1.20 respectively. It can be seen that the increase of the corner cell power overrides the geometry effect and reverses the direction of heat transfer by driving the heat back to the internal cell. On the other hand, the dimensionless clad temperature distributions of the corner and internal cells are shown in Figs. 12 and 13 respectively. The clad temperature of the corner cell shows least variation with the power tilting 1.1 : 0.9. However, all this depends heavily on the geometry and any change in P/D or $2W/D$ ratio and will certainly lead to completely

different conclusion. Finally the clad temperature of the internal cell is relatively more sensitive to the power tilting than to changes of the $2W/D$ ratio as shown earlier, but the variations remain small compared to those of the corner cell. Finally, Table 4 presents a summary of the parametric study done so far. It is interesting to see that for the case of 1.1/0.9 power tilting the net heat flow is still directed from the corner cell to the internal cell despite the fact that the mean coolant temperature of the internal cell is slightly higher. The fact it indicates is that the net heat transfer is not strictly governed by the mean coolant temperature difference but is rather a local effect which strongly depends on power difference and geometry.

The preliminary values of l_{ij}^* as given in Tables 3 and 4 should be taken with caution. Further studies are necessary and already underway to complete the whole picture and to confirm these data. Although the large variations in l_{ij}^* may surprise it is a well known fact that these variations are indeed occurring as was shown in [4] for a semi-infinite square array pattern.

Despite some doubts in the correct quantitative values it is quite obvious that using a single length scale to predict the energy transfer between the cells over a wide range of operating conditions will lead to substantial

errors.

4.2 Four-Cell Calculations

Finally a calculation for a symmetric section of a 19-pin bundle was made. The calculation consists of four characteristic unit cells namely, a corner cell, an edge cell, a four-sided internal cell and a small 30° internal cell. The calculation results are summarized in Fig. 14. Again the clad surface temperature of the small internal cell is very small compared to the other cells. The clad temperature of the corner cell from the four-cell calculation is also compared to the previous result of the two cell calculation in Fig. 15a. It can be seen that under the given geometry, the two-cell calculation is again not conservative as compared to the four-cell calculation. This is mainly due to the coupling with the side element and a more representative four-sided cell in the second row. What remains to be seen is, how influential the small internal cell is on the overall solution or whether this might be simply replaced by an adiabatic boundary condition at the left side boundary of the four-sided cell.

A parametric study is underway to clarify all the open questions. In summary, it should be pointed out that the solution of this case already contains all of the main ingredients of larger problems. Thus it is only a matter of convenient and effective data flow control in the code to take on larger bundles.

4.3 Verification of the Iterative Method

In order to verify the results of the iterative method, the special case of a two-cell calculation with a square-array pattern has been done and compared with the solution presented by France and Ginsberg [Fig. 4 in 16]. These authors considered a semi-infinite array which is bounded on the left side by the bundle wall. Due to the fact that they only analyzed the coolant region, the heat fluxes around the circumference of the pins was kept constant. The temperature fields were obtained by these authors by using the integral-boundary-least-square technique and compared against the result of a finite-difference solution by using the THTB-code. For the latter method several hundred points were used and it took 2598 iterations as well as 45 min on the CDC-3600 to get a solution within 2-3% accuracy. The series solution as obtained by France and Ginsberg ran about 2 min on the same machine for 20 terms in the series expansion, whereas the iterative scheme runs about 1 min on the IBM 360/65 for 24 terms. It is obvious from these numbers that the analytic methods clearly have an edge compared to the numerical ones.

Fig. 15b shows the comparison of the clad temperature distributions as obtained by the finite-difference solution and our iterative solution technique. The agreement is excellent. Additionally, this means that the iterative solution technique shows excellent agreement with the direct solution technique as used in [16].

By virtue of these findings, the confidence in all our other results presented thus far has naturally increased.

Therefore, further extensions and applications of the iterative method seem to be justified.

5. Internal Disturbances (L. Wolf)

As already mentioned in the introductory remarks, off-center positioning as well as cracking of the fuel pellet will introduce circumferential disturbances of the fuel-to-clad heat transfer coefficient thus leading to two-dimensional temperature fields in fuel and clad. Furthermore, any gradients in power generation through the pin, as expected in blanket pins, contribute to these variations, which add on top of the variations introduced by the coolant region asymmetries.

The major goal of modeling these phenomena is to look under what circumstances all these effects might add up. Furthermore, gap variations may play a major role for the reliable functioning of electrical heater elements which simulate fuel pins in experiments and for the interpretation of temperatures obtained by thermocouples embedded therein.

5.1 Modeling of the Disturbances

For the two-dimensional heat source distribution the following expression will be used:

$$q'''(r, \phi) = q_B'' (P_1 + q_1 r \cos \phi) e^{\xi_1 r^2} \quad (5)$$

This equation has proved to be flexible under quite different physical situations [5] and it is believed that it can successfully be applied in the LMFBR situation especially after it has been discovered that the hyperfine structure of the flux may very well play a major role in the design of blanket pins. The meaning of the individual terms is as follows:

i = number of row, where the fuel pin under consideration is located.

P_i = radial peaking factor of row i

$q_{B''}$ = average heat source density of bundle

q_i = tilt parameter of row i

ξ_i = depression parameter of row i

Fig. 16 shows the different kinds of heat source densities which can be simulated by setting one or two of the aforementioned parameters equal to zero. Obviously, the combination of tilted and depressed heat sources can be modeled, as well as each of them alone. Naturally, the empirical parameters have to be fitted either to experimental or calculated neutronic data.

For the case of gas-filled gaps between fuel and clad the validity of a zero-dimensional gap model has already been proven in [6], i.e., the variation of energy transfer across the gap can be simulated with an azimuthally

varying heat transfer coefficient $h_g(\phi)$. The following Fourier series was developed in [7] for this situation in terms of ϵ , the relative eccentricity.

$$h_g(\phi) = \frac{h_c}{\sqrt{1-\epsilon^2}} \left[1 + 2 \sum_{n=1}^{\infty} \left(\frac{\sqrt{1-\epsilon^2}-1}{\epsilon} \right)^n \cos n\phi \right] \quad \epsilon < 1 \quad (6)$$

An analytic solution for the fuel and clad temperature fields has been already established in [8] under consideration of Eq. (6) and Eq. (7). However, this solution is somewhat restricted by using either a constant or azimuthally varying heat transfer coefficient at the outside clad surface, which does not necessarily reflect the realistic thermal coupling of clad and cooling region. This restriction will now be removed by fully integrating the fuel pin model with disturbances into the multicell procedure as developed under TASK IV. A. Furthermore, the analysis will be extended by taking into account a contact between fuel and clad in case of a fully eccentric position of the pellet in the clad ($\epsilon=1$), because Eq. (6) is only valid for no contact between these two surfaces. Although, theoretically fuel and clad contact only at one point, in practice there should be always a contact zone of finite length which is characterized by an angle $\beta > 0$. Thus the complete formula in this

case becomes

$$h_{ecc}(\phi) = h_g(\phi) + h_{con} [Y(\phi - \pi + \frac{\beta}{2}) - Y(\phi - \pi - \frac{\beta}{2})] \quad (7)$$

where the function Y is described in terms of the argument x as

$$Y(x) = \begin{cases} 0 & \text{for } x \leq 0 \\ 1 & \text{for } x > 0 \end{cases} \quad (8)$$

and the contact heat transfer coefficient becomes in terms of the relative eccentricity

$$h_{con} = \begin{cases} 0 & \text{for } \epsilon < 1 \\ 1 & \text{for } \epsilon = 1 \end{cases} \quad (9)$$

The Fourier series and its associated coefficient for this extended version of the gap heat transfer coefficient will be derived soon.

For higher burnup the gap will presumably close. Thus the probability of an eccentric pellet position will sharply decrease, and a pure solid-to-solid contact problem situation will arise. On the other hand, power cycling leads to pellet cracking and subsequent dislocations of whole pellet sectors. Obviously, the gap heat transfer coefficient assumes quite different values in the regions of contact and dislocation. The height of the jump in the value of the gap heat transfer coefficient depends upon the contact pressure as well as the widths of the residual gaps formed by the dislocation.

The simplest model for this type of situation would start considering only one dislocation, as shown in Fig. 17. The circumferential extension of this dislocation is characterized by the angle α . Its associated gap width is δ^* which is assumed to be constant along α . Then the gap width around the whole circumference of the pin can be described as

$$\delta(\phi) = \begin{cases} \delta^* & \text{for } -\frac{\alpha}{2} \leq \phi \leq +\frac{\alpha}{2} \\ 0 & \text{for } -\frac{\alpha}{2} < \phi < 2\pi - \frac{\alpha}{2} \end{cases} \quad (10)$$

by taking symmetry into account. Thus the gap heat transfer coefficient for this situation becomes

$$h_{\text{crack}}(\phi) = \frac{k_g}{X + \delta^* [Y(\frac{\alpha}{2} - \phi) + Y(\phi - 2\pi + \frac{\alpha}{2})]} + h_{\text{con}} [Y(\phi - \frac{\alpha}{2}) - Y(\phi - 2\pi + \frac{\alpha}{2})] \quad (11)$$

where

k_g : thermal conductivity of the filling gas

X : minimum effective gap width

Obviously, this concept can be extended in order to account for more than one dislocation. However, it is felt that varying α and δ^* for one dislocation should give some kind of an upper limit of that effect on the temperature fields, because considering more dislocations would certainly relieve the severity of the jump in the gap heat transfer coefficient.

Having a dislocation on hand means that there exists discontinuities of circumferential heat transfer in at least the outer portion of the pellet due to the cracks. Thus the assumption of an isotropic thermal conductivity in the fuel is invalidated. No models have been developed thus far for accounting for this effect. Thus some reasonable engineering assumptions must be made.

5.2 Code Development

A fuel pin module will be constructed which accounts for the combined effects as described in the foregoing chapter. After completion of the stand alone version it will be integrated into the multicell computer code. Three options exist for coupling internal and external disturbances. First, the fuel pin analysis and coolant region analysis could be fully integrated in rederiving all major relationships between the sets of unknowns from scratch. Second, the fuel pin analysis could be completed by itself assuming a simplified thermal boundary condition, like $q''(\phi) = \text{const.}$, at the

outside clad surface. This would result in a specific temperature distribution which would then be used to drive the coolant region solution. In turn, this would result in a new heat flux distribution which is now a function of ϕ , and so forth. A third possibility would be to assume a Fourier series representation of the outside clad temperature where the coefficients would be iteratively fitted.

The last option is probably the most elegant one. However, no decision has been made yet. A successful development of an analytic fuel pin model incorporating all of these aforementioned effects would save a great amount of nodal points which are normally spent for nodalizing fuel and clad regions for use in the coolant region representation where they are desperately needed. The code FATHOM clearly shows that.

TASK IVB: 3-D SINGLE CELL SLUG FLOW HEAT TRANSFER
ANALYSIS (Chung Nin Wong)

Summary

All numerical difficulties were resolved in the meantime and the code was thoroughly checked against results from Meier's [9] original version. In a first step the experimental results done at Karlsruhe [10] were compared against our results and it was found that both compare quite favorably. Fortunately, the computer code always produces conservative results as compared to the experiment. Still existing discrepancies will be listed and explained. Furthermore, a study was made where a region wise thermal disturbance in the coolant temperature was initiated at the inlet section. The diffusional spread in radial, circumferential and axial direction was studied as well as its effects on the clad temperature field.

1. Code Modifications

As already indicated in the last report, the solution procedure and thus the code itself mainly consists of three parts: The first one is to handle the calculation of the fully developed temperature solution and is due to the fact that it only handles the coolant region a boiled-

down version of what is already used in the two-dimensional case; the second part solves the eigenvalue problem whereas the third one is to obtain the thermal entrance solution and combines it with the fully developed solution to form the complete set. Furthermore, the last part provides a solution for arbitrarily varying axial heat flux problems provided that the distribution is a well-behaved function.

Throughout the program, several changes have been made in order to upgrade the efficiency of the code. One of the problems we faced earlier in the verification is the searching of the eigenvalues with given accuracy. Upon changing the second part into double precision and using a three-point iteration scheme for the eigenvalue search, it improves the accuracy of the calculated eigenvalues from 10^{-4} to 10^{-10} at least. Furthermore, a new set of Bessel function subroutines were built into the code and resulted in a reduction in computation time by a factor of 10. In addition, three-nested DO-loops were abolished where possible in the third part. Moreover, in order to handle local perturbation in inlet temperature the respective parts of the code were streamlined.

2. Results for Side Cells

2.1 Uniform Inlet Temperature and Axially Uniform Heat Flux Profile

The code as it stands to date calculates 3-D temperature fields in corner and side cells of bare hexagonal

LMFBR bundles.

In order to confirm the validity of the results as obtained by the code, a comparison was made with recent measurements of 3-D clad temperature profiles in a sodium-cooled 19-pin bundle done by Möller et. al. [10]. For this purpose, it was decided to pick the following conditions.

$$P/D = 1.3 \quad ; \quad 2 W/D = 1.38 \quad ; \quad D=9\text{mm} \quad ; \quad Pe=372$$

and

$$T_{in} = 415^{\circ}\text{C} \quad \bar{q}''=104 \text{ W/cm}^2 \text{ (flat axial power distribution)}$$

Figs. 18 to 21 summarize our results with those of the experiment taken at different axial levels in the bundle.

In view of these figures two things are worth mentioning. First of all, it obviously seems that the calculated results miss the average temperature level at the specific heights although this difference steadily diminishes with increasing height. The explanation for this discrepancy is indeed very simple. The experimental results do not really represent outside clad temperatures but profiles as measured by the embedded thermocouples inside the clad. This fact was never disclosed in the figure captions of the original publications but was recently confirmed during a discussion [11]. As a result, the

experimental data must be corrected not only for radial but also for azimuthal heat conduction effects in the clad. Naturally, these corrections vary for both effects at different axial levels and must be accurately found by a true 3-D analysis. Certainly, a 1-D analysis which only accounts for radial heat conduction does not reflect reality. Thus, a temperature correction of $16^{\circ}\text{C} \pm 4^{\circ}\text{C}$ as quoted by Möller [11] does not hold equally well for all axial levels. In order to fully show the difficulties which are involved with the reduction of experimental data we did not try yet to either modify the original data or our own results. Modifications will follow in a second step later on. Obviously, the radial temperature gradient is largest at the inlet and smooths out at higher levels. On the other hand, the azimuthal temperature gradient increases with increasing axial direction from the channel inlet. What we learn from the foregoing discussion is that it is important to have reliable tools on hand to correct measurements. Having the aforementioned explanations in mind, our results compare quite favorable with the experiment.

The second point to mention concentrates on the azimuthal temperature variations. The following table shows that the analytical method always results in conservative values for the azimuthal temperature variations

at the outlet region. The conservatism increases with increasing axial distance from the inlet as is clearly demonstrated in the upper part of Fig. 22a.

Comparison Between Measured and Calculated
Azimuthal Temperature Differences for a Side Cell, Pe=372

<u>Axial</u> <u>height mm</u>	$(T_{\max} - T_{\min})$ °C <u>meas.</u>	$(T_{\max} - T_{\min})$ °C <u>calc.</u>	$(T_{\max} - T_{\min})$ <u>calc.</u>	$(T_{\max} - T_{\min})$ <u>meas.</u>
115	9.7	3.87	0.3988	
295	10.3	13.69	1.329	
475	18.3	22.77	1.244	
655	20.63	32.10	1.556	
thermally fully develop. --	--	121.45	--	

As can be seen from this table as well as Fig. 22a and Fig. 22b, the experimental results indicate larger azimuthal temperature variations in the inlet regime for low and medium Pe than calculated. This effect diminishes with increasing Pe-number. Furthermore, the difference between measured and calculated results decreases with increasing Pe. Thus, the calculation agrees quite well with the measurements for $Pe > 700$. The anomalies in the experimental results in the low Pe-range can be obviously attributed to already existing natural convection effects.

In the lower part of Fig. 22a, the calculational results are compared against the experimental results obtained by Möller [10] and Subbotin, et al [12]. In the first place it is surprising to see that the difference between the two measurements is of the same order as the difference between the calculation and the Karlsruhe measurement. An explanation for this discrepancy was given by Möller [10] who indicated that the Russian experiment was performed for $\bar{q}''$ being only $\bar{q}''=2-6 \text{ W/cm}^2$. Thus a large uncertainty has to be assigned to the results from these experiments. Furthermore, the thickness of the bundle wall was appreciably larger than normally used in reactors which leads to larger azimuthal conduction effects and thus decreasing coolant temperature asymmetries. From all of these figures it becomes quite clear that the thermally fully developed region is by far not reached, especially for higher Pe-numbers. This can also be seen from Fig. 23 where the Nu-number averaged over the cross sectional area is shown as function of the axial height. Nu still decreases and does not reach its asymptotic value Nu_{∞} throughout the whole core. As a result, 2-D, thermally fully developed analyses do result in too conservative values for azimuthal temperature differences as depicted in the last entry of the foregoing table. For the special case under consideration, the real temperature variation

is only 1/6 that of the result as calculated by a two-dimensional analysis. Thus, the 3-D analysis as used in this study is indeed a big step forward into the right direction. Fig. 24 clearly shows how good the agreement between experiment and calculation is for $Pe=622$. This agreement is indeed quite surprising when one realizes what effects really affect the measurement and what are neglected in the analysis. The following table gives an indication about these discrepancies.

Listing of Effects Which Are Considered
in Measurement and Analysis

<u>Effect</u>	<u>Measurement</u>	<u>Analysis</u>	<u>Nijsing, Eifler</u> <u>[13]</u>
conduction	yes	yes	yes
axial conduction	yes	no	no
conduction in walls	yes	no	no
turbulent convection	yes	no	yes
mixed convection at low Pe	yes	no	no
grid effects	yes	no	no
temperature dependent propert.	yes	no	no
coupling to neighboring cells	yes	no	no
perturbation due to TC_s	yes	no	no

This table includes a summary of the method by Nijsing and Eifler [13] which is the only known 3-D thermal analysis including turbulent effects. As already shown in [9] for $2W/D=1.6$ the conduction solution as presented herein compares

quite favorable to the results in [13]. The next report will include a thorough comparison to all of Nijing's results. This will definitely broaden the check of the validity of the slug flow solution.

2.2 Uniform Inlet Temperature and Axially Varying Heat Flux Profile

In order to check how an axially variable heat flux may change the results reported so far, the following function was used

$$q''(z) = \bar{q}'' \left(0.15 + 0.425 \pi \sin \frac{\pi z}{z_0} \right)$$

where z_0 is the total heated length. All other parameters were kept the same as used for the flat heat flux distribution.

Fig. 25 through 28 show a comparison of the azimuthal temperatures due to flat and chopped sinusoidal heat flux profiles, whereas Fig. 29 depicts the comparison of maximum temperature variations for both profiles as functions of the axial coordinate with Pe as parameter. As can be seen from the series of figures, the average temperature level for axially varying heat flux is lower than that of the flat profile in the inlet region. Further downstream the average temperature level for the sinusoidal profile slowly overtakes that of the flat profile but finally in the outlet region the average temperature level for the

flat profile is again higher than that of the sinusoidal profile. No experimental data exist to compare these findings. Fig. 29 summarizes the results with respect to the azimuthal temperature differences for both profiles and the experimental results for a flat heat flux profile. As can be seen from this figure, the difference is smaller in a certain inlet region but always larger than the difference for the flat distribution beyond that region for all Pe-numbers. The deviation between the results of both profiles decreases with increasing Pe-number. Fig. 30 shows the Nu-number versus the axial distance for both heat flux profiles. It is quite obvious that by taking asymptotic Nu-relationships for the calculation of clad temperatures in the inlet region might result in erroneous results because the Nu-number sharply decreases in this regime.

2.3 Non-uniform Inlet Temperature and Axially Uniform Heat Flux Profile

In order to check the flexibility of the code a perturbation of the dimensionless inlet temperature was introduced over a certain section of the cross section as shown in the sketch in Fig. 31. This can be thought of as introducing a hot lump of fluid into the side cell. Fig. 31 thru 34 show how this disturbance spreads around the circumference of the pin as a function of the axial distance. For easier comparison the solution for a uniform

inlet temperature distribution is given again. This comparison shows that the disturbance creates a rather severe clad temperature peak at the inlet. This peak is then reduced by the azimuthal conduction effect in the coolant. At the same time the temperature field of the coolant in the unperturbated section of the cell starts to develop. Thus, we have two competing effects on hand. Following the axial development of the temperature fields through the different axial levels one realizes that the perturbed temperature profile tends to the unperturbated temperature profile for larger axial distances. Naturally the higher temperature asymmetry throughout the cell reflects itself in a further decrease of the average Nu-number as shown in Fig. 35.

The study of inlet temperature disturbances will be continued in the future.

3. Results for Corner Cells

The geometry parameters and thermal input data for this study are the same as those already used for the side cell study. An additional feature of this study as compared to what has been done thus far elsewhere is that a rounded corner has been introduced in order to check its positive effect on local temperatures. A radius of $r_0 = 5.5\text{mm}$ has been chosen which coincides with the experimental setup at Karlsruhe [10].

Figures 36 thru 38 show the developments of the azimuthal

temperature distributions around the pin for the sharp-edged and the rounded corner cases. As can be seen from these distributions, the rounded corner lifts the temperature in the vicinity of the corner but what is more important, it reduces the maximum temperature at 180 degrees. Thus, it has indeed an overall beneficial effect which can be also seen from the table shown below

Comparison of Maximum Azimuthal Temperature
Differences of Sharp-Edged and Rounded Corner Cells

<u>Axial Height</u>	<u>$(T_{\max} - T_{\min})_{\text{sharp}}$</u>	<u>$(T_{\max} - T_{\min})_{\text{round}}$</u>	<u>$(T_{\max} - T_{\min})_{\text{exp}}$</u>
115 mm	3.02 °C	2.76 °C	6.5 °C
295 mm	11.19 °C	10.93 °C	13.2 °C
275 mm	19.26 °C	18.17 °C	18.1 °C
655 mm	27.49 °C	24.54 °C	21.7 °C
Thermally fully developed	81.75 °C	68.05 °C	

By comparing the results in this table to those of the table presenting the results for the side cells one notes that the latter are due to larger temperature variations.

Figures 39a and 39b compare the calculational results with the experimental evidence as reported by Möller, et.al. [10] and Subbotin [12]. Again, the agreement is quite satisfactory and becomes even better for higher Pe-numbers.

Finally, Fig. 40 shows the comparison of Nu-numbers for both corner shapes. Due to the fact that the sharp-edged corner creates larger coolant temperature asymmetry the resultant Nu-number is slightly less than that of the rounded corner.

TASK IVC: FULLY DEVELOPED MIXED CONVECTION (Jong-Yue Kim)

Summary

Work has been started on that subject on a low level of priority. Nevertheless, the set of momentum and energy equations has been set up and solved up to the point where only two sets of unknowns remain. These will be determined with a boundary least square method as used under TASK IVA.

1. Analysis

The problem under consideration will be solved by taking advantage of an analytic formulation presented by Iqbal, et. al [15]. These authors solved the single region mixed convection problem under simplified limiting thermal boundary conditions for a unit cell of an infinite triangular and square array.

This analysis will look at the more important corner and side cells of finite, hexagonal bundles. Furthermore it additionally takes the clad into account. Thus it solves two energy equations simultaneously together with the momentum equation. A constant heat flux has been assumed at the inner clad surface which basically reflects about the realistic conditions in UO_2 fueled pins.

All equations have been set up and the sets of unknowns were reduced to two by systematically substituting the

general solutions into the boundary and continuity conditions. Before writing the main program, it was decided that sufficient accurate methods for evaluating the Kelvin functions bei_n , ber_n , kei_n , and ker_n and their derivatives should be developed in the first place because these are not readily available in standard software packages. This subtask has been satisfactorily completed and the results were compared against values found in mathematical tables and the like. However, it has been realized that only a limited amount of reported data is available for this comparison. Especially no data are reported for higher orders and large arguments. Therefore, the method will rely even in this range on a procedure which fits best the reported data. Admittedly, this might lead to some errors. How severe these really are, can only be shown by a sensitivity study with the whole code.

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Table 1: Library of Unit Cells Considered in Analysis

Type	Unit Cell	β_i	α_i	a_i	Boundary Conditions
1		$0, \frac{\pi}{6}$	0	$\frac{P}{2}$	2
2		$0, \beta_2, \frac{5}{6}\pi, \pi$ $\beta_2 = \frac{\pi}{6} + \tan^{-1} \frac{P}{2W}$	$\frac{\pi}{6}, \frac{2}{3}\pi, \pi$	$W, \frac{P}{2}, \frac{P}{2}$	$0, 2, 2$
3		$0, \frac{\pi}{3}, \beta_3, \pi$ $\beta_3 = \frac{\pi}{2} + \tan^{-1} \frac{2W}{P}$	$\frac{\pi}{6}, \frac{\pi}{2}, \pi$	$\frac{P}{2}, \frac{P}{2}, W$	$2, 2, 0$
$\left\{ \begin{array}{l} 4 \\ 5 \end{array} \right.$		$0, \frac{\pi}{6}, \frac{\pi}{2}, \frac{5}{6}\pi, \pi$	$0, \frac{\pi}{3}, \frac{2}{3}\pi, \pi$	$\frac{P}{2}, \frac{P}{2}, \frac{P}{2}, \frac{P}{2}$	$2, 2, 0, 2$ $2, 2, 2, 2$
6		$0, \frac{\pi}{3}, \frac{2}{3}\pi, \pi$	$\frac{\pi}{6}, \frac{\pi}{2}, \frac{5}{6}\pi$	$\frac{P}{2}, \frac{P}{2}, \frac{P}{2}$	$2, 2, 2$
$\left\{ \begin{array}{l} 7 \\ 8 \end{array} \right.$		$0, \frac{\pi}{3}, \beta_3, \beta_4, \frac{5}{3}\pi, 2\pi$ $\beta_3 = \frac{\pi}{2} + \tan^{-1} \frac{2W}{P}$ $\beta_4 = 2\pi - \beta_3$	$\frac{\pi}{6}, \frac{\pi}{2}, \pi,$ $\frac{3}{2}\pi, \frac{11}{6}\pi$	$\frac{P}{2}, \frac{P}{2}, W, \frac{P}{2}, \frac{P}{2}$	$2, 2, 0, 2, 2$ $2, 2, 0, 0, 2$
$\left\{ \begin{array}{l} 9 \\ 10 \end{array} \right.$		$0, \frac{\pi}{3}, \frac{2}{3}\pi, \pi$ $\frac{4}{3}\pi, \frac{5}{3}\pi, 2\pi$	$\frac{\pi}{6}, \frac{\pi}{2}, \frac{5}{6}\pi, \frac{7}{6}\pi$ $\frac{3}{2}\pi, \frac{11}{6}\pi$	$\frac{P}{2}, \frac{P}{2}, \frac{P}{2}, \frac{P}{2}, \frac{P}{2}$ $\frac{P}{2}$	$2, 2, 2, 2, 2, 2$ $2, 2, 2, 2, 0, 2$

0 - adiabatic

2 - Dirichlet

or - Neumann

TABLE 2
Relations Between Index (m,n) and Cell Type

m	n	Type
0	0	1
1	1	4
$1 < m < M$	0	6
	1	10
	$1 < n < N$	9
	n	5
M	0	3
	1	8
	$1 < n < N$	7
	M	2

where

M - No. of the outer most row

N - Max. No. of cell within a row

Fig. 2. Indices for Neighboring Cells for a Typical Hexagonal Cell

Fig. 3. Geometric Parameters of a Unit Cell

Fig. 4. Symmetry Section of a 7-pin Bundle

Fig. 5. Dimensionless Clad Surface Temperature Behaviors of the Corner Cell During Iteration for a Fully Developed 7-pin Bundle

Fig. 6. Dimensionless Clad Surface Temperature Behaviors of the Internal Cell During Iteration for a Fully Developed 7-pin Bundle

Fig. 7. Dimensionless Heat Flux Distribution Behaviors at the Internal/Corner Cell Common Boundary During Iteration

Fig.8 Effect of the Wall to Radius Ratio on the Heat Flux Distributions at the Internal/Corner Common Boundary

Fig. 9 Effect of the Wall to Radius Ratio on the Clad Surface Temperature of the Corner Cell

Fig. 10 Effect of the Wall to Radius-Ratio on the Clad Surface Temperature of the Internal Cell

Fig. 11 Effect of Power Tilting on the Heat Flux Distribution at the Internal/Corner Common Boundary

Fig. 12 Effect of Power Tilting on the Clad Surface Temperature of the Corner Cell

Fig. 13 Effect of Power Tilting on the Clad Surface Temperature of the Internal Cell

Fig. 14 Dimensionless Temperature Distributions of the Four Unit Cells for a Fully Developed 19-Pin Bundle

Fig. 15a Effect of the Edge Cell on the Clad Temperature Behavior of the Corner Cell

Fig. 15b Comparison of Iterative Solution to Finite Difference Results

Fig. 16a: Schematic Hyperfine Heat Source Density Distribution in a 19-pin Bundle

Fig. 16b: Simulation of Different Heat Source Density Distributions by using Eq. (5)

Fig. 17a: Representation of an Eccentric Pellet Position with Finite Contact Angle β

Fig. 17b: Representation of Cracked Fuel

Fig.19 Temperature Distribution Around the Circumference of the Side Pin
(continued)

— calculated data
- - - experimental data [10]

Fig. 22a Maximum Temperature Variation around the Circumference of the Side Pin

Fig. 22b Maximum Temperature Variation Around the Circumference of Side Pin

Fig. 23 Nusselt Number vs Axial Distance for Side Channel

Fig. 24 Dimensionless Temperature Variation of Side Pin

Fig. 26 Temp. Dist. Around the
Circumference of Side Pin
(continued)

— $q'' = q''(z)$
 ---- $q'' = \text{constant}$

$$\frac{T - T_{in}}{q'' R_i} k$$

Fig. 27. Temperature Distribution Around the Circumference of Side Pin
(continued)

— $q'' = q''(z)$
 --- $q'' = \text{constant}$

$$\frac{T - T_{in}}{g'' R_c} k$$

Fig. 28 Temperature Distribution Around the Circumference of Side Pin (continued)

— $g'' = g''(\xi)$
 - - - $g'' = \text{constant}$

$$\Delta T^* = \frac{T_{max} - T_{min}}{q'' R_i} k$$

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Fig.29 Maximum Temp. Variation
Around the Circumference of
the Side Pin

Fig. 30 Nusselt Number vs Axial Distance for Side Channel
Heating Length = 700 mm.

$$\frac{T - T_{in}}{q'' R_1} k$$

Fig. 31 Dimensionless Temperature Distribution Around the Side Pin with $q'' = \text{constant}$ and Perturbed Inlet Temp.

— uniform inlet temp.
 - - - perturbed inlet temp.

Fig.34 Dimensionless Temperature Distribution
Around the Side Pin with
 $q'' = \text{constant}$ and Perturbed Inlet Temp.
(continued)

Fig. 35 Nusselt Number vs Axial Distance for Perturbed System.

Fig.36 Temperature Distribution Around the circumference of Corner Pin

Fig.37 Temperature Distribution Around the circumference of Corner Pin
(continued)

Fig.38 Temperature Distribution around the Circumference of Corner Pin
(continued)

Fig. 39a Maximum Temperature Variation around the Circumference of the Corner Pin

Fig. 39b Maximum Temperature Variation Around the Circumference of Corner Pin

Fig. 40 Nusselt Number vs Axial Distance for Corner Channel