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MESONS FROM DUAL RESONANCE MODELS

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MESONS FROM DUAL RESONANCE MODELS*

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ABSTRACT

We show that the generalized Veneziano model predicts the existence of quasi-stable high-mass mesons with definite masses and spins, which have their lowest-multiplicity decay-modes suppressed. Although the predicted degree of stability is less than experimentally required, proper introduction of internal quantum numbers into the model would likely enhance it. Besides $\psi(3.1)$ and $\psi'(3.7)$, relatively stable vector mesons of higher masses are expected, with the next one at a mass of ≈ 6 GeV.

The rule of Zweig et al.¹, currently used in the interpretation of narrow resonances, has connections of some sort with duality. It is intimately associated, in fact, with duality diagrams, which were originally introduced by Harari, Rosner and others² to represent solutions of SU(3) duality bootstraps. Actually, some of the predictions of the Zweig rule (suppressions

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of $\phi \rightarrow \rho\pi$ and $f' \rightarrow 2\pi$ decays) were independently obtained from such a dual approach.³ It should not be surprising, therefore, that duality when used in some other form still leads to decay suppression rules.⁴ This appears to be the case, in particular, when it is consistently enforced on multiparticle amplitudes, as it occurs in dual resonance models (DRM's). The existence of a suppression mechanism for the lower multiplicity decays in DRM's was realized rather early in the development of these models,⁵ but it found no phenomenological application at the time. We have recently called attention⁶ to its possible relevance in the interpretation of phenomena in which strong decay modes appear to be suppressed. Purpose of this note is to discuss the predictions of the generalized Veneziano model (GVM)⁷ concerning quasi-stable hadrons. We shall derive, more specifically, a rule for the values of the masses and spins of the most stable resonances in the model, which have their lowest multiplicity decay modes suppressed. The numerical results that we obtain (Table I) are a rigid prediction of the generalized Veneziano model,⁷ with no free parameters involved, except for the value of the universal Regge slope α' .

The theoretical results of Ref. 5 go rather far towards deriving the spectrum of the most stable hadrons expected in GVM, but they have to be somewhat modified in order to obtain a rule giving decay suppressed states with well defined masses. The resonance states of GVM can be represented by products of vector operators $b_{\mu}^{(n)}$ ($n = 1, 2, \dots, \infty$) applied to $|0\rangle$, the lightest particle state with angular momentum zero, and by linear combinations of these products.⁷ Although such operators are conventionally

associated in the DRM literature with oscillating modes of increasing frequency, one can equivalently consider them as representing hadron "constituents" with spin one and square masses $m_n^2 = n(1/\alpha')$, α' being the universal Regge slope.⁶ Constituent square masses (and not linear masses) simply add up to give the square mass of a hadron (this can be interpreted as simulating a binding energy effect), and since the $b_\mu^{(n)}$'s do not depend on space and time variables (they are not fields but just operators) the hadron spin is build up by the constituent spins only (there is no internal orbital motion). The decay amplitude of a resonance state represented by $\lambda(b)|0\rangle$, $\lambda(b)$ being a polynomial in the operators $b_\mu^{(n)}$'s, into k (lightest) particles is

$$\langle p, k | \lambda(b) | 0 \rangle = \int dx \phi(x, p) \lambda(P) \quad (1)$$

where $\phi(x, p)$ is the integrand of the $(k+1)$ - point dual amplitude, and $\lambda(P)$ is obtained from $\lambda(b)$ by the substitution

$$b_\mu^{(n)} \rightarrow p_\mu^{(n)} \equiv p_{0\mu} + (x_1)^\mu p_{1\mu} + \dots + (x_{k-2})^\mu p_{k-2,\mu} \quad (2)$$

where $p_{0\mu}$, $p_{1\mu}$, $\dots$, $p_{k-1,\mu}$ are the 4-momenta of the k decay particles, and $x_1, \dots, x_{k-2}$ are integration variables. Thus, resonance states described by $\lambda(b)$'s such that $\lambda(P) = 0$ cannot decay into k particles. In order to construct such states we shall basically use the results of Ref. 5, but with an important modification which allows to have decay suppressed states with well defined masses. Let us define a formal matrix β_k , each element of which is a $b_\mu^{(n)}$ operator according to the rule

$$(\beta_k)_{j\ell} = b_{\mu_\ell}^{(1j+h_\ell)} \quad (3)$$

with $j, \ell = 1, 2, \dots, k$ (not $k-1$)⁵, and where i_j, h_ℓ, μ_ℓ are arbitrary functions of the indices, but such that no two rows or columns are equal among themselves. The formal determinant $\det(\beta_k)$ is a coherent sum of hadronic states all having the same square mass

$$M^2 = (1/\alpha') \sum_{j=1}^k (i_j + h_j) = (1/\alpha') N. \quad (4)$$

Such a resonance state decouples from decay states with k or less particles. In fact, substituting $b_\mu^{(n)} \rightarrow p_\mu^{(n)}$ one obtains

$$\det(\beta_k) \rightarrow \sum_{\ell=1}^k p_{0\mu_\ell} M_\ell + D \quad (5)$$

where the M_ℓ 's and D are polynomial determinants of the same type as those constructed in Ref. 5, and which therefore do vanish.

Working in the resonance rest frame the time components of the $b_\mu^{(n)}$'s are to be disregarded (this has to be taken care of in GVM by appropriate Ward-like identities)⁸. The lightest hadron

which cannot decay into k or less particles is obtained for $i_j = j$, $h_{3m+1} = h_{3m+2} = h_{3m+3} = m$, $\mu_\ell = \text{mod}(\ell-1, 3) + 1$. Let us denote this hadron by Δ_k .

All other hadrons Δ_k^* that cannot decay into k or less particles (corresponding to β_k 's with more general configurations, or to $\det(\beta_k)$ times some polynomial in the $b_\mu^{(n)}$'s) are more massive than Δ_k and can decay into $\Delta_k + \text{something else}$, or $\Delta_{k'} + \Delta_{k''}$ with $k' + k'' = k$ (see below), etc. Therefore they are expected to be less stable than Δ_k . The most stable hadrons in GVM are therefore the states Δ_k for $k = 2, 3, \dots, \infty$. Because of their determinant form, their spin J is $J = 0$ if k is an integer multiple of 3, and otherwise $J = 1$. Spin and masses of the Δ_k 's (assuming $\alpha' = 1 \text{ GeV}^{-2}$) are listed in Table I. Concerning the decays $\Delta_k^* \rightarrow \Delta_{k'} + \Delta_{k''}$, $k' + k'' = k$, we like to remark

that, when possible, they are energetically favored with respect to $\Delta_k^* \rightarrow \Delta_k + \text{something else}$. For $k=5$, the Δ_5^* mass threshold for $\Delta_5^* \rightarrow \Delta_2(1730) + \Delta_3(2450)$ (Table I) lies about 60 MeV $[(\sqrt{3}+\sqrt{6}-\sqrt{17})/\sqrt{3}]$ above the mass of Δ_5 . If Δ_5^* is the ϕ -like member of a SU(3) nonet, Δ_2 and Δ_3 appearing in the decay must both carry strangeness. Although the decays of Δ_2 and Δ_3 with multiplicity less or equal to 2 and 3, respectively, are suppressed, in the real world they may have small non-zero branching fractions (as, e.g., the 2π decay mode of $\rho'(1600)$, see below). $\Delta_k^* \rightarrow \Delta_k + \pi$ decays, therefore, may look very similar to $D + \bar{D}^*$ decays in the charm scheme. We shall discuss the decay properties of the Δ_k^* 's at more length elsewhere.

It is hard to discuss the actual stability of the Δ_k 's without having some knowledge of their parity and internal quantum numbers. But unfortunately that is where GVM (and any other existing DRM) abandons us. Concerning parity, one may argue that in order to get the observed vector meson (ρ , etc.) and tensor meson (A_2 , etc.) multiplets, $|0\rangle$ must be a scalar and the $b_\mu^{(n)}$'s (negative parity) vectors. But then one misses (since there are no odd daughters) the pseudoscalar (π , etc.) and the observed axial vector mesons ($B(1235)$, etc.). In a realistic version one probably has to introduce ground states and vector operators of both parities, leaving unitarity corrections (loops) to take care of the necessary mass shifts. For our purposes, this means that in a realistic version of GVM Δ_k 's may have any parity, "a priori". As to internal quantum numbers, although Chan-Paton factors provide a simple recipe to combine any SU(N) symmetry with DRM's, it is obvious that they cannot tell the whole story.

If nothing else, in a complete theory, which must also include fermions, they prove inadequate to handle the $SU(3)$ octet and decuplet nature of baryons. On the other hand, the inclusion of internal quantum numbers is crucial for fully understanding decay suppressions in a duality framework. Besides thinking of the old result of Chiu and Finkelstein³ concerning $\phi \rightarrow \rho\pi$ and $f' \rightarrow 2\pi$, one has to keep in mind that a much larger number of decay suppressions (e.g. $\eta' \rightarrow \eta\pi\pi$, $K^{*}(1420) \rightarrow K\eta$, etc.) results from the enforcement of Veneziano-type duality on two-body amplitudes.⁴ In our case, one should know what is the generalization of this type of mechanism to multiparticle amplitudes. At the moment, the only thing that we can use from the existing knowledge of two-body duality bootstraps is that decay suppressions associated with quantum numbers mainly affect resonances or particles with least selective quantum numbers (e.g., $I=0$ instead of higher isospin values). The reason for that is that in a duality bootstrap such decay suppressions are needed to cope with factorization constraints, which of course are more numerous for couplings appearing in many amplitudes. That is the same reason why some degeneracy ($\omega - \phi$, etc.) is required in these cases.

Leaving open, therefore, the issue of how much the stability of the Δ_k 's would be improved by a consistent treatment of internal quantum numbers in DRM's, let us discuss Table I with reference to some experimental data. The vector Δ_2 at $M=1.7$ GeV with decay multiplicity 2 suppressed, if associated with $\rho'(1600)$, offers an understanding of why the 2π partial width of this resonance is so small.⁹ The vectors Δ_4 and Δ_5 are the

obvious candidates for identification with $\psi(3.1)$ and $\psi'(3.7)$ (masses can be made closer by a slight increase of α' , but of course this is not a relevant issue with the model at hand). It is clear that the suppressed multiplicities are not large enough to ensure stability if decay modes including pions are not somehow excluded. One would need something like $\phi \neq 3\pi$ but on a larger multiplicity scale. We believe that this is not hopeless, but is clearly left for a future generation of DRM's properly incorporating internal quantum numbers. The fact that for both ψ 's $I=0$ instead of 1 can be considered encouraging in this respect. The next vector in Table I is Δ_7 , with a mass ≈ 6 GeV, which looks interesting in view of the reported observation of some effect near this mass in both electron-positron¹⁰ and dimuon pairs.¹¹ As to the higher mass Δ_k vectors and the $J=0$ Δ_k 's there are no data at present to refer to.

In conclusion, DRM's, and more specifically the generalized Veneziano model,⁷ predict a spectrum of quasi-stable high-mass mesons which does not look unreasonable. Existing DRM's, deprived of internal quantum number information, are unable to give quantitatively satisfactory predictions about the degree of stability of these objects. They are only able, at this stage, to provide a list of the "most" stable high-mass resonance states. Taking into due account these limitations, one is nonetheless provided with detailed predictions for masses and spins and a unified framework in which phenomena like the suppression of the $\rho'(1600) \rightarrow 2\pi$ decay mode, the existence of the $\psi(3.1)$ and $\psi'(3.7)$ resonance states, the possible existence of other relatively

stable mesons with higher masses can be naturally correlated. As of now, one cannot tell whether by fully developing such an approach it will be eventually possible to reproduce the data, and in particular the observed stability of the ψ 's, without introducing charmed quarks. On the other hand, there is no theoretical problem "a priori" in thinking of some $SU(4)$ (or higher $SU(N)$ symmetric DRM, in which part of the decay suppressions have a symmetry origin and others a DRM origin.

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Table I. Masses (M) and spins (J) of the most stable resonance states (Δ_k) in the generalized Veneziano model (Ref.7); k is the highest decay multiplicity suppressed for a given state. Quoted masses in GeV result from assuming $\alpha' = 1 \text{ GeV}^{-2}$.

k	$N = M^2 \times \alpha'$	$M(\text{GeV})$	J
2	3	1.73	1
3	6	2.45	0
4	11	3.32	1
5	17	4.12	1
6	24	4.90	0
7	33	5.74	1
8	43	6.56	1
9	54	7.35	0
10	67	8.18	1
11	81	9.00	1
12	96	9.80	0
...	...	...	...