PPPL--2691

DE90 010280

Kinetic Theory of Geomagnetic Pulsations
I. Internal Excitations by Energetic Particles

Liu Chen

Plasma Physics Laboratory and Department of Astrophysical
Sciences, Princeton University, Princeton, NJ 08543

and

Akira Hasegawa

AT&T Bell Laboratories, Murray Hill, NJ 07974

ABSTRACT

Motivated by recent satellite observations, we have carried out a comprehensive theoretical analysis on the generation of hydromagnetic Alfvén waves in a realistic magnetospheric plasma environment consisting of a core (~ 100 eV) and an energetic (~ 10 keV) components. Our theoretical formulation employs the gyrokinetic equations and, thus, retains nonuniform plasma equilibria, anisotropy, finite Larmor radii, magnetic trapping as well as wave-particle interactions. A set of coupled equations for transverse and compressional magnetic perturbations is derived and analyzed for its stabilities assuming interchange stable equilibrium distribution functions. Our findings are (1) compressional and transverse shear Alfvén oscillations are generally coupled in realistic plasmas. (2) In the decoupled limit, for the compressional wave branch, one recovers the drift-mirror instability due to the Landau resonances and $\tau \equiv 1 + 4\pi(\partial P_{\perp}/B\partial B) < 0$. Here, $P_{\perp} = P_{\perp}(\psi, B)$ is the perpendicular pressure and ψ is the magnetic flux function. (3) For the decoupled transverse shear Alfvén branch, one obtains the drift Alfvén

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

ballooning instability due to the Landau resonances and free energy of the pressure gradient for $\tau > 0$. (4) For both branches, the most unstable modes have antisymmetric structures and propagate in the diamagnetic drift direction of the energetic ions. (5) Finite coupling can be shown to further enhance the drift Alfvén ballooning instabilities. Thus, we conclude that for $\tau \geq 0$, the coupled drift Alfvén ballooning-mirror instability constitutes an important internal generating mechanism of geomagnetic pulsations. The various predicted features of this instability are also found to be consistent with satellite observations.

I. INTRODUCTION

Recently, an increased number of satellite observations of compressional waves in the Earth's magnetosphere [c.f. the review by Takahashi, 1988] has stimulated renewed theoretical interest in the detailed generating mechanisms of such magnetospheric hydromagnetic waves or geomagnetic pulsations. At the same time, more precise measurements of the background plasmas have become available [c.f. Krimigis et al., 1985; Baumjohann et al., 1987] and have made such detailed studies on the generating mechanisms feasible. Near the geosynchronous orbit, which is of the primary interest here, a magnetospheric plasma has the following important general characteristics: (1) it consists of a core ($T_C \sim 100$ eV) and an energetic ($T_E \sim 10$ keV) components with the mass density ratio $\rho_C/\rho_E \sim 10$, and (2) the energetic component has a pressure profile which is interchange stable; i.e., $|L \partial \ln P(L)/\partial L| \lesssim 20/3$ [Gold, 1959]. Here, L is the invariant latitude; i.e., the equatorial distance of the magnetic field line. Characteristics (1) indicates that while the core component dominates the plasma inertia, free energy of the pressure gradient is carried by the energetic component. Characteristic (2), meanwhile, suggests that the energetic component possesses an equilibrium distribution function $\partial \hat{F}_{0E}/\partial \psi|_{\mu, J} \lesssim 0$, with μ and J being, respectively, the first and second adiabatic invariants. This is, thus, compatible to the equilibrium achievable by plasma injection while conserving μ and J ; i.e., adiabatic injection. $\partial \hat{F}_{0E}/\partial \psi|_{\mu, J} \lesssim 0$, furthermore, not only guarantees interchange stability [Liu, 1970] but is also consistent with available satellite data [Shultz and Lanzerotti, 1974; Cheng et al., 1985]. One

1974; Cheng et al., 1985]. One purpose of the present work is, therefore, to carry out a comprehensive study of plasma instabilities capable of generating long-period (Pc3 to Pc5) geomagnetic pulsations in a realistic magnetospheric plasma having the characteristics cited above.

Geomagnetic pulsations can be excited, in general, either by local field line resonances in response to external sources [Chen and Hasegawa, 1974a,b; Southwood 1974], or by local (internal) plasma instabilities. The former generally has small azimuthal wavenumbers [$m \lesssim O(10)$] and is observable on the ground, while the latter, which is the subject of the present work, tends to have large m numbers [$m \sim O(10^2)$] and is primarily observed by satellites [Takahashi, 1988].

Various authors have contributed to the theoretical studies of pulsations internally generated by local plasma instabilities [see, e.g., the review by Lanzerotti and Southwood, 1979]. One primary candidate for the instability mechanism is the drift-mirror instability [Hasegawa, 1969], which is an instability driven by an anisotropic plasma pressure in a $\beta_{\perp} \sim O(1)$ plasma. Here, $\beta_{\perp} \equiv 8\pi P_{\perp}/B^2$. While the threshold condition of the drift-mirror instability is the same as the classic mirror mode [Chandrasekhar, et al., 1958], i.e., $\tau \equiv 1 + 4\pi \partial P_{\perp}/\partial B \leq 0$, the instability picks up a real frequency of the order of diamagnetic drift frequency via coupling to the drift waves. Note that $P_{\perp} = P_{\perp}(\psi, B)$ and the dependence on B is due to the pressure anisotropy. Also, in contrast to the classic mirror mode, which is a reactive-type ideal magnetohydrodynamic (MHD) instability, the drift-mirror instability depends

critically on the Landau wave-particle interactions and thus may be regarded as a dissipative-type instability. While the observed wave characteristics are generally consistent with those of the drift-mirror instability, simultaneous satellite observations on the background plasma data, on the other hand, often fail to satisfy the $\tau \leq 0$ condition due to relative low- β values and/or mild anisotropy. Therefore, alternative candidates have been proposed such as coupling to the Alfvén ballooning mode [Pokhotelov et al., 1986] as well as excitations via wave-particle resonances [Southwood, 1976; Chen and Hasegawa, 1988]. These previous theories are, however, generally inadequate, when compared in detail with satellite observations, due to the neglect of either eigenmode structures along the field line or finite coupling between the compressional-mirror and transverse (shear) Alfvén ballooning modes. The inadequacies of existing theories further provide the motivation for this work.

Our theoretical formulation employs the low frequency gyrokinetic equation [Antonsen and Lane, 1980; Catto et al., 1981] and includes effects due to both radial and parallel plasma and magnetic field nonuniformities, pressure anisotropy, field line curvature, finite coupling between compressional and transverse magnetic field perturbations, finite Larmor radii, magnetic trapping as well as wave-particle interactions. Our theory is thus capable of delineating the significance of various physical effects and eigenmode structures, and provides a consistent picture of dominant instability generating mechanisms in the various plasma parameter regimes which are compatible with recent satellite observations.

II. THEORETICAL MODEL AND GYROKINETIC ANALYSIS

As discussed in Sec. I, we consider a magnetospheric plasma consisting of a core (C) and an energetic (E) component in an equilibrium magnetic field $\mathbf{B} = \nabla\psi \times \nabla\phi$ with ϕ being the azimuthal (East-West) symmetry coordinate. At the geosynchronous orbit of interest here, we have $T_C \sim O(100 \text{ eV})$, $T_E \sim O(10 \text{ keV})$ and, hence, $T_C/T_E \sim O(\delta^4)$ with $\delta^2 \equiv O(10^{-1})$ being the smallness parameter to be used in the following ordering scheme. Note that orderings are necessary here in order to simplify the coupled Vlasov-Maxwell system and, thereby, make further analytical progress.

Since we are interested in a high- β plasma with the pressure mainly residing in the energetic component, we order $\beta_E \sim O(1)$ and $\beta_C \sim O(\delta^2)$, which then imply $n_E/n_C \sim O(\delta^2)$. That is, the core component carries the inertia. To order the wave frequency, we have $\omega \sim O(v_A/L)$ corresponding to that of shear Alfvén ballooning mode. Here, v_A is the Alfvén speed. Noting that particles are magnetically trapped with typical bounce frequency $\omega_b \sim O(v/L)$, we then have $\omega/\omega_{b,E} \sim O(\delta)$ and $\omega/\omega_{b,C} \sim O(\delta^{-1})$. That is, while magnetic trapping of the core ions could be neglected dynamically, the energetic ions do bounce rapidly and magnetic trapping can be expected to affect the dynamics. With $(m_e/m_i)^{1/2} \sim O(\delta^3)$, we have $\omega/\omega_{be} \lesssim O(\delta^2)$ and all electrons bounce very rapidly. To order the wavelengths, we take $k_{\parallel}L \sim O(1)$ for the standing Alfvén waves. Furthermore, to investigate the coupling between drift-mirror and Alfvén ballooning modes, we order $\omega \sim v_A/L \sim \omega_{*E} \sim k_{\phi}\rho_{iE}v_{iE}/L$. Here, ω_* is the diamagnetic drift frequency, $\rho_{iE} = v_{iE}/\Omega_i$, and $\Omega_i = eB/m_i c$. We.

therefore, have $k_\phi \rho_{iE} \sim O(\delta)$, $k_\phi \rho_{ic} \sim O(\delta^3)$, and $k_\phi \rho_e \lesssim O(\delta^4)$. That is, while, due to the observed large azimuthal wavenumbers $m \sim O(10^2)$, it may be desirable to keep the finite Larmor radius effects of the energetic ions, those of the core ions and electrons are ignorable. To summarize, we have adopted the following ordering scheme in the present analysis:

$$\begin{aligned}
 \frac{T_C}{T_E} &\sim O(\delta^4 \equiv 10^{-2}), \quad \frac{n_E}{n_C} \sim O(\delta^2); \\
 \beta_E &\sim O(1), \quad \beta_C \sim O(\delta^2); \\
 \omega &\sim O\left(\frac{v_A}{L}\right) \sim O(\omega_{*iE}); \\
 \frac{\omega}{\omega_{b,iE}} &\sim O(\delta), \quad \frac{\omega}{\omega_{b,iC}} \sim O(\delta^{-1}), \quad \frac{\omega}{\omega_{be}} \lesssim O(\delta^2); \\
 k_{\parallel} L &\sim O(1); \\
 k_\phi \rho_{iE} &\sim O(\delta), \quad k_\phi \rho_{ic} \sim O(\delta^3), \quad k_\phi \rho_e \lesssim O(\delta^4).
 \end{aligned} \tag{1}$$

The above orderings, being consistent with the gyrokinetic orderings, then suggest that we employ the low-frequency linear gyrokinetic equations [Antonsen and Lane, 1980; Catto et al., 1981]. Thus, in terms of the guiding center variables, $\mathbf{V} \equiv \{\epsilon = v^2/2, \mu = v_\perp^2/2B, \alpha = \text{gyrophase angle}, \hat{\sigma} = \text{sgn}(v_{\parallel})\}$, and adopting WKB ansatz in the perpendicular direction, i.e.,

$$\delta f(\mathbf{x}, \mathbf{v}, t) = \delta F(\ell, \mathbf{V}) \exp[i(\int^{\mathbf{x}_\perp} \mathbf{k}_\perp \cdot d\mathbf{x}_\perp - \omega t)]$$

with ℓ being the coordinate along $\mathbf{B}$, the perturbed distribution function, δF , is given by

$$\delta F = \frac{q}{m} \left\{ \delta \phi \frac{\partial F_0}{\partial \epsilon} - \frac{Q}{\omega} F_0 J_0 \delta \psi e^{iL_k} + \frac{1}{B} \frac{\partial F_0}{\partial \mu} \right. \\ \left. \times \left[(\delta \phi + i \frac{v_{\parallel}}{\omega} \frac{\partial \delta \psi}{\partial \ell}) (1 - J_0 e^{iL_k}) - \frac{v_{\perp} J_1}{k_{\perp} c} \delta B_{\parallel} e^{iL_k} \right] \right\} + \delta K e^{iL_k} \quad (2)$$

where δK satisfies

$$[\partial |v_{\parallel}| \frac{\partial}{\partial \ell} - i(\omega - \omega_d)] \delta K = i \frac{q}{m} Q F_0 (\delta S_1 - i \partial \delta S_2) \quad (3)$$

$$\delta S_1 = J_0 (\delta \phi - \delta \psi) + \omega_d J_0 \delta \psi / \omega + v_{\perp} J_1 \delta B_{\parallel} / k_{\perp} c \quad (4)$$

$$\delta S_2 = -J_1 \frac{\partial \lambda}{\partial \ell} |v_{\parallel}| \delta \psi / \omega \quad (5)$$

$$Q F_0 = (\omega \frac{\partial}{\partial \epsilon} + \hat{\omega}_*) F_0 \quad (6)$$

$F_0 = F_0(\epsilon, \mu, \psi)$ is the equilibrium distribution function,

$$\hat{\omega}_* F_0 = (\mathbf{k} \times \mathbf{e}_{\parallel} / \Omega) \cdot \nabla F_0 \quad ,$$

$\mathbf{e}_{\parallel} = \mathbf{B}/B$, $J_0 (= J_0(\lambda))$, and $J_1 (= J_1(\lambda))$ are the Bessel functions $\lambda = k_{\perp} \rho$, $L_k = \mathbf{k} \cdot \mathbf{e}_{\parallel} \times \mathbf{v} / \Omega$, $\omega_d = \mathbf{k}_{\perp} \cdot \mathbf{v}_d$, and $\mathbf{v}_d = \mathbf{e}_{\parallel} \times [\mu \nabla B + v_{\parallel}^2 \kappa] / \Omega$ with $\kappa = \mathbf{e}_{\parallel} \cdot \nabla \mathbf{e}_{\parallel}$ is the magnetic (∇B and curvature) drift velocity. Since $J = J(\epsilon, \mu, \psi)$, we may express F_0 as $\hat{F}_0(\mu, J, \psi)$; that is, $F_0 = \hat{F}_0(\mu, J)$ for the adiabatically injected plasmas of interest here. As for the field variables, we have adopted vector and scalar potentials and the Coulomb gauge, i.e., $\delta \phi$, $i\omega \delta A_{\parallel} \equiv c \partial \delta \psi / \partial \ell$, and $\delta B_{\parallel}$. Given that particles are magnetically trapped with turning points at ℓ_1 and ℓ_2 , Eq. (3) can be readily solved and one finds [Rutherford and Frieman, 1968; Taylor and Hastie, 1968]

$$\begin{aligned} \delta K_s = \frac{1}{2} \sum_{\hat{\sigma}} \delta K(\hat{\sigma}) = - \frac{q}{m} QF_0 \{ \cot I_{\ell_1}^{\ell_2} c_{\ell_1}^{\ell} \tau_b \overline{\delta S_1 c_{\ell_1}^{\ell} - \delta S_2 s_{\ell_1}^{\ell}} \\ + c_{\ell_1}^{\ell} \tau_b \overline{\delta S_1 s_{\ell_1}^{\ell} + \delta S_2 c_{\ell_1}^{\ell}} + \int_{\ell_1}^{\ell} \frac{d\ell'}{|v_{\parallel}|} (\delta S_1 s_{\ell'}^{\ell} - \delta S_2 c_{\ell'}^{\ell}) \} . \end{aligned} \quad (7)$$

$$\begin{aligned} \delta K_d = \frac{1}{2} \sum_{\hat{\sigma}} \hat{\sigma} \delta K(\hat{\sigma}) = - i \frac{q}{m} QF_0 \{ \cot I_{\ell_1}^{\ell_2} s_{\ell_1}^{\ell} \tau_b \overline{\delta S_1 c_{\ell_1}^{\ell} - \delta S_2 s_{\ell_1}^{\ell}} \\ + s_{\ell_1}^{\ell} \tau_b \overline{\delta S_1 s_{\ell_1}^{\ell} + \delta S_2 c_{\ell_1}^{\ell}} - \int_{\ell_1}^{\ell} \frac{d\ell'}{|v_{\parallel}|} (\delta S_1 c_{\ell'}^{\ell} + \delta S_2 s_{\ell'}^{\ell}) \} . \end{aligned} \quad (8)$$

where

$$I_a^b = \int_a^b \frac{d\ell}{|v_{\parallel}|} (\omega - \omega_d) , \quad (9)$$

$\tau_b = 2 \int_{\ell_1}^{\ell_2} d\ell' / |v_{\parallel}|$ is the bounce period, $c_a^b = \cos I_a^b$, $s_a^b = \sin I_a^b$, and

$$\bar{A} = 2 \int_{\ell_1}^{\ell_2} (A/\tau_b) d\ell / |v_{\parallel}| \quad (10)$$

corresponds to bounce averaging. Note that, in Eq. (7) and (8),

$$\cot I_{\ell_1}^{\ell_2} = \sum_{K=-\infty}^{\infty} [\tau_b (\omega - \bar{\omega}_d - K\omega_b)]^{-1} , \quad (11)$$

where $\omega_b = 2\pi/\tau_b$ is the bounce frequency. δK_s and δK_d , therefore, do contain wave-particle interactions via the magnetic drift and bounce resonances. Note also that $|\bar{\omega}_d/\omega_b| \sim O(k_{\phi}\rho)$ and, hence, we need to keep the finite $k_{\phi}\rho$ effects when discussing the important magnetic drift-bounce resonances of the highly energetic (hot) ions [Chen and Hasegawa, 1988].

With the gyrokinetic orderings, Maxwell equations, meanwhile, are reduced

to (i) the quasi-neutrality condition $\sum_i q_i \langle \delta F \rangle_i = 0$ or, using Eq. (2),

$$\sum_i \left(\frac{q_i^2}{m} \right) \left\{ \left\langle \delta \phi \left[\frac{\partial F_0}{\partial \epsilon} - \frac{Q}{\omega} F_0 J_0^2 \delta \psi + \frac{m}{q} J_0 \delta K_s \right. \right. \right. \\ \left. \left. \left. + \frac{1}{B} \frac{\partial F_0}{\partial \mu} \left[\delta \phi (1 - J_0^2) + J_0 J_1 \frac{v_\perp \delta B_{||}}{k_\perp c} \right] \right] \right\rangle_i \right\} = 0, \quad (12)$$

where

$$\langle \dots \rangle = \int d^3 v (\dots) = 2\pi \sum_{\sigma} \int (\dots) B d\mu d\epsilon / |v_{||}|;$$

(ii) perpendicular Ampere's law

$$k_\perp^2 \delta B_{||} = - \frac{4\pi}{c} k_\perp \sum_i q_i \langle v_\perp J_1 \left\{ \frac{q}{m} \left[\frac{Q}{\omega} F_0 J_0 \delta \psi \right. \right. \right. \\ \left. \left. \left. + \left(J_0 \delta \phi + \frac{v_\perp J_1 \delta B_{||}}{k_\perp c} \right) \frac{1}{B} \frac{\partial F_0}{\partial \mu} \right] - \delta K_s \right\} \right\rangle_i; \quad (13)$$

and (iii) parallel Ampere's law or, alternatively, the following generalized vorticity equation

$$\frac{c^2}{4\pi\omega^2} B \frac{\partial}{\partial \ell} \left[\frac{k_\perp^2}{B} \sigma \frac{\partial}{\partial \ell} \right] \delta \psi = \sum_i q_i \left\langle \frac{\omega_d}{\omega} J_0 \delta K_s - \frac{i}{\omega} |v_{||}| J_1 \frac{\partial \lambda}{\partial \ell} \delta K_d \right. \\ \left. + \frac{q}{m} \left\{ \delta \phi \left[\frac{\partial F_0}{\partial \epsilon} - \frac{Q}{\omega} F_0 (J_0^2 \delta \phi + \frac{\omega_d}{\omega} J_0^2 \delta \psi + \frac{J_0 J_1 v_\perp}{k_\perp c} \delta B_{||}) \right. \right. \right. \\ \left. \left. \left. + \frac{1}{B} \frac{\partial F_0}{\partial \mu} [(1 - J_0^2) \delta \phi - \frac{J_0 J_1 v_\perp}{k_\perp c} \delta B_{||}] \right\} \right\rangle_i, \quad (14)$$

where

$$\sigma = 1 + (4\pi/k_{\perp}^2 c^2) \sum_j (q^2/m)_j \langle v_{\parallel}^2 (1 - J_0^2) \partial F_0 / B \partial \mu \rangle_j \quad (15)$$

is the firehose stability term including finite Larmor radius corrections. Note that $\partial F_0 / \partial \mu \neq 0$ corresponds to anisotropic velocity distribution.

III. REDUCED EIGENMODE EQUATIONS

While Eqs. (12) to (14) along with Eqs. (7) and (8) form a complete set of eigenmode equations for the eigenvalue ω and eigenfunctions $\delta\phi$, $\delta\psi$ and $\delta B_{\parallel}$, it is very difficult to solve them even numerically. One can, however, make significant analytical progress and gain physical insights on the stability properties by further simplifying the dynamics and, correspondingly, the eigenmode equations via subsidiary expansions suggested in the ordering scheme, Eq. (1). In carrying out the subsidiary expansions, we note the following essential features: (1) δK_d appears only in Eq. (14) along with the finite Larmor radius term $J_1 \partial \lambda / \partial \ell$ (2) Small Larmor radius expansions are valid except for those energetic hot ions ($T_{ih} \sim 10^2$ keV). (3) Electrons respond only to bounce averaged dynamics. (4) Core ions mainly respond to local dynamics along **B**. Small fraction of tail core ions do, however, experience $\omega \sim K \omega_{b,ic}$ resonances. (5) Energetic ions mainly respond to bounce averaged dynamics. Small fraction of warm ($T_{iw} \sim 1$ keV) and hot ($T_{ih} \sim 10^2$ keV) energetic ions, however, experience $\omega \sim K \omega_{b,iw}$ and $\bar{\omega}_{d,ih} \sim \omega_{b,ih}$ resonances, respectively. Leaving out the details, we find, for electrons $\delta K_{d,e} \simeq 0$ and

$$\delta K_{r,s} \simeq \left(\frac{q}{m} \right)_e \frac{Q F_{0e} \overline{\delta S_{1e}}}{\bar{\omega}_{de} - \omega}, \quad (16)$$

where

$$\delta S_{1e} = \delta \phi - \delta \psi + \omega_{de} \delta \psi / \omega + (\mu m / q)_e \delta B_{\parallel}.$$

For core ions, meanwhile, we have $\delta K_{d,1C} \simeq 0$ and

$$\delta K_{s,1C} \simeq - \left(\frac{q}{m} \right)_{iC} Q F_{0iC} \left[\frac{\delta S_1}{\omega} - i\pi \sum_K \delta(\omega - \bar{\omega}_d - K\omega_b) c_{\ell_1}^{\ell} \overline{\delta S_1 c_{\ell_1}^{\ell}} \right]_{iC}, \quad (17)$$

where

$$\delta S_{1,C} \simeq (\delta \phi - \delta \psi + \omega_d \delta \psi / \omega + \mu m \delta B_{\parallel} / q)_{iC}.$$

Finally, for the energetic ions, we obtain

$$\delta K_{d,1E} \simeq -\pi \left(\frac{q}{m} \right)_{iE} Q F_{0iE} \sum_K \left[\delta(\omega - \bar{\omega}_d - K\omega_b) s_{\ell_1}^{\ell} \overline{\delta S_1 c_{\ell_1}^{\ell} - \delta S_2 s_{\ell_1}^{\ell}} \right]_{iE}, \quad (18)$$

and

$$\begin{aligned} \delta K_{s,1E} \simeq & \left(\frac{q}{m} \right)_{iE} Q F_{0iE} \left[\overline{\delta \phi - \delta \psi + \omega_d \delta \psi / \omega + \mu m \delta B_{\parallel} / q} P(\bar{\omega}_d - \omega)^{-1} \right. \\ & \left. + i\pi \sum_K \delta(\omega - \bar{\omega}_d - K\omega_b) c_{\ell_1}^{\ell} \overline{\delta S_1 c_{\ell_1}^{\ell} - \delta S_2 s_{\ell_1}^{\ell}} \right]_{iE}. \end{aligned} \quad (19)$$

Here, P stands for the principal value.

As to the field equations, Eqs. (12) to (14), we may apply the same subsidiary expansions and use the simplified expressions of δK 's, Eqs. (16) to (19), to derive

a set of reduced eigenmode equations. Specifically, the quasi-neutrality condition, Eqs. (12), reduces to the ideal MHD approximation $\delta E_{\parallel} \simeq 0$ or

$$\delta\phi \simeq \delta\psi. \quad (20)$$

Physically, $\delta E_{\parallel} \simeq 0$ results from the existence of the core component, by which the plasma possesses large parallel conductivity and negligible finite ion Larmor radius effects. Meanwhile, perpendicular Ampere's law, Eq. (13), and the generalized vorticity equation, Eq. (14), become, respectively,

$$\begin{aligned} \tau \hat{B}_{\parallel} &= \frac{4\pi}{B^2} (\mathbf{e}_k \cdot \tilde{\nabla} P_{\perp}) \delta\psi + 4\pi \sum_{j=e,i} m_{jE} < QF_0 P (\omega - \bar{\omega}_d)^{-1} \mu \bar{\Omega}_d \delta\psi + \mu \delta \hat{B}_{\parallel} >_{jE} \\ &- 4i\pi^2 \sum_{j=e,i} m_j < \sum_K QF_0 \delta(\omega - \bar{\omega}_d - K\omega_b) S_B \overline{S_{\psi} \delta\psi + S_B \delta \hat{B}_{\parallel}} >_j, \end{aligned} \quad (21)$$

and

$$\begin{aligned} B \frac{\partial}{\partial \ell} \left(\frac{k_{\perp}^2}{B} \sigma \frac{\partial \delta\psi}{\partial \ell} \right) + \frac{\omega^2 k_{\perp}^2}{v_A^2} \delta\psi &= - \frac{4\pi}{B^2} (\mathbf{e}_k \cdot \tilde{\nabla} P_{\perp}) (\delta \hat{B}_{\parallel} + \Omega_B \delta\psi) \\ &- \frac{4\pi}{B^2} (\mathbf{e}_k \cdot \tilde{\nabla} P_{\parallel}) \Omega_{\kappa} \delta\psi - 4\pi \sum_{j=e,i} m_{jE} < QF_0 P (\omega - \bar{\omega}_d)^{-1} \Omega_d \overline{\Omega_d \delta\psi + \mu \delta \hat{B}_{\parallel}} >_{jE} \\ &+ 4i\pi^2 \sum_{j=e,i} m_j < \sum_K QF_0 \delta(\omega - \bar{\omega}_d - K\omega_b) S_{\psi} \overline{S_{\psi} \delta\psi + S_B \delta \hat{B}_{\parallel}} >_j; \end{aligned} \quad (22)$$

where $\delta \hat{B}_{\parallel} = \omega \delta B_{\parallel} / c$, $\sigma \simeq 1 + 4\pi (P_{\perp} - P_{\parallel}) / B^2$, $\mathbf{e}_k = \mathbf{k} \times \mathbf{e}_{\parallel}$,

$\tilde{\nabla} P_{\perp} = \nabla \psi \partial P_{\perp} / \partial \psi$, $\Omega_d = \mu \Omega_B + v_{\parallel}^2 \Omega_{\kappa} / B$, $\Omega_B = \mathbf{e}_k \cdot \nabla \ln B$, $\Omega_{\kappa} = \mathbf{e}_k \cdot \boldsymbol{\kappa}$,

$\boldsymbol{\kappa} = (\mathbf{e}_{\parallel} \cdot \nabla) \mathbf{e}_{\parallel}$ is the curvature of B,

$$S_w = \Omega_d J_0 c_{\ell_1}^{\ell} + (q|v_{\parallel}|/mc)(J_1 \partial \lambda / \partial \ell) s_{\ell_1}^{\ell} , \quad (23)$$

$$S_B = 2(\mu J_1 / \lambda) c_{\ell_1}^{\ell} . \quad (24)$$

and electrons contribute only to the $K = 0$ term. Note also that, in deriving Eqs. (21) and (22), we have used the ideal MHD equilibrium conditions

$$B \frac{\partial P_{\parallel}}{\partial B} = P_{\parallel} - P_{\perp} , \quad (25)$$

and

$$\tau \nabla_{\perp} \ln B + \frac{4\pi}{B^2} \tilde{\nabla} P_{\perp} = \sigma \kappa . \quad (26)$$

Equations (21) and (22) are the desired set of reduced eigenmode equations describing coupling between the compressional, $\delta \hat{B}_{\parallel}$, and transverse, $\delta \psi$, magnetic perturbations due to finite pressure nonuniformity, $\partial P_{\perp} / \partial \psi \neq 0$. One can readily observe that both modes are affected by bounce averaged dynamics and wave-particle interactions. Equation (22) further indicates the well-known fact that the transverse shear Alfvén mode in a $\beta \sim O(1)$ plasma could be strongly modified by the $|\tilde{\nabla} P_{\parallel} \Omega_{\kappa}|$ and $|\tilde{\nabla} P_{\perp} \Omega_B|$ ballooning/interchange terms.

IV. QUADRATIC FORMS AND "DISPERSION RELATION"

Since the reduced eigenmode equations have both differential and integral operators, numerical solutions are required in general. Significant insights to the stability properties can, however, be obtained from the following quadratic

forms readily derivable from Eqs. (21) and (22). Denoting,

$$[[A]] \equiv \int_{\ell_S}^{\ell_N} A d\ell / B \quad (27)$$

with ℓ_S and ℓ_N being the end points of the field line and $\delta\psi(\ell = \ell_S, \ell = \ell_N) = 0$, we have

$$F(\omega) = -\omega^2 \delta I + \delta W_I + \delta W_{KT} + i\delta W_R = 0, \quad (28)$$

where

$$\delta I = [[k_1^2 |\delta\psi|^2 / v_A^2]], \quad (29)$$

$$\begin{aligned} \delta W_I = & [[\tau] \delta \hat{B}_{\parallel} - \frac{4\pi}{B^2 \tau} (\mathbf{e}_k \cdot \tilde{\nabla} P_{\perp}) \delta\psi|^2]] + [[k_1^2 \sigma \left| \frac{\partial \delta\psi}{\partial \ell} \right|^2]] \\ & - 4\pi [[\frac{1}{B^2} \Omega_k |\delta\psi|^2 \mathbf{e}_k \cdot (\tilde{\nabla} P_{\parallel} + \frac{\sigma}{\tau} \tilde{\nabla} P_{\perp})]], \end{aligned} \quad (30)$$

$$\delta W_{KT} = (4\pi)^2 \sum_{j=e,i} << m Q F_0 P (\bar{\omega}_d - \omega)^{-1} \tau_b \left| \Omega_d \delta\psi + \mu \delta \hat{B}_{\parallel} \right|^2 >>_j, \quad (31)$$

$$\delta W_R = \omega \delta W_{\omega} + \delta W_{*}, \quad (32)$$

$$\left\{ \frac{\delta W_{\omega}}{\delta W_{*}} \right\} = 16\pi^3 \sum_K \sum_{j=e,i} << m \left\{ \frac{\partial F_0 / \partial \epsilon}{\hat{\omega}_* F_0} \right\} \delta(\omega - \bar{\omega}_d - K\omega_b) \tau_b \left| \overline{S_{\psi} \delta\psi + S_B \delta \hat{B}_{\parallel}} \right|^2 >>_j, \quad (33)$$

and

$$<< A >> \equiv \int d\mu \int d\epsilon (A). \quad (34)$$

δI , thus, corresponds to the inertia of core ion transverse motion. δW_I is the well-known δW of the ideal MHD energy principle [Bernstein et al., 1958]. The first term in δW_I , Eq. (30), corresponds to perpendicular compression and its

sign depends on τ . For $P_{\perp} \gtrless P_{\parallel}$, as of interest here, $\sigma > 0$ and the second term in δW_i represents the positive-definite field line bending effect. The third term in δW_i , meanwhile, is the well-known ballooning/interchange effect. As to δW_{KT} , we note that

$$\frac{QF_0}{\bar{\omega}_d - \omega} = - \frac{\partial F_0}{\partial \epsilon} + \frac{\mathbf{e}_k \cdot \nabla \psi}{\Omega(\bar{\omega}_d - \omega)} \frac{\partial \hat{F}_0}{\partial \psi}. \quad (35)$$

For adiabatically injected energetic ions, $\partial \hat{F}_0 / \partial \psi \approx 0$, we obtain, with $\partial F_{0E} / \partial \epsilon \lesssim 0$ as typically the case,

$$\delta W_{KT} \geq 0. \quad (36)$$

δW_{KT} thus corresponds to the positive-definite trapped-particle compression effect [Kruskal and Oberman, 1958; Rosenbluth and Rostoker, 1959; Van Dam et al., 1982]. This effect can also be readily observed to be rather general by noting that, for $|\omega| > |\epsilon \hat{\omega}_*| \sim |\bar{\omega}_d|$, $P[QF_0/(\bar{\omega}_d - \omega)] \simeq -\partial F_0 / \partial \epsilon \gtrless 0$ and, for $|\omega| < |\epsilon \hat{\omega}_*| \sim |\bar{\omega}_d|$, $P[QF_0/(\bar{\omega}_d - \omega)] \simeq \hat{\omega}_* F_0 / \bar{\omega}_d \gtrless 0$ for typically inward pressure gradient at, e.g., the outer edge of the ring current. Finally, δW_R corresponds to the effects of wave-particle interactions and could be regarded as the dissipative (anti-Hermitian) part of the "dielectric constant" $F(\omega)$. Here, we emphasize again that electrons only contribute to the ($K=0$) $\omega = \bar{\omega}_d$ magnetic drift resonance. Equation (28), $F(\omega) = 0$, thus may be conveniently viewed as the "dispersion relation".

Since δW_R is typically small, it is instructive to consider the system to be near marginal stability. Let $\omega = \omega_r + i\gamma$ and $|\gamma/\omega_r| < 1$, the "dispersion

relation", Eq. (28), becomes

$$-\omega_i^2 \delta I + \delta W_i + \delta W_{KT} - \gamma \delta W_\omega = 0, \quad (37)$$

and

$$-2\gamma\omega\delta I + \omega_i\delta W_\omega + \delta W_* = 0 \quad (38)$$

V. GENERAL STABILITY PROPERTIES

To delineate the underlying physics, we shall first analyze the stability properties of Eqs. (37) and (38) assuming the compressional and transverse modes are decoupled, and then consider the effects of finite coupling.

V.1 The decoupled limit.

Consider first the compressional mode. Here, $\delta\psi \rightarrow 0$ and, hence, $\delta I \rightarrow 0$. Equations (37) and (38) then reduce to

$$\gamma_D = [(\delta W_i + \delta W_{KT})/\delta W_\omega]_D, \quad (39)$$

and

$$\omega_{iD} = -[\delta W_*/\delta W_\omega]_D. \quad (40)$$

Here, the subscripts D on the right hand sides denote those with $\delta\psi \rightarrow 0$. Thus, noting Eq. (30),

$$(\delta W_i + \delta W_{KT})_D = [|\tau| \delta \hat{B}_\parallel|^2] + \delta W_{KTD}. \quad (41)$$

With $\delta W_{\omega D} < 0$, Eq. (39) indicates that the instability sets in for

$[[\tau|\delta\hat{B}_{||}|^2]] + \delta W_{\text{KTD}} < 0$; that is, the most unstable mode corresponds to perturbations which minimize $(\delta W_i + \delta W_{\text{KT}})_D$. From Eq. (36), we note that $\delta W_{\text{KTD}} \geq 0$ and, hence, is stabilizing. Since δW_{KTD} involves the bounce averaging of $\delta\hat{B}_{||}$ [c.f. Eq. (31)], it is then obvious that while $\delta W_{\text{KTD}} > 0$ for the symmetric (even) $\delta\hat{B}_{||}$, $\delta W_{\text{KTD}} = 0$ for the antisymmetric (odd) $\delta\hat{B}_{||}$. The odd- $\delta\hat{B}_{||}$ modes, therefore, corresponds to the most unstable perturbations. The instability conditions then reduces to

$$[[\tau|\delta\hat{B}_{||}|^2]] < 0. \quad (42)$$

Equation (42) represents the drift-mirror instability criterion generalized to realistic plasmas. It shows that the drift-mirror instabilities tend to be antisymmetric odd- $\delta\hat{B}_{||}$ modes localized within the $\tau \leq 0$ region of the flux tube. These features agree with those of the numerical solutions [Cheng and Lin, 1987]. Equation (40), meanwhile, shows that ω_{rD} is of the order of the diamagnetic drift frequency of the dominant resonant particles. For odd- $\delta\hat{B}_{||}$ modes, electrons do not contribute to δW_w and δW_* , and we have $\omega_{rD} \sim \omega_{*iE}$. Assuming, furthermore, that the mode localization width $\Delta_r \ll \ell_T$ with ℓ_T being a typical magnetic turning point, we can then approximate the energetic ions with straight-line orbits and Eq. (39) recovers the expression of the growth rate derived by Hasegawa [1969],

$$\gamma_D \approx -\tau_0 |v_{iE}/\Delta_r| (T_{||}/T_{\perp}\beta_{\perp})_{0E}, \quad (43)$$

where the subscript 0 denotes equatorial ($\ell = 0$) value. Again, we emphasize that, in contrast to the classical mirror mode, the presence of δW_w and δW_*

indicate that wave-particle resonances play a crucial role in the dynamics of the drift-mirror instability. Equation (43), furthermore, shows that γ_D does not depend sensitively on the azimuthal wavenumber m .

Next, we let $\delta B_{\parallel} \rightarrow 0$ and consider the transverse shear Alfvén mode. Equations (37) and (38) then reduce to

$$\omega_{rA}^2 \simeq [(\delta W_I + \delta W_{KT})/\delta I]_A, \quad (44)$$

and

$$\gamma_A = [(\delta W_{\omega} + \delta W_{*}/\omega_r)/2\delta I]_A. \quad (45)$$

Here, the subscripts A on the right hand sides denote those with $\delta \hat{B}_{\parallel} \rightarrow 0$. Equation (45) demonstrates that the transverse Alfvén mode may be destabilized via wave-particle interactions even for $r \geq 0$ [Southwood, 1976; Chen and Hasegawa, 1988]. In fact, noting that $\delta W_{\omega A} < 0$, instabilities then correspond to perturbations which maximize $(\delta W_{*}/\omega_r \delta I)_A > 0$. Several interesting properties then follow. (1) The most unstable modes tend to have minimal values of ω_{rA} and, hence, only lower-order harmonic modes are of interest here. (2) With $\partial \hat{F}_{0E}/\partial \psi|_{\mu, J} \lesssim 0$, the $K = 0$ ($\omega = \bar{\omega}_d$) resonance does not contribute to the destabilization. (3) Instability mechanisms are then dominated by the $K = \pm 1$ wave-particle resonances of energetic ions. Therefore, we have $\omega_{rA} \omega_{*iE} > 0$. Furthermore, it suggests that the second-harmonic odd- $\delta \psi$ antisymmetric mode to be the most unstable solution [Chen and Hasegawa, 1988]. (4) More precisely, the $K = -1$ ($\bar{\omega}_d \simeq \omega_b$)_{ih} magnetic drift-bounce resonance of the hot energetic ion [$T_{ih} \sim O(10^2 \text{ keV})$] is

destabilizing, while the $K = 1$ ($\omega_{rA} \approx \omega_{b,1w}$) wave-bounce resonance of the warm energetic ion, [$T_{1w} \sim O(1 \text{ keV})$] is often stabilizing [Chen and Hasegawa, 1988].

(5) With $\delta W_* \propto k_\phi^3$ and $\delta I \propto k_\perp^2$, the instabilities tend to have $|k_\phi| \gg |k_\psi|$; i.e., $|\delta B_\psi| > |\delta B_\phi|$, and large East-West azimuthal wavenumbers, $m \sim O(10^2)$ [Southwood, 1976; Chen and Hasegawa, 1988]. Note $\nabla\psi$ is in the radial direction. Finally, (5) with $|k_\phi| \gg |k_\psi|$, we also maximize the effect of the ballooning term ($\propto k_\phi^2$) and, thereby, enhance the instabilities by further lowering ω_{rA} . In this respect, we may term the instabilities as drift Alfvén ballooning modes.

V.2 The coupled case.

As noted in Sec. III, $\delta\hat{B}_\parallel$ and $\delta\psi$ are always coupled in realistic plasmas due to $|\tilde{\nabla}\tilde{P}_\perp| \neq 0$; which, of course, is also the interesting case due to the finiteness of the ballooning driving free energy; c.f. Eq. (30). The detailed nature of the coupling depends, however, on the mirror-mode parameter, τ_0 . Here, $\tau_0 \equiv \tau(\ell=0)$ and is typically the minimal value.

First, we consider $|\tau_0| \sim O(1)$. For the drift-mirror mode, Eq. (43) suggests that we may assume

$$|\omega| \sim |\gamma_D| \sim |k_\parallel v_{iE}| \gg |k_\parallel v_A| ,$$

Equation (22) then yields

$$\delta\psi \simeq - \frac{4\pi v_A^2}{\omega^2 k_\perp^2 B^2} (\mathbf{e}_k \cdot \tilde{\nabla} P_\perp) \delta\hat{B}_\parallel . \quad (46)$$

Substituting Eq. (46) into Eq. (21), we readily find that the $\delta\psi$ term is $O(|\beta_{\perp} v_A / L_P \omega|^2)$ smaller than the $\delta\hat{B}_{\parallel}$ term, where $L_P^{-1} \equiv |\nabla P_{\perp} / P_{\perp}|$. We then essentially recover the decoupled result. Equation (46), furthermore, shows that

$$\left| \frac{\delta B_{\psi}}{\delta B_{\parallel}} \right| \sim \left| \beta_{\perp} \frac{k_{\parallel} v_A^2}{\omega^2 L_P} \right| \ll 1. \quad (47)$$

Because $\delta B_{\parallel}$ is odd and δB_{ψ} is even, Eq. (47) should be understood as the ratio between peak amplitudes.

As to the drift Alfvén ballooning modes, let us concentrate on the antisymmetric odd- $\delta\psi$ modes since they correspond to the more unstable perturbations in the decoupled limit. We then have, from Eq. (21), $\delta\hat{B}_{\parallel} = \delta B_{\parallel}^{(0)} + \delta B_{\parallel}^{(1)}$,

where

$$\delta B_{\parallel}^{(0)} = \frac{4\pi}{\tau B^2} \left(\mathbf{e}_k \cdot \bar{\nabla} P_{\perp} \right) \delta\psi, \quad (48)$$

and

$$\begin{aligned} \delta B_{\parallel}^{(1)} = & -i \frac{4\pi^2}{\tau} m_{iE} < \sum_{K \neq 0} Q F_0 \delta(\omega - \bar{\omega}_d - K\omega_b) S_B \\ & \times \overline{S_{\psi} \delta\psi + S_B \delta B_{\parallel}^{(0)}} >_{iE}. \end{aligned} \quad (49)$$

Substituting Eqs. (48) and (49) into Eq. (22), we arrive at a single eigenmode equation for $\delta\psi$. Again, we may gain analytical insight via the corresponding quadratic form. Letting $\omega = \omega_{Ac} + i\gamma_{Ac}$ and $|\gamma/\omega|_{Ac} < 1$, we readily derive

$$\omega_{Ac}^2 = [\delta W_f / \delta I]_{Ac} , \quad (50)$$

and

$$\gamma_{Ac} = [(\delta W_\omega + \delta W_*/\omega) / 2\delta I]_{Ac} ; \quad (51)$$

where the subscripts Ac on the right hand sides denote those with $\delta \hat{B}_\parallel = \delta B_\parallel^{(0)}$.

In particular, we have, from Eq. (30),

$$\delta W_{fAc} = [|k_\perp^2 \sigma| \frac{\partial \delta \psi}{\partial \ell}|^2] - 4\pi [[\frac{1}{B^2} \Omega_e |\delta \psi|^2 \mathbf{e}_k \cdot (\tilde{\nabla} P_\parallel + \frac{\sigma}{\tau} \tilde{\nabla} P_\perp)]] \quad (52)$$

Comparing Eqs. (50) and Eq. (52) with Eq. (44) and Eq. (30) of the decoupled limit, we first note that $\delta W_{KT} = 0$ due to the odd- $\delta \psi$ perturbations and, more importantly, $\delta W_{fAc} < \delta W_{fA}$ for $\tau > 0$, since $\delta B_\parallel^{(0)}$ nullifies the stabilizing effect of the perpendicular compression. That is, for $\tau > 0$, we have $\omega_{Ac} < \omega_{rA}$ and, thus, $\gamma_{Ac} > \gamma_A$. Finite coupling to the compressional $\delta B_\parallel$ component, therefore, further enhances the drift Alfvén ballooning instability. The general stability properties are, however, similar to those discussed in the decoupled limit. Meanwhile, Eq. (48) shows that

$$\left| \frac{\delta B_\psi}{\delta B_\parallel} \right| \sim \left| \frac{k_\parallel L_p \tau}{\beta_\perp} \right|. \quad (53)$$

Thus, while for $\beta_\perp \ll 1$ the instability mainly consists of δB_ψ perturbations, the $\delta B_\parallel$ and δB_ψ perturbations become comparable in $\beta_\perp \sim O(1)$ plasmas.

Up to now, we have considered drift Alfvén ballooning instabilities excited via wave-particle resonances. Such instability mechanisms are effective in low- β ,

$\beta \sim O(10^{-1})$, plasmas. However, as β increases, one can readily see from Eqs. (50) and (52) that there exists a critical β_{10} and/or τ_0 value at which $\omega_{Ac}^2 \rightarrow 0$ and the instability assumes the nature of the fast-growing reactive-type ideal MHD ballooning mode. Since, with $k_{\perp} \simeq k_{\phi}$, ω_{Ac}^2 is independent of k_{ϕ} , this reactive-type MHD ballooning instability, therefore, depends only weakly on the azimuthal wavenumber m ; in contrast to excitations via wave-particle interactions which peak at large m , $m \sim O(10^2)$. The critical value of β_{10} , β_B , depends on τ and the steepness of the pressure profile. For an isotropic pressure ($\tau=1$) with the pressure profile of $P(\psi) = P_0\psi^{20/3}$, it has been found $\beta_B \simeq 2.5$ [Chan, 1990]. Pressure anisotropy tends to lower β_B , while smoother pressure profiles tend to raise β_B . Detailed numerical results of more general cases will be reported in a future publication. Let us now comment on the reactive-type ideal MHD ballooning stability of the fundamental even- $\delta\psi$ symmetric mode. From Eq. (28), we have, neglecting wave-particle interactions, $\omega^2\delta I = \delta W_I + \delta W_{KT}$ where δW_I and δW_{KT} are given, respectively by Eqs. (30) and (31). With $\delta\psi$ being even, we then have finite trapped-particle compression, $\delta W_{KT} > 0$; which exerts a strongly stabilizing effect over the destabilizing ballooning term in δW_I . In fact, taking $\partial\hat{F}_{0E}/\partial\psi|_{\mu,J} \approx 0$ and further assuming $\delta\psi$ and $\delta\hat{B}_{||}$ are nearly constant in the β carrying region, we can see, more readily from Eqs. (21) and (22), that $\delta\hat{B}_{||} \approx 0$ and the ballooning term is almost completely cancelled by the compression term. This is, of course, expected since $\partial\hat{F}_{0E}/\partial\psi|_{\mu,J} \approx 0$ corresponds to a sufficient condition for the interchange stability [Liu, 1970]. In practical situations, while the cancellation

may not be as complete, we could still expect sufficient $\delta W_{KT} > 0$ stabilization such that the fundamental symmetric mode is more stable than the second-harmonic anti-symmetric mode; that is, the former has a higher critical $\beta_{\perp 0}$ value.

Next, we consider $|\tau_0| \ll 1$. In this regime, we can see from Eq. (21) that $|\delta \hat{B}_{\parallel}| \gg |k_{\perp} \delta \psi / L|$, i.e., $|\delta B_{\psi} / \delta B_{\parallel}| \ll 1$. Equation (22), for antisymmetric modes, can then be approximated as

$$B \frac{\partial}{\partial \ell} \left(\frac{k_{\perp}^2}{B} \sigma \frac{\partial}{\partial \ell} \delta \psi \right) + \frac{\omega^2 k_{\perp}^2}{v_A^2} \delta \psi \approx - \frac{4\pi}{B^2} (\mathbf{e}_k \cdot \tilde{\nabla} P_{\perp}) \delta \hat{B}_{\parallel}. \quad (54)$$

For the drift-mirror mode, $|\omega| \sim |k_{\parallel} v_{th} \tau| \ll |k_{\parallel} v_A|$ and

$$\delta \psi \approx 4\pi (\mathbf{e}_k \cdot \tilde{\nabla} P_{\perp}) \delta \hat{B}_{\parallel} / (B^2 k_{\parallel}^2 k_{\phi}^2 \sigma). \quad (55)$$

Here, $|k_{\parallel}| \sim 1/\Delta_r$ with Δ_r being the narrow mode localization width where $\tau \leq 0$. Equation (55) into Eq. (21), we readily derive $\omega_{TD} \sim \omega_{*iE}$ and the following local instability threshold condition

$$\tau_0 - \frac{1}{\sigma_0} \left(\frac{4\pi \mathbf{e}_k \cdot \tilde{\nabla} P_{\perp}}{B^2 k_{\parallel} k_{\phi}} \right)_0^2 \approx 0. \quad (56)$$

Since the second term in Eq. (56) is of the order $(\beta_{\perp}/k_{\parallel} L_p)^2 \ll 1$, the finite coupling has a very mild destabilizing effect.

As to the drift Alfvén ballooning mode, we note that, for $|\tau_0| \ll 1$ and antisymmetric modes, $\omega_{Ac}^2 \propto \delta W_{IAc}$ and

$$\delta W_{IAc} \simeq -4\pi \left[\left(\frac{\sigma}{\tau B^2} \Omega_K (\mathbf{e}_K \cdot \tilde{\nabla} P_\perp) |\delta\psi|^2 \right) \right], \quad (57)$$

and we, thus, expect strong reactive-type MHD instabilities to set in. Assuming $|\omega| \gg |k_\parallel v_A|$, Eq. (54) reduces to

$$\delta\psi \simeq -\frac{4\pi v_A^2}{B^2 \omega^2 k_\phi^2} (\mathbf{e}_K \cdot \tilde{\nabla} P_\perp) \delta \hat{B}_\parallel. \quad (58)$$

Substituting Eq. (58) into Eq. (21) and noting that $|\omega| \gg |\omega_{*E}|$, we obtain the following quadratic form

$$\left[\left[\left(\tau + \left(\frac{4\pi \mathbf{e}_K \cdot \tilde{\nabla} P_\perp}{B^2 \omega k_\phi} \right)^2 \right) |\delta \hat{B}_\parallel|^2 \right] \right] = -i\omega \delta W_{\omega D}. \quad (59)$$

Noting that $|\delta W_{\omega D}| \sim |k_\parallel v_{iE}|^{-1}$, we obtain, for $1 \gg |\tau_0| \gg |\omega/k_\parallel v_{iE}|$,

$$\omega_{Ac}^2 \simeq -\frac{1}{\tau_0} \left(\frac{4\pi \mathbf{e}_K \cdot \tilde{\nabla} P_\perp v_A}{B^2 k_\phi} \right)_0^2. \quad (60)$$

Recalling the MHD equilibrium condition $\sigma \kappa_\psi \simeq 4\pi \tilde{\nabla} P_\perp / B^2$ for $|\tau| \ll 1$, Eq. (60) then agrees with that given by δW_{IAc} of Eq. (57), as it should. On the other hand, for $1 \gg |\omega/k_\parallel v_{iE}| \gg |\tau|$, Eq. (59) yields, noting $\delta W_{\omega D} < 0$,

$$\begin{aligned} \omega_{Ac} &\approx i \left\{ \left[\left[\left(\frac{4\pi \mathbf{e}_K \cdot \tilde{\nabla} P_\perp v_A}{B^2 k_\phi} \right)^2 \right] \right] / (-\delta W_{\omega D}) \right\}^{1/3} \\ &\sim i \left(v_A^2 k_\parallel v_{iE} / L_p^2 \right)_0^{1/3} \left(\beta_\perp T_\parallel / T_\perp \right)_{0iE}^{1/3}. \end{aligned} \quad (61)$$

The instability thus remains strong and essentially purely growing. Equation (61) further indicates that the instability has characteristics similar to

those of the drift-mirror instability except that the free energy is provided by the finite pressure gradient via coupling to the transverse component. With this perspective, we shall term this instability as the drift Alfvén ballooning-mirror instability.

In Fig. 1, we have sketched the growth rates of the drift-mirror (DM) and the drift Alfvén ballooning-mirror (DABM) instabilities versus β_{10} . Here, β_{WP} , β_B and β_M correspond to the β_{10} values for instability excitations via wave-particle resonances, ballooning driving term and mirror instabilities, respectively. Typically, we have $\beta_{WP} \sim O(10^{-1})$, $\beta_B \gtrsim 1$, and $\beta_M > 1$. Finally, we have also sketched in Fig. 2 the relative magnitude of peak values of δB_ψ and $\delta B_\parallel$ versus β_{10} . We note that while the drift-mirror mode is essentially a compressional wave, the drift Alfvén ballooning-mirror instability evolves gradually from being a predominantly transverse wave in the $\beta_\perp \sim O(10^{-1})$ regime to being a predominantly compressional wave in the $\beta_\perp \gtrsim 1$ regime.

VI. SUMMARY AND DISCUSSION

In this work, we have developed a comprehensive kinetic theory on the generating mechanisms of hydromagnetic Alfvén waves in a $\beta \sim O(1)$, anisotropic magnetospheric plasma consisting of a core (~ 100 eV) and an energetic (~ 10 keV) components. Our theory includes effects due to finite ion Larmor radii, plasma and magnetic nonuniformities, pressure anisotropy, curved magnetic field lines, magnetically trapped particles, as well as wave-particle interactions. A general set of coupled eigenmode equations, Eqs. (21) and (22),

is then derived which demonstrates that, due to pressure nonuniformities, the compressional and transverse (shear) magnetic perturbations are always coupled.

We have constructed the corresponding quadratic forms, Eq.(28), and assuming $\partial \hat{F}_{0E}/\partial \psi|_{\mu,J} \approx 0$, found the following general and yet interesting stability properties. (i) While the drift-mirror instability dominates for $\tau = 1 + 4\pi \partial P_{\perp}/B \partial B < 0$, the drift Alfvén ballooning-mirror instability dominates for $\tau \geq 0$. (ii) Both instabilities have antisymmetric odd $\delta B_{\parallel}$ perturbations and propagate in the energetic ion diamagnetic drift direction. (iii) $\delta B_{\parallel} \gg \delta B_{\perp}$ for drift-mirror instability. (iv) $\delta B_{\psi} > \delta B_{\phi}$ for the drift Alfvén ballooning-mirror instability. While $\delta B_{\parallel} < \delta B_{\psi}$ in the $\beta \ll 1$ regime, the instability becomes more compressional in the $\beta \gtrsim 1$ regime. (iv) For low β values and $\tau \sim O(1)$, the drift Alfvén ballooning-mirror instability is excited via wave-particle interactions and tends to have a large East-West azimuthal wavenumber. (vi) For $\beta \gtrsim 1$ and/or $1 \gg \tau > 0$, the drift Alfvén ballooning-mirror instability becomes a strong reactive-type instability and depends weakly on the azimuthal wavenumber. We note that the above instability features are consistent with available satellite observations [Takahashi et al., 1985; Takahashi, 1988].

While the present analysis has set up a proper theoretical framework, obviously it will be interesting to examine the theoretical predictions in sufficient details employing realistic magnetic field geometries, plasma equilibria, distribution functions, finite ionospheric effects, etc. Such analyses

will involve numerical solutions of the coupled eigenmode equations, and the results will not only check the stability features presented here but also provide quantitative comparisons with available satellite measurements.

Acknowledgements

The work done at Princeton is supported by NSF grant ATM-86-09585 and U.S. Department of Energy contract DE-AC02-76-CH03073.

References

- Antonsen, T. M., Jr., and B. Lane, Kinetic equations for low frequency instabilities in inhomogeneous plasmas, *Phys. Fluids* **23**, 1205, 1980.
- Baumjohann, W., N. Sekopke, J. Labelle, B. Klecker, H. Lür and K. H. Glassner, Plasma and field observations of a compressional Pc5 event, *J. Geophys. Res.* **92**, 12203, 1987.
- Bernstein, I. B., E. A. Frieman, M. D. Kruskal, and R. M. Kulsrud, An energy principle for hydromagnetic stabilities, *Proc. Roy. Soc. (London)* **A244**, 17, 1958.
- Catto, P. J., W. M. Tang and D. E. Baldwin, Generalized gyrokinetics, *Plasma Physics* **23**, 639, 1981.
- Chan, A. A., Ph.D Thesis, Princeton University, 1990.
- Chandrasekhar, S., A. N. Kaufman, and K. M. Watson, The stability of the pinch, *Proc. Soc. A*, **245**, 435, 1958.
- Chen, L. and A. Hasegawa, A theory of long-period magnetic pulsations, 1. Steady state excitations of field line resonance, *J. Geophys. Res.* **79**, 1024, 1974a.
- Chen, L. and A. Hasegawa, A theory of long-period magnetic pulsations, 2. Impulse excitation of surface eigenmode, *J. Geophys. Res.* **79**, 1033, 1974b.
- Chen, L. and A. Hasegawa, On magnetospheric hydromagnetic waves excited by

- energetic ring-current particles, *J. Geophys. Res.* **93**, 8763, 1988.
- Cheng, A. F. and S. M. Krimigis, Near equality of ion phase space densities at Earth, Jupiter and Saturn, *J. Geophys. Res.* **90**, 526, 1985.
- Cheng, C. Z. and C. S. Lin, Eigenmode analysis of compressional waves in the magnetosphere, *Geophys. Res. Lett.* **14**, 884, 1987.
- Gold, T., Motion in the magnetosphere of the Earth, *J. Geophys. Res.* **64**, 1219, 1959.
- Hasegawa, A., Drift mirror instability in the magnetosphere, *Phys. Fluids* **12**, 2642, 1969.
- Krimigis, S. M., G. Gloeckler, R. M. McEntire, T. A. Potemra, F. L. Scarf and E. G. Shelley, Magnetic storm of September 4, 1984: a synthesis of ring current spectra and energy densities measured with AMPTE/CCE, *Geophys. Res. Lett.* **12**, 329, 1985.
- Kruskal, M. D. and C. R. Oberman, On the stability of plasma in static equilibrium, *Phys. Fluids* **1**, 275, 1958.
- Lanzerotti, L. J. and D. J. Southwood, Hydromagnetic waves, in solar system plasma physics, Vol. III, p. 111, North Holland, 1979.
- Liu, C. S., Low-frequency drift instabilities in the ring current belt, *J. Geophys. Res.* **75**, 3789, 1970.
- Pokhotelov, O. A. V. A. Pilipenko, Yu. M. Nezlina, J. Woch, G. Kremser, A. Korth and E. Amata, Excitations of high- β plasma instabilities at the

- geostationary orbit; theory and observations, *Planet. Space Sci.*, 34, 695, 1986.
- Rosenbluth, M. n. and N. Rostaker. Theoretical structure of plasma equations, *Phys. Fluids* 2, 23, 1959.
- Rutherford, P. H. and E. A. Frieman, Drift instabilities in general magnetic field configurations, *Phys. Fluids* 11, 569, 1968.
- Schultz, M. and L. J. Lanzerotti, *Particle Diffusion in the Radiation Belt*, Springer-Verlag, New York, 1974.
- Southwood, D. J., Some features of field line resonances in the magnetosphere, *Planet Space Sci.* 22, 483, 1974.
- Southwood, D. J., A general approach to low-frequency instability in the ring current plasma, *J. Geophys. Res.* 81, 3340, 1976.
- Takahashi, K., P. R. Higbie and D. N. Baker, Azimuthal propagation and frequency characteristics of compressional Pc5 waves observed at geostationary orbit, *J. Geophys. Res.* 90, 1473, 1985.
- Takahashi, K., Multisatellite studies of ULF waves, *Adv. Space Res.* 8, 427, 1988.
- Taylor, J. B. and R. J. Hastie, Stability of general plasma equilibria, I. Formal theory, *Plasma Phys.* 10, 479, 1968.
- Van Dam, J. W., M. N. Rosenbluth and Y. C. Lee, A generalized kinetic energy principle, *Phys. Fluids* 25, 1349, 1982.

Figure Captions

- Fig. 1. Sketch of growth rates of the drift-mirror (DM) and the drift Alfvén ballooning-mirror (DABM) instabilities versus β_{10} . The various characteristic β_{10} values are defined in the text.
- Fig. 2. Sketch of the relative magnitudes between the peak amplitudes of the radial and compressional components of the magnetic perturbations, $|\delta B_\psi / \delta B_\parallel|$, versus β_{10} for both the DM and DABM instabilities.

FIGURE 1

FIGURE 2