THE EFFECT OF FLUID VISCOSITY
ON COUPLED TUBE/FLUID VIBRATIONS

by

Tsyh-tyan Yeh and Shoei-sheng Chen

Components Technology Division

U of C-AUA-USERDA

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Base Technology

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ABSTRACT

In many reactor system components, in order to accommodate thermal expansion and to facilitate assembly, very frequently there exist small gaps among various structural and mechanical components. The gaps are usually containing flowing viscous fluid to maintain temperature in a specific range. The fluid in the gap will affect the component dynamic characteristics. Thus, its response will be different from that without the coupling of viscous fluid. The role of fluid viscosity on the coupled structural/fluid vibration is still not well understood. The objective of this report is to provide some additional insights into the effect of fluid viscosity on coupled vibration.

Two coaxial tubes separated by a viscous fluid are studied theoretically. The fluid forces acting on the tubes associated with small tube displacements are obtained in closed form based on the linearized Navier Stokes equation. Once the fluid force coefficients are calculated, free and forced vibrations of the coupled tube/fluid system can then be analyzed. Numerical results are presented for several cases to illustrate the effect of various system parameters.

Based on the results of this study, several general conclusions can be drawn:

(1) The fluid force coefficient matrix for two coaxial tubes separated by a viscous fluid can be obtained from the single coefficient for a single tube vibrating in a viscous fluid annulus.

(2) The real part of fluid force coefficients are insensitive to oscillation Reynolds number Re while the imaginary part varies with Re significantly.

(3) Except for very small gaps, natural frequencies of coupled modes can be calculated from the potential flow theory as a good approximation.

(4) The natural frequency of out-of-phase mode is lower than either of the uncoupled modes while the in-phase mode higher than either of the uncoupled modes.

(5) In a system in which the structural damping is very small, the modal damping of out-of-phase mode is higher than the damping of uncoupled modes while the damping of in-phase mode is lower than either of the uncoupled modes.

(6) In a system with relatively large structural damping, there is a minimum at certain fluid gaps for total modal damping of the out-of-phase mode; that is, decreasing fluid gap does not always increase system damping.

(7) In scaled-model testing, oscillation Reynolds number has to be scaled for small Re ; otherwise, the scaled model may not be conservative. Specifically, a reduced scaled model always yields higher damping than the original system.

(8) Several simplified expressions are provided in the report to facilitate the evaluation of the effect of fluid viscosity on coupled vibration.

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NOMENCLATURE

A_i, a_i	Constants
α_{ij}	Fluid force coefficient matrix defined in Eq. (16)
b_{ij}	Matrix defined in Eq. (39)
β_i	$k R_i$
c_i	Structural damping coefficient
c_{ij}	Fluid damping matrix defined in Eq. (24)
E_i	Young's modulus
ϵ_i	$\epsilon_1 = 1, \epsilon_2 = -1$
e_{ij}	Orthonormal matrix defined in Eq. (49)
F_n	n th order Bessel functions of the first kind
f_i	Excitation force per unit length
f_{in}	Generalized force defined in Eq. (37)
G	$\frac{R_2 - R_1}{R_1}$
G_n	n th order Bessel function of the second kind
$H_n^{(1)}, H_n^{(2)}$	Hankel function
$h_i, \bar{h}_i$	Modal amplitude
I_i	Moment of inertia of a rod
J_n	First kind Bessel function
j	$\sqrt{-1}$
k	$(-j\omega/v)^{1/2}$
ℓ_i	Length of cylinder
m_i	Structure mass per unit length
m_{ij}	Added mass matrix defined in Eq. (24)
n	Axial modal number
p	Fluid pressure
R_i	Radius of interface
$\text{Re}(\cdot)$	Real part of $(\cdot)$

NOMENCLATURE (Contd.)

Re	$\frac{\omega R_1^2}{\nu}$, oscillation Reynolds number
r	Radial coordinate
t	Time
τ_{rr}, τ_r	Fluid stresses
U_1	Velocity amplitude of oscillating tube
u	Fluid velocity in radial direction
v	Fluid velocity in θ direction
ω	Circular frequency
$\bar{\omega}_1, \bar{\omega}_{in}$	Circular natural frequency in vacuo
ω_1^*	Natural frequency of uncoupled modes
X_1	Resultant fluid force in the x acting on cylinder
$\bar{X}_1$	Resultant fluid force in the x acting on fluid
Y_1	Resultant fluid force in the y acting on cylinder
$\bar{Y}_1$	Resultant fluid force in the y acting on fluid
Y_n	Second kind Bessel function
$y_1, \bar{y}_1$	Lateral displacement
z	Axial coordinate
ρ	Fluid density
μ	Absolute fluid viscosity
μ_i	$\rho \pi R_i^2 / m_i$
ν	μ / ρ kinematic viscosity
ψ, ψ_i	Velocity potential
γ	R_1 / R_2
ϕ_{in}, ϕ_n	Orthonormal function
λ_n	$\ell \frac{\phi_n''''}{\phi_n}^{1/4}$, modal constant
σ_i	$\bar{h}_1 / \bar{h}_2$ at $\omega = \omega_i$

NOMENCLATURE (Contd.)

(•)	Time derivative
ζ_f	Modal damping ratio due to fluid viscosity
ζ_{si}	$c_i/2m_i\bar{\omega}_i$, the structural modal damping ratio in vacuo
ζ_{ti}	Total system damping ratio of coupled modes
ζ_{ti}^*	Total system damping ratio of uncoupled modes

1. INTRODUCTION

In many reactor system components, in order to accommodate thermal expansion and to facilitate assembly, very frequently there exist small gaps among various structural and mechanical components. Usually, the gaps contain flowing fluid to maintain component temperature. The fluid in the gap has important effects on structural response: (1) fluid inertia coupling makes the multiple structural elements as an integrated body; and (2) fluid viscosity modifies system damping characteristics. Thus, if a component containing viscous fluid is subjected to an excitation, its response will be very different from that without viscous fluid.

The effect of viscous fluid on structural responses has been studied previously for a cylindrical rod vibrating in a viscous fluid annulus [1] and two coaxial shells containing viscous fluid [2]. In these studies, it was shown that the fluid viscosity contributes significantly to damping, and the natural frequencies can be obtained using the ideal fluid as a good approximation for large gaps, but the effect of fluid viscosity has to be included for small gaps. The objective of this study is to provide some additional insights into the role of fluid viscosity on system response.

In this report, the dynamics of two coaxial tubes separated by a viscous fluid is studied theoretically. First, the fluid forces associated with the tube motion are obtained in closed form based on the linearized Navier Stokes equation. Then, natural frequency and damping of the coupled tube/fluid vibration are analyzed; both exact and approximate solutions are obtained. Finally, several numerical examples are presented to illustrate the system characteristics.

2. FLUID FORCE COEFFICIENT MATRIX

2.1 Formulation

Consider two concentric circular cylinders separated by an incompressible viscous fluid annulus as shown in Fig. 1, in which R_1 and R_2 are the interface radii between fluid and cylinders. It is convenient to express the Navier-Stokes equations, which govern the motion of viscous fluids, in a circular cylindrical coordinate (r, θ, z) system. For most applications, the length of cylinders, l , is much larger than the interface radii R_i ($i = 1, 2$), the fluid field is considered to be two dimensional. That is, the axial motion of the fluid is neglected. For non-steady, small-amplitude oscillations, the equations of motion can be linearized and the governing equations in two-dimensional polar coordinates are:

$$\begin{aligned} \frac{\partial u}{\partial r} + \frac{u}{r} + \frac{1}{r} \frac{\partial v}{\partial \theta} &= 0, \\ \rho \frac{\partial u}{\partial t} &= -\frac{\partial p}{\partial r} + \mu \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial v}{\partial \theta} \right), \\ \rho \frac{\partial v}{\partial t} &= -\frac{\partial p}{r \partial \theta} + \mu \left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} - \frac{v}{r^2} + \frac{1}{r^2} \frac{\partial^2 v}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial u}{\partial \theta} \right), \end{aligned} \quad (1)$$

where ρ is fluid density, μ is absolute fluid viscosity, p is fluid pressure, and u and v are the velocity components in the r and θ directions respectively.

Now let the i th cylinder be oscillating with the velocity $U_i e^{j\omega t}$ along $\theta = 0$, where $j = \sqrt{-1}$. At the interfaces between the cylinders and fluid, the following non-slip conditions must be satisfied:

$$\begin{aligned} u|_{r=R_i} &= U_i \cos \theta e^{j\omega t}, \\ v|_{r=R_i} &= -U_i \sin \theta e^{j\omega t}. \end{aligned} \quad (2)$$

Fig. 1. Schematic and coordinate system of two concentric tubes containing a viscous fluid

Equations (1) and (2) completely specify the fluid field. From the first equation of Eqs. (1), it is seen that a scalar function $\psi(r, \theta, t)$ exists such that

$$u = -\frac{\partial \psi}{r \partial \theta} \quad , \quad \text{and } v = \frac{\partial \psi}{\partial r} \quad . \quad (3)$$

Inserting Eq. (3) into Eq. (1), and then eliminating p from the resulting equations, we obtain

$$\nabla^4 \psi - \frac{1}{v} \frac{\partial}{\partial t} \nabla^2 \psi = 0 \quad , \quad (4)$$

where

$$v = \mu / \rho \quad ,$$

and

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \quad .$$

The general solution of ψ is

$$\psi = \psi_1 + \psi_2 \quad , \quad (5)$$

where ψ_1 and ψ_2 satisfy the following equations:

$$\nabla^2 \psi_1 = 0 \quad ,$$

and

$$\left(\nabla^2 - \frac{1}{v} \frac{\partial}{\partial t} \right) \psi_2 = 0 \quad . \quad (6)$$

The general solution of the fluid velocity potential may be assumed as follows:

$$\psi_1 = \bar{\psi}_1(r) \sin \theta e^{j\omega t} \quad . \quad (7)$$

Substitution of Eq. (7) into Eq. (6) yields

$$\begin{aligned} \bar{\psi}_1'' + \frac{1}{r} \bar{\psi}_1' - \frac{1}{r^2} \bar{\psi}_1 &= 0 \quad , \\ \bar{\psi}_2'' + \frac{1}{r} \bar{\psi}_2' + \left(k^2 - \frac{1}{r^2} \right) \bar{\psi}_2 &= 0 \quad , \end{aligned} \quad (8)$$

where $k = (-j\omega/\nu)^{1/2}$ and the prime denotes differentiation with respect to r .

The general solutions of Eqs. (8) are

$$\begin{aligned}\bar{\psi}_1 &= A_1 \frac{R_1^2}{r} + A_2 r, \\ \bar{\psi}_2 &= R_1 [A_3 F_1(kr) + A_4 G_1(kr)],\end{aligned}\tag{9}$$

where A_i are the constants to be determined from the boundary conditions, Eqs. (2), and F_n and G_n are the n th order Bessel functions. They can be either the first and second kind Bessel functions, J_n and Y_n , or the Hankel functions, $H_n^{(1)}$ and $H_n^{(2)}$. The selection of the functions mainly depends on the computational consideration. In general the function J_n and Y_n can be used for a small argument, while $H_n^{(1)}$ and $H_n^{(2)}$ are recommended for a large argument.

Substituting Eqs. (3), (5), (7) and (9) into the interface conditions, Eqs. (2), gives four linear, algebraic equations:

$$\begin{pmatrix} -1 & -1 & -F_1(\beta_1) & -G_1(\beta_1) \\ -\gamma^2 & -1 & -\gamma F_1(\beta_2) & -\gamma G_1(\beta_2) \\ 0 & -2 & -\beta_1 F_o(\beta_1) & -\beta_1 G_o(\beta_1) \\ 0 & -2 & -\beta_1 F_o(\beta_2) & -\beta_1 G_o(\beta_2) \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{pmatrix} = \begin{pmatrix} U_1 \\ U_2 \\ 2U_1 \\ 2U_2 \end{pmatrix},\tag{10}$$

where $\gamma = R_1/R_2$ and $\beta_1 = kR_1$.

The constant A_1 can be determined from Eq. (10) and the solution of the fluid medium can be expressed in terms of cylinder velocity U_1 . Now we are in the position to calculate the hydrodynamic forces acting on the cylinders. The resultant forces acting on the i -th cylinder per unit length of a cylinder are:

$$X_1 = \varepsilon_1 R_1 \int_0^{2\pi} (\tau_{rr}|_{r=R_1} \cos\theta - \tau_{r\theta}|_{r=R_1} \sin\theta) d\theta, \quad (11)$$

$$Y_1 = \varepsilon_1 R_1 \int_0^{2\pi} (\tau_{rr}|_{r=R_1} \sin\theta - \tau_{r\theta}|_{r=R_1} \cos\theta) d\theta,$$

where $\varepsilon_1 = 1$ and $\varepsilon_2 = -1$ and X_1 and Y_1 are the force components parallel to and perpendicular to the direction of the cylinder movement respectively; and the fluid stresses are:

$$\begin{aligned} \tau_{rr} &= -p + 2\mu \frac{\partial u}{\partial r}, \\ \tau_{r\theta} &= \mu \left(\frac{\partial v}{\partial r} - \frac{v}{r} + \frac{1}{r} \frac{\partial u}{\partial \theta} \right). \end{aligned} \quad (12)$$

The last equation of Eq. (1) can be used to calculate the fluid pressure. Substituting Eqs. (3), (7) and (12) into Eq. (11) and using the relation of Eq. (8) gives

$$\begin{aligned} X_1 &= \rho \pi R_1^2 j\omega e^{j\omega t} (U_1 + 2A_1), \\ X_2 &= -\rho \pi R_2^2 j\omega e^{j\omega t} (U_2 + 2\gamma^2 A_1), \\ Y_1 &= Y_2 = 0. \end{aligned} \quad (13)$$

Eq. (13) states that the movement of a cylinder produces no resultant force in the direction normal to the direction of the cylinder movement. It also shows that X_1 depends only on the constant A_1 , which can be solved from Eq. (10).

$$A_1 = - \frac{\begin{vmatrix} U_1 & 1 & F_1(\beta_1) & G_1(\beta_1) \\ U_2 & 1 & \gamma F_1(\beta_2) & \gamma G_1(\beta_2) \\ 2U_1 & 2 & \beta_1 F_o(\beta_1) & \beta_1 G_o(\beta_1) \\ 2U_2 & 2 & \beta_1 F_o(\beta_2) & \beta_1 G_o(\beta_2) \end{vmatrix}}{\begin{vmatrix} 1 & 1 & F_1(\beta_1) & G_1(\beta_1) \\ \gamma^2 & 1 & \gamma F_1(\beta_2) & \gamma G_1(\beta_2) \\ 0 & 2 & \beta_1 F_o(\beta_1) & \beta_1 G_o(\beta_1) \\ 0 & 2 & \beta_1 F_o(\beta_2) & \beta_1 G_o(\beta_2) \end{vmatrix}}. \quad (14)$$

or $A_1 = a_1 U_1 + a_2 U_2$. Thus Eq. (13) can be rewritten as

$$X_i = -\rho\pi(j\omega e^{j\omega t})R_i \sum_{\ell=1}^2 \alpha_{i\ell} R_\ell U_\ell, \quad (i = 1, 2) \quad (15)$$

where

$$\begin{aligned} \alpha_{11} &= -(1 + 2a_1), \\ \alpha_{12} &= -2\gamma a_2, \\ \alpha_{21} &= 2\gamma a_1, \\ \alpha_{22} &= 1 + 2\gamma^2 a_2. \end{aligned} \quad (16)$$

Equation (14) shows that the expression, $A_i = a_1 U_1 + a_2 U_2$, is valid for any value of U_i . Furthermore, when $U_1 = U_2$, it yields $A_1 = 0$. This means $a_1 = -a_2$. Thus all the coefficients $\alpha_{i\ell}$ can be expressed in terms of one constant a_1 and Eqs. (16) reduce to

$$\begin{aligned} \alpha_{11} &= -(1 + 2a_1), \\ \alpha_{12} &= \alpha_{21} = 2\gamma a_1 = -\gamma(1 + \alpha_{11}), \\ \alpha_{22} &= 1 - 2\gamma^2 a_1 = 1 + \gamma^2(1 + \alpha_{11}), \end{aligned} \quad (17)$$

where

$$a_1 = - \frac{\begin{vmatrix} 1 & 1 & F_1(\beta_1) & G_1(\beta_1) \\ 0 & 1 & \gamma F_1(\beta_2) & \gamma G_1(\beta_2) \\ 2 & 2 & \beta_1 F_o(\beta_1) & \beta_1 G_o(\beta_1) \\ 0 & 2 & \beta_1 F_o(\beta_2) & \beta_1 G_o(\beta_2) \end{vmatrix}}{\begin{vmatrix} 1 & 1 & F_1(\beta_1) & G_1(\beta_1) \\ \gamma^2 & 1 & \gamma F_1(\beta_2) & \gamma G_1(\beta_2) \\ 0 & 2 & \beta_1 F_o(\beta_1) & \beta_1 G_o(\beta_1) \\ 0 & 2 & \beta_1 F_o(\beta_2) & \beta_1 G_o(\beta_2) \end{vmatrix}}. \quad (18)$$

It is noted that the coupled problem of two concentric tubes separated by viscous fluids reduces to that of a rod vibrating in confined viscous fluids [1].

The relationship among the force coefficient can also be shown from a physical consideration. Let $\bar{X}_1$ and $\bar{X}_2$ be the net forces acting on the fluid due to the unit movement of the inner cylinder and the outer cylinder respectively. Considering the annular fluid as a free body system, one has

$$\begin{aligned}\bar{X}_1 &= -\rho\pi R_1^2, \\ \bar{X}_2 &= \rho\pi R_2^2.\end{aligned}\tag{19}$$

On the other hand, from Eq. (15), these forces are also given by

$$\bar{X}_i = \rho\pi R_i \{ \alpha_{i1} R_1 + \alpha_{i2} R_2 \}.\tag{20}$$

Equations (19) and (20) yield the following relationship:

$$\alpha_{21} = -\gamma(1 + \alpha_{11}),$$

and (21)

$$\alpha_{12} = \frac{1}{\gamma} (1 - \alpha_{22})$$

It is easy to see that Eqs. (16) satisfy the required relationship of Eq. (21).

As discussed in Ref. [2], the coefficient α_{ii} is proportional to the resultant dynamic fluid forces acting on a cylinder due to its own movement; while the others, $\alpha_{i\ell}$, $i \neq \ell$, are proportional to the resultant dynamic fluid forces acting on the i -th cylinder due to the movement of the ℓ -th cylinder.

The fluid forces acting on the cylinders are linear functions of the cylinder motions. In general, the coefficients $\alpha_{i\ell}$ are complex. The fluid forces can be separated into two components: one proportional to real part of $\alpha_{i\ell}$, $\text{Re}(\alpha_{i\ell})$, is in phase with the cylinder accelerations and is related to the added mass effect, while the other, proportional to the imaginary part of $\alpha_{i\ell}$, $\text{Im}(\alpha_{i\ell})$, is opposing to the movement of the cylinder and is related to damping mechanism. If the fluid is inviscid and incompressible, the second component of the force opposing cylinder motion will be zero.

For an oscillatory motion, $U_\ell = j\omega\bar{y}_\ell$, where $\bar{y}_\ell$ is the oscillation amplitude of the ℓ -th cylinder. Equation (15) can be written as

$$X_i = \rho \pi \omega^2 e^{j\omega t} R_i \sum_{\ell=1}^2 \alpha_{i\ell} R_\ell \bar{y}_\ell \quad (22)$$

Equations (15) and (22) are obtained for a system performing sinusoidal oscillations. The hydrodynamic forces can also be expressed in terms of cylinder acceleration and velocity

$$X_i = - \sum_{\ell=1}^2 m_{i\ell} \ddot{y}_\ell - \sum_{\ell=1}^2 c_{i\ell} \dot{y}_\ell \quad (23)$$

where $y_\ell(t)$ is the instantaneous displacement of the ℓ -th cylinder and the dot denotes differentiation with respect to time, and

$$m_{i\ell} = \rho \pi R_i R_\ell \operatorname{Re}\{\alpha_{i\ell}\} \quad (24)$$

and

$$c_{i\ell} = \rho \pi R_i R_\ell \omega \operatorname{Im}\{-\alpha_{i\ell}\}$$

$m_{i\ell}$ and $c_{i\ell}$ are the added mass and fluid damping of the system respectively. Although Eq. (23) is derived for a system performing sinusoidal oscillation, it can be considered as an approximation for a general vibration problem in which the motion is not sinusoidal.

The results presented above are for cylinders moving in the same direction and frequency. However, because of the linearity of the problem, the total resultant force associated with the movements of the two cylinders moving in the direction θ_ℓ , frequency ω_ℓ , and phase angle ϕ_ℓ can be expressed

$$X_i = \rho \pi R_i \sum_{\ell=1}^2 \alpha_{i\ell}(\omega_\ell) R_\ell \omega_\ell^2 e^{j(\omega_\ell t + \phi_\ell)} \cos \theta_\ell \bar{y}_\ell \quad (25)$$

$$Y_i = \rho \pi R_i \sum_{\ell=1}^2 \alpha_{i\ell}(\omega_\ell) R_\ell \omega_\ell^2 e^{j(\omega_\ell t + \phi_\ell)} \sin \theta_\ell \bar{y}_\ell$$

It is convenient to define an oscillation Reynolds number $Re = \frac{\omega R_1^2}{\nu}$ so that $\beta_1^2 = -jRe$ and $\beta_2^2 = -jRe/\gamma^2$. Thus the fluid force coefficient matrix is a function of γ and Re .

2.2 Special Cases

It can be seen from Eqs. (17) and (18) that the force coefficient α_{1l} depends on the oscillation Reynolds number Re and radius ratio γ in a very complicated way. Theoretically α_{1l} must be computed from Eqs. (17) and (18), approximate solutions can be obtained in special cases. Approximate solutions are desirable not only for simplicity but also for avoiding difficulty in numerical calculations in some parameter ranges. To obtain simplified results, one must first know the values of the argument β_1 and the behavior of the Bessel functions. The value of β_1 are, in general, within the fourth quadrant in the complex plane, i.e., $-\pi/2 < \arg(\beta_1) < 0$.

For a large fluid viscosity or small oscillation Reynolds number, Bessel functions J_n and Y_n are recommended. On the other hand for a large oscillation Reynolds number, $|\beta_1| \gg 1$, the Hankel functions $H_n^{(1)}$ and $H_n^{(2)}$ are recommended.

As shown in Eq. (17), all α_{1l} are related to α_{11} . Therefore only the values of α_{11} will be presented.

a. Viscous fluid with very large radius ratio

For $\gamma \rightarrow 0$ and $|\beta_2| \gg 1$,

$$\alpha_{11} = 1 - \frac{4H_1^{(2)}(\beta_1)}{\beta_1 H_0^{(2)}(\beta_1)} \quad (26)$$

Furthermore, if $|\beta_1| \gg 1$,

$$\alpha_{11} = 1 - \frac{4j}{\beta_1} \quad (27)$$

b. Viscous fluid with large value of Re (e.g., $Re > 1000$)

$$\alpha_{11} = -1 + \frac{\left(16\beta_1^2 - \frac{73 - 578\gamma + 9\gamma^2}{8}\right)\sin(G\beta_1) - 2\beta_1(1-\gamma)(16 + \sqrt{\gamma})\cos(G\beta_1)}{(1-\gamma^2)\left(8\beta_1^2 - \frac{9 + 30\gamma + 9\gamma^2}{16}\right)\sin(G\beta_1) + \beta_1(1+\gamma)(1 + 14\gamma + \gamma^2)\cos(G\beta_1) - 32\beta_1\gamma\sqrt{\gamma}} \quad (28)$$

where

$$G = \frac{R_2 - R_1}{R_1} = \frac{1 - \gamma}{\gamma}$$

c. Re >> 1 and $G^2 \text{Re} \ll 1$

The leading term of Eq. (28) yields

$$\alpha_{11} = - \frac{12j}{G^2 \text{Re}} \quad (29)$$

d. Re >> 1 and moderate gap (e.g., $G > 0.02$)

Eq. (28) reduces to

$$\alpha_{11} = \frac{\beta_1(1+\gamma^2)\sin(G\beta_1) - 2(2-\gamma+\gamma^2)\cos(G\beta_1) + 4\gamma\sqrt{\gamma}}{\beta_1(1-\gamma^2)\sin(G\beta_1) + 2\gamma(1+\gamma)\cos(G\beta_1) - 4\gamma\sqrt{\gamma}} \quad (30)$$

e. Re >> 1 and $G^2 \text{Re} \gg 1$

Eq. (28) or (30) reduces to

$$\alpha_{11} = \frac{\beta_1(1+\gamma^2) - j2(2-\gamma+\gamma^2)}{\beta_1(1-\gamma^2) + j2\gamma(1+\gamma)} \quad (31)$$

Furthermore, when $G \gg 1$, Eq. (30) or (31) reduces to Eq. (27).

f. Re >> 1, $G^2 \text{Re} \gg 1$ and $G \ll 1$ (e.g., $G < 0.1$)

Eq. (28) reduces to

$$\alpha_{11} = \frac{1+\gamma^2}{1-\gamma^2} + \frac{\sqrt{2}}{G^2 \text{Re}^{1/2}} \left[1 - j \left(1 - \frac{4\sqrt{2}}{\text{Re}^{1/2}} \right) \right] \quad (32)$$

g. $\nu \rightarrow 0$, $\text{Re} \rightarrow \infty$

Eqs. (28), (30), (31) and (32) reduce to that of inviscid fluid

$$\alpha_{11} = \frac{1+\gamma^2}{1-\gamma^2} \quad (33)$$

Although the general form of α_{11} , Eq. (24), is very complicated, many practical applications belong to one of the special cases discussed above. Because the typical value of Re is usually much larger than 1, Eqs. (30) and (31) are most often used.

It is noted that Eqs. (26) and (29) are the same as those in Ref. [1] and Ref. [6] respectively. Eq. (28) is slightly different from that in Ref. [1]; the latter is not accurate for $G \rightarrow 0$. However, for most applications, both equations give practically the same results. The imaginary part of α_{11} in Eq. (32) is different from that of Ref. [6] by a factor of G .

2.3 Numerical Results

The analytical method to evaluate the hydrodynamic force coefficients is programmed on a digital computer. The program can be used to calculate the force coefficients α_{ij} for two concentric cylinders separated by viscous fluid for any given set of input parameters γ and Re .

As mentioned in Section 2.1 the Bessel functions $F_n(\beta)$ and $G_n(\beta)$, in principle, can be either the first and second kind of Bessel functions, J_n and Y_n , or the Hankel functions $H_n^{(1)}$ and $H_n^{(2)}$. However, in practical applications a proper selection of the function is required because of the limitations imposed by a digital computer and required accuracy.

In general, the functions J_n and Y_n can be used for a small argument (e.g., $|\beta| < 10$), while $H_n^{(1)}$ and $H_n^{(2)}$ are recommended for a large argument. However, in many practical applications, it requires only a simple algebraic calculation to obtain α_{ij} without the need of the Bessel functions (e.g., Eqs. (30) and (31)).

Some numerical results of the force coefficients were obtained and are shown in Figs. 2 to 4. Although the circular frequency ω (and so the Reynolds number Re) is in general a complex number, numerical examples are shown only for real numbers of ω . The figures show that α_{11} and α_{12} decrease as the Reynolds number Re increases. It is also interesting to note that the sensitivity of all $Re\{\alpha_{ij}\}$ with respect to Re is much smaller than that

Fig. 3. Fluid force coefficient α_{12} as a function of fluid gap

Fig. 2. Fluid force coefficient α_{11} as a function of fluid gap

Fig. 4. Fluid force coefficient α_{11} as a function of oscillation Reynolds number Re for $G = 0.08$

of $\text{Im}\{\alpha_{ij}\}$, although both $\text{Re}\{\alpha_{ij}\}$ and $\text{Im}\{\alpha_{ij}\}$ increase with decreasing Re . Therefore in most cases, in computing the added mass (i.e., $\text{Re}\{\alpha_{ij}\}$) the fluid may be considered inviscid as a first approximation. Indeed it is a very good approximation. The results also indicate that $\text{Im}\{\alpha_{ij}\}$ can be much larger than $\text{Re}\{\alpha_{ij}\}$ when Re and fluid gap, G , are small. Thus, the system damping due to the annular fluid will be very large.

Since the fluid force is not only a function of fluid gap, G , but also a function of the oscillation Reynolds number, Re , which is a function of model scale, the simulation of the Reynolds number in a scaled-model test should be included when fluid viscosity effect is important. In a reduced-scale model test, the geometrical simulation commonly employed tends to overestimate the fluid damping and thus the test result will not be conservative. Example of the scaling effect will be given in a later section.

3. VIBRATION OF TWO COAXIAL TUBES

It has been demonstrated that the hydrodynamic force matrix can be calculated for two concentric cylinders separated by an incompressible viscous fluid. The hydrodynamic force matrix is needed in the study of the free and forced vibrations of two coaxial cylinders.

3.1 Equation of Motion

The two components of the tube displacement are along the x-axis and y-axis (Fig. 1). Since the hydrodynamic forces in the two directions are independent of each other, tube vibrations in the x-z and y-z planes can be considered separately. For the y-z plane motion the equation of motion are as follows:

$$E_i I_i \frac{\partial^4 y_i}{\partial z^4} + m_i \frac{\partial^2 y_i}{\partial t^2} + c_i \frac{\partial y_i}{\partial t} = X_i + f_i, \quad (i = 1, 2) \quad (34)$$

where $E_i I_i$ is flexural rigidity, m_i is mass per unit length, c_i is the damping coefficient of cylinder, f_i is external excitation force, and X_i is the hydrodynamic force defined in Eq. (23).

It should be noted that the hydrodynamic forces given in Eq. (23) are based on the two dimensional theory. In the case of two cylinders vibrating in fluid, the fluid field is not two dimensional. However, the three dimensional effect is very small for large wave lengths [3]. Consequently, the hydrodynamic forces based on the two dimensional theory are applicable.

Substituting Eq. (23) into Eq. (34) yields

$$E_1 I_1 \frac{\partial^4 y_1}{\partial z^4} + (m_1 + m_{11}) \ddot{y}_1 + (c_1 + c_{11}) \dot{y}_1 + m_{12} \ddot{y}_2 + c_{12} \dot{y}_2 = f_1, \quad (35)$$

and

$$E_2 I_2 \frac{\partial^4 y_2}{\partial z^4} + (m_2 + m_{22}) \ddot{y}_2 + (c_2 + c_{22}) \dot{y}_2 + m_{21} \ddot{y}_1 + c_{21} \dot{y}_1 = f_2.$$

Let the displacement be

$$y_i = \sum_{n=1}^{\infty} h_{in}^{(t)} \phi_{in}(z) , \quad (36)$$

where $\phi_{in}(z)$ is the n th orthonormal function of the i -th cylinder in vacuo, and n is the axial mode number. Substituting Eq. (36) into Eq. (35) yields

$$\begin{aligned} (m_1 + m_{11})\ddot{h}_{1n} + (c_1 + c_{11})\dot{h}_{1n} + m_1\omega_{1n}^{-2}h_{1n} \\ + m_{12} \sum_{m=1}^{\infty} \ddot{h}_{2m}\beta_{nm} + c_{12} \sum_{m=1}^{\infty} \dot{h}_{2m}\beta_{nm} = f_{1n} , \\ (m_2 + m_{22})\ddot{h}_{2n} + (c_2 + c_{22})\dot{h}_{2n} + m_2\omega_{2n}^{-2}h_{2n} \\ + m_{21} \sum_{m=1}^{\infty} \ddot{h}_{1m}\beta_{mn} + c_{21} \sum_{m=1}^{\infty} \dot{h}_{1m}\beta_{mn} = f_{2n} , \end{aligned} \quad (37)$$

$$(n = 1, 2, \dots, \infty)$$

where

$$\beta_{nm} = \frac{1}{\ell} \int_0^{\ell} \phi_{1n} \phi_{2m} dz ,$$

$$f_{in} = \frac{1}{\ell} \int_0^{\ell} f_i \phi_{in} dz ,$$

$$\omega_{in}^{-2} = \frac{E_i I_i}{m_i} \frac{\phi_{in}''''}{\phi_{in}} .$$

Equation (37) consist of an infinite number of coupled ordinary differential equations. However, only a finite number of equations are taken in any particular case according to the desired accuracy.

When two cylinders are of the same length ℓ and have the same type of boundary conditions, the orthonormal functions are identical for both cylinders; i.e., $\phi_{1n} = \phi_{2n} = \phi_n$. Equations (37) reduce to

$$\begin{bmatrix} m_1 + m_{11} & m_{12} \\ m_{21} & m_2 + m_{22} \end{bmatrix} \begin{Bmatrix} \ddot{h}_{1n} \\ \ddot{h}_{2n} \end{Bmatrix} + \begin{bmatrix} c_1 + c_{11} & c_{12} \\ c_{21} & c_2 + c_{22} \end{bmatrix} \begin{Bmatrix} \dot{h}_{1n} \\ \dot{h}_{2n} \end{Bmatrix} + \begin{bmatrix} \omega_{1n}^{-2} & 0 \\ 0 & \omega_{2n}^{-2} \end{bmatrix} \begin{Bmatrix} h_{1n} \\ h_{2n} \end{Bmatrix} = \begin{Bmatrix} f_{1n} \\ f_{2n} \end{Bmatrix}$$

$$(n = 1, 2, \dots, \infty) \quad (38)$$

where

$$f_{in} = \frac{1}{\ell} \int_0^{\ell} f_i \phi_n dz ,$$

and

$$\bar{\omega}_{in}^{-2} = \frac{E_i I_i}{m_i} \frac{\phi_n''''}{\phi_n} = \frac{I_i I_i}{m_i} \frac{\lambda_n^4}{\ell^4} .$$

Here $\bar{\omega}_{in}$ and λ_n are the n -th natural frequency and the modal constant of the i -th cylinder in vacuo and f_{in} is the generalized external force. Note that Eq. (38) can be applied to all values of n . For each n , there are two coupled equations. However, there is no coupling among the equations for different n . This is true for two cylinders with the same type of boundary conditions and of the same length. Therefore, in the following discussion the subscript n will be omitted.

3.2 Frequency Equation

3.2.1 Exact solution

Letting $h_i = \bar{h}_i e^{j\omega t}$, $f_i = 0$, and substituting them into Eq. (38) yields

$$\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{Bmatrix} \bar{h}_1 \\ \bar{h}_2 \end{Bmatrix} = 0 , \quad (39)$$

where

$$b_{ii} = \bar{\omega}_i^{-2} - \omega^2 (1 + \mu_i \alpha_{ii}) + 2j\zeta_{si} \bar{\omega}_i \omega , \quad i = 1, 2 ,$$

$$b_{12} = -\frac{\omega^2 \mu_1 \alpha_{12}}{\gamma} , \quad b_{21} = -\omega^2 \gamma \mu_2 \alpha_{21} , \quad \text{and } \mu_i = \frac{\rho \pi R_i^2}{m_i} .$$

Here $\zeta_{si} = C_i / 2m_i \bar{\omega}_i$ is the structural modal damping ratio in vacuo.

The frequency equation of the coupled fluid/cylinder system is obtained by setting the determinant of the coefficient b_{ij} in Eq. (39) equal to zero, it can be written as $|b_{ij}| = 0$ or in the function form as

$$F(\omega, \gamma, \text{Re}, \bar{\omega}_i, \mu_i, \zeta_{si}) = 0 \quad (40)$$

As pointed out before, the hydrodynamic force coefficient for a given system is not a constant but a function of the system frequency, ω , which in general is a complex number. Therefore, in order to determine the natural frequency and the damping ratio of the coupled/rod system, an iteration procedure is required.

Once the system natural frequency is obtained, the corresponding mode shape can be calculated from Eq. (39),

$$\bar{h}_1/\bar{h}_2 = -b_{12}/b_{11} \quad (41)$$

3.2.2 Approximate solution

It is straightforward to obtain the frequencies from the exact frequency equation (40). However, it is instructive to examine approximations to the exact frequency. The following approximate method is applicable when the damping ratio of the system is not very large.

First, consider the uncoupled vibrations: i.e., one of the cylinders is rigid. The equation of motion (38) becomes

$$(m_i + m_{ii})\ddot{h}_i + (c_i + c_{ii})\dot{h}_i + m_i \bar{\omega}_i^2 h_i = f_i \quad (i = 1, 2) \quad (42)$$

The uncoupled natural frequency, ω_i^* , and the corresponding modal damping ratio ζ_{ti}^* are

$$\begin{aligned} \omega_i^* &= \sqrt{\frac{m_i}{m_i + m_{ii}}} \bar{\omega}_i = \frac{\bar{\omega}_i}{\sqrt{1 + \mu_i \text{Re}\{\alpha_{ii}\}}} , \\ \zeta_{ti}^* &= \frac{c_i + c_{ii}}{2\sqrt{(m_i + m_{ii})m_i} \bar{\omega}_i} = \frac{\zeta_{si}}{\sqrt{1 + \mu_i \text{Re}\{\alpha_{ii}\}}} + \frac{\mu_i \text{Im}\{-\alpha_{ii}\}}{2[1 + \mu_i \text{Re}\{\alpha_{ii}\}]} , \end{aligned} \quad (43)$$

For a general coupled vibration, by neglecting damping term, Eq. (39) becomes

$$\begin{bmatrix} \omega_1^{-2} - \omega^2 (1 + \mu_1 \text{Re}(\alpha_{11})) & -\omega^2 \frac{\mu_1}{\gamma} \text{Re}(\alpha_{12}) \\ -\omega^2 \mu_2 \gamma \text{Re}(\alpha_{21}) & \omega_2^{-2} - \omega^2 (1 + \mu_2 \text{Re}(\alpha_{22})) \end{bmatrix} \begin{Bmatrix} \bar{h}_1 \\ \bar{h}_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (44)$$

The frequency equation is

$$A\omega^4 - B\omega^2 + C = 0, \quad (45)$$

where

$$A = (1 + \mu_1 \text{Re}(\alpha_{11}))(1 + \mu_2 \text{Re}(\alpha_{22})) - \mu_1 \mu_2 \text{Re}^2(\alpha_{12})$$

$$B = \{(1 + \mu_1 \text{Re}(\alpha_{11}))\omega_1^{-2} + (1 + \mu_2 \text{Re}(\alpha_{22}))\omega_2^{-2}\},$$

$$C = \omega_1^{-2} \omega_2^{-2}.$$

Eq. (45) gives two coupled natural frequencies

$$\omega_1 = \left[\frac{B - \sqrt{B^2 - 4AC}}{2A} \right]^{1/2},$$

and

$$\omega_2 = \left[\frac{B + \sqrt{B^2 - 4AC}}{2A} \right]^{1/2}. \quad (46)$$

It can be shown that $\omega_1 < \omega_1^*, \omega_2^*$ and $\omega_2 > \omega_1^*, \omega_2^*$, where ω_1^* and ω_2^* are the frequencies of the uncoupled modes given in Eqs. (43). Thus the frequencies of the uncoupled modes are always within the range of those of the coupled modes.

Since $\text{Re}\{\alpha_{ij}\}$ is not a constant but a function of the system frequency, theoretically the system frequencies, ω_1^* and ω_2^* , given in Eqs. (43) or (46), have to be obtained through an iteration procedure. However, because of the insensitivity of the natural frequencies on the value of $\text{Re}\{\alpha_{ij}\}$, fairly accurate results can be obtained either with no iteration or only a few iterations. The value of $\text{Re}\{\alpha_{ij}\}$ at $\omega = \bar{\omega}_1$ or $\bar{\omega}_2$ can be used for the first approximation. The corresponding mode shapes are

$$\frac{\bar{h}_1}{\bar{h}_2} = \frac{\omega_1^2 \frac{\mu_1}{\gamma} \operatorname{Re}(\alpha_{12})}{\omega_1^2 - \omega_1^2 [1 + \mu_1 \operatorname{Re}(\alpha_{11})]} = \sigma_i \quad (47)$$

In many practical situations, the damping matrix [C] may be assumed to be proportional to the stiffness matrix [K] or the mass matrix [M]. Again the values of [M] can be calculated based on those of $\omega = \bar{\omega}_1$ or $\bar{\omega}_2$.

Let

$$[E]^T [M] [E] = [I] \quad ,$$

and

$$[E] = \begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{bmatrix} \quad , \quad (48)$$

where [E] is an orthonormal matrix with respect to [M]. The elements of [E] are therefore expressed as

$$e_{2i} = 1 / \sqrt{\sigma_i^2 (m_1 + m_{11}) + \sigma_i (m_{21} + m_{12}) + (m_2 + m_{22})} \quad , \quad (49)$$

and

$$e_{1i} = \sigma_i e_{2i} \quad (i = 1, 2)$$

Thus Eq. (38) can be reduced to a set of uncoupled modal equations:

$$\ddot{q}_1 + 2\omega_1 \zeta_{t1} \dot{q}_1 + \omega_1^2 q_1 = \bar{F}_1 \quad , \quad (50)$$

and

$$\ddot{q}_2 + 2\omega_2 \zeta_{t2} \dot{q}_2 + \omega_2^2 q_2 = \bar{F}_2 \quad ,$$

where

$$\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = [E]^{-1} \begin{Bmatrix} \bar{h}_1 \\ \bar{h}_2 \end{Bmatrix} \quad ,$$

$$\begin{Bmatrix} \bar{F}_1 \\ \bar{F}_2 \end{Bmatrix} = [E]^T \begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} \quad ,$$

and $\zeta_{ti} = \frac{d_i}{2\omega_i}$ is the modal damping ratio of the coupled modes. d_i is defined as

$$d_i = \{e_{1i} \ e_{2i}\} \begin{bmatrix} c_1 + c_{11} & c_{12} \\ c_{21} & c_2 + c_{22} \end{bmatrix}_{\omega=\omega_i} \begin{Bmatrix} e_{1i} \\ e_{2i} \end{Bmatrix}, \quad i=1,2 \quad (51)$$

Usually the damping matrix [C] is neither proportional to [K] nor [M], and the resultant damping matrix [D] is not a diagonal matrix. In this case, the method cannot be used to reduce Eq. (38) into a set of uncoupled modal equations and different method [4] should be used. Nevertheless, in most practical situations, one can still neglect the damping coupling to decouple the equations.

3.3 Numerical Examples

For specific numerical examples, consider two concentric steel circular cylindrical tubes separated by water at room temperature. The full-scale dimensional values are shown in Table 1. Several scaled models were used to examine the scaling effect. The fluid gap, $G = (R_2 - R_1)/R_1$, was varied from 0.01 to 10. The rods were assumed to be simply supported at both ends. Calculations were performed both with and without structural damping.

The frequencies of the system depend on the axial mode number n . In this report, unless specified, only the data associated with the lowest axial mode number (i.e., $n = 1$) were given.

For comparison, calculations were included for several related cases: 1) inner tube in vacuo; 2) outer tube in vacuo; 3) an uncoupled tube system with rigid outer tube; 4) an uncoupled tube system with rigid inner tube; and 5) a coupled cylindrical tube system.

Both the exact and approximate methods were used in calculations. It is found that the results obtained by these methods were practically

Table 1. Dimensional Values of the Numerical Examples for a Full-Scale Model

- 1) Inner interface radius: $R_1 = 5 \text{ cm (1.97 in.)}$
- 2) Wall thickness ratios: $\delta_1 = \Delta R/R_1 = 0.125$ and
 $\delta_2 = \Delta R/R_2 = 0.1$
- 3) Steel mass density: $\rho_1 = \rho_2 = 7.47 \times 10^3 \text{ kg/m}^3 \text{ (.270 lbm/in.}^3\text{)}$
- 4) Young's modulus: $E_1 = E_2 = 1.93 \times 10^{11} \text{ PA (2.8} \times 10^7 \text{ psi)}$
- 5) Rod length: $\ell_1 = \ell_2 = 150 \text{ cm (59 in.)}$
- 6) Structural damping ratio ζ_s .
 Two cases: (a) $\zeta_{s1} = \zeta_{s2} = 0.01$; (b) $\zeta_{s1} = \zeta_{s2} = 0$
- 7) Water density: $\rho = 1 \times 10^3 \text{ kg/m}^3 \text{ (0.036 lbm/in.}^3\text{)}$
- 8) Kinematic viscosity of water: $\nu = 1 \times 10^{-6} \text{ m}^2/\text{s (1} \times 10^{-5} \text{ ft}^2/\text{sec)}$
- 9) Fluid gap, $G = (R_2 - R_1)/R_1$: $0.01 \leq G \leq 10$
- 10) End condition: simple-simple

identical except at the very small fluid gap region where the fluid viscosity effect and so the fluid damping were very large (e.g., $\zeta_t > 0.8$). For most practical applications, the approximate solution is as good as the exact one. Fig. 5 shows the system natural frequency as a function of fluid gap for $\zeta_{s1} = \zeta_{s2} = 0$.

The lower natural frequency is associated with the mode in which two tubes move out-of-phase, while the higher frequency is associated with in-phase mode. The frequencies of the uncoupled modes (indicated by cases 3 and 4) are always in between those of the coupled modes (case 5). The out-of-phase natural frequency is lower than either of the coupled modes; while the in-phase frequency is always higher than either of the uncoupled modes.

As the fluid gap, G , increases, the effect of the existence of fluid decreases. Also the natural frequency of the out-of-phase mode approaches that of the uncoupled mode of a tube in an infinite fluid, and the frequency of the in-phase mode approaches that of the uncoupled mode of a tube containing a fluid.

As expected the fluid affects significantly both the out-of-phase modal frequency and the uncoupled modal frequencies as the fluid gap decreases. However, the in-phase modal frequency is practically unchanged as the fluid gap decreases.

Figs. 6(a) and (b) show the system damping ratios as a function of fluid gap for $\zeta_{s1} = \zeta_{s2} = 0.01$ and $\zeta_{s1} = \zeta_{s2} = 0$ respectively. Except for the in-phase mode, the annulus fluid contributes significant damping to the system for all the cases especially at small fluid gaps. The in-phase modal damping ratio is not shown in Fig. 6(b) because it is too small. When $\zeta_{s1} = \zeta_{s2} = 0$, the modal damping ratios for coupled modes are higher (out-of-phase mode) or lower (in-phase mode) than either of the

Fig. 5. Natural frequencies of a fluid/tube system as a function of fluid gap G (full scale model)

Fig. 6. Modal damping ratio ζ_t of a fluid/tube system as a function of fluid gap (full scale model): (a) $\zeta_s = 0.01$ (b) $\zeta_s = 0$

uncoupled modes. Usually the system damping ratio due to the fluid viscosity decreases as fluid gap increases as shown in Fig. 6(b). It is also interesting to point out that the total system damping ratio ζ_t has a local minimum as Fig. 6(a) shows. This behavior can be seen more clearly in Fig. 8.

Fig. 7 shows the numerical results for a 1/4-scale model.

Fig. 8 shows the effect of scale model on the system properties. For comparison, the data for a four-time scale and for potential flow solution are also included. The potential flow solution can be interpreted as that of an infinite-time scale model. Fig. 8(a) shows that the natural frequency ratios (or added mass effect) are practically independent of the modal scale. However, the modal damping ratios are significantly different. This is because the fluid viscosity, ν , or the oscillation Reynolds number, Re , has a small effect on $Re\{\alpha_{ij}\}$ and has a large effect on $Im\{\alpha_{ij}\}$. The change of natural frequency is due to fluid inertia effect, which is proportional to $Re\{\alpha_{ij}\}$, while the increase in damping is mainly attributed to fluid drag, which is proportional to $Im\{\alpha_{ij}\}$. The total system damping ratio ζ_t consists of $\sqrt{\frac{m_s}{m_s+m_f}} \zeta_s$ and ζ_f , where m_s , m_f , ζ_s and ζ_f are the structure mass, added mass, structural damping ratio and fluid damping ratio respectively. Usually $\sqrt{\frac{m_s}{m_s+m_f}} \zeta_s$ will decrease and ζ_f will increase as fluid gap decreases. Therefore the value of ζ_t could be an increasing function or a decreasing function of the gap depending on the values of structural damping and fluid damping.

Except for the potential flow solution in which the damping ratio always decreases as the gap decreases, all other cases in Fig. 8(b) show that there are local minimum values of damping ratio at some value of G . This behavior of minimum damping ratio seems to disappear as the modal scale decreases.

Fig. 7. Natural frequencies and modal damping ratio of a fluid/tube system as a function of fluid gap for a 1/2-scale model

Fig. 8. Comparison of natural frequency and modal damping ratio of coupled modes for different scale models

Figure 8 also shows that as the scale is reduced, the modal damping ratio always increases; i.e., in a reduced scale model, the modal damping will always be larger than that of the prototype. This is very important in practice. In particular, in larger system components such as nuclear reactor components, very frequently a reduced scale model is used to study the system characteristics. In those cases, the scale model tends to produce higher damping. This effect should be properly accounted for in the analysis or testing.

The strong dependence of the modal damping ratio on model scale indicates that the results from scale models, frequently used in practice for design evaluations, may not be conservative if the oscillation Reynolds number is not simulated. It should also be noted that the frequently used potential flow solution predicts the system frequency very well, but underestimates the damping ratio.

Fig. 9 shows the modal shape of a coupled fluid/tube system as a function of fluid gap. Although the modal damping ratios for different model scales are quite different, the mode shapes are practically identical.

Some of the data for the second axial mode ($n = 2$) are also calculated. Fig. 10 shows the coupled system frequencies and the modal damping ratio for the first two axial modes. It shows that the out-of-phase modal frequency for $n = 2$ is very close to or lower than the in-phase modal frequency for $n = 1$. For the out-of-phase mode, the damping ratio for $n = 2$ is always lower than that for $n = 1$, and the damping ratio for the in-phase mode is usually negligible.

Fig. 9. Mode shape of a coupled fluid/tube system as a function of fluid gap

Fig. 10. Comparison of natural frequencies of coupled modes for the first two axial wave numbers

4. CONCLUDING REMARKS

In this report, coupled vibration of two coaxial tubes separated by a viscous fluid is studied analytically. The fluid forces acting on the tubes associated with tube oscillations are obtained in closed form based on the following assumptions: (1) tube displacements are small; and (2) fluid motion is two dimensional and satisfies the linearized Navier-Stokes equation of motion. It is found that the fluid force coefficient matrix can be obtained from the coefficient for a single cylinder vibrating in a viscous fluid annulus [1]. Once the fluid forces are known, natural frequencies, modal damping and forced response of coupled tube/fluid vibration can readily be calculated.

Numerical results for fluid force coefficients are presented for fairly large ranges of system parameters. It is shown that the fluid force coefficient is always a decreasing function of oscillation Reynolds number Re , and fluid gap, G . The sensitivity of Re on the added mass coefficient is much smaller than that on the damping coefficient.

An analysis is also presented for coupled vibrations of two concentric tubes separated by a viscous fluid. The natural frequencies of the uncoupled modes are always in between those of coupled modes. The out-of-phase modal frequency, which is the lowest system frequency, is always lower than those of the uncoupled modes; while the in-phase modal frequency is higher than those of the uncoupled modes. The effect of the fluid viscosity on the system natural frequencies and mode shape are negligibly small in most practical applications. However, fluid viscosity contributes significantly to the modal damping. For a coupled tube/fluid system the viscous effects are mostly pronounced for the out-of-phase modes, but these effects are considered to be negligible for the in-phase mode.

Finally it should be pointed out that results from scale models, frequently used in practice for design evaluation, may not be conservative if the oscillation Reynolds number is not simulated. Specifically, a reduced scale model always yields higher damping than the original system. Also, the potential flow solution always underestimates the system damping.

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