LIMITATION ON RESONANT EXTRACTION DUE TO RANDOM FIELD*
 ERRORS IN THE ES/D MAGNETS AT FNAL

M. Month
 Brookhaven National Laboratory
 Upton, New York 11973

Abstract

Resonant extraction from a superconducting accelerator is considered. In particular, the effects of random field errors, induced by errors in the positioning of the superconducting coils, on third integer extraction from the ES/D ring is discussed. Without a special correction scheme, the tolerable error in the coil positioning is unrealistically small. A detailed study on possible dynamic corrections is required. It is suggested that a reevaluation of the magnet geometry be undertaken, with special consideration being given to the "slow" extraction process.

MASTER

*Research carried out under the auspices of the U.S. Energy Research and Development Administration under Contract No. EE(30-1)-16

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

NOT EE(30-1)-16

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

29

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

Resonant extraction is accomplished by including nonlinear lenses in the magnet lattice. They provide the necessary resonant harmonic. Particles are "slowly" squeezed out of a stable central region within the magnet aperture. Their betatron amplitudes grow at a rate increasing with amplitude. A septum device placed at the machine aperture limit will collect particles and transport them into the external beam lines.

If, however, there are nonlinear error fields present, they tend to inhibit the orderly extraction. Random errors in the positioning of the coils in superconducting magnets introduce nonlinear fields which increase rapidly as the coils are reached. But they also extend far into the aperture. The result is a limit to how close to the coils the extraction septum can be placed. This of course has implications with regard to the aperture size, the coil location (i.e. the magnet size) and also the care with which the magnets must be constructed (i.e. the precision of the coil construction and positioning within the magnets).

To appreciate the nature of the problem, we will take rough numbers for the FNAL ring and the ES/D magnets. These are listed in Table I.

One mechanism by which nonlinear error fields can disrupt the resonant extraction is through an amplitude dependent tune shift. A particle whose small amplitude tune is in resonance will grow in amplitude. As amplitude growth proceeds the error fields induce a tune shift off resonance. This means a stable separatrix will begin to grow behind the particle moving out. If the separatrix grows at a fast enough rate, it will retrap particles at some amplitude. If this amplitude is within the septum aperture, this will stabilize the particles and prevent their extraction. Since the separatrix size is a strong function of tune, small tune changes will cause rapid separatrix growth so that this phenomenon is a general area of concern for the resonant extraction process.

There are other possible mechanisms arising out of this sort of error field that could affect the extraction process. For example, the effect of certain higher multipoles can distort the resonant amplitude function at "large amplitudes". Also coupling resonances are excited and

an amplitude dependent verticle tune shift could force particles onto these. We will however not deal with these possibilities, but restrict ourselves to the rather simple case of self-retrapping by a tune shift as described above.

Since the calculation presented here is meant to be "order-of-magnitude", we will not attempt to include a superimposed correction system. It is clear that such a system is complicated and will be difficult to implement. If the ES/D project is designed so that correction is required, it is suggested that a detailed analysis be made to demonstrate whether an effective, feasible system can be devised.

Table I: Parameters

Parameter	Symbol	Value
Machine Average Radius	R	1 km.
Tune	ν	19 1/3
Superconducting Coil Radius	R_c	3 cm.
Septum Amplitude	A_s	2 cm.
Integrated Sextupole Strength	g	0.32 m^{-2}
rms Coil Positioning Error	ϵ_{rms}	Length
Particle Betatron Amplitude	A	Length
Number of Ring Magnets	N_B	774

Tune Shift With Amplitude

The random field error due to coil positioning errors can be roughly expressed by

$$\left(\frac{\Delta B}{B_0}\right)_{\text{rms}} = \left(\frac{\epsilon_{\text{rms}}}{4R_c}\right) \frac{1}{(1-x/R_c)^{3/2}}, \quad (1)$$

where x is the particle displacement, which in terms of amplitude, A , and phase, φ , is given by

$$x = A \cos \varphi. \quad (2)$$

The tune shift with amplitude due to this field error can be evaluated, giving

$$\Delta \nu = \frac{\beta \epsilon_{\text{rms}}}{8\pi R_c^2 \sqrt{N_B}} \frac{1}{\alpha} \int_0^{2\pi} d\varphi \frac{\cos \varphi}{[1-\alpha \cos \varphi]^{3/2}}, \quad (3)$$

where we have written

$$\alpha = A/R_c, \quad (4)$$

and

$$\beta = R/\nu \quad (5)$$

is the usual average β function.

Separatrix Amplitude

The separatrix amplitude for third integer resonant extraction is expressed in terms of the amplitude of the three unstable fixed points

$$A_{\text{fp}} = \frac{8\pi(\Delta \nu)}{\beta g}. \quad (6)$$

Substituting $\Delta \nu$ from (3) and writing

$$\alpha_{\text{fp}} = A_{\text{fp}}/R_c, \quad (7)$$

we have

$$\alpha_{fp} = \frac{k}{\alpha} \int_0^{2\pi} d\varphi \frac{\cos \varphi}{[1 - \alpha \cos \varphi]^{3/2}}, \quad (8)$$

with

$$k = \frac{\epsilon_{rms}}{g R_c^3 \sqrt{N_B}}. \quad (9)$$

It should be pointed out that the expression (8) is not valid close to $\alpha = 0$, but should be adequate in the region $\alpha > 0.1$.

If we define

$$U(\alpha) = \frac{1}{\alpha^2} \int_0^{2\pi} d\varphi \frac{\cos \varphi}{[1 - \alpha \cos \varphi]^{3/2}}, \quad (10)$$

then (8) becomes

$$\frac{\alpha_{fp}}{\alpha} = kU(\alpha). \quad (11)$$

Retrap Condition

The separatrix amplitude is a function of the particle amplitude, given in (11). Thus a given particle is retrapped when its amplitude equals the fixed point amplitude; that is, when

$$\alpha = \alpha_{fp}(\alpha). \quad (12)$$

Without superimposed correction, the required condition for extraction is

$$\alpha \gg \alpha_{fp}(\alpha), \quad (13)$$

or,

$$kU(\alpha) \ll 1. \quad (14)$$

Limitations

The inequality (14) is a constraint which puts either an upper limit on k or a lower limit on α . The former means a maximum coil positioning error, ϵ_{rms} , while the latter means either an upper limit on the amplitude, A , or a lower limit on the coil radius, R_c . Dealing with the limit on α leads to a rather detailed discussion on the optimum septum location and overall aperture questions which will not serve any useful purpose at this point. We will therefore take the amplitude to be the septum amplitude, A_s , and also take R_c as given, i.e. as in the proposed EE/D magnet design. (See table I). Thus, (14) simply gives the tolerable error in coil positioning through the relation

$$k \ll U(\alpha_s)^{-1}, \quad (15)$$

where

$$\alpha_s = A_s / R_c. \quad (16)$$

Taking the values from Table I, this becomes

$$\epsilon_{rms} \ll 2.4 \times 10^{-4} U\left(\frac{2}{3}\right)^{-1} \text{ (m)}, \quad (17)$$

$$\text{with } U\left(\frac{2}{3}\right) = 4.5 \int_0^\pi \frac{\cos \varphi}{[1 - 2/3 \cos \varphi]^{3/2}} d\varphi. \quad (18)$$

Computing $U(2/3)$, which has the value 13.0, we have for the tolerable positioning error

$$\epsilon_{rms} \ll 1.8 \times 10^{-5} \text{ m},$$

or,

$$\epsilon_{rms} \ll 0.7 \text{ mils.}$$

This is clearly not acceptable. A detailed study on possible correction schemes is required. Also a reevaluation of the magnet parameters, taking into consideration the "slow" extraction requirements, appears to be in order.