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FILAMENT-REINFORCED CYLINDRICAL PRESSURE VESSELS: ANALYSIS AND NUMERICAL EVALUATION

M. MOSS

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Marvin Moss

ABSTRACT

The response to internal pressure of a cylindrical vessel consisting of a metal liner reinforced with a circumferentially wound filament-epoxy composite has been analyzed. Expressions for pressures required to produce the onset of yielding and complete yielding in the liner, failure of the composite, and efficiencies are derived for all combinations of three liner materials (HP-9-4-20 steel, 6061-T6 Al, and Ti-6Al-4V), and three composites (Kevlar 49-epoxy, Thornel 400-epoxy, and Thornel 75-epoxy). Failure pressures are highest for the Kevlar 49-reinforced vessels. At lower failure pressures, 6061-T6 Al/Kevlar 49-epoxy vessels are most efficient as measured by the product of failure pressure and interior volume divided by vessel mass.

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INTRODUCTION

A pressure analysis of a composite-reinforced metal cylinder has been performed and applied to several materials systems in common use. The objective was to supply engineering data useful in the selection of a vessel for particular pressure requirements. Long-term performance is not addressed. The analysis is similar to Gerstle's [1,2] treatment of filament-reinforced spherical pressure vessels, his quasi-isotropic composite laminate being replaced by the cylindrically anisotropic geometry of a circumferentially, filament-wound structure. Aspects of this problem have been analyzed by Lekhnitskii [3,4].

As in Gerstle's analysis, composite response is assumed to be linear-elastic to failure, and the liner is considered an elastic-perfectly plastic material (no work hardening at the small strains characteristic of composite fracture). Plane strain is assumed and the Tresca yield criterion is imposed. Laminate properties are developed from a micro-mechanical treatment of lamina behavior. Liner yielding and composite failure are numerically evaluated for all combinations of three metals and three composites over a range of liner and composite thicknesses. Vessels are order ranked in efficiency (failure pressure times interior volume divided by vessel mass).

ANALYSIS

The governing equations for the composite-reinforced cylindrical pressure vessel have been developed elsewhere [5]. In this section, the analytical results for yielding of the liner and failure of the composite are presented. As in Gerstle's analysis for spherical pressure vessels [2], the maximum-strain criterion for vessel failure is applied. Numerical values of internal pressure at the onset of liner yield, p_Y , for a fully plastic liner, p_T , and at composite failure, p_F , as well as pressure vessel efficiency, e , are given for nine combinations of liner materials and composites.

Yielding of Liner

The vessel is shown in Fig. 1. The onset of yielding may be determined from the general expression for internal pressure, p , where $p_Y \leq p \leq p_T$ [5]:

$$p = Y \left[\frac{1}{2 + 4(\lambda + \mu)k} + \ln \frac{r_p}{r_1} \right], \quad (1)$$

where Y is the yield strength and λ and μ are the Lamé constants of the liner. The radius of the elastic-plastic boundary is denoted by r_p , and k is a constant composed of liner and composite properties.

At the onset of yielding ($r_p = r_1$)

$$p_Y = \frac{Y}{2 + 4(\lambda + \mu)k_Y}, \quad (2)$$

where

$$k_Y \equiv \frac{1}{2(\lambda + \mu)(\alpha^2 - 1) + 2cd \alpha^2 (\lambda + 2\mu)(\beta^{2n} - 1)} \cdot \frac{1}{[d(2\mu + c) + c\beta^{2n}(2\mu - d)]}$$

The quantities c and d are composite laminate stiffness relations [5], and $n = (E_\theta/E_r)^{\frac{1}{2}}$, the anisotropy. E_θ and E_r are the circumferential and radial composite elastic moduli, respectively.

For total yielding of the liner ($r_p = r_2$),

$$p_T = Y \left[\frac{1}{2 + 4(\lambda + \mu)k_T} + \ln \alpha \right], \quad (3)$$

where

$$k_T \equiv \frac{[d(2\mu + c) + c\beta^{2n}(2\mu - d)]}{2cd (\lambda + 2\mu)(\beta^{2n} - 1)}$$

Figure 1. Cylindrical pressure vessel consisting of metal liner and reinforcing composite.

For the particular case of an unreinforced isotropic cylinder ($\beta = 1$),

$$p = Y \left[\left(\frac{r_2^2 - r_p^2}{2r_2^2} \right) + \ln \frac{r_p}{r_1} \right] \quad (4)$$

Therefore,

$$p_Y = \frac{Y(\alpha^2 - 1)}{2\alpha^2}, \quad \beta=1 \quad (5)$$

and

$$p_T = Y \ln \alpha, \quad \beta=1 \quad (6)$$

Composite Failure

Failure of the vessel is assumed to occur when the maximum circumferential strain, K_θ , which the composite can withstand is attained [5]. The pressure required to cause failure is,

$$p_F = \frac{K_\theta c(\beta^{2n} - 1)}{(1 + \frac{c}{d} \beta^{2n})} + Y \ln \alpha \quad (7)$$

For the unreinforced cylinder, Eq. 7 reduces to Eq. 6. The failure pressure of a vessel with an infinitely thick composite is

$$p_F = d K_\theta + Y \ln \alpha, \quad \beta = \infty \quad (8)$$

A useful measure of pressure vessel efficiency is the product of failure pressure and interior volume, V , divided by vessel mass, m ,

$$e \equiv \frac{p_F V}{m} = \frac{p_F}{\rho_\ell (\alpha^2 - 1) + \rho_c \alpha^2 (\beta^2 - 1)}, \quad (9)$$

where ρ_ℓ and ρ_c are the respective densities of liner and composite.

The efficiency of a composite vessel with no liner ($\alpha=1$) is

$$e = \frac{K_{\theta} c (\beta^{2n} - 1)}{\rho_c (\beta^2 - 1) (1 + \frac{c}{d} \beta^{2n})} \quad (10)$$

The efficiency approaches its largest value for $\alpha = 1$ and as $\beta \rightarrow 1$:

$$e_{\max} = \lim_{\substack{\alpha=1 \\ \beta \rightarrow 1}} e = \frac{n K_{\theta} c}{\rho_c (1 + \frac{c}{d})} \quad (11)$$

Numerical Evaluation and Conclusions

Combinations of several common liner and composite materials in cylindrical pressure vessel configurations were numerically evaluated. Values of p_Y , p_T , p_F , and e were calculated on a CDC 6600 computer for $1 \leq \alpha \leq 1.1$ and $1 \leq \beta \leq 2$, as well as $\beta = \infty$.

Data for the liner materials are given in Table I. HP-9-4-20 is a high-performance steel made by Republic Steel Corporation. It has high toughness and weldability, and is used commonly in pressure vessels. The numbers in its designation refer to percent contents of nickel and cobalt, and hundredths of a percent of carbon, respectively. The other two materials are well-known aluminum and titanium alloys. The Lamé constants, λ and μ , for all these materials were calculated from E and ν by the usual relations [6],

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}, \quad \mu = \frac{E}{2(1+\nu)}, \quad (12a,b)$$

where E is the liner elastic modulus in tension and ν , Poisson's ratio.

Table I. Liner Properties

	$\frac{Y}{(MPa)}$	$\frac{E}{(GPa)}$	$\frac{\nu}{}$	$\frac{\rho}{(g/cc)}$
HP-9-4-20	1210	197	0.29	7.86
6061-T6 Al	255	69	0.33	2.71
Ti-6Al-4V	830	110	0.34	4.43

The composites were those examined by Gerstle [1,2]; his PRD-49 is the old name for Kevlar 49 filament. Table II lists typical properties which cover a wide range of anisotropy and failure strain. K_0 is to be considered a short-term value.

Tables III a-c give the results for all combinations of liner materials and reinforcing composites. Fig. 2 shows p_Y , p_T , and p_F vs. β for the particular case of HP-9-4-20 steel reinforced with Kevlar 49-epoxy where $\alpha = 1.05$. Fig. 3 shows p_Y and p_F vs. β for all liner-composite

Table II. Composite Properties

Composite	V_f	c (GPa)	d (GPa)	n	$K_0^\dagger$	dK_0 (GPa)
Kevlar 49*-epoxy	0.65	32.0	25.6	3.03	0.018	0.461
Thornel 400†-epoxy	0.55	33.9	28.2	4.00	0.012	0.338
Thornel 75†-epoxy	0.50	46.8	41.5	6.18	0.0046	0.191

* Du Pont aramid filament

† Union Carbide graphite filaments. Thornel 400 is no longer available and has been superseded by Thornel 300 which has a slightly higher tensile modulus and lower elongation at failure.

‡ Short-term estimates from manufacturers' literature.

combinations for $\alpha = 1.05$; the superiority of Kevlar 49-epoxy as a pressure vessel reinforcing material is evidenced by a consistently higher failure pressure compared with the Thornel composites. This is true for all values of α for a particular liner material (Tables III a-c).

The reason for this may be seen from the expression for p_F (Eq. 7); the important parameters are more evident from the simple case of an infinitely thick composite (Eq. 8). The value of d for Kevlar 49-epoxy is smaller than for the Thornel composites, due mainly to a lower elastic anisotropy, n . However, its large strain to failure, K_0 , is the dominant parameter, being four-fold greater than the Thornel 75 composite fracture strain.

The higher yield pressures associated with the Thornel-reinforced vessels result from the large d values for these composites; Kevlar 49-epoxy suppresses yielding the least. Thornel 75-epoxy's relatively large d will generally cause this low failure-strain material to fail before the onset of yielding in the thinner steel and titanium alloy liners; Fig. 3, curve T-75 illustrates this phenomenon.

Pressure vessel efficiency for HP-9-4-20/Kevlar 49-epoxy is plotted against β for several values of α in Fig. 4. As is true for all material combinations considered, efficiency increases with decreasing liner wall thickness to its greatest value at the limit of zero thickness of both liner ($\alpha=1$) and composite ($\beta=1$).

Tables III(a-c). Values of p_Y , p_T , p_F (MPa), and e (MPa-cc/g) as functions of α and β for all liner combinations.

(a)

HP-9-4-20/Kevlar 49-epoxy

 $\alpha=1.00$

β	p_Y	p_T	p_F	e
1.00	-	-	0.0	575.1
1.05	-	-	74.0	534.9
1.10	-	-	138.7	492.3
1.20	-	-	244.0	410.7
1.30	-	-	315.9	339.1
1.50	-	-	395.0	234.1
2.00	-	-	449.4	111.8
∞	-	-	461.7	.0

1.01

1.00	11.9	12.0	12.0	76.3
1.05	34.3	34.9	86.0	287.5
1.10	53.7	54.7	151.7	339.2
1.20	83.9	85.4	256.3	335.1
1.30	104.1	106.1	327.9	235.9
1.50	125.9	129.3	407.0	216.5
2.00	140.5	143.3	461.4	107.5
∞	143.3	146.6	473.7	.0

1.05

1.00	56.1	58.8	58.8	73.0
1.05	76.3	81.7	132.9	138.7
1.10	94.3	101.5	185.5	177.5
1.20	122.7	132.3	262.8	207.3
1.30	141.4	152.9	374.7	204.5
1.50	161.5	175.1	453.8	170.2
2.00	175.1	190.1	508.2	96.4
∞	178.2	193.5	520.5	.0

1.10

1.00	104.7	114.9	114.9	69.6
1.05	123.5	137.8	189.3	103.9
1.10	139.3	157.6	254.6	127.7
1.20	165.3	198.4	358.0	151.5
1.30	192.4	259.9	437.8	155.1
1.50	200.3	231.2	509.9	138.1
2.00	213.1	246.2	564.3	86.1
∞	215.9	249.6	576.6	.0

HP-9-4-20/Thornel 400-epoxy

 $\alpha=1.00$

β	p_Y	p_T	p_F	e
1.00	-	-	0.0	492.7
1.05	-	-	70.0	455.2
1.10	-	-	138.1	413.3
1.20	-	-	217.6	329.7
1.30	-	-	269.5	260.3
1.50	-	-	315.0	168.0
2.00	-	-	336.0	74.7
∞	-	-	338.4	.0

1.01

1.00	11.9	12.0	12.0	76.3
1.05	43.5	44.3	82.0	260.4
1.10	69.9	71.2	142.1	296.4
1.20	107.3	109.1	229.6	276.2
1.30	129.4	130.8	281.5	231.9
1.50	145.7	149.6	327.0	157.9
2.00	155.1	158.1	348.0	73.3
∞	156.3	159.1	350.4	.0

1.05

1.00	56.1	58.8	58.8	73.0
1.05	85.3	91.1	128.3	132.1
1.10	109.7	118.0	188.9	163.9
1.20	144.1	155.9	276.5	186.3
1.30	163.9	177.7	328.3	168.6
1.50	190.9	196.4	373.3	130.1
2.00	198.5	204.9	394.8	68.5
∞	199.4	205.9	397.2	.0

1.10

1.00	104.7	114.9	114.9	69.6
1.05	131.3	147.2	184.9	100.7
1.10	153.5	174.1	245.0	120.6
1.20	194.9	212.0	332.6	135.8
1.30	202.9	233.3	384.4	132.4
1.50	218.3	252.5	429.9	109.7
2.00	225.4	261.0	450.9	63.6
∞	226.2	262.0	453.3	.0

HP-9-4-20/Thornel 75-epoxy

 $\alpha=1.00$

β	p_Y	p_T	p_F	e
1.00	-	-	0.0	419.8
1.05	-	-	59.2	331.2
1.10	-	-	103.3	331.8
1.20	-	-	156.3	236.5
1.30	-	-	177.3	172.5
1.50	-	-	189.5	131.2
2.00	-	-	190.9	42.7
∞	-	-	193.3	.0

1.01

1.00	11.9	12.0	12.0	76.3
1.05	79.3	80.7	70.2	223.8
1.10	128.9	131.4	115.9	242.7
1.20	183.1	186.6	163.3	203.6
1.30	203.9	207.9	189.3	156.9
1.50	214.9	219.0	200.6	97.5
2.00	217.3	221.3	202.9	43.0
∞	217.1	221.3	202.9	.0

1.05

1.00	56.1	58.8	58.8	73.0
1.05	113.4	127.6	117.1	120.2
1.10	164.4	178.3	162.7	141.9
1.20	214.5	233.5	215.2	140.3
1.30	233.7	254.7	236.2	121.8
1.50	243.8	265.8	247.4	86.5
2.00	245.9	268.1	249.7	43.6
∞	245.9	268.2	249.3	.0

1.10

1.00	104.7	114.9	114.9	69.6
1.05	151.5	183.7	173.2	94.4
1.10	203.3	234.4	218.3	107.8
1.20	249.0	290.6	271.3	111.3
1.30	266.5	310.8	292.3	131.0
1.50	275.7	321.9	303.5	77.7
2.00	277.6	324.2	305.8	43.3
∞	277.7	324.3	305.9	.0

6061-T6 Al/Kevlar 49-epoxy

 $\alpha=1.00$

β	P_Y	P_T	P_F	e
1.00	-	-	6.0	575.1
1.05	-	-	74.3	534.9
1.10	-	-	139.7	492.3
1.20	-	-	244.3	410.7
1.30	-	-	315.9	339.1
1.50	-	-	395.3	234.1
2.00	-	-	449.4	111.0
∞	-	-	461.7	.0

1.01

1.00	2.5	2.5	2.5	46.6
1.05	15.4	15.7	75.6	391.3
1.10	26.4	26.9	142.2	413.9
1.20	42.7	43.5	246.5	373.3
1.30	53.3	54.4	313.4	316.9
1.50	64.5	65.7	397.5	223.9
2.00	71.8	73.2	451.9	108.0
∞	73.4	74.9	464.2	.0

1.05

1.00	11.9	12.4	12.4	44.8
1.05	23.8	25.6	86.5	210.9
1.10	33.9	36.8	152.1	257.7
1.20	49.0	53.4	256.4	274.9
1.30	58.9	64.3	328.3	251.6
1.50	69.2	75.6	407.4	190.6
2.00	76.0	83.1	461.8	97.4
∞	77.5	84.8	474.1	.0

1.10

1.00	22.1	24.3	24.3	42.7
1.05	33.0	37.5	98.3	133.5
1.10	42.2	48.6	164.0	179.8
1.20	56.3	65.3	268.3	219.7
1.30	65.0	76.2	340.2	200.6
1.50	74.4	87.5	419.3	160.6
2.00	86.5	95.0	473.7	86.6
∞	81.9	96.6	486.0	.0

(b)

6061-T6 Al/Thornel 400-epoxy

 $\alpha=1.00$

β	P_Y	P_T	P_F	e
1.00	-	-	6.3	492.7
1.05	-	-	70.3	455.2
1.10	-	-	132.1	413.0
1.20	-	-	217.6	329.7
1.30	-	-	269.5	260.3
1.50	-	-	315.0	168.0
2.00	-	-	336.3	74.7
∞	-	-	338.4	.0

1.01

1.00	2.5	2.5	2.5	46.6
1.05	20.7	21.0	72.5	343.2
1.10	35.2	35.9	132.6	352.9
1.20	54.9	55.9	225.2	302.5
1.30	65.7	67.0	272.0	245.0
1.50	74.8	76.3	317.5	161.4
2.00	78.9	80.5	338.5	72.9
∞	79.4	80.9	340.9	.0

1.05

1.00	11.9	12.4	12.4	44.8
1.05	28.5	30.9	82.4	134.3
1.10	42.1	45.8	142.5	228.0
1.20	60.3	65.8	230.1	228.8
1.30	70.3	76.9	281.9	198.7
1.50	78.8	86.2	327.4	136.6
2.00	82.5	90.4	348.4	66.5
∞	83.0	90.8	350.8	.0

1.10

1.00	22.1	24.3	24.3	42.7
1.05	37.4	42.8	94.3	124.9
1.10	49.7	57.6	154.4	162.5
1.20	66.3	77.7	241.9	176.9
1.30	75.4	88.8	293.8	161.3
1.50	83.1	98.1	339.3	119.6
2.00	86.5	102.2	360.3	59.9
∞	86.9	102.7	362.7	.0

6061-T6 Al/Thornel 75-epoxy

 $\alpha=1.00$

β	P_Y	P_T	P_F	e
1.00	-	-	6.1	419.8
1.05	-	-	58.2	331.2
1.10	-	-	103.4	331.8
1.20	-	-	156.3	238.5
1.30	-	-	177.3	172.5
1.50	-	-	188.6	101.2
2.00	-	-	190.9	42.7
∞	-	-	190.9	.0

1.01

1.00	2.5	2.5	2.5	46.6
1.05	40.3	41.0	68.8	239.0
1.10	66.0	67.3	106.4	284.6
1.20	92.3	94.1	158.9	219.7
1.30	101.3	103.9	179.9	163.1
1.50	106.9	109.0	191.1	97.8
2.00	107.9	110.0	193.4	41.9
∞	107.9	110.0	193.5	.0

1.05

1.00	11.9	12.4	12.4	44.8
1.05	46.8	50.9	70.7	138.4
1.10	75.6	77.2	116.3	196.7
1.20	94.9	104.0	168.3	168.7
1.30	103.8	113.8	189.8	134.5
1.50	108.4	118.9	201.0	86.2
2.00	109.3	119.9	203.3	39.1
∞	109.4	119.9	203.4	.0

1.10

1.00	22.1	24.3	24.3	42.7
1.05	54.0	62.8	82.5	135.5
1.10	75.7	89.1	128.1	135.2
1.20	97.3	115.9	180.6	132.6
1.30	105.9	125.7	201.7	111.2
1.50	110.1	130.7	212.9	75.4
2.00	110.9	131.8	215.2	36.0
∞	111.0	131.8	215.3	.0

(c)

Ti-6Al-4V/Kevlar 49-epoxy

 $\alpha=1.00$

β	P_Y	P_T	P_F	e
1.00	-	-	6.3	575.1
1.05	-	-	74.3	534.9
1.10	-	-	139.7	492.8
1.20	-	-	244.3	413.7
1.30	-	-	315.9	339.1
1.50	-	-	395.3	234.1
2.00	-	-	449.4	111.0
∞	-	-	461.7	.0

1.01

1.00	8.1	8.2	8.2	92.4
1.05	34.6	35.2	82.2	357.3
1.10	57.4	58.5	147.9	391.1
1.20	92.4	94.1	252.2	362.9
1.30	115.6	117.8	324.1	311.9
1.50	143.4	143.1	403.2	222.7
2.00	156.9	160.0	457.5	138.4
∞	153.7	163.8	469.3	.0

1.05

1.00	38.4	40.3	40.3	88.9
1.05	62.9	67.4	114.5	188.5
1.10	84.0	90.6	180.1	234.9
1.20	116.4	126.2	284.3	256.4
1.30	137.8	149.9	356.2	240.5
1.50	160.3	175.2	435.4	186.1
2.00	176.1	192.1	489.7	99.6
∞	179.5	195.9	502.4	.0

1.10

1.00	71.8	78.8	78.8	34.7
1.05	94.1	105.8	152.4	139.2
1.10	113.3	129.1	218.5	171.6
1.20	142.8	164.7	322.5	175.7
1.30	152.3	188.4	354.7	191.8
1.50	183.2	213.7	473.5	159.4
2.00	197.2	230.6	528.2	90.6
∞	200.3	234.4	540.5	.0

Ti-6Al-4V/Thornel 400-epoxy

 $\alpha=1.00$

β	P_Y	P_T	P_F	e
1.00	-	-	9.3	492.7
1.05	-	-	70.4	455.2
1.10	-	-	130.1	413.0
1.20	-	-	217.6	329.7
1.30	-	-	269.5	260.3
1.50	-	-	315.0	168.0
2.00	-	-	336.3	74.7
∞	-	-	338.4	.0

1.01

1.00	8.1	8.2	8.2	92.4
1.05	45.4	46.3	78.2	316.1
1.10	76.2	77.6	138.3	337.0
1.20	118.9	121.2	225.9	296.3
1.30	143.2	146.0	277.7	242.6
1.50	163.9	167.1	323.2	161.5
2.00	173.2	176.6	344.2	73.6
∞	174.3	177.7	346.6	.0

1.05

1.00	38.4	40.3	40.3	88.9
1.05	73.0	78.4	110.3	176.9
1.10	101.4	109.7	170.4	212.7
1.20	140.9	153.4	258.3	218.3
1.30	163.4	178.1	309.8	194.2
1.50	182.5	199.2	355.3	140.9
2.00	191.2	208.8	376.3	69.5
∞	192.2	209.8	378.7	.0

1.10

1.00	71.8	78.8	78.8	34.7
1.05	103.2	116.9	148.8	133.3
1.10	129.1	148.2	208.9	159.3
1.20	165.2	191.8	296.4	171.5
1.30	185.6	216.6	348.3	159.6
1.50	203.1	237.7	393.8	123.1
2.00	210.9	247.2	414.8	65.1
∞	211.8	248.3	417.2	.0

Ti-6Al-4V/Thornel 75-epoxy

 $\alpha=1.00$

β	P_Y	P_T	P_F	e
1.00	-	-	6.3	419.8
1.05	-	-	58.2	351.2
1.10	-	-	103.3	331.8
1.20	-	-	150.3	238.5
1.30	-	-	177.3	172.5
1.50	-	-	188.6	111.2
2.00	-	-	192.9	42.7
∞	-	-	192.9	.0

1.01

1.00	8.1	8.2	8.2	92.4
1.05	87.0	88.7	66.5	271.4
1.10	143.8	146.6	112.0	274.5
1.20	204.4	208.4	164.6	217.2
1.30	227.3	231.8	185.6	163.1
1.50	239.3	244.0	196.8	98.9
2.00	241.7	246.5	199.1	42.8
∞	241.9	246.6	199.2	.0

1.05

1.00	38.4	40.3	40.3	88.9
1.05	111.4	120.8	99.5	158.4
1.10	154.0	178.7	144.2	180.4
1.20	220.3	240.5	166.7	167.1
1.30	241.2	263.9	217.7	137.1
1.50	252.3	276.1	228.9	91.3
2.00	254.5	278.6	231.2	43.0
∞	254.6	278.7	231.3	.0

1.10

1.00	71.8	78.8	78.8	34.7
1.05	138.3	159.3	137.0	122.9
1.10	186.1	217.2	182.6	139.5
1.20	237.2	279.0	235.2	136.4
1.30	256.5	302.4	258.2	117.8
1.50	266.6	314.6	267.4	84.0
2.00	268.7	317.1	269.7	42.5
∞	268.7	317.1	269.8	.0

Figure 2. Onset of yield, total yield, and failure pressures as functions of β for HP-9-4-20/Kevlar 49-epoxy cylindrical pressure vessel ($\alpha = 1.05$). The crosses indicate respective asymptotic values ($\beta = \infty$).

Figure 3. Onset of yield (dashed) and failure pressures (solid) as functions of β for all liner-composite combinations ($\alpha = 1.05$). HP-9-4-20, 6061-T6 Al, and Ti-6Al-4V denoted by S, A, and T, respectively. Kevlar 49-epoxy, ThorneI 400-epoxy, and ThorneI 75-epoxy denoted by 49, 400, and 75, respectively.

Figure 4. Pressure vessel efficiency for HP-9-4-20/Kevlar 49-epoxy as a function of β for several values of α .

Interpolation of the data of Tables III a-c enables one to select the most efficient vessel at any failure pressure. Table IV lists failure pressures from 100 to 500 MPa, and indicates which liner-composite combination performs best at each pressure for $\alpha = 1.05$. Kevlar 49-epoxy outperforms the other composites. At the higher failure pressures the higher-yield strength liners perform better than 6061-T6 Al in combination with this composite.

Table IV. Highest pressure vessel efficiencies
for various short-term failure pressures

P_F (MPa)	Best Vessel	e (MPa-cc/g)
100	A-49	218
150	A-49	257
200	A-49	272
250	A-49	277
300	A-49	265
350	T-49	245
400	T-49	215
450	S-49	172
500	S-49	115

SYMBOLS

r, θ, z	cylindrical coordinates
$C_{rr}, C_{\theta r}$	laminate stiffnesses [5]
c	$nC_{rr} + C_{\theta r}$
d	$nC_{rr} - C_{\theta r}$
E	linear elastic modulus in tension
e	pressure vessel efficiency
K_{θ}	maximum circumferential strain
m	mass of pressure vessel
n	$(E_{\theta}/E_r)^{\frac{1}{2}}$, anisotropy
p	internal pressure
p_Y	internal pressure at onset of liner yield
p_T	internal pressure for fully plastic liner
p_F	internal pressure at composite (or vessel) failure
r_1	inner radius of liner
r_2	outer radius of liner (or inner radius of composite)
r_3	outer radius of composite
r_p	radius of elastic-plastic boundary in liner
V	interior volume
V_f	volume fraction of filaments in composite
Y	yield strength in tension
α	r_2/r_1
β	r_3/r_2
λ, μ	Lamé constants
ν	Poisson's ratio
ρ_l	density of liner
ρ_c	density of composite

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