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ENERGY DISTRIBUTIONS
of
ELECTRONS
in
ELECTRON BEAM PRODUCED
NITROGEN PLASMAS

MASTER

by
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THESIS

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ABSTRACT

A theory was developed which predicts the equilibrium electron energy distributions resulting from the injection of an electron beam into molecular nitrogen. The results were highly non-Maxwellian with a depletion region existing near 2.5 eV. Using these distributions, fractional power transfers to various excitation processes were calculated. The theory was verified experimentally by using Langmuir probes to measure the electron energy distributions produced by a beam generated by a cold cathode discharge in low pressure nitrogen. The distributions were measured in absolute units and compared directly with theory. All of the major features of the theory were found to be present in the measurements.

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1. INTRODUCTION

The traditional method of exciting a gas laser system is an electrical discharge, in which power is supplied to the system by an electrical field within the plasma. High energy electron beams, however, are now commonly being used to directly excite high pressure laser systems. Fluxes of high energy particles from nuclear reactions have also been used recently for laser excitation,¹ and in this case, the particles are believed to generate a secondary flux of electrons, which ultimately excites the gas.^{2,3} The mechanisms of electrical discharges are fairly well understood, and electron energy distribution theories have been advanced relating discharge parameters to excitation rates useful in laser systems.^{4,5} The processes of direct electron beam excitation have not been developed, however, and it is the purpose of this study to generate a simple distribution and excitation efficiency theory for electron beam excited molecular nitrogen. Experiments designed to verify the distribution theory were also conducted by using Langmuir probes in a low pressure molecular nitrogen plasma generated by an electron beam.

Electron energy distributions for electrical discharges decrease rapidly in the regions of inelastic excitation, and consequently the calculations need only be performed at low energies, where many inelastic cross sections are well known. In the case of electron beam excitation, the distribution remains at a low but sig-

nificant level up to the beam energy. High energy inelastic cross-sections are therefore needed, and since they were not well known experimentally, semi-empirical cross sections in analytical form were used for the processes of ionization and electronic excitation. The method of calculation and some resulting distributions for molecular nitrogen are given in Section III, and various excitation and ionization rates are also presented. In Section IV, experimental work used to verify the calculations is described, and it essentially involves an electron beam which is injected into low pressure N_2 . Langmuir probes are then used to measure the resulting distributions, in order to ascertain the validity of applying the theory to a practical situation.

II. PRELIMINARY DISCUSSION OF BOLTZMANN PHYSICS

All of the methods used to determine the electron energy distribution function are encompassed by the Boltzmann equation, given in the general form by:

$$0 = \frac{\partial f}{\partial t} + \vec{v} \cdot \nabla_r f + \frac{\vec{E}}{m} \cdot \nabla_v f + \left(\frac{\partial f}{\partial t} \right)_{coll} \quad (1)$$

Here f is the distribution function, v represents velocity, r is the spatial coordinate, E is an external force, and m is the electron mass. The Boltzmann equation describes a continuity of f within the dimensions of time, space and velocity, and the first three terms respectively represent the rate of change of f within these dimensions. The last term is grouped separately since it represents the rate of change of f due to collisions, which are considered discrete and instantaneous, as compared to the other processes which are smoothly analytical.

In order to solve the Boltzmann equation, simplifying assumptions must be made. The plasma is usually considered as infinite, eliminating spatial coordinates, and a steady state condition, such as a dc discharge, is also usually assumed. Two of the velocity components are next eliminated by having an isotropic distribution combined with a spherical harmonic expansion about a

preferred axis. Only the first order expansion term is retained, so that the distribution now consists of an isotropic part plus a first order term oriented along the axis.

With the Boltzmann equation represented as only a variable of velocity, and consequently energy, the various well known solutions can be obtained. If inelastic collisions are neglected, the solution may be either Maxwellian or Druyvesteyn, depending upon the form of the elastic collisional rate. The inelastic collisional term was first derived by Holstein,⁶ and applied to molecular gases by Carleton and Megill,⁷ and Frost and Phelps.⁸ The inelastic techniques were later used by other workers^{4,5} in laser related studies, in which not only distributions, but also excitation and ionization efficiencies were obtained. Although an efficiency calculation is trivial in comparison to determining the distribution, it is of considerable importance since it permits a linkage between abstract theory and useful engineering applications.

In a plasma generated by an electron beam, the only term remaining in the Boltzmann equation is due to collisions. The distribution must also be isotropic in velocity space, since external forces are not present. The remaining collisional term can be interpreted more easily when formulated in terms of a differential continuity equation. This is accomplished through multiplication by the differential velocity volume element, giving the result

$$4\pi v^2 dv \left(\frac{\partial f(v)}{\partial t} \right)_{\text{Coll}} = 4\pi \frac{\partial}{\partial t} \left(v^2 f(v) dv \right)_{\text{Coll}} \quad (2)$$

In this form, the collisional term represents the differential rate of change of the electron density at the velocity v due to the various collisional processes, and has the units of $(\text{cm}^{-3} \text{ sec}^{-1})$. It is also easier and more meaningful to work in terms of electron energy rather than velocity. For an isotropic distribution, the conversion to energy units can be made, with the total normalization given by:

$$4\pi \int_0^{\infty} v^2 f(v) dv = 4\pi \sqrt{2} \left(\frac{e}{m}\right)^{1/2} \int_0^{\infty} u^{1/2} f(u) du = n \quad (3)$$

Here n is the total electron density, $\frac{e}{m}$ is the electron charge to mass ratio, and u is the energy in eV.

Any collisional process can be represented in terms of the cross section $\sigma(u)$, for the given process. The differential collisional rate for the j^{th} process at the energy u is then given by:

$$N_g 8\pi \left(\frac{e}{m}\right)^2 u f(u) \sigma_j(u) du = N_g \phi(u) \sigma_j(u) du \quad (4)$$

Electron neutral collisions are assumed in this formulation, where N_g is the neutral gas density, and $\phi(u)$ is an isotropic electron flux in units of $(\text{cm}^{-2} \text{ eV}^{-1} \text{ sec}^{-1})$, which has been defined mainly for convenience of notation.

For inelastic collisions, the electron loses an energy of Δu_j for the j^{th} inelastic process. Due to the energy loss, these electrons must exit from their energy interval at u , however, electrons are also entering this same interval due to the j^{th} inelastic collisions at $u = \Delta u_j$. These electrons at the energy $(u + \Delta u_j)$ collide at the inelastic excitation rate and enter the interval at the energy u . The inelastic differential collisional term is, therefore:

$$\sum_j [\phi(u + \Delta u_j) \sigma_j(u + \Delta u_j) - \phi(u) \sigma_j(u)] du \equiv dG_{\text{inelastic}}^{(5)}$$

where all the various discrete inelastic processes are summed, and where the plus sign indicates an addition of electrons, and the negative a loss.

If elastic collisions were lossless, the elastic collisional term would be zero. An electron loses the fraction $(2m/M)$ of its energy per elastic collision however, where M is the neutral mass, so that electrons leave the energy interval at u at the differential elastic collisional rate at u , and they enter at the differential elastic collisional rate at u' , where u' is given by:

$$u' = \left(1 - \frac{2m}{M}\right)^{-1} u \approx \left(1 + \frac{2m}{M}\right) u. \quad (6)$$

The differential elastic collisional rate at u' is therefore:

$$N_g \phi(u') \sigma_e(u') du' \\ \approx N_g \left(1 + \frac{2m}{M}\right) \left\{ \phi(u) \sigma_e(u) + \frac{2m}{M} u \frac{d}{du} [\phi(u) \sigma_e(u)] \right\} du, \quad (7)$$

where $\sigma_e(u)$ is the elastic cross section, and (mu/M) is considered small so that the first order expansion is valid. The differential collisional term for elastic collisions at u is therefore the sum of the differential collisional rate at u' minus the rate at u , and is given by

$$N_g \phi(u') \sigma_e(u') du' - N_g \phi(u) \sigma_e(u) du \\ \approx N_g \frac{2m}{M} \left\{ \phi(u) \sigma_e(u) + u \frac{d}{du} [\phi(u) \sigma_e(u)] \right\} du \quad (8) \\ = N_g \frac{2m}{M} \frac{d}{du} [u \phi(u) \sigma_e(u)] du \equiv dG_{\text{elastic}}.$$

In this formulation, it is also assumed that the gas temperature is low, so that neutrals do not impart any appreciable energy to the electrons.

The remaining major electron neutral collision process is ionization, in which the primary electron loses not only the ionization energy, but it also divides the remaining energy between the secondary and the primary. Since the secondary and primary electrons are indistinguishable after the collision, the secondary is arbitrarily

considered to have the least energy. The maximum secondary energy is therefore $(u_p - I)/2$, where I is the ionization potential. The differential ionization cross section relates the differential probability of the secondary having a given energy for a specific primary energy by the relationship

$$dP(u_s, u_p) = \frac{\sigma_i(u_s, u_p) du_s}{\int_0^{(u_p - I)/2} \sigma_i(u_s, u_p) du_s} = \frac{\sigma_i(u_s, u_p) du_s}{\sigma_I(u_p)} \quad (9)$$

Here $dP(u_s, u_p)$ is the differential probability, $\sigma_i(u_s, u_p)$ is the differential ionization cross section in units of $(\text{cm}^2 \text{ eV}^{-1})$, u_p is the incident primary energy, u_s is the secondary energy and $\sigma_I(u)$ is the total ionization cross section. The total ionization cross section is simply the integral of the differential ionization cross section over all possible secondary energies.

The ionization collisional quantity has several contributions, and is represented by:

$$N_g \left[\int_{2u+I}^{\infty} \sigma_i(u, u_p) \phi(u_p) du_p + \int_{u+I}^{2u+I} \sigma_i(u_p - I - u, u_p) \phi(u_p) du_p - \phi(u) \sigma_I(u) \right] du \\ = dG_{\text{ionization}} \quad (10)$$

The last term of this quantity represents electrons leaving the interval at u at the total ionization rate, and it is similar to the corresponding expression for inelastic collisions. The first term enters electrons at u due to secondaries being formed at this energy. With the secondary energy being u , the minimum primary energy must be larger than $(2u + I)$, in order that the secondary energy will be smaller than the primary energy after collision. Contributions from all possible primary energies is then summed by the integral in the first term, with the infinite upper limit representing the maximum extent over which the flux distribution extends. In the case of primary electrons losing energy to ionization and dropping back to the energy interval at u , the energy of the primary after collision is now determined as u . The sum of the primary energy after collision with the ionization potential and secondary energy must equal the incident primary energy, so that the secondary energy is constrained to be $(u_p - I - u)$. The minimum possible primary energy for this case is $(u + I)$, in which the incident primary loses only the ionization potential, and the maximum energy is $(2u + I)$, in which both the secondary and the primary after collision have the same energy u , and these values then determine the integration limits over possible primary energies for the second term. This term is also similar to the corresponding discrete inelastic collisional term, except that the energy loss is variable.

All necessary collisional terms are now available, with the exception of a process linking the electron beam with the plasma. This can be accomplished by assuming the electron beam has a density loss rate as the beam passes through the gas. The differential electron beam term may therefore be represented by:

$$S_1 \delta(u_B) du = dG_{E-Beam} \quad (11)$$

where $\delta(u_B)$ is the impulse function and indicates that the beam electrons are entering at the beam energy, and S_1 is the injection source rate in $(\text{cm}^{-3} \text{ sec}^{-1})$. The injection rate formulation is also equivalent to fixing the value of the distribution at the beam energy as an initial condition, which would then require an external source rate of electrons in order to maintain the distribution at this level. In either case, it is necessary to assume that the injection rate, and consequently the electron beam be spatially isotropic, in order that the initial value of the flux distribution will be isotropic.

All the necessary collisional terms have now been formulated, and to solve for the flux density, the various differential collisional terms are added and set equal to zero, as required by the Boltzmann equation. The resulting equation is finite differenced, and must be solved numerically by dividing the range of the distribution into appropriate cells, and consecutively solving for the distribution at each interval, beginning at the highest energy. The result, under

the assumptions of the presently formulated terms, is dependent only upon beam energy, and has the remarkable property of linearly scaling with respect to the ratio of source rate to neutral density. An unfortunate property of the linear solution however, is the existence of an infinite density at zero energy. This behavior is a result of not including any particle loss mechanisms to balance the injection and ionization rates, so that the electron density becomes infinite at zero energy. Most physical particle loss processes, such as recombination and diffusion depend upon several additional parameters, and would complicate the present simple solution. The calculation of the flux at low energy would also involve processes whereby the electron can gain energy upon collision, such as collisions with energetic neutrals. Unfortunately such processes enter into the Boltzmann equation as second order differentials and require two initial conditions. The first initial condition is the injection rate, or the value of the flux distribution at the beam energy, and the second would necessarily be the total electron density. The solutions would, therefore, be dependent upon the four parameters of electron density, neutral density, source rate and beam energy, and would not be nearly so simple and appealing as the simple result. A worthwhile course of action therefore, is to limit the solution to higher energies, where the simple results apply. This restricts our attention to energies greater than a few tenths of an eV, however as will be shown for N_2 , only the higher energy portion of the distribution is of any consequence in this project.

III. THEORY

A. Basic Formulation

The essential problem is to determine the equilibrium distribution of electrons resulting from the degradation of a high energy electron beam in molecular nitrogen. In order to produce a tractable problem, numerous restrictive assumptions concerning the theoretical model were required. An infinite medium was assumed so that diffusion effects could be neglected, and the plasma was required to be weakly ionized so that electron neutral collisions dominated over plasma coulomb collisions. All of the power necessary to sustain the equilibrium distribution was assumed to come directly from the electron beam, which required the absence of internal electric fields and a low neutral temperature in order that electrons could not gain appreciable energy from elastic collisions with high speed neutrals. A further restriction is that the electron beam must be spatially isotropic and have a discrete beam energy, and it is also assumed that one knows a priori the effective source rate produced by the electron beam, which is the rate of electron beam particles lost per unit volume as the beam passes through the gas.

Instead of the usual electron energy distribution function, calculations were carried out in terms of an isotropic flux $\phi(u)$, with the total normalization such that:

$$n = \int_0^{\infty} \left(\frac{2eu}{m} \right)^{1/2} \phi(u) du \quad (12)$$

Here n is the total electron density, e and m are the electron charge and mass, and u is the energy in eV. The advantages of using the electron flux instead of the traditional distribution function are that electronic charge and mass units are eliminated from the calculations, and the convolution integral of $\phi(u)$ with inelastic cross sections over the appropriate energy range produces the excitation rate for a given process, which is the result ultimately desired in many calculations.

The equilibrium isotropic flux distribution is given by the solution of the dc Boltzmann equation. Since the medium is infinite, and the distribution isotropic and time independent, the only variable is the energy. Under these conditions, the Boltzmann equation represents a balance of the flow of particles in energy at every energy point. That is, within every energy interval du centered at the energy u , the flow of particles into this region by a given process must be balanced by the outward flow of particles by other competing processes in order to maintain the equilibrium flux density at the energy u .

The calculations are initiated by assuming a spatially uniform injection of electrons at the electron beam energy u_B , and degrading these electrons in energy, plus secondary electrons produced through ionization, by the processes of inelastic and elastic collisions. The steady state form of the Boltzmann equation describing this process is given by:

$$\begin{aligned}
0 = & N_g \sum_j \left[\phi(u + \Delta u_j) \sigma_j(u + \Delta u_j) - \phi(u) \sigma_j(u) \right] \\
& + N_g \left(\frac{2m}{M} \right) \frac{d}{du} \left[\sigma_c(u) u \phi(u) \right] + S_i \delta(u_B) \\
& + N_g \left[\int_{2u+I}^{u_p} \phi(u_p) \sigma_i(u, u_p) du_p + \int_{I+u}^{I+2u} \phi(u_p) \sigma_i(u_p - I - u, u_p) du_p - \phi(u) \sigma_I(u) \right].
\end{aligned} \tag{13}$$

Here N_g is the neutral gas density, M is the N_2 molecular mass, $\sigma_c(u)$ is an equivalent momentum transfer cross section which includes the effects of rotational energy loss, $\sigma_j(u)$ represents an inelastic cross section for the process j in which an energy Δu_j is lost, S_i is the source rate of injected primary electrons at the beam energy u_B , $\delta(u_B)$ is the unit impulse function at the energy u_B , $\sigma_i(u_s, u_p)$ is the differential ionization cross section for a process in which an incoming primary electron at energy u_p produces a secondary at energy u_s , $\sigma_I(u)$ is the total ionization cross section, and I is the ionization potential.

The first term is the standard representation for inelastic processes, which is present in most electrical field driven distribution studies.^{4,5} The product $(N_g \phi(u) \sigma_j(u))$ gives the derivative of the excitation rate for the j^{th} inelastic process, and since an energy of Δu_j is lost during such collisions, these electrons leave the energy region centered at u , as indicated by the negative sign. Electrons at energy $(u + \Delta u_j)$ however, lose the excitation energy during an inelastic collision, and drop back into the energy region at u , as indicated by the plus sign.

The second term represents elastic processes, and it can produce either a gain or loss of electrons at u , depending upon the sign of the differentiated quantity. The effects of a finite gas temperature were neglected, and the equivalent momentum transfer cross section, which includes the effects of rotational losses in the continuous approximation, is given by⁵

$$\sigma_c(u) = \sigma_m(u) + \frac{M}{mu} C \quad (14)$$

In this formula, $\sigma_m(u)$ is the ordinary momentum transfer cross section, and C is given by:

$$C = \frac{16\pi B_0 Q^2 a_0^2}{15} \quad (15)$$

where a_0 is the Bohr radius, $B_0 = 2.43 \times 10^{-4}$ eV is the rotational constant for N_2 ,⁹ and $Q = 1.04$ in units of $e a_0^2$ is the quadrupole moment for nitrogen.⁵ The electron flux distribution was solved explicitly for energies greater than 0.3 eV, and it can be shown that the continuous approximation is quite valid in N_2 for energies greater than 0.3 eV.¹⁰

The third term of Eq. (13) is the injection source rate, where S_1 represents the number density rate of electrons entering the

plasma at the beam energy u_B , due to the external electron beam. All of the externally supplied power enters the distribution via this quantity.

The last quantity of the Boltzmann equation represents differential ionization. In a single ionization event, the primary and secondary electrons share various amounts of the original energy in excess of the ionization potential, and since the two electrons are indistinguishable after the collision, the primary is considered by convention to have the greater energy.¹¹ The largest possible secondary energy is therefore, $(u_p - I)/2$, where u_p is the original primary energy. The total ionization cross section is related to the differential form by integration over all possible secondary energies, so that

$$\sigma_I(u) = \int_0^{(u_p - I)/2} \sigma_i(u_s, u_p) du_s \quad (16)$$

The quantity representing ionization contains terms producing both gain and loss rates of electrons as a function of energy. The third term of this quantity is similar to the inelastic collision term, where electrons are leaving the region at u at a rate determined by the total ionization. The first term enters electrons by the process

of secondaries being formed at the energy u . In order to produce a secondary with an energy u , the primary must have had an energy of at least $(2u + I)$, and the integral extends from this point to the maximum extent of the distribution, at the electron beam energy u_B . The second term represents primary electron losing energy and dropping back to the energy u . For this case, the energy of the secondary electron must be such that when added to the energy of the primary after ionization and the ionization potential, the sum must equal the original primary energy. This defines the secondary energy of $(u_p - I - u)$. The limits of integration over the possible primary energies are $(I + u)$, which corresponds to the secondary being produced with zero energy, in which the primary drops back the ionization potential, and the maximum energy is $(I + 2u)$, which corresponds to the possibility of both the secondary and the primary electrons entering the same energy interval at u . Energies greater than the upper limit would not be allowed within the definition of the differential ionization cross section, since the secondary would now have more energy than the primary has after collision. This entire ionization quantity can be shown as identical to the corresponding term of Lo and Miley,² where the integral, rather than the differential form of the Boltzmann equation was solved.

B. Cross Sections

Although N_2 has been extensively studied, good reliable experimental cross sections do not exist at the higher energies needed in this project, which necessitated the use of semi-empirical cross sections. For electronic excitation and ionization, the cross sections of Stolarski, Dulack, Watson and Green¹² were used, which are analytical and of the following form:

$$\sigma(u) = \frac{qA}{X^2} \left(1 - \frac{X}{u}\right)^{\nu} \left(\frac{X}{u}\right)^{-\Omega} \quad (17)$$

Here X is the excitation energy, ν , Ω and A are constants given in Table I, and $q = 6.51 \times 10^{-14} \text{ cm}^2 \text{ eV}^2$.¹³ For the differential ionization cross section, the same form as Eq. (17) is used with the additional factor of $(1/X)^{1.5}$ multiplying the equation, where for this case $X = u_s + I$, which is the sum of the secondary and ionization energies. The u of Eq. (17) then refers to the energy of the incident primary electron.

These cross sections essentially come from the Born approximation coupled with experimental data,¹⁴ as more fully explained in Appendix A. In order to simplify the calculations, the differential ionization cross section was taken to be a sum of the five ionization states given by Reference 12, where the ionization potential of 17.0 eV is an average of the five ionization potentials weighted by the magnitude of the cross sections.

TABLE I
 N_2 parameters for electronic transitions and ionization

State	χ (eV)	A	ν	Ω
$A^3\Sigma_u^+$	6.14	0.609	1.5	4.0
$B^3\Pi_g$	7.30	1.080	1.5	4.0
$a^1\Pi_g$	9.10	0.140	2.0	1.0
$C^3\Pi_g$	11.03	0.541	1.5	3.0
12.85 eV peak	12.85	1.600	3.0	0.75
14.00 eV peak	14.00	1.052	3.0	0.75
Ionization	17.00	0.863	2.0	0.75

For ionization, $p = 1.5$ and $\chi = u_g + I$

The remaining cross sections were experimental. The momentum transfer cross sections are those of Frost and Phelps,⁸ as tabulated in Appendix B, along with the vibrational cross sections. The low energy vibrational cross sections of Phelps¹⁴ were used for energies below 1.5 eV, while the first eight vibrational states of Shulz¹⁵ were used at higher energies, and these were adjusted in magnitude at $2.0 \times 10^{-16} \text{ cm}^2$ the values given, in approximate agreement with later published normalization values.¹⁶ In the region from 4.0 to 6.5 eV, vibrational data was not available, however, it was vitally necessary to have inelastic losses in this region. As a consequence, the $v = 1$ cross section was assumed to decrease linearly from its value at 3.8 eV to a value of $3.0 \times 10^{-18} \text{ cm}^2$ at 6.5 eV.¹⁷ A presumed second vibrational resonance at 5.0 eV, reported in earlier studies,¹⁸ is now believed to be non-existent, and was not used in the present tabulation.

C. Calculational Procedure

The flux densities were calculated in two separate regions, a high energy part above ionization to the electron beam energy, and a part below ionization extending to 0.3 eV. In the high energy region, elastic collisions were neglected, and a numerical energy interval of 5.0 eV, which is smaller than the smallest discrete electronic loss, was used. The equation is solved by beginning at the input energy, and proceeding downward at each energy interval. In the low energy region, the energy interval was much finer, at 0.1 eV, and elastic and vibrational collisions were included, however, the computational procedure was basically the same, starting at the highest energy and proceeding an interval at a time to the lowest energy.

The calculations were launched at the electron beam energy by a delta function, and this would not, in a strict sense, produce an analytical solution. By virtue of the 5.0 eV calculational interval, however, this problem did not arise, since delta functions are now spread over 5.0 eV, and the solution rapidly becomes smooth in only a few intervals from the beam energy.

The only variable is the electron beam energy, since the ratio of (S_1/N_g) merely scales the solutions. It was found, however, that the shape of the low energy part below ionization was nearly invariant to the injection energy, and that the beam energy mainly produced linear scaling, along with the injected source rate and neutral density. A new normalization factor dependent upon the beam

energy was therefore devised which allowed the low energy flux density to become independent of beam energy. The new normalization factor is the total source rate, which includes the injected source rate plus electrons produced by ionization. It is given by:

$$S_T = S_i (1 + u_B / W(u_B)) , \quad (18)$$

where S_T is the total source rate, and $W(u_B)$ is the W-value for ionization, which is defined as the total number of electron - ion pairs a primary electron of energy u_B can produce as it is thermalized. Consequently, electrons are produced at the total source rate, and since the medium is infinite, electrons are also produced below ionization at the total source rate. With the shape of the flux density being rather invariant in this region, S_T merely scales the results. The solutions were, therefore, normalized with respect to (S_T/N_g) .

D. Results and Discussion

Flux densities were calculated and normalized to a neutral density of one particle cm^{-3} and a total source rate of one electron ($\text{cm}^{-3} \text{sec}^{-1}$) for several input electron beam energies, and several high energy flux distributions are shown in Fig. 1. It can be seen that under total source rate normalization, the various curves do become

Fig. 1. High energy flux densities for various primary energies in N_2 .

similar at lower energies. A minimum is attained in the flux densities at about one-half the beam energy. This effect is due to the addition of secondaries to the lower half of the distribution, and to the decrease in cross sections at higher energies. The sharp increase in the flux in this region is mainly a result of including the initial delta function into the averaging. These results are quite similar to those obtained in helium in other studies.^{2,3}

A low energy flux distribution is shown in Fig. 2, where only one example is given since the results are nearly independent of the electron beam energy. The maximum near 4.0 eV is due to the small size of the vibrational cross sections, the minimum near 2.5 eV is also due to vibrational excitations which peak in this region, and the rapid increase at low energies results from the decrease in the vibrational cross sections, and elastic losses become more important in this region. The calculations were not extended below 0.3 eV, and the total source rate was used for normalization.

The flux density calculations cover the entire range of inelastic loss processes, so that the power losses to the various mechanisms can be obtained. Shown in Fig. 3 are the percentage power losses to ionization and electronic triplet and singlet excitations as a function of primary energy. The electronic excitation processes were not subdivided into individual states due to the poor quality of the cross sections, however, it is found that at reasonably high electron beam energies, the excitation rates are nearly constant, with about half the energy producing ionization. For

Fig. 2. Low energy flux density in N_2 for 1000 eV primaries.

Fig. 3. Calculated power loss to ionization, singlet and triplet excitation in N_2 at various primary energies.

electronic excitation, about twice as much power is lost to the singlet states as compared to triplet excitation. The remainder of the power loss produces vibrational excitation and elastic losses, in which the rotational losses are also included. Figure 4 shows the allocation of power to the vibrational states, which remains fairly constant at about 6 percent, and the elastic losses also include the region below 0.3 eV, where the flux density was not explicitly calculated. W-values for ionization are also given in Fig. 4, which is simply the ionization potential divided by the percentage power loss to ionization. These calculated W-values agree fairly well with measured values,¹⁹ and they are also similar to the results of other energy degradation studies.²⁰

Fig. 4. Calculated energy loss parameters for N_2 .

IV. EXPERIMENTAL

A. Purpose

The experimental intent is to verify the calculated distributions by direct measurement using Langmuir probing techniques. The theory, however, is based upon many restrictive assumptions not fully met in a physical situation, and difficulties also exist with the use and interpretation of probing results. Due to these limitations, deviations between theory and experiment can be expected. In order to help compensate for the inaccuracies of probe measurements, the data was obtained in absolute units and compared directly with theory. Two dimensional comparisons between theory and experiment is therefore possible, and ensuing correlations will gauge the validity of applying the theory to a given physical situation.

B. Experimental Apparatus

The experimental apparatus is shown schematically in Fig. 5, and it consists basically of a 4.0 cm diameter discharge tube fitted with a movable planar cold cathode, which is used to produce an electron beam. The ability of a cold cathode operated in the abnormal region to produce a low energy, uniform electron beam is well-known,²¹ and recently such devices have also been used at much higher energies on a pulsed basis.²² The cathode material is stainless steel, and it was designed to fit loosely inside the tube. Attached

Fig. 5. Cold cathode and Langmuir probe assembly.

to the back of the cathode is a plug of soft iron, so that the electrode could be moved externally by means of a magnet, with a loose spring producing electrical contact.

The discharge apparatus also consisted of a single Langmuir probe introduced through a port in the wall of the tube, and the probe was constructed of a tungsten wire with a ceramic sheath used for support. A ceramic sheath was used in order to be able to heat the probe to incandescence during probe cleaning by ion bombardment. The anode consisted of a hollow aluminum cylinder which fit loosely inside the discharge tube, and this design minimized the obstruction of the electron beam and gas flows by the anode structure. The gas was continually flowing, with both the gas supply inlet and outlet located on the anode end of the tube. Commercial grade N_2 gas was used, in conjunction with a dry ice-acetone cold trap used to remove water vapor.

C. Electronic Differentiation Technique

It is well-known that the second derivative of the current with respect to the applied voltage of a Langmuir probe operated in the electron retardation region is proportional to the electron energy distribution, or more precisely,

$$\frac{d^2 I_p}{dV^2} = -2\pi A_p e \left(\frac{e}{m}\right)^2 f(u) \quad u = -eV_p \quad (19)$$

Here I_p is the probe current, V is the probe voltage and A_p is the area of the probe. The derivative is also evaluated at the energy $-eV_p$, where V_p is the potential difference between zero, or plasma potential, and the potential being measured. It is, therefore, necessary to accurately obtain the second derivative of the probe current, which is usually quite small and often accompanied by considerable plasma noise.

Graphical differentiation of probe curves was first used by Druyvesteyn, and later improved upon by Medicus.²³ Unfortunately large errors resulted from this method, and it was replaced by various harmonic methods employing synchronous detection.²⁴ A simpler form of differentiation was used in this study however, in which the probe voltage was swept linearly in time, and the second derivative was obtained using RC differentiating networks. A similar method was used by others,²⁵ for nitrogen positive column measurements, however only a single differentiation could be obtained in those cases due to plasma noise. The present cold cathode discharge proved to be quiet enough to directly obtain both the first and second derivatives, and they could be obtained in a relatively short sweep time of a few milliseconds. The results could, therefore, be displayed in real time, which is the main advantage of this method. For a noisier plasma however, the bandpass of the system would have to be narrowed, resulting in longer sweep times and eventually the signal could no longer be displayed on

video. At this point, the RC differentiation method would probably have no advantages over the harmonic methods.

A block diagram of the circuit appears in Fig. 6. The positive sawtooth from the oscilloscope is conveniently used as a ramp function, automatically producing synchronization between signal and oscilloscope trace. The sawtooth is passed through an emitter follower amplifier circuit in which the output amplitude can be varied, and which has a low output impedance. A bias voltage of approximately the plasma potential is added to the ramp signal in the RC coupler circuit, and the result is applied directly to the probe.

The probe current is added to the anode current and passed through an anode resistor, the value of which must be small enough so that the probe current will create only a negligible shift in the plasma potential with respect to ground. The resistor potential is then differentiated by the first RC differentiator stage and amplified by a PAR 110 amplifier before the second differentiation. The amplifier not only produces gain, but also proper impedance isolation between the differentiator stages. The signal is then applied to a Tektronix 1A7A oscilloscope amplifier, which has a variable bandpass, which is used to eliminate noise components beyond the signal frequency range.

Fig. 6. Electronic circuit for differentiating probe curves.

For a unit amplification factor, the distribution function is given by:

$$f = -V_o / (2\pi A_p e (\frac{e}{m})^2 R_a S^2 (RC)^2), \quad (20)$$

where R_a is the anode resistance, S is the ramp speed in (volts sec⁻¹) applied to the probe, V_o is the output of the final differentiator stage, and RC is the resistance capacitance product of a single differentiator stage, which are assumed here to be identical. In the actual circuit, $(RC)^2$ was measured by applying sinusoidal signals to the first differentiator, and comparing the input to V_o . The basic restriction on the values of the differentiator components needed to produce a true differentiation is that $(RC \omega_m) \ll 1$, where ω_m is the maximum radial frequency contained in the signal being differentiated. Larger values of RC will produce greater signal strengths, however, so that for a given signal to be differentiated, a compromise must be reached on the value of RC .

D. Results and Discussion

The general shape of the measured distribution functions were found to be somewhat invariant over the range of parameters studied. A detailed parameter study of existing variations was not conducted, however, due to the lack of probing reliability. Spatial effects, for instance, were not investigated, and the probing position was arbitrarily fixed at the center of the tube and the cathode position was adjusted such that the probe was situated in the glow region at a distance of 1.0 cm from the cathode dark space boundary. At this position, the electron beam current density was near its maximum value, and the plasma should be more directly produced by the electron beam than at positions further from the cathode, where diffusion and mobility effects become more important.

Various probe materials were tested, with no detectable differences in the probe curve or differentiated probe curves, however, tungsten was selected due to its refractory properties. The effects of probe contamination are well known,²⁶ and the probe was cleaned before each measurement through ion bombardment by connecting the probe to a negative high voltage supply and heating the probe to incandescence. The probe was then allowed to cool in order to avoid shifts in contact potential,²⁷ and it was found that the probe data was fairly reproducible for several minutes, before the probe again became contaminated.

It was assumed that one knows a priori the injection source rate of primary electrons into the plasma. In the present case only the total cathode current is known, and the electron beam current would then be dependent upon the quantitatively uncertain processes of ionization within the cathode dark space and the electron production efficiency at the cathode.²⁸ This difficulty was overcome by measuring the total source rate, which is the production plus injection rate of electrons, and normalizing the distributions with respect to this quantity. The theoretical results are already normalized with respect to the total source rate, so that a direct comparison can be made.

The total source rate was equated to the loss processes of diffusion and recombination, and these loss rates were then indirectly measured. Due to the low gas pressure, N_2^+ was assumed to be the dominant ion, and dissociative recombination and ambipolar diffusion were used for this ion species. For the diffusion term, radial diffusion was assumed to be dominant since the axial range of the electron beam was much greater than the tube radius, and a zero order Bessel function was arbitrarily assigned as the spatial electron density pattern. Under these conditions, the total source rate is given by:

$$S_T = (2.4048/R_0)^2 \mu_+ \left(\frac{kT_+}{e} + \frac{kT_e}{e} \right) n + \alpha_R n^2 \quad (21)$$

Here R_0 is the discharge tube radius, μ is the N_2^+ ion mobility, T_+ is the ion temperature, assumed to be 300°K , T_e is the electron temperature, and α_R is the N_2^+ dissociative recombination coefficient. The value of μ was taken to be $1.7 \text{ cm}^2 \text{ V}^{-1} \text{ sec}^{-1}$ at 0°C and 760 torr,²⁹ and α_R was taken to be $2.7 \times 10^{-7} \text{ cm}^3 \text{ sec}^{-1}$ at 300°K .³⁰ The experimental quantities to be measured for a given distribution are, therefore, the electron density and temperature.

At a constant pressure, the probe characteristics remained essentially invariant and scaled fairly linearly with cathode current. Shown in Fig. 7 are the probe characteristics for 2.0 ma discharges at pressures of 0.05 and 0.2 torr. The probe was at a distance of 1.0 cm from the cathode dark space boundary, and the dimensions of the probe were 0.010 inch diameter and 0.05 inch long. The cathode voltage drop was 1090 volts for the 0.05 torr case and 580 volts for the 0.2 torr curve.

By assuming a Maxwellian dependence for the low energy part of the distribution, the current in the electron retardation region is given by:

$$I = -A_p e \left(\frac{e}{m}\right)^{1/2} n (2\pi)^{-1/2} \left(\frac{kT_e}{e}\right)^{1/2} e^{\frac{eV_p}{kT_e}}, \quad (22)$$

where V_p is negative and is the difference between plasma potential and the measured potential. The probe curves of Fig. 7 are fairly

Fig. 7. Probe curves for 0.05 and 0.2 torr N_2 at 2.0 ma discharge current.

Maxwellian in the retardation region, with a transition to electron acceleration occurring at plasma potential. Plasma potential was determined by the zero crossing of the second derivative curve, in compliance with the prediction of simple probe theory. By using this procedure, it was found that the electron density and (kT/e) temperature for the 0.05 torr case was $1.05 \times 10^{10} \text{ cm}^{-3}$ and 0.08 eV, and $2.13 \times 10^{10} \text{ cm}^{-3}$ and 0.10 eV for the 0.2 torr case. The reliability of probe density measurements was also tested by comparing probe and microwave density measurements in a similar apparatus of smaller diameter, so that the shift in frequency of a small microwave cavity operating at frequencies much higher than the plasma frequency could be used. The results indicated that the probe density measurements were somewhat lower than the cavity measurements, but within a factor of two. Details of this measurement are given in Appendix C.

The low energy part of the probe curves were then differentiated and normalized with respect to the total source rate and neutral density, and are shown in Fig. 8. Under this normalization, the distribution is effectively produced by a total source rate of one electron ($\text{cm}^{-3} \text{ sec}^{-1}$) in a neutral density of one molecule cm^{-3} . Although the calculations were not carried out for energies less than 0.3 eV, the distribution is nearly Maxwellian in this region, which would appear as a decaying exponential. The discrepancy from a Maxwellian at lower energies, which is not extremely noticeable on the undifferentiated probe curve, is probably a result of probing

Fig. 8. Low energy measured distribution function at 0.05 and 0.2 torr in N_2 .

errors. The rounding of the differentiated probe curves for energies on the order of (kT/e) is a well-known effect, and can be attributed to finite probe size, collisional effects and ion currents.³¹ In some studies, the low energy part was adjusted to become more Maxwellian;³² however the low energy portion was left untouched in this work.

In order to provide sufficient signal sensitivity, a larger probe of 0.010 inch diameter and 0.5 inch length was used to obtain the distributions at higher energies. It was found that distortion of the probe curves resulted when the probe current attained an appreciable fraction of the anode current, however, a large probe could be used at higher energies where the probe current was very small. This distortion may be a result of shifts in the anode to plasma potential as the probe alters the anode current. It was also discovered that increasing the total anode surface area decreased the distortion, which is the reason for the large cylindrical probe used in this project.

Shown in Fig. 9 are the measured distributions at energies greater than 0.3 eV for the 0.05 and 0.2 torr discharges at 2.0 ma, which have been normalized with respect to the total source rate and neutral density, and compared to theory. It can be seen that the major features of the measurements and theory agree, such as the minimum at 2.5 eV, the maximum around 4.0 eV, the rapid decrease at electronic excitation energies, and the rapid increase at lower energies. The depletion at 2.5 eV would appear to be more pronounced

Fig. 9. Measured distribution functions at 0.05 and 0.2 torr in N_2 , and compared to theory.

for the 0.2 torr case, which may be due to diffusion; however the actual depth of this depletion region is in doubt, since it was also dependent upon probe diameter. Various size probes from 0.001 inch to 0.030 inch diameter were tested, and it was found that all sections of the distribution except for the depletion region scaled approximately with probe area; however at 2.5 eV, smaller diameter probes gave a shallower minima than larger diameter probes. The decrease in depth was most apparent for very small probes, and this may be due to the inability of simple orbital probe theory to describe the physical situation as the probe radius becomes smaller than the Debeye length. The present diameter of 0.010 inch was chosen since it is large enough to place the probe electron collection in a free fall condition, yet it is still much smaller than the mean free path for collisions. For all probe diameters and all energies, it was found that the measurements scaled linearly with probe length, provided the probe current was small in comparison to the anode current.

V. SUMMARY

An electron energy distribution theory was developed based upon the concept of high energy injected electrons supplying the necessary sustaining power for the distribution, instead of the usual electric field source of power. A remarkable feature of the theory is the insensitivity of the shape of the distributions to the injected primary energy, and as a consequence of this invariance, the calculated inelastic excitation rates are nearly constant for reasonably high primary energies. In general, it was determined that about half of the input power produces ionization, around 30 percent produces singlet excitations, and the remainder goes to triplet, vibrational and elastic losses. The distributions can also be linearly scaled with respect to the injected source rate and neutral density, making the theoretical results universal for any source rate and pressure. By obtaining close experimental agreement in an electron beam discharge using Langmuir probes, the validity of applying the theory to a practical laboratory situation was illustrated. The agreement was also obtained in absolute units, further confirming the correctness of the theory in predicting reality. A most unusual feature of the distributions, shown both theoretically and experimentally, is a deep depletion region at 2.5 eV, which is due to large vibrational cross sections in this region. This depletion makes the present distributions probably the most non-Maxwellian and unusual ever reported.

If appropriate cross sections are available, the present calculations could be extended to electron beam excited gas lasers. Many lasing gases have now been excited by relativistic electron beams, and some have been quite efficiently pumped, such as the 1730 Å xenon laser with 6 percent energy conversion.³³ The measured W-values of most gases indicate that roughly half of the electron beam energy produces ionization, and a large fraction of the remainder would then be available for inelastic excitation. The present type of calculations could be used to determine the excitation rates, although at relativistic energies, the machine calculations would become time consuming. The excitation rates are fairly constant at higher energies, however, so that the results at lower energies could probably be extrapolated to higher energies, provided higher energy processes are not initiated.

APPENDIX A

ELECTRONIC AND IONIZATION CROSS SECTIONS FOR N_2

The electronic and differential ionization cross sections used in this study are semi-empirical and based upon many sources. The high energy shape is derived from the Born Approximation coupled with experimental oscillator strength data, while the low energy portion results from a generalization of experimental electron impact cross sectional data of helium.¹⁴ The excitation energies from the various states are also experimentally based upon spectroscopic data.

The triplet electronic cross sections are shown graphically in Fig. 10, while the singlet cross sections are given in Fig. 11. The triplet states are characteristically sharply peaked at lower energies, and they also decrease more rapidly at higher energies than the singlet states. These cross sections are in reasonable agreement with the experimental results of Brinkmann and Trajmar,³⁴ although the shapes of the triplet states are more complex experimentally than the simple analytic form, and the analytical singlet states are also the sum of many individual cross sections.

For a given primary electron of a given energy, a spectrum of secondary electron energies result, the amplitude of which represents the probability of the secondary electron having this energy after an ionizing collision. The differential ionization cross section is also proportional to this probability, and shown in Fig. 12 is the dif-

Fig. 10. Triplet system electronic cross sections for N_2 .

Fig. 11. Singlet system electronic cross sections for N_2 .

Fig. 12. Secondary electron distribution for 1000 eV primaries compared to the measurement of Opal, Peterson and Beatty.³⁵

ferential ionization cross section for 1000 eV primaries, and a comparison is also made to the experimental measurements of Opal, Beatty and Peterson.³⁵ The excellent agreement indicates the validity of the analytical form, which is composed of the sum of the five ionization states given in Reference 7, all of which have the same shape. The ionization potential of 17.0 eV represents an arithmetic average of the five states, weighted by the amplitude of the states. The integral of the differential ionization cross section over all possible secondary energies gives the total ionization cross section, the magnitude of which was adjusted to fit experimental data.¹²

APPENDIX B

TABULATED CROSS SECTIONS FOR N_2

The following list of momentum transfer and vibrational cross sections represent the explicit values used in the present calculations. The momentum transfer cross sections are those of Frost and Phelps,⁸ and the vibrational cross sections for energies less than 1.5 eV are those of Phelps.¹⁴ At higher energies, the first eight vibrational states of Shulz¹⁵ were used, which were multiplied by a factor of $2.0 \times 10^{-16} \text{ cm}^2$, in approximate compliance with later published normalization values.¹⁶ From 3.8 to 6.5 eV, the $v = 1$ cross section was assumed to decrease linearly to a recently measured value of $3.0 \times 10^{-18} \text{ cm}^2$ at 6.5 eV.¹⁷ The sum of the vibrational cross sections is also shown in Fig. 13, which illustrates the resonant behavior near 2.5 eV.

MOMENTUM TRANSFER CROSS SECTIONS FOR NITROGEN
(cm^2)

ENERGY	CMT	ENERGY	CMT
0.00	.4531E-15	4.00	.1088E-14
0.10	.7086E-15	4.10	.1056E-14
0.20	.8443E-15	4.20	.1032E-14
0.30	.9470E-15	4.30	.1018E-14
0.40	.9820E-15	4.40	.1006E-14
0.50	.9940E-15	4.50	.1005E-14
0.60	.9820E-15	4.60	.1004E-14
0.70	.9702E-15	4.70	.1003E-14
0.80	.9700E-15	4.80	.1002E-14
0.90	.9880E-15	4.90	.1001E-14
1.00	.9940E-15	5.00	.1000E-14
1.10	.1012E-14	5.10	.1001E-14
1.20	.1043E-14	5.20	.1002E-14
1.30	.1082E-14	5.30	.1003E-14
1.40	.1115E-14	5.40	.1003E-14
1.50	.1142E-14	5.50	.1004E-14
1.60	.1266E-14	5.60	.1005E-14
1.70	.1328E-14	5.70	.1006E-14
1.80	.1455E-14	5.80	.1007E-14
1.90	.1651E-14	5.90	.1008E-14
2.00	.2053E-14	6.00	.1009E-14
2.10	.2181E-14	6.10	.1010E-14
2.20	.2234E-14	6.20	.1010E-14
2.30	.2491E-14	6.30	.1011E-14
2.40	.2598E-14	6.40	.1012E-14
2.50	.2598E-14	6.50	.1013E-14
2.60	.2506E-14	6.60	.1014E-14
2.70	.2476E-14	6.70	.1015E-14
2.80	.2181E-14	6.80	.1016E-14
2.90	.1841E-14	6.90	.1017E-14
3.00	.1797E-14	7.00	.1017E-14
3.10	.1682E-14	7.10	.1018E-14
3.20	.1499E-14	7.20	.1019E-14
3.30	.1428E-14	7.30	.1020E-14
3.40	.1361E-14	7.40	.1021E-14
3.50	.1266E-14	7.50	.1022E-14
3.60	.1192E-14	7.60	.1023E-14
3.70	.1142E-14	7.70	.1023E-14
3.80	.1115E-14	7.80	.1024E-14
3.90	.1100E-14	7.90	.1025E-14

MOMENTUM TRANSFER CROSS SECTIONS FOR NITROGEN
(cm^2)

ENERGY	CMT	ENERGY	CMT
7.90	.1025E-14	11.50	.1056E-14
8.00	.1026E-14	11.60	.1057E-14
8.10	.1027E-14	11.70	.1058E-14
8.20	.1028E-14	11.80	.1059E-14
8.30	.1029E-14	11.90	.1060E-14
8.40	.1030E-14	12.00	.1061E-14
8.50	.1030E-14	12.10	.1062E-14
8.60	.1031E-14	12.20	.1063E-14
8.70	.1032E-14	12.30	.1063E-14
8.80	.1033E-14	12.40	.1064E-14
8.90	.1034E-14	12.50	.1065E-14
9.00	.1035E-14	12.60	.1066E-14
9.10	.1036E-14	12.70	.1067E-14
9.20	.1036E-14	12.80	.1068E-14
9.30	.1037E-14	12.90	.1069E-14
9.40	.1038E-14	13.00	.1069E-14
9.50	.1039E-14	13.10	.1070E-14
9.60	.1040E-14	13.20	.1071E-14
9.70	.1041E-14	13.30	.1072E-14
9.80	.1042E-14	13.40	.1073E-14
9.90	.1043E-14	13.50	.1074E-14
10.00	.1043E-14	13.60	.1075E-14
10.10	.1044E-14	13.70	.1076E-14
10.20	.1045E-14	13.80	.1076E-14
10.30	.1046E-14	13.90	.1077E-14
10.40	.1047E-14	14.00	.1078E-14
10.50	.1048E-14	14.10	.1079E-14
10.60	.1049E-14	14.20	.1080E-14
10.70	.1050E-14	14.30	.1081E-14
10.80	.1050E-14	14.40	.1082E-14
10.90	.1051E-14	14.50	.1083E-14
11.00	.1052E-14	14.60	.1083E-14
11.10	.1053E-14	14.70	.1084E-14
11.20	.1054E-14	14.80	.1085E-14
11.30	.1055E-14	14.90	.1086E-14
11.40	.1056E-14		

VIBRATIONAL CROSS SECTIONS FOR NITROGEN

(cm²)

ENERGY	V1	ENERGY	V1
0.00	.0	4.00	.2949E-16
0.10	.0	4.10	.2843E-16
0.20	.0	4.20	.2737E-16
0.30	.1818E-18	4.30	.2631E-16
0.40	.3102E-18	4.40	.2525E-16
0.50	.3743E-18	4.50	.2419E-16
0.60	.4064E-18	4.60	.2313E-16
0.70	.4492E-18	4.70	.2207E-16
0.80	.5027E-18	4.80	.2101E-16
0.90	.6203E-18	4.90	.1995E-16
1.00	.7700E-18	5.00	.1889E-16
1.10	.8984E-18	5.10	.1783E-16
1.20	.1433E-17	5.20	.1677E-16
1.30	.3102E-17	5.30	.1572E-16
1.40	.6417E-17	5.40	.1466E-16
1.50	.8727E-17	5.50	.1360E-16
1.60	.1490E-16	5.60	.1254E-16
1.70	.1616E-16	5.70	.1148E-16
1.80	.3529E-16	5.80	.1042E-16
1.90	.1098E-15	5.90	.9358E-17
2.00	.3059E-15	6.00	.8298E-17
2.10	.1098E-15	6.10	.7239E-17
2.20	.8235E-16	6.20	.6179E-17
2.30	.2753E-15	6.30	.5119E-17
2.40	.2149E-15	6.40	.4060E-17
2.50	.1153E-15	6.50	.3000E-17
2.60	.1984E-15	6.60	.1940E-17
2.70	.5412E-16	6.70	.8807E-18
2.80	.7059E-16	6.80	.0
2.90	.5020E-16	6.90	.0
3.00	.5412E-16	7.00	.0
3.10	.7686E-16	7.10	.0
3.20	.5961E-16	7.20	.0
3.30	.4706E-16	7.30	.0
3.40	.3765E-16	7.40	.0
3.50	.3451E-16	7.50	.0
3.60	.3373E-16	7.60	.0
3.70	.3267E-16	7.70	.0
3.80	.3161E-16	7.80	.0
3.90	.3055E-16	7.90	.0

VIBRATIONAL CROSS SECTIONS FOR NITROGEN
(cm^2)

ENERGY	V2	ENERGY	V3
0.00	.0	0.00	.0
0.10	.0	0.10	.0
0.20	.0	0.20	.0
0.30	.0	0.30	.0
0.40	.0	0.40	.0
0.50	.0	0.50	.0
0.60	.0	0.60	.0
0.70	.0	0.70	.0
0.80	.0	0.80	.0
0.90	.0	0.90	.0
1.00	.0	1.00	.0
1.10	.0	1.10	.0
1.20	.0	1.20	.0
1.30	.0	1.30	.0
1.40	.0	1.40	.0
1.50	.0	1.50	.0
1.60	.0	1.60	.0
1.70	.0	1.70	.0
1.80	.3137E-17	1.80	.0
1.90	.2667E-16	1.90	.0
2.00	.1694E-15	2.00	.7843E-16
2.10	.2165E-15	2.10	.1412E-15
2.20	.9725E-16	2.20	.1725E-15
2.30	.3137E-16	2.30	.1161E-15
2.40	.1867E-15	2.40	.1569E-17
2.50	.1569E-15	2.50	.5961E-16
2.60	.4941E-16	2.60	.9255E-16
2.70	.6745E-16	2.70	.5255E-16
2.80	.8784E-16	2.80	.7059E-17
2.90	.2353E-16	2.90	.4392E-16
3.00	.5334E-16	3.00	.2274E-16
3.10	.4157E-16	3.10	.1255E-16
3.20	.2353E-16	3.20	.3137E-16
3.30	.2118E-16	3.30	.7843E-17
3.40	.1804E-16	3.40	.0
3.50	.1569E-16	3.50	.0
3.60	.9412E-17	3.60	.0
3.70	.6274E-17	3.70	.0
3.80	.0	3.80	.0
3.90	.0	3.90	.0

VIBRATIONAL CROSS SECTIONS FOR NITROGEN
(cm^2)

ENERGY	V4	ENERGY	V5
0.00	.0	0.00	.0
0.10	.0	0.10	.0
0.20	.0	0.20	.0
0.30	.0	0.30	.0
0.40	.0	0.40	.0
0.50	.0	0.50	.0
0.60	.0	0.60	.0
0.70	.0	0.70	.0
0.80	.0	0.80	.0
0.90	.0	0.90	.0
1.00	.0	1.00	.0
1.10	.0	1.10	.0
1.20	.0	1.20	.0
1.30	.0	1.30	.0
1.40	.0	1.40	.0
1.50	.0	1.50	.0
1.60	.0	1.60	.0
1.70	.0	1.70	.0
1.80	.0	1.80	.0
1.90	.0	1.90	.0
2.00	.0	2.00	.0
2.10	.2902E-16	2.10	.0
2.20	.1004E-15	2.20	.4235E-16
2.30	.1490E-15	2.30	.7529E-16
2.40	.8627E-16	2.40	.9647E-16
2.50	.2980E-16	2.50	.8941E-16
2.60	.3922E-17	2.60	.3922E-16
2.70	.7372E-16	2.70	.6274E-17
2.80	.4549E-16	2.80	.2980E-16
2.90	.7059E-17	2.90	.3922E-16
3.00	.2196E-16	3.00	.6274E-17
3.10	.2039E-16	3.10	.9412E-17
3.20	.9412E-17	3.20	.1961E-16
3.30	.0	3.30	.6274E-17
3.40	.0	3.40	.0
3.50	.0	3.50	.0
3.60	.0	3.60	.0
3.70	.0	3.70	.0
3.80	.0	3.80	.0
3.90	.0	3.90	.0

VIBRATIONAL CROSS SECTIONS FOR NITROGEN

 (cm^2)

ENERGY	V6	ENERGY	V7
0.00	.0	0.00	.0
0.10	.0	0.10	.0
0.20	.0	0.20	.0
0.30	.0	0.30	.0
0.40	.0	0.40	.0
0.50	.0	0.50	.0
0.60	.0	0.60	.0
0.70	.0	0.70	.0
0.80	.0	0.80	.0
0.90	.0	0.90	.0
1.00	.0	1.00	.0
1.10	.0	1.10	.0
1.20	.0	1.20	.0
1.30	.0	1.30	.0
1.40	.0	1.40	.0
1.50	.0	1.50	.0
1.60	.0	1.60	.0
1.70	.0	1.70	.0
1.80	.0	1.80	.0
1.90	.0	1.90	.0
2.00	.0	2.00	.0
2.10	.0	2.10	.0
2.20	.0	2.20	.0
2.30	.0	2.30	.0
2.40	.7137E-16	2.40	.4706E-17
2.50	.1035E-15	2.50	.2039E-16
2.60	.1027E-15	2.60	.3843E-16
2.70	.4078E-16	2.70	.6039E-16
2.80	.1647E-16	2.80	.3922E-16
2.90	.1255E-16	2.90	.1098E-16
3.00	.2823E-16	3.00	.3137E-17
3.10	.2412E-16	3.10	.7843E-17
3.20	.0	3.20	.1176E-16
3.30	.0	3.30	.2353E-17
3.40	.0	3.40	.1569E-17
3.50	.0	3.50	.0
3.60	.0	3.60	.0
3.70	.0	3.70	.0
3.80	.0	3.80	.0
3.90	.0	3.90	.0

VIBRATIONAL CROSS SECTIONS FOR NITROGEN
(cm^2)

ENERGY	V8
0.00	.0
0.10	.0
0.20	.0
0.30	.0
0.40	.0
0.50	.0
0.60	.0
0.70	.0
0.80	.0
0.90	.0
1.00	.0
1.10	.0
1.20	.0
1.30	.0
1.40	.0
1.50	.0
1.60	.0
1.70	.0
1.80	.0
1.90	.0
2.00	.0
2.10	.0
2.20	.0
2.30	.0
2.40	.0
2.50	.0
2.60	.2353E-17
2.70	.1725E-16
2.80	.1725E-16
2.90	.1882E-16
3.00	.1412E-16
3.10	.2353E-17
3.20	.1569E-17
3.30	.7843E-17
3.40	.3137E-17
3.50	.0
3.60	.0
3.70	.0
3.80	.0
3.90	.0

Fig. 13. Sum of vibrational cross sections for N_2 .

APPENDIX C

COMPARISON OF PROBING AND MICROWAVE DENSITY MEASUREMENTS

Electron density measurements in a discharge tube are commonly made by inserting the tube through small holes centered in the cavity, and measuring the shift in cavity resonant frequency created by the plasma. If the collisional frequency is much less than the radial cavity frequency, the shift in cavity frequency is given by³⁶

$$\Delta f \approx (4.03 \times 10^7) f_0^{-1} \frac{\int n |E|^2 dv}{\int |E|^2 dV} \quad (23)$$

In this equation, the numerator is integrated over the plasma volume and the denominator includes the total internal cavity volume. The cavity resonant frequency without the plasma is f_0 , E is the cavity field, and n is the electron density in cm^{-3} . This equation results from perturbation theory, and has the restriction that the plasma frequency must be much less than the cavity resonant frequency. To determine the density at the center of the discharge tube, a density pattern is assumed for the plasma, so that the equation may be integrated. The cavity field is usually assumed to be that of the cavity unperturbed by the holes where the tube enters, which is correct within 10 percent for the TM_{010} mode if the ratio of the cavity height to hole radius exceeds four.³⁷

For a variety of reasons, only the lowest order cavity modes can be used for density measurements in most cases, and since the resonant frequency must be much greater than the plasma frequency in order to apply perturbation theory, the maximum diameter of the cavity is predetermined for a given situation. The diameter of a cavity needed to measure the density in the large discharge tube used in this project proved to be too large to meet all the restrictions necessary for cavity density measurements, and consequently, a miniature version of the discharge apparatus was built. The diameter of the small tube was 0.85 cm, a probe of dimensions 0.010 inch diameter and 0.05 inch length was placed 1.0 cm from the cathode dark space boundary, and a cylindrical anode was used. The only difference is that the probe entered the plasma axially through the cylindrical anode, instead of transversely through a port in the wall of the tube. The cathode current was also smaller, since the surface area of the cathode was smaller.

A zero order Bessel function was assumed for the electron density pattern, the cavity had dimensions of 4.8 cm diameter and 1.3 cm height, and was operated in the TM_{010} mode. The electric cavity field is in the Z direction, and can be easily integrated over the plasma. The resulting electron density measurement at the center of the tube was then compared to the probing density obtained by using Eq. (11). The resulting comparisons for various pressures and cathode currents is given in Table II.

The data of Table II indicates that probing density measurements are consistently lower than the microwave measurements, however, for pressures of 0.2 torr, they are within a factor of two. Collisional effects may be responsible for the lower probe measurements, however another possibility is that the spatial electron density is flatter than a Bessel function, which would lower the measured microwave density. If the density were constant, for instance, the measurements would be reduced by a factor of 2.2. At lower pressures, collisional effects will decrease, and the density pattern should become more of a zero order Bessel function as diffusion losses increase in comparison to recombination. It may, therefore, be concluded that the probe and microwave measurements are within a factor of two for the pressure range of 0.05 to 0.2 torr used in this project.

TABLE II

Comparison of probe and microwave electron
density measurements

Pressure (torr)	Cathode Current(μ a)	n_o -microwave (cm^{-3})	n_o -probe (cm^{-3})	$\frac{n_o\text{-microwave}}{n_o\text{-probe}}$
0.2	100	5.55×10^9	3.61×10^9	1.54
0.2	200	1.67×10^{10}	9.50×10^9	1.75
0.2	300	2.22×10^{10}	1.12×10^{10}	1.98
0.3	100	8.89×10^9	4.39×10^9	2.02
0.3	200	2.67×10^{10}	1.32×10^{10}	2.02
0.3	300	4.22×10^{10}	1.86×10^{10}	2.26

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