112-26
LA-6291-MS

Informal Report

UC-28

Reporting Date: March

Issued: April 1976

Thermal Stress Analysis of Tungsten Collimators Heated by High-Energy Proton Beams

by

Robert W. Meier
Kenneth H. Duerre

MASTER

An Affirmative Action/Equal Opportunity Employer

This work was supported by the US Energy Research and Development Administration, Division of Physical Research.

Printed in the United States of America. Available from
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road
Springfield, VA 22151
Price: Printed Copy \$3.50 Microfiche \$2.25

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights.

THERMAL STRESS ANALYSIS OF TUNGSTEN COLLIMATORS HEATED BY HIGH-ENERGY PROTON BEAMS

by

Robert W. Meier and Kenneth H. Duerre

ABSTRACT

An analytical and numerical investigation of the effect of a large quantity of high-energy protons on a tungsten alloy collimator is undertaken herein. Three different modes of failure are analyzed: thermal stress, thermal shock, and temperature in excess of the maximum allowable.

Three analyses are developed to study the first beam pulse at 1-mA beam current. A quasi-steady-state analysis is applied to study the effect of multiple beam pulses at 0.1-mA beam current.

The results for a 1-mA beam indicate that the yield stress is exceeded after 200 μ s, but the ultimate stress is not reached during the first beam pulse. The analytical solution is found to give good results in the elastic range. The numerical solution for 0.1-mA beam current shows that the yield stress is reached after 0.23 s and the ultimate stress is exceeded after 0.48 s.

1. INTRODUCTION

This study was undertaken to investigate the interaction of high-energy proton beams with collimators. The Clinton P. Anderson Meson Physics Facility is a proton linear accelerator which is capable of accelerating protons to energies up to 800 MeV at an average current of 1 mA.

Ideally, the beam moves directly down the center of the annular-shaped collimators and strikes the target. At times, however, the beam may stray off target and deposit its energy into a collimator. The energy deposited into a collimator when a large quantity of protons strike the collimator causes thermal stresses to develop. The purpose of this study was to investigate the effect of deposition of highly concentrated beam energy into a collimator.

Several possible failure modes exist. One possible mode of failure is that the maximum allowable temperature of the material will be exceeded. Another possibility is that localized heating of

the collimator will establish thermal stresses which exceed the yield or ultimate stress of the material. A third possible mode of failure is that shock waves might be generated in the material by the pulsating nature of the beam energy deposition.

Attention was focused on the first collimator in the beam line which is designated collimator A-1. The collimator is machined from a tungsten alloy composed of 95% W, 3.5% Ni, and 1.5% Fe. The geometry of the collimator is shown in Fig. 1.

Fig. 1. Collimator A-1.

The beam current, and therefore the power deposited in the collimator by the beam, is a periodic step function, as shown in Fig. 2. The frequency of the cycle is 120 Hz, with a positive beam current for 500 μ s and a zero current during the remainder of the cycle.

The spatial distribution of beam current is a two-dimensional Gaussian normal to the beam. The current density is described by

$$J = \frac{1}{2\pi\sigma_x\sigma_y} e^{-\frac{1}{2}\left[\left(\frac{x}{\sigma_x}\right)^2 + \left(\frac{y}{\sigma_y}\right)^2\right]} \quad (1)$$

The worst beam condition was expected to have an average current of 1 mA and an aspect ratio of 10, for which the standard deviations of the current distribution were $\sigma_x = 0.43$ mm, $\sigma_y = 4.3$ mm.

The power deposited by the beam in the collimator is described by

$$\dot{q} = \frac{dW}{dz} J \quad (2)$$

In tungsten, $\frac{dW}{dz}$ is approximately $11.5 \frac{\text{GeV}}{\text{g}\cdot\text{cm}^2}$ for 600-MeV protons.

As the proton beam enters the collimator the power deposited per unit depth increases from its value at the surface, reaches a maximum, and decreases more gradually. In the following analyses the power deposited was assumed to be independent of depth so that the thermal and stress solutions would be two-dimensional. For collimators thicker than the one considered here this would be a poor

assumption and the third dimension should be considered.

11. ANALYTICAL SOLUTION

An analytical solution was obtained for the temperature and resultant stress field for the case where the beam strikes near the center of the material rather than on the inner or outer rims of the collimator. Under the assumption of zero thermal conductivity, all of the energy deposited in the collimator by the beam remains within the beam area and is given by

$$Q = \dot{q} V = \rho V C_p \Delta T \quad (3)$$

Assuming constant properties and that the energy is deposited uniformly in the axial direction, the temperature change is given as a function of time by

$$\Delta T = \dot{q}/C_p \quad (4)$$

If a thin circular disc at uniform temperature undergoes a sudden temperature change ΔT throughout a comparatively small central portion of elliptical shape, the maximum stress is the tangential stress at the ends of the ellipse,¹

$$S_t = \alpha E \Delta T / (1-b/a) \quad (5)$$

If the heated portion is circular, the radial and compressive stresses within the heated portion are given by

$$S_r = S_t = \frac{1}{2} \alpha E \Delta T \quad (6)$$

At points outside the heated area and distance r from the center of the disc, the compressive and tensile stresses are

$$S_r = S_t = \frac{1}{2} \alpha E \Delta T \left(\frac{a}{r}\right)^2 \quad (7)$$

At the edge of the heated portion, the shear stress is given by

Fig. 2. Beam current as a function of time.

$$S_{MS} = \frac{1}{2} \alpha E \Delta T \quad (3)$$

The state of stress in the material is indicated in Fig. 3. The stress decreases rapidly with distance from the heated area. At a distance of one radius away from the heated area, the stress is already reduced to one-ninth its maximum.

Substitution of Eq. (4) into Eq. (5) yields a relation for the maximum tangential stress at the ends of an elliptically shaped beam as a function of time.

$$S_t = \alpha E \dot{q} t / C_p (1 + \frac{b}{a}) \text{ for } S \leq S_y \quad (9)$$

Equation (9) applies until the yield stress is reached. In the plastic range, corresponding strains produce relatively lower stresses. In this range, the stress was approximated by use of the plastic modulus, which produced the expression

$$S_t = S_y + \frac{\alpha E \dot{q} t (t - t_y)}{C_p (1 + \frac{b}{a})} \text{ for } S_y < S_t \leq S_u \quad (10)$$

Data that were supplied indicated an effective beam size of 1 mm x 10 mm for the beam of aspect ratio 10. Based on this assumption, for 1-mA average beam current, one obtains from Eqs. (1) and (2) a beam power deposition of 225 kW/g (Ref. 2).

The tangential stress as a function of time was determined from Eqs. (9) and (10). Thermal and mechanical properties were assumed constant, equal to their values at 629.8 K. Thermal and mechanical properties are shown in the Appendix. The results are plotted in Fig. 4 for a beam of aspect ratio 10.

Fig. 3. Radial and tangential stresses inside and in the vicinity of a small circular area which is suddenly heated.

Fig. 4. Analytical solution for maximum tangential stress for an elliptical beam of aspect ratio 10.

The yield and ultimate tensile stresses were superimposed on the compressive stress results by calculating the temperature within the beam area as a function of time from Eq. (4). The yield and tensile stresses at a given time were then determined from the temperature-dependent properties data in the Appendix, assuming an initial temperature of 310.9 K.

The analytical results indicate that yielding of the collimator will occur after 165 μs and the ultimate tensile stress will be reached after 445 μs.

III. NUMERICAL SOLUTION

Numerical solutions to the collimator heating problem were obtained by use of the thermal and stress analysis code TSAAS.³ TSAAS is a finite element code capable of solving transient problems under axisymmetric thermal and mechanical loads.

A. Mesh

After experimenting with several different nodal point meshes, a mesh was chosen which achieved a fine enough discretization to obtain significant results

in the vicinity of the beam and still stay within the maximum number of nodal points allowed by the code, which is 1600. If the beam contacts the collimator in the vertical or horizontal directions, the problem is symmetrical and only half of the collimator need be considered. Also, since conduction was anticipated to be of relatively minor importance for time scales of the order of several beam pulses, it was anticipated that the stresses would be concentrated in the vicinity of the beam. Therefore, a 30° annular segment, as shown in Fig. 5, was chosen for analysis.

The point of contact of the beam with the collimator was along boundary No. 4. A fine mesh was chosen in this area in order to obtain a high resolution of stresses near the beam. Boundary No. 2 was chosen to be far enough away from the beam that conditions along the boundary were not affected by the beam during several beam pulses. Boundaries Nos. 1 and 3 correspond to the inner and outer rims of the collimator. Boundary No. 5 was introduced in order to achieve a fine mesh in the vicinity of the beam. The nodal point spacing along boundaries Nos. 2, 4, and 5 were equally spaced, while the

Fig. 5. Material boundaries and nodal point mesh.

nodal points along boundaries Nos. 1 and 3 were allowed to be unequally spaced. The boundaries in i, j nodal point space are shown in Fig. 6.

B. Initial and Boundary Conditions

The temperature of the collimator before being contacted by the beam was assumed a uniform 310.9 K for purposes of determination of properties. Boundary No. 4 was treated as a zero heat flux boundary because of the symmetry condition. Boundary No. 2 was positioned far enough from the beam that it would not affect the solution and was also specified as having zero heat flux. The collimator is surrounded by a stainless steel cooling jacket; therefore, boundary No. 3 was specified to be a constant temperature of 310.9 K.

Boundary No. 1 is actually described by a radiative condition because a vacuum exists inside the collimator. TSAAS, however, is not capable of handling a radiative boundary condition. For the analysis of one pulse, the actual boundary condition specified was not expected to affect the results to any great extent, since the heat transferred from the surface would not be very great. For longer times, however, the heat radiated from the boundary would be greater, although it was expected to be less than that conducted away through the collimator. The condition at boundary No. 1 was specified as zero heat flux because this is a much better

Fig. 6. Material boundaries in i, j nodal point space.

approximation to the radiative condition than is a condition of constant temperature.

The collimator was free to expand in the axial direction, so plane stress conditions were assumed. Stress and displacement along the collimator boundaries were unspecified other than zero displacement along boundary No. 2. The boundary conditions are shown in Fig. 7.

C. Analysis of First Pulse at 1-mA Beam Current

The primary problem of interest was to determine whether or not the collimator would fail during the first beam pulse at 1-mA beam current. If the collimator did not fail during the first pulse, then the temperature and stresses would decrease until the next pulse arrived. Thus 8 ms of time would be assured in which action could be taken to prevent damage to the collimator. Therefore, attention was focused on the state of the collimator during the first pulse.

1. Uniform Deposition Beam. TSAAS is not capable of treating a continuously variable rate of internal heat generation. Therefore, the Gaussian distribution of heat deposition was first approximated by a nearly rectangular area of constant heat

generation. The postulated worst case that was analyzed in Section II was modeled. That is, a beam area measuring 1 mm x 10 mm with a uniform rate of heat deposition of 225 kW/g was programmed into TSAAS, using the nodal point mesh and boundary conditions previously described.

Initial calculations were directed toward determining the beam location on the collimator which produced the earliest failure. A beam located adjacent to the outer rim resulted in the lowest temperatures and stresses because the constant temperature boundary condition on the outer rim resulted in high rates of heat removal from the collimator. When the beam was located adjacent to the inner rim, the stresses were slightly higher than when the beam was in the center of the material. Therefore, in the remainder of the numerical solutions the beam was located adjacent to the inner rim. The location and orientation of the beam is shown in Fig. 8.

Since boundary No. 4 is a line of symmetry, the model beam width in the tangential direction is one-half of that being modeled. In order to conform to the nodal point mesh, it was necessary to specify the beam to be 7 elements wide x 10 elements high

Fig. 7. Boundary conditions applied to collimator segment.

Fig. 8. Location and orientation of uniform deposition beam.

when the beam was adjacent to the inner rim. This gave a model beam which was approximately rectangular in shape, 0.5263 mm x 9.921 mm. This configuration modeled a beam of aspect ratio 9.425 with an area of 10.44 mm².

The effect of beam aspect ratio on the stresses developed within the beam area was investigated. As indicated by the analytical results, the stresses were found to be slightly higher for higher aspect ratios. So that the worst probable case could be considered, the highest expected aspect ratio of 10 was used in subsequent studies.

The effect of plastic-to-elastic modulus ratio on the stresses was investigated by considering ratios of 0.005 and 0.013. The stresses differed by less than 1%; therefore, a plastic-to-elastic modulus ratio of 0.005 was used in subsequent calculations.

The effect of the thermal boundary condition on the inner rim was investigated by running cases for which the inner rim was insulated, or alternatively constant temperature. As expected, the temperatures and stresses were little changed because of the short time involved in the analysis.

Results obtained from this model are shown in Figs 9 through 11. Tensile, compressive, and shear stresses are plotted as a function of distance from the vertical line of symmetry, going out through the beam area after 500 μ s of heating. The results agree qualitatively with the analytical results. The tensile stress attains a maximum at the edge of the beam and decreases approximately as $\frac{1}{r^2}$ outside of the beam area. The compressive stress also reaches a maximum at the edge of the beam and is higher than the tensile stress inside the beam area.

Fig. 9. Maximum normal stress as a function of distance.

Fig. 10. Minimum normal stress as a function of distance.

Fig. 11. Maximum shear stress as a function of distance.

The shear stress also attains a maximum at the beam edge.

The maximum temperature in the beam area as a function of time is shown in Fig. 12. The analytical solution is also shown for comparison. It can be seen that the temperature-time relationship departs very little from linearity and is very similar to the results obtained from the analytical solution. Part of the nonlinearity is due to the variable properties which were incorporated into the code, but were not accounted for in the analytical solution. The results show that very little heat is conducted away from the beam area during the first pulse.

The maximum compressive stress within the beam area is plotted as a function of time in Fig. 13. The compressive stress was chosen for analysis because it was higher than the maximum tensile stress, as indicated by the analytical solution and Figs. 9 and 10. In addition, the yield stress in compression is the same as that in tension for this tungsten alloy.

Fig. 12. Maximum temperature in beam area for uniform deposition beam at 1-MA beam current.

Superimposed on the stress curve are plots of the yield stress and ultimate tensile stress of the collimator. The yield and ultimate stress curves were obtained by plotting the stress corresponding to the maximum temperature at each time increment. Temperatures were obtained from Fig. 12, and yield and tensile stresses were obtained from Fig. A-1 in the Appendix.

This manner of interpreting the data is conservative in that the maximum compressive stress does not correspond to the point of maximum temperature. To be precisely accurate, the compressive, yield, and ultimate stresses at a variety of points within the beam would have to be considered. In this manner, the point where the failure first occurs could be determined. This procedure would result in a somewhat longer time to failure. The worst condition would generally correspond to the point of maximum temperature because the yield and ultimate stresses decrease significantly with increasing temperature (see Fig. A-1 in the Appendix), while the compressive

Fig. 13. Maximum compressive stress in beam area for uniform deposition beam at 1-MA average beam current.

stress decreases relatively little in going from the point of maximum stress to that of maximum temperature inside the beam area.

Figure 13 shows that the yield stress is reached after 185 μs. After this point, the stress increases slightly and then relaxes significantly. This is due largely to the fact that the yield stress decreases rapidly with increasing temperature and the stress resulting from similar strains is correspondingly reduced. The ultimate strength of the material is just reached after 500 μs.

2. Composite Beam. Analyses were made with a biareal beam to achieve a better approximation to the Gaussian distribution of heat deposition. The beam was divided into an inner section with a uniform rate of heat generation equal to the peak of the Gaussian power deposition and an outer section in which the rate of heat generation was also uniform and one-half of that in the inner section. Thus, the same power would be deposited in the collimator, but the deposition would occur over a larger area. With this configuration, it was expected that the outer section would allow greater expansion of the

material surrounding the inner section and would thus decrease the stresses in the beam area.

Substitution of Eq. (1) into Eq. (2) yields an expression for the power density of the beam,

$$\bar{q} = \frac{\frac{dW}{dz} I}{2\pi \sigma_x \sigma_y} e^{-\frac{1}{2} \left[\left(\frac{x'}{\sigma_x} \right)^2 + \left(\frac{y'}{\sigma_y} \right)^2 \right]} \quad (11)$$

In tungsten, $\frac{dW}{dz}$ is approximately $13.5 \frac{\text{GeV}}{\text{g} \cdot \text{cm}^2}$ for 600-MeV protons. For the beam with aspect ratio 10, $\sigma_x = 0.43 \text{ mm}$ and $\sigma_y = 4.3 \text{ mm}$. Thus, for an average current of 1 mA, Eq. (11) yields a Gaussian power deposition distribution where the peak power deposition is

$$\bar{q}_0 = \frac{\frac{dW}{dz} I}{2\pi \sigma_x \sigma_y} \quad (12)$$

or $193.7 \frac{\text{kW}}{\text{g}}$.

The total power deposited in the collimator is proportional to $\bar{Q}$, where

$$\bar{Q} = \iint_{-\infty}^{\infty} \bar{q} \, dx' \, dy' \quad (13)$$

Substitution of Eq. (11) into Eq. (13) and evaluation of the integral gives the result

$$\bar{Q} = \frac{dW}{dz} I \quad (14)$$

For the case being considered, the rate of energy deposition is directly proportional to $2.25 \frac{\text{MW} \cdot \text{cm}^2}{\text{g}}$. Elimination of $\frac{dW}{dz}$ from Eqs. (12) and (14) yields

$$\bar{Q} = 2\pi \sigma_x \sigma_y \bar{q}_0 \quad (15)$$

Equating the sum of the heat generation in the two areas to that generated by the Gaussian beam,

$$A_1 \bar{q}_0 + A_2 \frac{\bar{q}_0}{2} = 2\pi \sigma_x \sigma_y \bar{q}_0 \quad (16)$$

The areas of the two sections are given by

$$A_1 = l_{x1} l_{y1} \quad (17)$$

$$A_2 = l_{x2} l_{y2} - l_{x1} l_{y1} \quad (18)$$

For a beam of aspect ratio 10 and an outer section having dimensions twice that of the inner section

$$l_{y1} = 10 l_{x1}$$

$$l_{y2} = 10 l_{x2}$$

(19)

$$l_{y2} = 2 l_{y1}$$

$$l_{x2} = 2 l_{x1}$$

Substitution of Eqs. (19) into Eqs. (17) and (18), which are then substituted into Eq. (16), yields

$$l_{x1} = \frac{\sqrt{2\pi \sigma_x \sigma_y}}{5} \quad (20)$$

From Eqs. (19) and (20) the desired dimensions of the areas were found to be

$$l_{x1} = 0.6817 \text{ mm}$$

$$l_{y1} = 6.817 \text{ mm}$$

(21)

$$l_{x2} = 1.363 \text{ mm}$$

$$l_{y2} = 13.63 \text{ mm}$$

The power deposition and the resulting approximation to it for the y-direction are shown in Fig. 14. Since boundary No. 4 is a line of symmetry, the ideal model beam width in the tangential direction is one-half that indicated for l_{x1} and l_{x2} in Eqs. (21).

Fig. 14. Distribution of beam power deposition in y -direction and two-sectioned approximation.

To conform to the nodal point mesh used in the code, the inner area was 4 elements wide $\times$ 7 elements high and the outer area was 9 elements wide $\times$ 14 elements high. This produced approximately rectangular areas with dimensions of 0.3109 mm $\times$ 6.947 mm for the inner section and 0.7076 mm $\times$ 13.89 mm for the outer section, and corresponding aspect ratios of 11.17 and 9.815. Thus, the power deposited in the composite model was $2.322 \frac{\text{MW} \cdot \text{mm}^2}{\text{g}}$ as compared to $2.350 \frac{\text{MW} \cdot \text{mm}^2}{\text{g}}$ in the uniform deposition model. The orientation and location of the composite beam is shown in Fig. 15.

The maximum temperatures attained in each of the beam sections are shown as functions of time in Fig. 16. The temperature increase in both sections is very nearly linear, but the slope of both of the curves is less than that for the uniform deposition model. This is caused by the fact that a lower rate of energy deposition was applied over a larger area in the composite model.

The maximum compressive stress within the inner section of the beam is plotted as a function of time in Fig. 17. Also shown are the yield and ultimate stresses of the collimator corresponding to the maximum temperature attained within the inner beam section at each time. The yield stress is reached after 200 μs at 1-mA beam current. This is a slightly longer time to yield than the uniform deposition analysis produced. The stress continues to increase after yielding until it reaches a maximum at 250 μs where it falls off rapidly, never approaching the

Fig. 15. Geometry and location of composite beam.

Fig. 16. Maximum temperature in beam area of composite beam.

Fig. 17. Maximum compressive stress in inner section of composite beam.

ultimate stress during the first beam pulse. The more rapid decrease of stress after reaching the yield stress as compared to the uniform deposition model can be attributed to increased expansion of the material surrounding the inner section.

The maximum compressive stress within the outer section of the beam is shown in Fig. 18, along with the yield and ultimate stresses corresponding to the maximum temperature attained within the outer section at each time. It can be seen that the stress increases more slowly than in the inner section and reaches the yield stress after 295 μ s. After this time, the stress continues to increase to a level slightly higher than that reached in the inner section. However, the temperature is lower in the outer section. Therefore, the ultimate stress remains much higher than the compression stress.

D. Analysis of 0.1 mA Pulses

Another problem of interest is the condition of the collimator under lower levels of beam current, 0.1 mA, for times up to 1 s. Under these

Fig. 18. Maximum compressive stress in outer section of composite beam.

conditions the stresses developed in the collimator at a given time would be expected to be much lower because the power deposition is one-tenth of that for the previous case. In addition, since the collimator would be expected to survive far beyond one beam pulse, conduction of thermal energy away from the beam would become significant. Conduction would thus reduce the stresses in the vicinity of the beam.

Since the length of one pulse would be an insignificant time interval compared to the expected time to failure, a quasi-steady-state assumption was made. In this treatment the beam was assumed to be continuous rather than of a pulsatile nature. The power deposition for the continuous beam was taken to be 6% of that from the pulsatile beam so that the same amount of energy would be deposited over the pulse period.

The calculations were based on data which indicated a peak instantaneous power deposition of $225 \frac{\text{kW}}{\text{g}}$ over an effective area of 10 mm^2 at 1-mA

average beam current.² Thus, the power deposition for the 0.1-mA beam was $1.35 \frac{\text{kW}}{\text{g}}$.

The beam was located adjacent to the inner rim and had uniform power deposition. However, in this case the beam had an aspect ratio of 2. For an aspect ratio of 2,

$$\ell_y = 2 \ell_x \quad (22)$$

and

$$A = 2 \ell_x^2. \quad (23)$$

From Eqs. (22) and (23) one obtains

$$\ell_x = 2.236 \text{ mm}, \quad (24)$$

$$\ell_y = 4.472 \text{ mm}.$$

Since boundary No. 4 is a line of symmetry, the ideal model beam width in the tangential direction is one-half of that indicated in Eq. (24). In order to conform to the nodal point mesh, the beam was 8 elements wide x 5 elements high. This produced a model beam approximately rectangular in shape, 1.133 mm x 4.950 mm, which modeled a beam of aspect ratio 2.184.

The maximum temperature within the beam area as a function of time is shown in Fig. 19. The collimator reaches a much higher temperature than in the previous case because of the longer heating time. The temperature reaches 1.366 kK, which is the maximum serviceable temperature of the tungsten alloy, after 0.57 s.

The maximum compressive stress developed inside the beam area is shown in Fig. 20 and the stresses developed outside of the beam are shown in Fig. 21. The stress outside the beam area reaches a higher value, but, due to the lower temperature, the yield and ultimate stresses are exceeded at slightly later times than inside the beam area. The yield stress is exceeded after 0.23 s and the ultimate stress is exceeded after 0.48 s.

Fig. 19. Maximum temperature in beam area for 0.1 average beam current.

Fig. 20. Maximum compressive stress inside beam at 0.1 mA average beam current.

Fig. 21. Maximum compressive stress outside beam at 0.1 mA average beam current.

IV. THERMAL SHOCK ESTIMATES

Pulse heating can cause the temperature of the material to rise faster than its ability to respond by thermal expansion, giving rise to dynamic stresses. This subject has been investigated for pulse neutron heating of reactor fuel elements^{4,5} and aerodynamic heating.⁶ Generally, it was found that theoretical calculations overestimate deflection and stress compared with experiments in both elastic and elastic-plastic formulation, but the calculated vs observed vibrational frequency was reasonably close.

Considering the collimator first as a cylindrical shell, the period of natural vibration is found to be

$$T_p = \frac{R_m}{\sqrt{E/\rho}} = 1.45 \times 10^{-5} \text{ s} . \quad (25)$$

Burgreen and others found that for ratios of heating time to natural period greater than 3, the expansion follows the temperature rise and there is no dynamic effect. For the collimator, the ratio is $500 \times 10^{-6} / 1.45 \times 10^{-5}$, or $T_h/T_p = 34.5$.

As a check, the wave propagation time is $(R_o - R_i)/C = \tau$, and $\tau = 1.36 \times 10^{-5} \text{ s}$. The ratio of heating time to wave propagation time is 36.8.

For longitudinal waves, the expression for natural period becomes $T_p = 2L/C = 1.75 \times 10^{-5} \text{ s}$. The ratio in this case is 28.6.

Considering the collimator to be a plate of length $2\pi R_m$ and width ΔR , Boley finds for a step temperature input on one face that the dynamic effects are negligible for values of the nondimensional parameter B_1 greater than 3.5.

$$B_1 = h \left(\frac{D}{h\rho} \right)^{1/4} / \Delta R \sqrt{K} \quad (26)$$

where

$$D = \frac{Eh^3}{12(1-\nu)} .$$

B_1 for the collimator is 51.2 compared to 3.5 which gives less than 10% error in deflection.

Based on the above values, it is felt that shock amplification of stresses is improbable and that quasi-static solutions such as obtained from TSAAS are adequate.

CONCLUSIONS

The numerical solutions obtained with the composite beam model indicate that the yield stress of the tungsten alloy collimator will be reached after 200 μs at 1-mA average beam current. The ultimate stress, however, will not be reached during one beam pulse. Thus, if permanent deformation of the collimator can be allowed, the results indicate that the collimator will survive one beam pulse.

The numerical results achieved with the uniform deposition beam model indicate the same qualitative results as the composite beam. Yielding occurs slightly earlier at 185 μs and the stress in the plastic range does not decrease quite as much, so that the ultimate stress is just reached at the end of the first pulse.

The biareal model, however, is a better approximation to the beam power deposition. The uniform deposition model deposits a relatively high amount of power over a relatively small area. This

produces higher temperatures and somewhat higher stress. The higher temperatures lead to reduced yield and ultimate stresses, and, therefore, an earlier time of failure.

The analytical solution yields a good approximation to the temperature and stress that develops during the first beam pulse. Agreement with the numerical results of the uniform deposition model is very good in the elastic range. Relaxation of stress in the plastic range is not predicted by the analytical solution, however. Also, the assumption of zero thermal conductivity produces an overestimate of the temperature by 328 K at the end of the pulse. The temperature overestimate leads to a significantly lower ultimate stress at the end of the pulse.

The temperature and stress predicted by the analytical treatment are directly proportional to beam power deposition density. If data for an approximation to the Gaussian beam power deposition derived in a manner similar to that used in the composite beam analysis were used in the analytical solution, such as a lower power deposition density over a larger area, the results would be expected to agree well with the composite analysis in the elastic range.

Results of the numerical analysis of the collimator at 0.1-mA average beam current show that the yield stress is reached after 0.23 s and the ultimate stress is exceeded after 0.48 s. In addition, the maximum allowable temperature of the tungsten alloy is attained after 0.57 s.

APPENDIX THERMAL AND MECHANICAL PROPERTIES

Thermal and mechanical properties were extrapolated from numerous references for tungsten and tungsten alloys.^{7,8,9} Yield and ultimate tensile stresses for 95 W-3.5 Ni-1.5 Fe from room temperature to 1.366 kK are shown in Fig. A-1. The maximum allowable temperature of the alloy is regarded to be 1.366 kK. The dashed portions of the curves indicate extrapolated or interpolated values.

The modulus of elasticity is shown in Fig. A-2. The modulus was extrapolated at constant slope from known data. Figures A-3 through A-5 show the coefficient of thermal expansion, thermal conductivity, and specific heat. The specific heat was extrapolated at the same slope as tungsten from a known value for the alloy at room temperature. Variable properties were incorporated into TSAAS by inputting data at 0.2932, 0.5332, 0.8109, 1.089, and 1.366 kK.

The effective plastic-to-elastic modulus ratio for calculations in the plastic range was obtained from data at room temperature. The plastic modulus was determined by assuming a linear stress-strain relation between the yield stress at 0.2% offset to the ultimate tensile stress at 15% elongation in 50.8 mm as shown in Fig. A-6. The ratio of plastic-to-elastic moduli was thus found to be 0.005031. This ratio was held constant with temperature.

Fig. A-1. Yield and ultimate tensile stresses of 95 W-3.5 Ni-1.5 Fe.

Fig. A-2. Modulus of elasticity of 95 W-3.5 Ni-1.5 Fe.

Fig. A-3. Coefficient of thermal expansion of 95 W-3.5 Ni-1.5 Fe.

Fig. A-4. Thermal conductivity of 95 W-3.5 Ni-1.5 Fe.

Fig. A-5. Specific heat of 95 W-3.5 Ni-1.5 Fe.

Fig. A-6. Determination of effective plastic-to-elastic modulus ratio.

ACKNOWLEDGMENTS

The authors wish to express their appreciation to R. V. Browning, WX-3, and C. A. Anderson, R-4, for their helpful explanations of the intricacies of TSAAS.

REFERENCES

1. R. J. Roark, Formulas for Stress and Strain (McGraw-Hill, 1965).
2. R. J. Macek, personal communication, May 1975.
3. R. V. Browning, D. G. Miller, and C. A. Anderson, "TSAAS: Finite Element Thermal and Stress Analysis of Axisymmetric Solids with Orthotropic Temperature-Dependent Material Properties," Los Alamos Scientific Laboratory report LA-5599-MS (May 1974).
4. J. A. Reuscher, "Analysis of Internal Heating Shock Effects in Reactor Fuel Components," Nuclear Engineering Design **18**, 213-251 (1972).
5. D. Burgreen, "Thermoelastic Dynamics of Rods, Thin Shell, and Solid Spheres," Nuclear Science and Engineering **12**, 203-217 (1962).
6. B. A. Boley and J. H. Weiner, Theory of Thermal Stress (John Wiley and Sons, 1960).
7. Materials Properties Data Book, Report 2275, Revision of November 17, 1970, Contract SNP-1, Aerojet Nuclear Systems Company, Division of Aerojet-General.
8. F. C. Holtz and R. J. Van Thyne, "Development and Evaluation of High-Temperature Tungsten Alloys, Summary Report," AEC Research and Development report, Contract AT(33-3)-4 Task 1, November 12, 1959.
9. High Density Metals, Mallory Metallurgical Company, Form 6-278 SM 12-68, pp. 4-5.

NOMENCLATURE

- a Semi-major axis of ellipse.
- A Area of beam in model.
- b Semi-minor axis of ellipse.
- c Speed of sound.
- C_p Specific heat at constant pressure.
- E Modulus of elasticity.
- E_p Effective stress-strain modulus in plastic range.
- h Thickness of plate.
- i, j Nodal point indices.
- I Average beam current.
- J Average beam current per unit area normal to beam.
- k Thermal conductivity.

- K Thermal diffusivity.
- ℓ Length of beam area.
- q Heat transfer rate per unit area.
- $\dot{q}$ Beam power deposition per unit mass.
- $\dot{q}_0$ Maximum beam power deposition per unit mass.
- Q Total beam energy deposited in collimator.
- r Radial distance from center of disc.
- R_i Inside radius of collimator.
- R_m Mean radius of cylinder.
- R_o Outside radius of collimator.
- S_{MN} Minimum normal stress.
- S_{MS} Maximum shear stress.
- S_{MX} Maximum normal stress.
- S_r Radial stress.
- S_t Tangential stress.
- S_u Ultimate stress.
- S_y Yield stress.
- t Time from beginning of beam pulse.
- t_y Time at which yield stress is reached.
- T Temperature.
- T_p Period of natural vibration.
- T_h Heating time.
- V Volume.
- W Kinetic energy of proton.
- x Cartesian coordinate normal to beam in direction of narrowest beam dimension, with origin at centerline of collimator.
- y Cartesian coordinate normal to beam in direction of largest beam dimension, with origin at centerline of collimator.
- z Cartesian coordinate in direction of proton beam.
- α Coefficient of thermal expansion.
- c Material strain.
- ν Poisson's ratio.
- ρ Material density.
- σ Standard deviation of Gaussian distribution.
- τ Wave propagation time.

Subscripts:

- x, y Component of the symbol in the x and y directions, respectively.
- 1 Denotes the inner section of a two-section approximation to the Gaussian curve.
- 2 Denotes the outer section of a two-section approximation to the Gaussian curve.

Superscript:

- Denotes Cartesian coordinate system with origin at center of proton beam.