Theory of Pion-Nucleus Scattering Lengths, II

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Abstract:

We calculate the scattering lengths for pions on light nuclei, in a multiple scattering theory given earlier. We include all effects through second order in the pion-nucleon scattering amplitudes, using a simple nuclear model. The resulting scattering lengths are smaller than those obtained from measurements of pi-atomic level shifts. The size of the difference is consistent with the contribution of pion absorption, which we estimate.

[NUCLEAR REACTIONS ^4He , ^{11}B , ^{12}C , ^{14}N , ^{16}O ; calculated pion scattering lengths.]

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In an earlier paper (denoted here as I) Moyer and Koltun¹ presented a theory for pion-nucleus scattering lengths based on multiple scattering of the pion on nucleons in the target. (See also ref. 2.) In the present paper we update and improve that work, correct some errors, and recalculate the scattering lengths for several light nuclei, using a newer analysis³ of the low energy pion nucleon scattering data. More recent pionic atom data⁴ has provided a more accurate and more complete set of 1s level shifts and widths from which "experimental" scattering lengths may be obtained. This analysis has been done by Hüfner et al.⁵ These authors have shown that the scattering lengths may be obtained from an α -particle model of the nuclear targets, using π - α scattering as the elementary process in a multiple scattering series. We shall remark on this approach.

The theory of (I) follows the method of Watson, in which the π -nucleus scattering amplitude T is expanded in terms of the free π -N amplitude $t(j)$ for the j -th nucleon. As in (I) we write the multiple scattering series in the form

$$T = T_{ss} + \Delta T_2 + \Delta T_B + \Delta T_{ms} \quad (1)$$

$$T_{ss} = \sum_{j=1} t(j) \quad (1a)$$

$$\Delta T_2 = \sum_{j \neq l} t(j) G t(l) \quad (1b)$$

$$T_B = \sum_j t(j) [G - g(j)] t(j) \quad (1c)$$

G is the propagator for the pion in the nucleus, $g(j)$ the propagator for a free π -N system. T_{ss} is the single scattering term in the impulse approximation, ΔT_2 represents double scattering from different nucleons,

and ΔT_B is the term for double scattering on the same bound nucleon. The contribution of all higher orders is included in ΔT_{ms} .

The propagator G is a many-body operator, which includes excitation of the target nucleus, as well as propagation of the pion. We consider a single-particle model of the target. Then, we evaluate (1b) and (1c), ignoring the Pauli Principle for intermediate nuclear states. As discussed in (I), this choice has no effect on the sum of the terms $\Delta T_2 + \Delta T_B$, as long as the ground state is appropriately antisymmetric, but it does simplify the evaluation of each term. In a single-particle model, without requiring the Pauli Principle, the binding correction term (1c) can be evaluated as a restricted three-body problem, of the pion scattering from a single nucleon bound in a potential. Also, the double scattering term (1b) takes a particularly simple form (see Eq. 5) since excited nuclear states do not contribute to ΔT_2 , in this model. The only approximation in this approach is the use of a single-particle model; the particular treatment of the Pauli Principle is exact as long as all terms through a given order in $t(j)$ are retained.

The π -nucleus scattering length is given by

$$a(i_3, Z, N) = -(\mu/2\pi)g^{-1}(A) \langle T \rangle \quad (2)$$

where $\langle T \rangle$ is the expectation value of (1) in the target (Z, N) ground state, for a pion of charge i_3 . The pion mass is μ , and $g(A) = (1 + \mu/AM)$ is a reduced-mass factor for a nucleus of mass AM ; M is the nucleon mass, and $A = Z + N$.

As in (I), we use a scattering length approximation for the s-wave π -N amplitudes. In terms of the scattering lengths for π -N

isospin 1/2, 3/2, we use the isoscalar length $a_0 = \frac{1}{3} (a_{1/2} + 2a_{3/2})$, and the isovector length $a_1 = \frac{1}{3} (a_{3/2} - a_{1/2})$.

In (I), a correction for p-wave π -N scattering due to nucleon motion in the target (recoil) was included in the calculation of T_{ss} . Moyer later estimated² the p-wave contribution to T_2 ; we shall include this as well.

One result of (I) was that the binding term (1c), was comparable to the double scattering term (1b). We now find that the binding correction was not correctly derived. This term depends on the difference between the bound and free propagators, G , and $g(j)$, respectively. In a single-particle model, in which the target nucleon has binding energy E_λ in a potential U , we may write (see I: (4.2) (4.8) (5.2))

$$G^{-1} = g^{-1} - U + E_\lambda - E_K \quad (3)$$

where E_K is the kinetic energy of a free nucleon. The effect of U was correctly calculated in (I) (although the sign was given incorrectly in the tables). The effect of the energy shift, $E_\lambda - E_K$ was neglected. This approximation is inconsistent, since the shift induces a correction of second order in $a_{\pi N}$. This was discovered by Myhrer and Silbar,⁶ who showed the connection of this effect to unitarity.

In fact, the unitarity correction tends to cancel the potential effect, since $U < 0$, while $E_K - E_\lambda > 0$. (For the ground state of the target, $\langle E_\lambda - E_K - U \rangle \approx 0$.) We have recalculated the total binding effect to second order in a model similar to (I), but including the energy shift (unitarity correction).

For completeness we give expressions for the leading contributions

for (1) and (2). We write

$$a(i_3, Z, N) = a_{ss} + a_B + a_2 \quad (4)$$

where the single scattering is given as an impulse term plus p-wave (recoil) correction:

$$a_{ss} = a_{IA} + a_1(P)$$

$$a_{IA} = (g(1)/g(A)) (Aa_0 + a_1 \langle 2i \cdot I \rangle) \quad (5)$$

$$a_1(P) = (g(1)/g(A)) \mu^2 (\mu + M)^{-2} \langle q^2 \rangle (A c_0 + c_1 \langle 2i \cdot I \rangle) .$$

We use $\underline{i}$ and $\underline{I}$ to denote the pion and nuclear isospin vectors, respectively; the ground-state expectation value $\langle 2i \cdot I \rangle = \pm(Z-N)$ for $\pi^\pm$. In $a_1(P)$, $\langle q^2 \rangle$ is the mean squared nucleon momentum, and c_0, c_1 are the isoscalar and isovector p-wave scattering volumes (b_0, b_1 in (1)).

Double scattering may be divided into contributions from two s-wave scatterings, one s- and one p-wave, and two p-wave scatterings:

$$a_2 = a_2(SS) + a_2(SP) + a_2(PP) \equiv a_2(SS) + a_2(P) \quad (6)$$

$$a_2(SS) = (g^2(1)/g(A)) \langle \frac{1}{r} \rangle [A(A-1)a_0^2 - 2Aa_1^2$$

$$+ \langle 2i \cdot I \rangle (2[A-1]a_0 a_1 + a_1^2)$$

$$+ \langle (2i \cdot I)^2 \rangle a_1^2]$$

where $\langle \frac{1}{r} \rangle$ is the mean inverse separation between scatterers. For estimates of $a_2(SP)$ and $a_2(PP)$, see Ref. 2. For evaluation of a_B , see Refs. 1, 2, and 6.

We have evaluated the contributions (4) to the scattering length (in 10^{-3} fermis) for the case of π^- on a number of light nuclei, mostly $N=2$ ($T=0$) cases. We have used the values of a_0 , a_1 given by the recent analysis by Bugg et al³:

$$a_0 = -0.0047 \pm 0.0017 \mu^{-1} \quad (7)$$

$$a_1 = -0.0873 \pm 0.0013 \mu^{-1}$$

and the values of c_0 , c_1 used in (I):

$$\begin{aligned} c_0 &= 0.208 \mu^{-3} \\ c_1 &= 0.180 \mu^{-3} \end{aligned} \quad (8)$$

The contributions of the individual terms are listed in Table I.

The single scattering contributions a_{ss} from s-wave and p-wave almost cancel (for $T=0$ nuclei). The total binding effect, including the unitary correction, is now very small in contrast to the results of (I), from which the unitarity correction was omitted. The major remaining term is the double scattering in s-waves, $a_2(ss)$. The other double scattering terms are quite small, as found in Ref. 2. The sum (SS+DS) of multiple scattering through second order (4) is listed in the sixth column. Higher order s-wave multiple scattering is estimated^{1,2} to be of the order 10-20% of $a_2(ss)$ for light nuclei.

The sensitivity of the sum SS+DS to the input π -N data is seen in Eqs. 5 and 6. The largest uncertainty is due to the uncertainty in a_0 , which is given in Ref. 3 as $\pm 35\%$; a_1 , c_0 , c_1 are supposedly known to a few percent. A change in the value of a_0 would show up largely in a_{IA} , less in $a_2(ss)$, and negligibly in other terms contributing to SS+DS.

As an example, if we choose $a_0 = -0.0100 \mu^{-1}$, keeping the other πN data unchanged, we obtain the results shown in parentheses in Table I. Thus, changing a_0 by a factor of 2 (about three standard deviations) changes SS+DS by about 1.5. Uncertainties in the nuclear model have little effect on the results.

For comparison with experiment, we have used the scattering lengths reduced from π -atomic data by Hüfner et al.⁵; we list their results ($a^{(1)}$) in the column labeled "Expt." The imaginary part ($\text{Im } a^{(1)}$) comes from pion absorption, which is not included in the multiple scattering series. Therefore, the calculated values of (SS+DS) should be compared to $\text{Re } a^{(1)}$.

The experimental scattering lengths, $\text{Re } a^{(1)}$, as a function of target mass number A , form two approximately linear plots, one for $T=0$, and one for $T=1/2$ nuclei. These trends are approximately given by (SS+DS), but the magnitudes are too small, by a factor of 2/3 to 1/2. The terms a_{IA} and $a_1(P)$ are linear in A , but, as noted, cancel approximately. The dominant term $a_2(SS)$ increases roughly as $A^{2/3}$, as can be seen from the formula (Eq. 6), with $a_0^2 \ll a_1$, and $\langle \frac{1}{r} \rangle \propto A^{-1/3}$.

By contrast, the multiple scattering series for pions on α -clusters in the targets, gives results much closer to the experiment $a^{(1)}$. Of course, the experimental scattering length for ${}^4\text{He}$ is fitted in that approach (as opposed to using π -N scattering lengths, as in our). For point α -particles (non-overlapping) the multiple scattering is relatively unimportant, so the model gives the approximately linear behavior of $a^{(1)}$. Also, since pion absorption is included in the $\pi - {}^4\text{He}$ data, complex scattering lengths are obtained for all nuclei. Similarly, other clusters have also been considered.⁷

The effect of absorption has not been included in our π -N multiple scattering theory, since single (free) nucleons don't absorb pions. Now there is a contribution to the real part of a ($\text{Re } a$) from the process of absorption, followed by re-emission, of a pion. This effect is automatically included, empirically, by Hüfner et al. Early work by Brueckner⁸ and by Thouless⁹ estimated this contribution to be comparable with the absorptive part, $\text{Im } a$, and negative (repulsive). There are recent realistic calculations^{10,11} of this term, but only for the deuteron, for which, indeed, the absorptive contribution to $\text{Re } a$ is approximately $-\text{Im } a$. We may use this relation to estimate the order of the absorptive effect on $\text{Re } a$ for the nuclei of Table I, by simply adding $-\text{Im } a^{(1)}$ to the term $(\text{SS}+\text{DS})$. The results are listed in the last column, and are in somewhat closer agreement with $\text{Re } a^{(1)}$ than are the $(\text{SS}+\text{DS})$ results alone. The uncertainty in the use of $-\text{Im } a^{(1)}$ for the absorptive contribution, is, however, rather large. Furthermore, a correct treatment of the absorption-emission contribution to $\text{Re } a$ requires a modification of the recoil term $(a_1(P))$.¹¹

In conclusion, it seems that a theory of pion-nucleus scattering lengths based on π -N multiple scattering, may be successful in reproducing the experimental results, if the effect of absorption and emission of pions can be included theoretically. This calculation is not necessary in the extreme α -cluster model used by Hüfner et al. However, it remains a strong possibility that one may explain the same data without an explicit clustering assumption.¹²

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TABLE I

Contributions to π^-A Scattering Lengths, in 10^{-3} fm.*

Target	a_{IA}	a_B	$a_1(P)$	$a_2(SS)$	$a_2(P)$	SS + DS	Expt ($a^{(1)}$)	Total Real
^4He	-29 (-63)	-12	38	-94 (-92)	-13	-110 (-142)	-139 + i43	-153 (-185)
^{11}B	-222 (-316)	-16	118	-172 (-163)	13	-279 (-364)	-502 + i127	-406 (-491)
^{12}C	-90 (-192)	-18	125	-204 (-192)	15	-172 (-262)	-453 + i133	-305 (-395)
^{14}N	-105 (-225)	-20	135	-227 (-211)	16	-201 (-305)	-505 + i125	-326 (-430)
^{16}O	-120 (-257)	-23	144	-239 (-221)	16	-222 (-341)	-548 + i154	-376 (-495)

* With π -N data given in (Eq. 7,8). Entries in parentheses are for $a_0 = -0.01 \mu^{-1}$, other values unchanged.