190
8-11-77

Dr # 1302

LA-6875-MS

Informal Report

UC-20F

Issued: July 1977

**A Design for Coordinated Measurements of
Faraday Rotation and Line-of-Sight Electron
Density Using Heterodyne Techniques**

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A DESIGN FOR COORDINATED MEASUREMENTS OF FARADAY
ROTATION AND LINE-OF-SIGHT ELECTRON DENSITY
USING HETERODYNE TECHNIQUES

by

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ABSTRACT

This report proposes a device which can overcome certain of the compromises of conventional Faraday rotation methods and at the same time measure the optical phase as well as the polarization. This would be useful for unfolding the Faraday rotation signal using the line-of-sight density along exactly the same path. Preliminary design parameters using a CO₂ laser are presented.

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I. INTRODUCTION

Optical Faraday rotation measurements¹⁻³ are characterized by the half-shade angle θ_c between polarizer and analyzer. An unfortunate compromise is necessary in choosing θ_c . For small Faraday rotations α , one can obtain linear response and high sensitivity with a large θ_c ($\sim 45^\circ$); but in many situations the large common mode signal^{1,2} changes with time irregularly due to refraction. The alternative is to operate near extinction ($\theta_c \sim 0$). In this case the sensitivity to α is small, the response may be nonlinear, but there is reduced susceptibility to refraction. Jarboe³ has shown that in this regime the optimum

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half-shade angle is $\theta_0 \approx \sin^{-1} \sqrt{f}$, where f is the extinction ratio of the analyzer.

Whereas the Faraday rotation techniques to date have utilized the high brightness available from lasers, they have not made use of the laser's other principle feature, namely coherence. By coherent detection with a portion of the laser output power serving as a local oscillator, the Faraday rotation α can be measured linearly even near extinction, but with a high sensitivity (characteristic of $\theta_c \approx 45^\circ$ for conventional systems). Thus the coherent detection technique offers simultaneously the advantages both of sensitive, linear detection and of relative immunity to refractive bending.

As described so far, the proposed device would confuse the effects of Faraday rotation α and plasma phase ϕ . As ϕ changes, there would be temporal fringes in the detected signal. The solution to this problem is to frequency-shift the local oscillator by some intermediate frequency, using a Bragg cell. The detected signal will now be an IF carrier, amplitude-modulated by the Faraday rotation effect and phase-modulated by the electron refractivity effect. By opening up the half-shade angle θ_0 to be a few times larger than the maximum Faraday rotation, the AM and PM can be separately demodulated, with acceptably low interaction between the two.

The combined interferometer/Faraday rotation concept is shown schematically in Fig. 1. If $\theta_0 = 0$ and $\alpha = 0$, only reference light reaches the detector. If $\theta_0 \neq 0$ and $\alpha = 0$, a constant amount of scene light reaches the detector. The phase of the IF carrier is the density information. If a magnetic field is then turned on so that $0 < |\alpha| < |\theta_0|$, the IF carrier's AM will be the Faraday rotation amplitude and sign information.

II. DETECTION PRINCIPLES

The reference beam arrives at the analyzer in s-polarization with respect to the Brewster plate. The scene beam arrives at the analyzer rotated by an angle $\theta_0 + \alpha$ out of pure p-polarization. (In addition, the scene beam has some residual ellipticity which can be caused by such effects as stress birefringence and internal reflections in discharge tube windows.) The s-polarization components of each beam are reflected, with power efficiency r_s , and impinge on a diffuser in front of the detector. The polarizations are shown in Fig. 2.

Fig. 1.
Conceptual layout of the heterodyne device.

Fig. 2.
Polarization states of the scene and reference beams going into the analyzer. Openness of either ellipse is exaggerated.

Define

- $f \equiv$ extinction ratio of analyzer
- $\theta_0 \equiv$ half-shade angle ($\theta_0 \ll 1$)
- $\alpha \equiv$ Faraday rotation angle* ($\alpha < \theta_0$)
- $P_{ro} \equiv$ product of analyzer reflectance r_s , times the s-polarization reference power reaching the analyzer
- $P_{so} \equiv$ product of analyzer reflectance r_s , times the total scene power reaching the analyzer
- $a/b \equiv$ ratio of minor to major diameter of scene electric field polarization ellipse ($a/b \ll 1$)
- $\phi \equiv$ interferometer phase due to electron refractivity
- $\kappa \equiv$ interference contrast factor ($\kappa \leq 1$)
- $\omega_B \equiv$ Bragg cell radian frequency
- $\epsilon \equiv$ fraction of diffused light that is detected.

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* α must be corrected for the effect of unequal s- and p-transmissions of windows and beamsplitters.

The detected power will be approximately

$$P_o \simeq \varepsilon(P_{ro} + f P_{so}) + \varepsilon(\theta_o + \alpha)^2 P_{so} + 2K\varepsilon \sqrt{P_{ro} P_{so}} \left[(\theta_o + \alpha) \cos(\omega_B t + \phi) + \frac{a}{b} \sin(\omega_B t + \phi) \right] .$$

The first term is essentially DC. The second term corresponds to the conventional Faraday rotation signal. The third term is the important heterodyne signal. Provided that the scene beam ellipticity can be suppressed to satisfy $a/b \ll |\theta_o + \alpha|$, the quadrature term may be ignored and the heterodyne signal is

$$P_{HET} \simeq 2K\varepsilon \sqrt{P_{ro} P_{so}} (\theta_o + \alpha) \cos(\omega_B t + \phi) \\ \simeq P_{HO} (1 + M_\alpha) \cos(\omega_B t + \phi) ,$$

where

$$M_\alpha = \frac{\alpha}{\theta_o} = \text{modulation index}$$

and

$$P_{HO} = 2K\varepsilon \sqrt{P_{ro} P_{so}} \theta_o .$$

III. REFERENCE DESIGN USING A 3-WATT CO₂ LASER

The layout is shown in Fig. 3. The drive power for the Bragg cell will be only tens of milliwatts (rf), since the local oscillator power on the detector should not exceed a couple of milliwatts. In the power budget to follow, the reference beam will be ignored, since it is such a small fraction of the total power available.

The Bragg cell insertion loss is $\leq 12\%$. With a two-plate polarizer using germanium plates, the efficiency is about 60%. The beamsplitter should be chosen to transmit as much as possible of the scene power; correspondingly, the

reflectance of reference light will be poor, but this is not a significant waste of power. Assuming the scene beam transmission through the discharge tube windows and the beamsplitter is 30% net, the scene power reaching the analyzer is about 0.5 watt. If the analyzer is also germanium and two-element, then $P_{SO} = 0.3$ watts.

Assuming $\epsilon = 0.33$, $P_{PO} = 3$ mW will assure that 1 mW of LO reaches the detector. Assuming $\kappa = 0.6$, the heterodyne amplitude is $P_{ho} \sim 12 \text{ mW} \times \theta_o$. Therefore with $\theta_o = 25$ milliradians, $P_{ho} = 300 \text{ } \mu\text{W}$.

The (Hg,Cd) Te photovoltaic detector should have a responsivity of $\geq 1 \text{ } \mu\text{A}/\text{ } \mu\text{W}$. Thus the heterodyne electric current amplitude out of the detector will exceed 300 μA . A Faraday rotation angle of 1×10^{-4} radian will then account for at least 1.2 μA , which should be detectable.

The half-shade angle θ_o cannot be chosen too large. If the laser AM noise modulation index is M_n , then this imposes a lower limit of resolution $\alpha_{\min} \sim \theta_o \cdot M_n$. For the GTE 941 laser, $M_n \lesssim 0.01$ over a 500- μs time scale, so in this case $\alpha_{\min} \sim 0.25$ milliradian.

The electronic signal processing is shown in Fig. 4. The photodiode capacitance, cable capacitance, and an external inductance form a tank circuit tuned to the 40-MHz carrier. The IF strip features a wide-range linear detector output to record AM on the carrier. The strip is manually set to a fixed gain in this application. The amplified IF output will also be amplitude modulated

Fig. 3.

Schematic for CO_2 system. The dots indicate s-polarization and the arrows p-polarization, in each case with respect to the analyzer.

Fig. 4.

Signal-processing electronics.

by the Faraday rotation. To remove this AM prior to measuring the PM due to electron density, the IF output is forced through a very high-speed voltage comparator.

The AM-to-PM conversion, which should be minimized, may be estimated by means of the variation of the voltage comparator's propagation delay with changes in the input level. For an input signal of ± 500 mV, the variation of the AM 685 propagation delay with respect to AM index is specified as

$$\frac{\partial \tau_{\text{delay}}}{\partial M} \sim 0.2 \text{ ns} \quad .$$

The AM index M will thus cause a phase-shift of the comparator output equal to

$$\phi_{\text{AM}} \sim \omega_B \frac{\partial \tau_{\text{delay}}}{\partial M} M \quad .$$

Assuming $M \leq 0.5$ (i.e., $|\alpha| \leq 0.5|\theta_0|$), the maximum AM-to-PM conversion amounts to 0.025 radians, which almost always will be small compared to the plasma phase.

The voltage comparator output feeds into a quadrature phase comparator. The local input to the quadrature phase comparator is derived from the Bragg cell drive. The two outputs are $\sin\phi$ and $\cos\phi$ and provide complete information on the plasma phase.⁴

The success of the technique proposed here will hinge on how small the scene beam ellipticity a/b can be kept. The discharge tube windows are likely to have the most important effect here. The effects of overall scene beam ellipticity are discussed in the Appendix.

REFERENCES

1. I. S. Falconer and S. A. Ramsden, "Measurement of the Faraday Rotation of a Laser Beam Passing Through a θ -Pinch Plasma," J. Appl. Phys. 39, 3449 (1968).
2. R. F. Gribble, E. M. Little, R. L. Morse, and W. E. Quinn, "Faraday Rotation Measurements on the Scylla IV Theta Pinch," Phys. Fluids 11, 1221 (1968).

3. T. R. Jarboe, "Measurement of Faraday Rotation in the Implosion Heating Experiment," J. Appl. Phys. 48, 557 (1977).
4. A. R. Jacobson, "A Novel Interferometer for the Measurement of Plasma Density," Los Alamos Scientific Laboratory report LA-6727-MS (1977).

APPENDIX

EFFECTS OF OVERALL SCENE BEAM ELLIPTICITY

The full heterodyne signal is

$$\begin{aligned}
 P_{\text{HET}} &\simeq P_{\text{HO}} \left\{ \left(1 + \frac{\alpha}{\theta_o} \right) \cos(\omega_B t + \phi) + \frac{a/b}{\theta_o} \sin(\omega_B t + \phi) \right\} \\
 &= P_{\text{HO}} \left(1 + \frac{\alpha}{\theta_o} \right) \sqrt{1 + \left(\frac{a/b}{\theta_o} \right)^2 \frac{1}{\left(1 + \alpha/\theta_o \right)^2}} \\
 &\quad \cos \left\{ \omega_B t + \phi - \tan^{-1} \left(\frac{a/b}{\theta_o} \frac{1}{1 + \alpha/\theta_o} \right) \right\} .
 \end{aligned}$$

The arctan term causes some AM-to-PM conversion in the presence of scene beam ellipticity. This apparent phase shift $\phi_{a/b}$ is shown in Fig. A-1 as a function of the Faraday angle for various amounts of ellipticity.

The square root factor causes nonlinear AM in response to changes in the Faraday angle α . The total AM versus the Faraday angle is plotted in Fig. A-2 for various amounts of ellipticity.

Fig. A-1.
Apparent phase shift due to AM-to-PM
conversion caused by scene beam ellipticity.

Fig. A-2.
Total AM due to Faraday rotation for
various scene beam ellipticities.