MECHANICAL BEHAVIOR OF MATERIALS AND
STRUCTURAL ELEMENTS AT ELEVATED TEMPERATURES

Progress Report
for Period
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R.H. Lance and S. Mukherjee
Cornell University
Ithaca, New York

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ABSTRACT

This report summarizes the research performed between June 1, 1975 and May 31, 1976 on the project Mechanical Behavior of Materials and Structural Elements at Elevated Temperatures. The analytical portion of the research performed includes solutions of several boundary value problems for structural components subjected to steady loads with and without steady thermal gradients using Hart's constitutive relations for the material. Study of crack propagation in a ductile material has been initiated. The experimental portion of the research, designed to perform creep experiments on structural components at room and elevated temperatures, has been started. Hardware is being built and measuring equipment is being purchased.

1. Analytical and numerical research on creep of structural elements using Hart's theory.

During the period June 1976 to date we have solved a number of problems involving creep of structural elements at elevated temperatures using Hart's constitutive relations [1-6]. We have developed a general method of three-dimensional inelastic analysis using stress and displacement rates [3] and applied this method to a number of problems. Recently, we have been successful in the application of the boundary-integral equation method to the analysis of creep of metallic structural elements obeying newer constitutive relations [4]. Some solutions of relaxation of tension-torsion in solid and hollow cylinders have also been obtained and study of crack propagation in a ductile metal is in progress.

- a. Relatively Steady loads - Uniform temperature.

Spheres, cylinders and rotating discs

We have analysed a variety of problems involving thick spheres, cylinders and rotating discs (Fig. 1) subjected to a variety of steady loads at uniform elevated temperatures. For example, we have considered creep of a long thick cylinder under internal and external pressures, axial load and torque (neglecting the effects of the ends) and creep of a thin disc rotating at a constant angular velocity. The numerical results clearly show the effect of initial state (hardness) on subsequent stress redistribution and creep.

Beams and plates.

The mechanical equation of state proposed by Hart has been established in uniaxial tensile tests, and its application to structural components under bending must be examined.

The general bending of beams is formulated first. Then the problems of rectangular beams under constant and variable moments with different boundary conditions are solved numerically by a computational routine. The case of stationary creep under pure bending is obtained in a closed form. All the results compare favorably with those of the classical creep analyses - plus the effect of the state variable upon subsequent deformation [5].

The same approach is used in a biaxial state of stress, namely, the bending of thin plates [6]. The problems of clamped and simply-supported circular plates under uniform loading are analysed and extension to other loading conditions will be carried out.

An approximation to Hart's equations is proposed and the corresponding transformed equations relating moment and curvature rate are formulated. Great simplification will be achieved in the analysis of beams and plates by incorporating the new constitutive equations in terms of global variables, such as moment and curvature, as is commonly done in technical theories of structural behavior.

b. Relatively steady loads - including steady thermal gradients.

We have considered the creep of spheres, cylinders and discs under a variety of steady mechanical loading conditions and in the presence of steady thermal gradients [3]. This class of problems can be solved very efficiently by the general method of analysis (see [3]) in terms of stress and displacement rates. The presence of thermal stresses and strains completely alters the numerical results obtained [Fig. 2] but the effect of initial state on subsequent creep and relaxation is once again clearly evident [see Figs. 2-4].

c. Time varying (including discontinuous) loads.

A generalized version of Hart's theory [7,8] including anelastic strains is now available for the analysis of creep of structural elements subjected to time varying load and thermal histories. We have started using these generalized relations to calculate the response of uniaxial specimens to time varying loads. During June-August 1976 we hope to continue this analysis and begin investigating the response of axisymmetric structural components (i.e. cylinders, discs etc.) in creep to time varying loads.

d. The boundary-integral equation method applied to creep problems.

The boundary-integral equation method is a powerful technique that has been used by other researchers to solve elasticity, plasticity and linear viscoelasticity problems. We have recently been successful in applying this method to creep analysis of structural components [4]. The method is an alternative to finite element techniques and appears extremely promising since it essentially reduces the dimensions of a problem by one. We have shown that the BIE method can be used conveniently if the creep problem is formulated in terms of rates and the constitutive laws have the general form

$$\dot{\epsilon}^n = f(\underline{g}, \underline{q}_i, T) \quad (1)$$

$$\dot{q}_i = g_i(\underline{g}, \underline{q}_i, T) \quad (2)$$

where $\dot{\epsilon}^n$ is the nonelastic strain rate tensor, $\underline{g}$ is the stress tensor, $\underline{q}_i$ are suitably defined state variables and T is the temperature. Hart's equations fit into this general format and the state variables he uses are

are the scalar hardness σ^* and the anelastic strain tensor ϵ^a .

Several examples involving simple geometry - e.g. inflation of spheres and cylinders and torsion of circular shafts have been solved (Fig. 5) by this method. We plan to carry out the computer implementation of this method for more complicated geometries during the next year i.e., September '76 to August '77.

e. Exact solutions

Initial-boundary value problems for simple geometries and load histories have been studied using Hart's constitutive theory in an effort to understand the interaction between the elastic compressibility and inhomogeneity and the incompatibility of the inelastic strain. Solutions have been obtained for the relaxation of tension-torsion in solid and hollow cylinders which yield relationships between the plastic strain and the boundary tractions which are insensitive to the details of the creep law. For thin walled cylinders a relationship between the longitudinal force and the axial moment is obtained.

This summer the analytical work will be concentrated on obtaining bounding and asymptotic estimates from the three dimensional theory for other simple multiaxial problems.

In progress is a study of crack propagation in a ductile metal according to a simplified version of Hart's theory. The differential equations governing the steady propagation of a mode III crack in an infinite medium are subjected to similarity transformations. Similar solutions which give the elastic solution as the far field limit indicate the modification of the stress singularity at the crack tip due to the inelastic deformation.

2. Experimental Program

The purposes of our experimental investigation are to study the creep behavior of structures under multiaxial loads, to provide a data base in order to evaluate the modern and the classical theories of creep behavior, and to develop inhouse structural experimental expertise as a basis for further more detailed and realistic experimental studies. The experiments we intend to conduct include creep and relaxation tests on small-scale beams and plates, and thin walled tubes, at room and elevated temperatures. During the first stage of our study, we are looking into the simple bending of a beam at room and elevated temperatures.

At present, particular attention is being given to creep as observed in four-point bending of beams. A simply supported beam is loaded by two equal constant line loads at equal distance from each other and the displacements of the beam center and the other two symmetric points are measured as functions of time to study creep phenomena in structures under pure bending. The load applied to the beam can be programmed to be a step function of time by using a special loading device. In the first period of development of the experimental program necessary testing equipment is being designed and built. Such equipment can be used for investigation of more complex cases later.

The testing equipments consist of four modules. They are (1) the load applying system (the drum device)(2) the loading rig and the load frame (3) the temperature chamber and controller, and (4) the measuring devices and data aquisition system. A brief description of these modules follows (see Figs. 6 and 7).

- 1) The "drum device" is an aluminum cylinder with two sections of diameters 4" and 12" respectively. The drum is supported by a steel shaft and

free to rotate about its horizontal centerline. On the 12" diameter portion of the drum, several weights are hung on both sides; and there is a blade tangent to the 4" section to transmit the load to the loading rig. If the total weight, A, is bigger than B, then the balance of rotation of the drum requires that $T = 3(A-B)$, thus a load of $3(A-B)$ is transmitted to the loading rig. Here it is required that A be bigger than B, unless it is desired that no force be transmitted to the loading rig. On the other hand, if we take off one load on the A side, say A_1 , providing $A = A_2 + A_3 > B$, then T is going to reduce by an amount $3A_1$ because the moment applied to the drum must be balanced. Thus the step load history, which is illustrated in Fig. 6(b), may be applied.

- 2) Fig. 7 shows the load frame and the testing configuration. The drum device is mounted onto the frame and the temperature chamber also sits on the frame as shown (Fig. 7). The loading rig is also shown in Fig. 7. In this device, the load is transmitted from the drum device and distributed equally to the points of application by use of two pins which are designed for reduction of friction at the contact surfaces. Similar pins are used to support the beam. A load cell at the top of the frame is connected to the load jig. The level and alignment of the system is crucial; it is designed so that the load cell, the temperature chamber, and the drum may be leveled independent of one another.
- 3) The temperature chamber and controller are required for the study of creep at elevated temperatures. The temperature chamber, shown in Fig. 7 is intended to be an Instron temperature chamber which is available for our use in the materials laboratory at Cornell. The temperature controller will be bought in order to perform the tests at elevated temperatures.

- 4) The transducers used in the measurements are linear variable differential transformers (DC-LVDTs) for the beam displacements, thermocouples for temperatures along the beam, and a standard load cell for measuring the applied load. Strain gages will also be used to measure the surface strain of the specimen, especially for more complex structures.

To record all these measured quantities as functions of time, we shall use AUTO-DATA NINE manufactured by the VIDAR CO.. This is a digital data acquisition system with integrating digital voltmeter (IDVM). It has a high resolution (120,000 counts for 0.001% of range) and can be expanded to achieve fast reading up to 24 readings per second continuously by using the 8400 data cassette or a 9-track magnetic tape. It can also be used in connection with a remote computer or terminal site. When the step function loading is applied, the fast speed reading is desirable to record data of the transient periods while load changes. This system can be mated with the Hewlett-Packard 900-series or the Tektronix 31 programmable calculator. We would then have an on-site data acquisition and processing system.

At the present time, the loading rig, the load frame and the drum device are being built; the transducers are on order; and the AUTO DATA NINE is being considered. The temperature controller will not be used this year. Once the loading system is built, we will conduct room temperature creep tests of four point bending of aluminum specimens.

The results of the tests will be correlated with analytical work already completed and described in previous sections.

RESEARCH SCHEDULE

References

1. Kumar, V., and Mukherjee, S., "Creep of a thick-walled spherical shell - An equation of state approach," ERDA Report No. COO-2733-1, Department of Theoretical and Applied Mechanics, Cornell University, Ithaca, N.Y., July 1975.
2. Kumar, V., and Mukherjee, S., "Creep analysis of structures using a new equation of state type constitutive relation," ERDA Report No. COO-2733-2, Department of Theoretical and Applied Mechanics, Cornell University, Ithaca, N.Y., September 1975. Journal of Computers and Structures (to appear).
3. Kumar, V., and Mukherjee, S., "Inelastic analysis of metallic structures in the presence of thermal gradients using newer constitutive relations," ERDA Report No. COO-2733-3, Department of Theoretical and Applied Mechanics, Cornell University, Ithaca, N.Y., February 1976. Journal of Pressure Vessel Technology, Trans. ASME (to appear).
4. Kumar, V., and Mukherjee, S., "A boundary-integral equation formulation for time-dependent inelastic deformation in metals," ERDA Report No. COO-2733-4, Department of Theoretical and Applied Mechanics, Cornell University, Ithaca, N.Y., April 1976. Submitted for publication.
5. Waung, Y.C. and Lance, R.H., "Creep of beams using a state variable approach" (under preparation).
6. Waung, Y.C. and Lance, R.H., "Creep of plates using a state variable approach", (under preparation).
7. Hart, E.W., Li, C.Y., and Yamada, H., "Phenomenological theory - a guide to constitutive relations and fundamental deformation properties," in "Constitutive Equations in Plasticity", Argon, A.S., ed., M.I.T. Press, Cambridge, Mass., 1976, pp. 149-197.
8. Hart, E.W., "Constitutive relations for the non-elastic deformation of metals," Journal of Engineering Materials and Technology, Trans. ASME (to appear).

Figure 1. Structural elements with loads

(a) Sphere (a,b) Cylinder (c) Disc

(Fig. 1 from [3])

Figure 2. Redistribution of tangential stress in a rotating disc under creep ($N = 10,000$ rpm ($\omega = 1047.2$ rad/s), $\bar{\sigma}_0^*(\xi) = 38.14$ (i.e. 10% cold work), $\kappa = 3$, $a = 1$ in. (2.54 cm)).
 (a) $T_a = 482^\circ\text{F}$ (250°C), $T_b = 437^\circ\text{F}$ (225°C) (b) $T_a = T_b = 482^\circ\text{F}$.
 (Fig. 9 from [3])

Figure 3. Variation of tangential stress in a creeping cylinder. (a) Stress redistribution (b) Effect of initial hardness on stress relaxation ($p = 1000$ psi (6.895 M Pa), $T_a = 482^\circ\text{F}$ (250°C), $T_b = 437^\circ\text{F}$ (225°C), $\kappa = 2$). (Fig. 3 from [3])

Figure 4. Radial displacement at the inner radius of a creeping sphere ($\kappa=2$) with time for the cases:

- (I) $p = 3000$ psi (20.684 M Pa), $\bar{\sigma}_0^*(\xi) = 3.53$ (i.e. 10% cold work),
- (II) $p = 2000$ psi (13.790 M Pa), $\bar{\sigma}_0^*(\xi) = 5.30$ (i.e. 10% cold work),
- (III) $p = 1500$ psi (10.342 M Pa), $\bar{\sigma}_0^*(\xi) = 2.0 \bar{\sigma}(\xi, 0)$,
- (IV) $p = 1500$ psi (10.342 M Pa), $\bar{\sigma}_0^*(\xi) = 7.07$ (i.e. 10% cold work),
 $T_a = 482^\circ\text{F}$ (250°C), $T_b = 437^\circ\text{F}$ (225°C).

(Fig. 6 from [3])

(a) SPHERE

(b) CYLINDER AND DISC

(c) TORSION

Figure 5. Boundary-integral formulations
(Fig. 1 from [4])

Figure 6(a). The Drum Device

Figure 6(b). Time history of load

Figure 7

Time devoted by principal investigator:

Professor R.H. Lance has devoted one month, i.e., 6.67% of
his time on this contract so far.

List of Contract Reports to Date

1. Kumar, V., and Mukherjee, S., "Creep of a thick-walled spherical shell - An equation of state approach," ERDA Report No. COO-2733-1, Department of Theoretical and Applied Mechanics, Cornell University, Ithaca, N.Y. July 1975.
2. Kumar, V., and Mukherjee, S., "Creep analysis of structures using a new equation of state type constitutive relation," ERDA Report No. COO-2733-2, Department of Theoretical and Applied Mechanics, Cornell University, Ithaca, N.Y., September 1975.
3. Kumar, V., and Mukherjee, S., "Inelastic analysis of metallic structures in the presence of thermal gradients using newer constitutive relations," ERDA Report No. COO-2733-3, Department of Theoretical and Applied Mechanics, Cornell University, Ithaca, N.Y., February 1976.
4. Kumar, V., and Mukherjee, S., "A boundary-integral equation formulation for time-dependent inelastic deformation in metals," ERDA Report No. COO-2733-4, Department of Theoretical and Applied Mechanics, Cornell University, Ithaca, N.Y., April 1976.

Report of Equipment Purchased or Fabricated

<u>Equipment</u>	<u>Status</u>	<u>Cost</u>
Drum devise	being fabricated	\$1,000 (estimate)
Load frame and rig	being fabricated	\$1,000 (estimate)
Load cell	ordered	\$ 440
Power supply (2)	ordered	\$ 380
L.V.D.T. (3)	to be ordered	\$ 900
		<hr/>
		\$3,720