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SEMI-EMPIRICAL FORMULAS FOR HEAVY-ION STRIPPING DATA*

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ABSTRACT

All available charge state measurements for heavy ions in dilute gases and carbon foils at equilibrium or near-equilibrium conditions have been analyzed to improve semi-empirical formulas for the distribution parameters. Each experimental distribution was fit to an asymmetric function $F_q = F_m \cdot \exp\{-0.5\ t^2/(1 + \varepsilon t)\}$, where $t = (q - q_0)/\rho$ and q_0 is the maximum intensity charge value. Subsequent fits of the resultant distribution parameters q_0, ρ, and ε to empirical functions of projectile charge and velocity yielded rms deviations of about 0.5 for q_0, 4% for q_0/Z, 5-7% for ρ, and 0.03 for ε/ρ. The asymmetric distribution gives substantial improvement over previous expressions for prediction of small F_q values for heavy ions in dilute gases.

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Accurate predictions of charge state distributions for heavy ions in gaseous and solid media are particularly of interest for the design of new accelerators and for estimation of output beam intensities. Five years ago a comprehensive review[1] of charge-changing processes for heavy ions in gases and solids and two extensive compilations[2,3] of equilibrium charge state distribution measurements appeared. In the review by Betz[1] several semi-empirical formulas for the mean charge, $\overline{q}$, and width, d, of the distribution were discussed. In the past five years several new experimental results for highly stripped ions have been reported, but there has been relatively little development of new empirical representations of the data. We have utilized recent experimental measurements and data from previous compilations[2,3] to obtain improved semi-empirical formulas for the distribution parameters.

Inspection of the data reveals differences of 0.5-1.0 unit in the values of mean charge reported by different experimenters for identical projectile energies and target materials. These variations may be attributed to effects such as the angular acceptance of the apparatus, insufficient target thickness for equilibrium, and the "density effect" in gases. Such ambiguities make it difficult to develop completely general formulas and consequently only the two most commonly used types of stripping media, carbon foils and dilute light gases, will be considered here.

Each experimental charge state distribution was fit to an asymmetric function $F_q = F_m \cdot \exp\{-0.5t^2/(1 + \varepsilon t)\}$, where $t = (q - q_o)/\rho$ and q_o is the maximum intensity charge value. Values of $F_q < 0.1\%$ were excluded from the fits. When the asymmetry parameter $\varepsilon \to 0$, the distribution function reduces to the usual Gaussian of width ρ, $q_o \to \overline{q}$, $\rho \to d$, and $F_m\rho \to 1/\sqrt{2\pi}$.

The moments $T_n \equiv \int dt\, t^n F_q / \int dt\, F_q$ may be evaluated numerically with integration limits chosen so that $\varepsilon t > -1$. Then $\bar{q} = q_0 + \rho T_1$; $(d/\rho)^2 = T_2 - T_1^2$; and $(d/\rho)^3 s = T_3 - 3T_2T_1 + 2T_1^3$, where d and s are parameters defined by Betz.[1]

Fits with asymmetric and Gaussian distributions to experimental F_q values[3] for 15 MeV ^{127}I ions in dilute Ar gas are presented in Fig. 1. Clearly the inclusion of the ε parameter produces a significantly better fit, although small F_q values at high q are still underestimated somewhat. In Fig. 2, F_q values for 58 experimental distributions for heavy ions ($Z \geq 16$) in dilute gases are compared with the asymmetric fit values by plotting $(F_q/F_m)^{1+\varepsilon t}$ vs. t. The curve represents the asymmetric distribution function. The agreement between experimental and fit values is quite satisfactory. An additional parameter to improve the fits at high q is felt to be unwarranted because the high q intensities can be very sensitive to shell effects and to experimental conditions such as the angular acceptance of the apparatus and the target thickness.

The distribution parameters q_0, ρ and ε were fit to semi-empirical functions of projectile charge, Z, and velocity, βc. The results of the fits for q_0/Z, ρ and ε/ρ are presented in Figs. 3, 4, and 5, respectively. Each figure contains a set of data for carbon strippers, a set for dilute gaseous strippers, curves corresponding to the least squares fits to the two data groups, and the appropriate semi-empirical expressions. In each figure the upper abscissa should be used for carbon and the lower abscissa for dilute gases.

Our q_0/Z expression differs from that used by Betz *et al.*[4] for gases in that the exponent of β is a free parameter rather than unity. A Nikolaev-Dmitriev[5] q_0 formula with a β^δ term gave statistically similar fits in certain cases, but our relation fit the shape of the q_0/Z vs. $Z^\alpha \beta^\delta$ curve better over a wide range of q_0/Z. However, neither previous formulas nor the present formula represent the data adequately at both low and high ionization. For the fits shown in Fig. 3, therefore, the range of experimental ionizations was restricted to $0.07 \leq q_0/Z$ for gases and to $0.15 \leq q_0/Z \leq 0.95$ for carbon. These ranges cover most cases of practical interest. The rms deviations in q_0 were 0.4 for gases and 0.5 for carbon, and the rms deviations in q_0/Z were 4.0% for gases and 3.3% for carbon.

With the assumption of a Gaussian distribution Nikolaev and Dmitriev[5] used the Bohr criterion and found $d^2 \sim d\bar{q}/d \ln \beta$. If we assume $\rho^2 \sim dq_0/d \ln \beta$, then $\rho^2 \sim Z^\alpha \beta^\delta (1-q_0/Z)$. We chose the slightly different expression $\rho^2 \sim Z^\alpha \{(q_0/Z)(1-q_0/Z)\}^\gamma$ for the fits shown in Fig. 4. The data are rather well represented by this expression; the rms deviations in ρ are 4.5% for gases and 6.7% for carbon. In Fig. 6 predictions from the present formula (solid curves) and the simple expression $d = 0.27 \sqrt{Z}$ (dashed lines) proposed by Betz *et al.* are compared with experimental ρ values for ^{32}S, ^{79}Br, and ^{127}I ions in dilute gases. It can be seen that inclusion of energy dependence via q_0/Z in the ρ formula yields predictions much more consistent with the data than those given by the simpler formula.

In Fig. 5 the ε/ρ data are compared with fits to the formula $\varepsilon/\rho = A + BZ + C\beta$. Although there is considerable scatter in the data, the much larger asymmetry for gases is clearly evident. The rms deviation in ε/ρ is only 0.03. As expected, the parameter ε is negative at high β and positive at low β.

Predicted intensities, especially those for high q, are a principal concern of the potential accelerator user. These high q intensities may be extremely sensitive to the functional form of the empirical distribution.

Examples are presented in Fig. 7, where experimental intensities for charges 10, 13, and 16 are plotted as a function of energy for ^{79}Br ions in N_2, O_2, and Ar. The solid curves were obtained with our asymmetric distribution, and the dashed curves with the $\bar{q}$ and d formulas of Betz *et al.*[4] for the Gaussian distribution $F_q \sim \exp\{-.5(q-\bar{q})^2/d^2\}$. It should be noted that Betz *et al.* obtained a different set of parameters for their q_0/Z formula for each combination of projectile and target whereas our single set of parameters is based on fits to all data for ions with $Z \geq 16$ in dilute N_2, O_2, air, and Ar.

For the cases shown in Fig. 7 both predictions agree reasonably well with each other and with the data near the peak intensity. However, small F_q values are underestimated substantially by the Gaussian assumption while the asymmetric predictions are generally consistent with experiment. Similar results were obtained for other projectiles.

Consideration of recent experimental data, use of an asymmetric distribution function, and inclusion of additional velocity dependence have led to semi-empirical formulas that give more accurate predictions of equilibrium charge state distributions than previous expressions.[4,5] Substantially better agreement with experiment was obtained for small F_q values for heavy ions in dilute gases. Extreme caution should be used, however, in predictions for the heaviest, highly stripped ions where the data is scarce, and in extrapolation outside the indicated ranges of q_0/Z.

1. H. D. Betz, Revs. Mod. Phys. 44, 465 (1972).
2. A. B. Wittkower and H. D. Betz, Atomic Data 5, 113 (1973). We are indebted to Dr. Betz for providing a tape containing data reported in the above compilation.
3. S. Datz, C. D. Moak, H. O. Lutz, L. C. Northcliffe, and L. B. Bridwell, Atomic Data 2, 273 (1971).
4. H. D. Betz, G. Hortig, E. Leischner, Ch. Schmelzer, B. Stadler, and J. Weihrauch, Phys. Lett. 22, 643 (1966).
5. V. S. Nikolaev and I. S. Dmitriev, Phys. Lett. 28A, 277 (1968); I. S. Dmitriev and V. S. Nikolaev, Sov. Phys. JETP 20, 409 (1965).

FIGURE CAPTIONS

1. Plot of charge state intensities in % vs. q for 15 MeV ^{127}I in Ar. The solid line is a least squares fit to the experimental values (Ref. 3) with $F_m = 23.3$, $q_o = 5.07$, $\rho = 1.67$, $\varepsilon = 0.31$, and $t = (q-q_o)/\rho$. The dashed line corresponds to $\varepsilon = 0$, i.e. a pure Gaussian.

2. Plot of the values $(F_q/F_m)^{1+\varepsilon t}$ vs. t for 58 experimental distributions for heavy ions ($Z \geq 16$) in dilute gas strippers. The curve corresponds to the asymmetric distribution $F_q/F_m = \exp\{-.5t^2/(1+\varepsilon t)\}$.

3. Plot of q_o/Z vs. reduced velocity functions for heavy ions ($Z \geq 16$) in carbon (upper abscissa) and dilute gas (lower abscissa) strippers. The curves represent least squares fits using the formula $q_o/Z = 1 - k \cdot \exp\{-AZ^{\alpha}\beta^{\gamma}\}$.

4. Plot of ρ vs. functions of the form $AZ^{\alpha}\{(q_o/Z)(1-q_o/Z)\}^{\gamma}$ for carbon (upper abscissa) and dilute gas (lower abscissa) strippers. The curves represent least squares fits.

5. Plot of ε/ρ vs. functions of the form $A + B \cdot Z + C \cdot \beta$ for carbon (upper abscissa) and dilute gas (lower abscissa) strippers. The curves represent least squares fits.

6. Plot of ρ vs. energy for ^{32}S ions (squares), ^{79}Br ions (circles), and ^{127}I ions (triangles) in dilute gases. The dashed curves correspond to $\rho = 0.27\sqrt{Z}$, and the solid curves to $\rho = 0.35\,Z^{.55}\{(q_o/Z)(1-q_o/Z)\}^{.27}$.

7. Charge state intensities vs. energy for ^{79}Br in N_2, O_2, and Ar. The squares, circles, and triangles correspond to q = 10, 13, and 16, respectively. The solid and dashed curves are predictions with asymmetric and Gaussian distributions, respectively.

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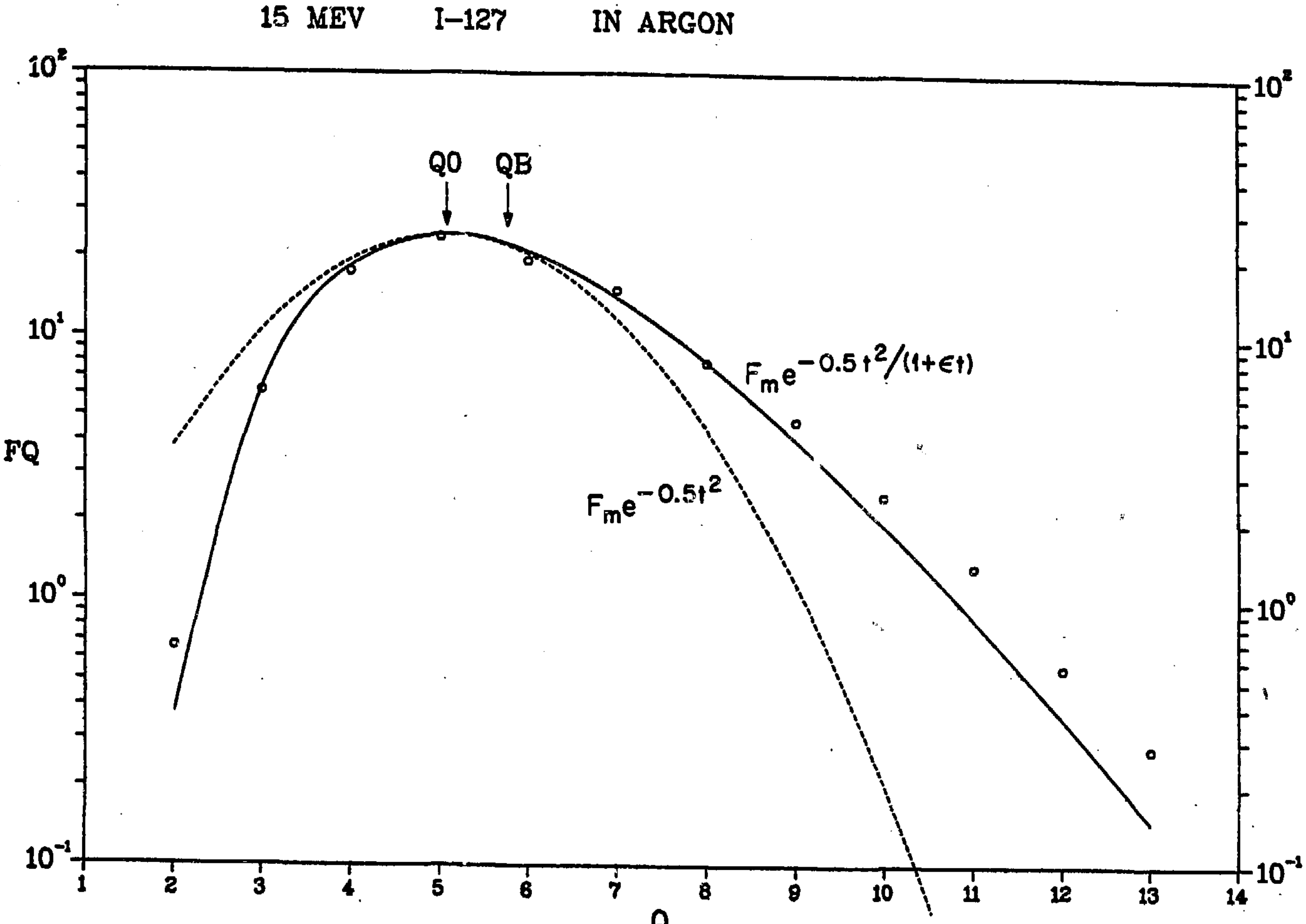

FIGURE 1

ORNL-DWG 77-9974

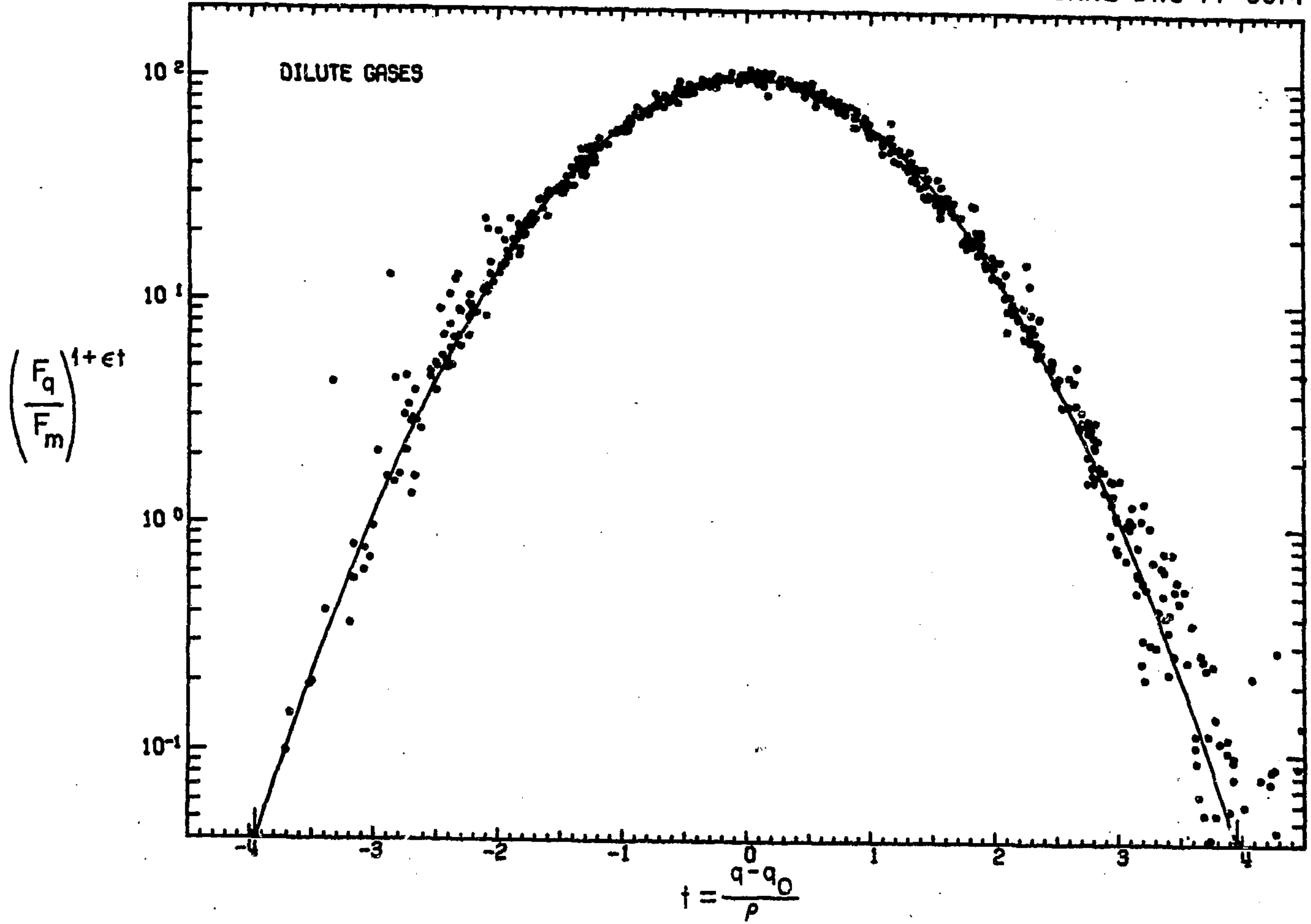

FIGURE 2

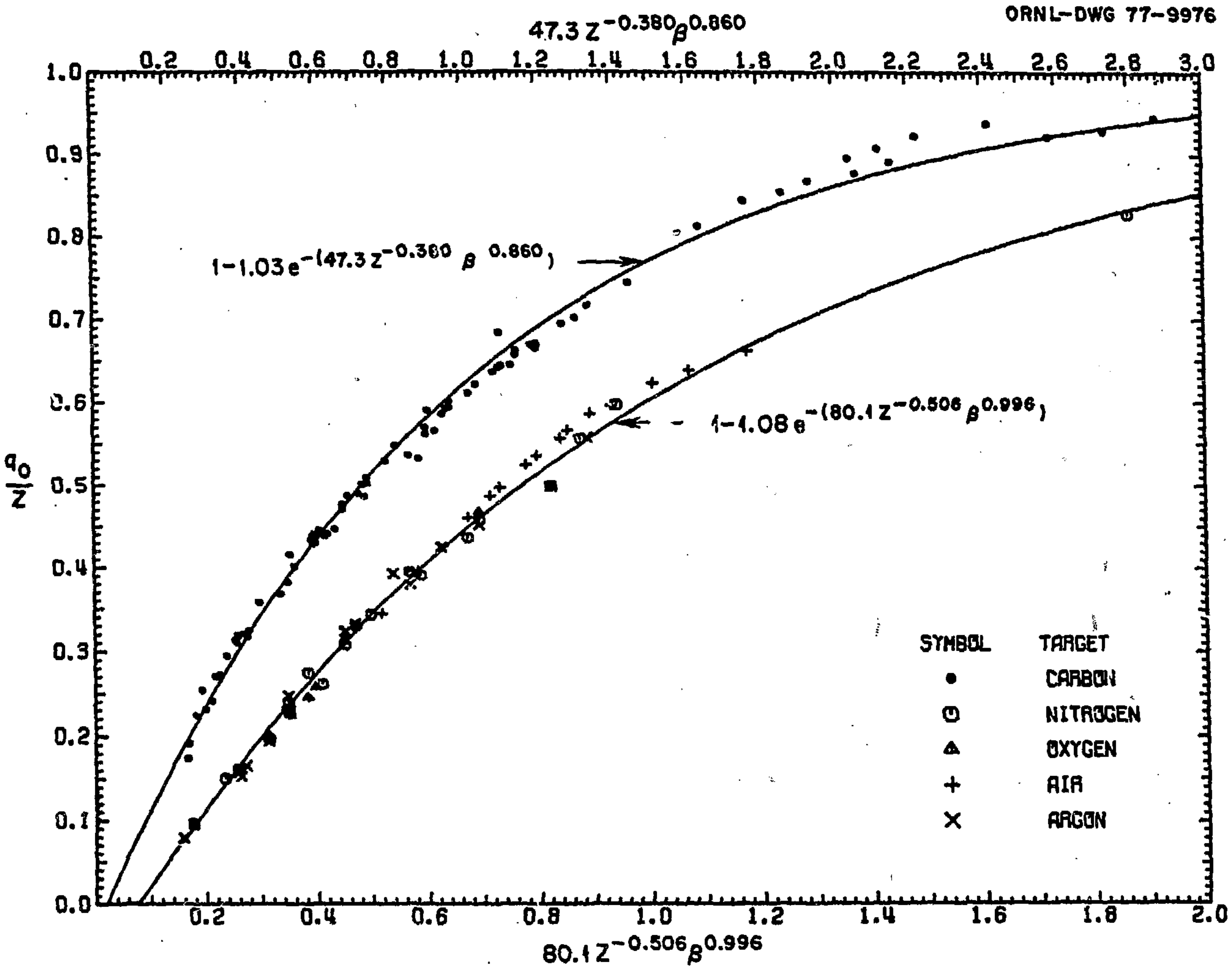
ORNL-DWG 77-9976
47.3 Z^-0.380 β^0.860
0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0 2.2 2.4 2.6 2.8 3.0
q_0/Z
1.0 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0.0
1-1.03 e^-(47.3 Z^-0.380 β^0.860)
1-1.08 e^-(80.1 Z^-0.506 β^0.996)
SYMBOL TARGET
● CARBON
Φ NITROGEN
▲ OXYGEN
+ AIR
× ARGON
0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0
80.1 Z^-0.506 β^0.996

FIGURE 3

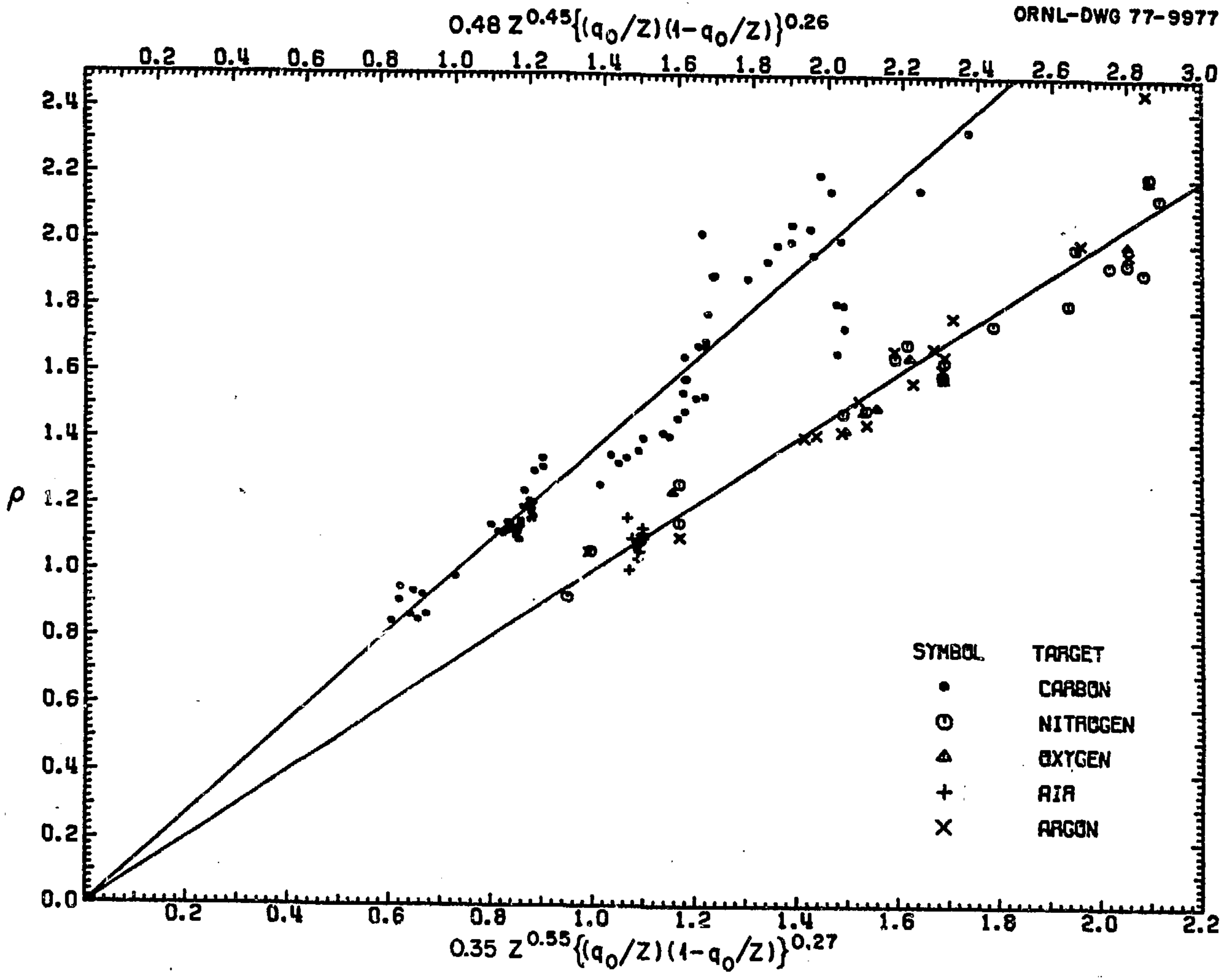
ORNL-DWG 77-9977
$0.48\,Z^{0.45}\{(q_0/Z)(1-q_0/Z)\}^{0.26}$
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
ρ
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
$0.35\,Z^{0.55}\{(q_0/Z)(1-q_0/Z)\}^{0.27}$
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
SYMBOL TARGET
● CARBON
ʘ NITROGEN
△ OXYGEN
+ AIR
× ARGON

FIGURE 4

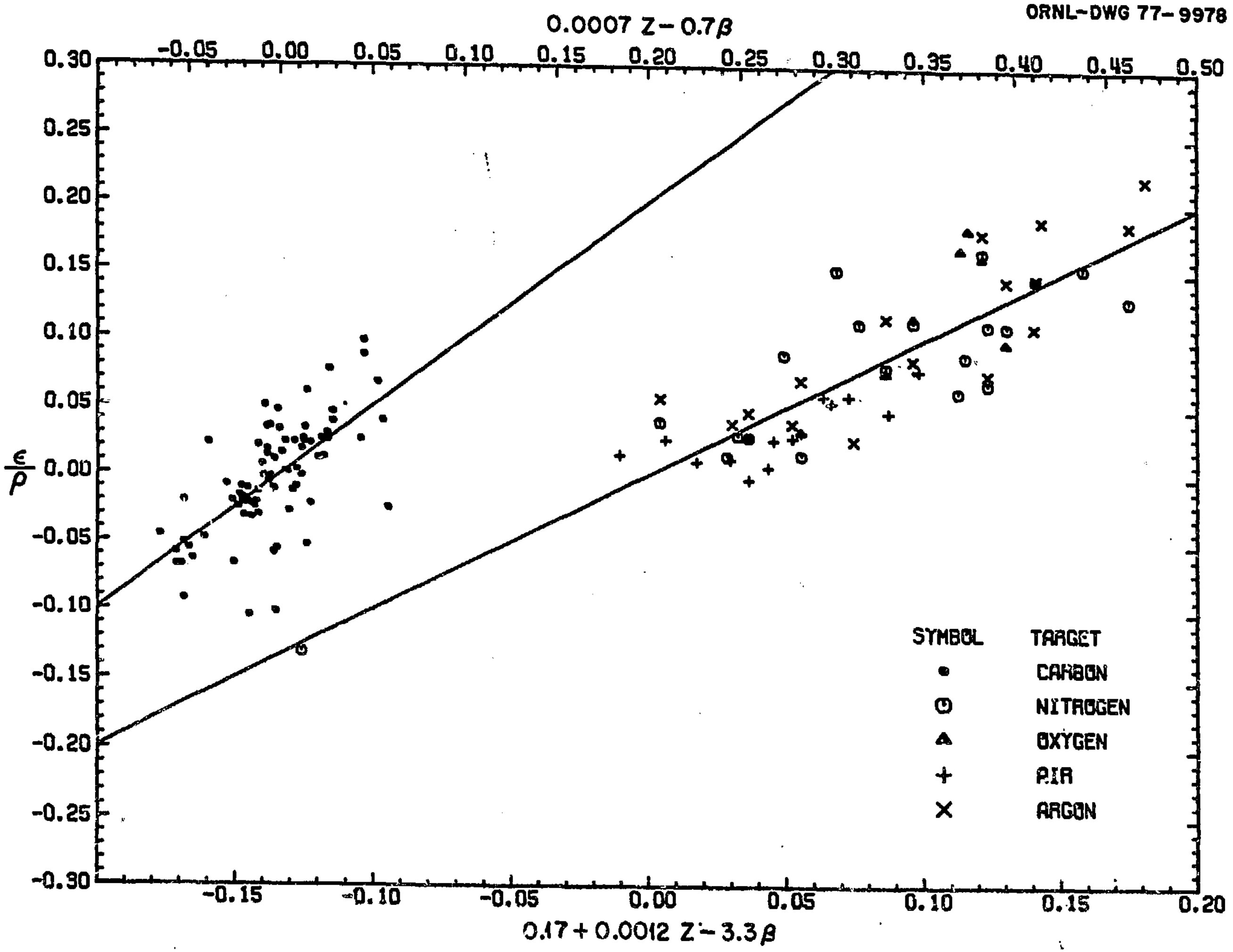
ORNL-DWG 77-9978
0.0007 Z − 0.7β
-0.05
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
ε/ρ
0.30
0.25
0.20
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
-0.25
-0.30
SYMBOL
TARGET
● CARBON
ʘ NITROGEN
▲ OXYGEN
+ AIR
× ARGON
-0.15
-0.10
-0.05
0.00
0.05
0.10
0.15
0.20
0.17 + 0.0012 Z − 3.3β

FIGURE 5

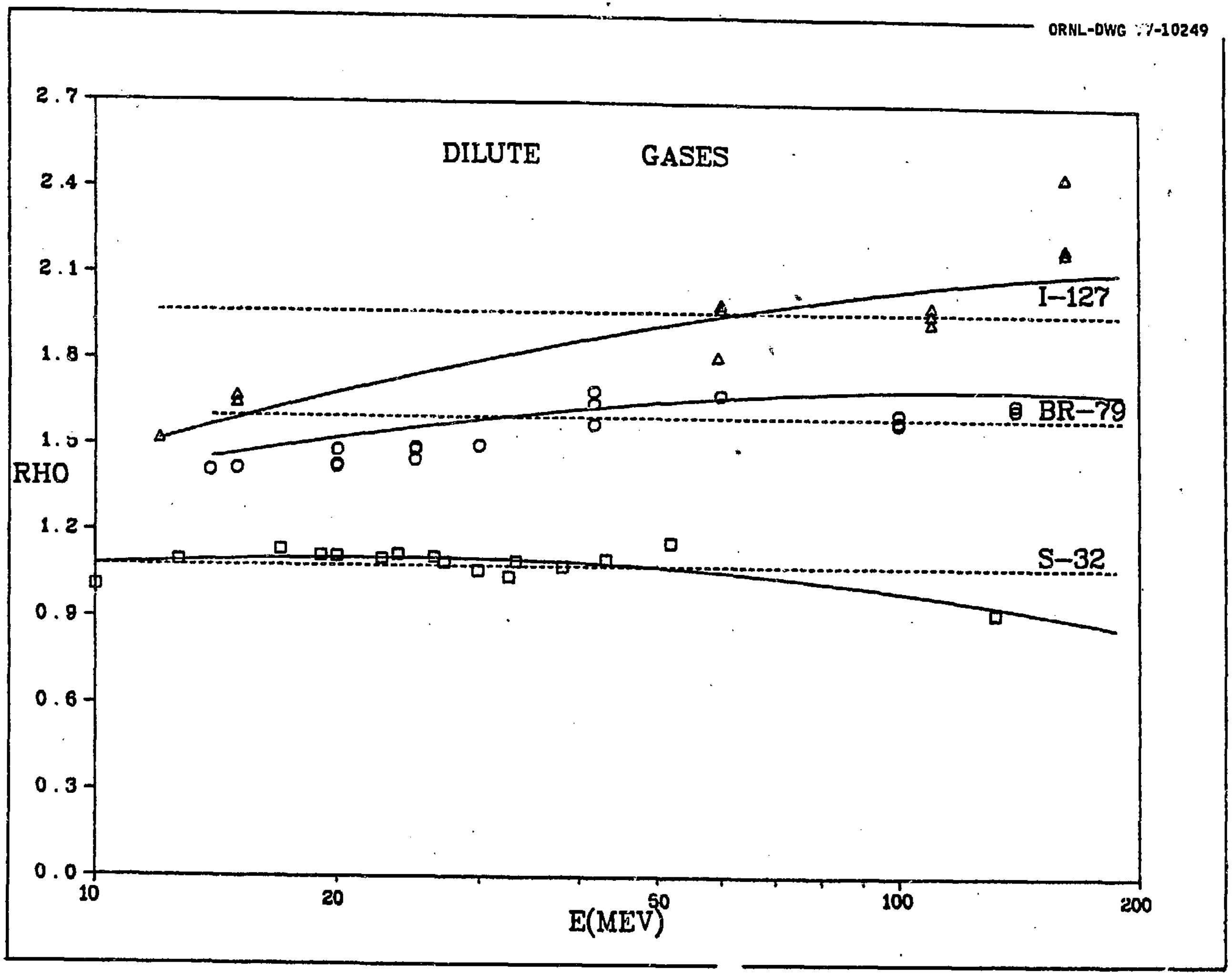
ORNL-DWG 77-10249
DILUTE GASES
I-127
BR-79
S-32
RHO
2.7
2.4
2.1
1.8
1.5
1.2
0.9
0.6
0.3
0.0
10
20
50
100
200
E(MEV)

FIGURE 6

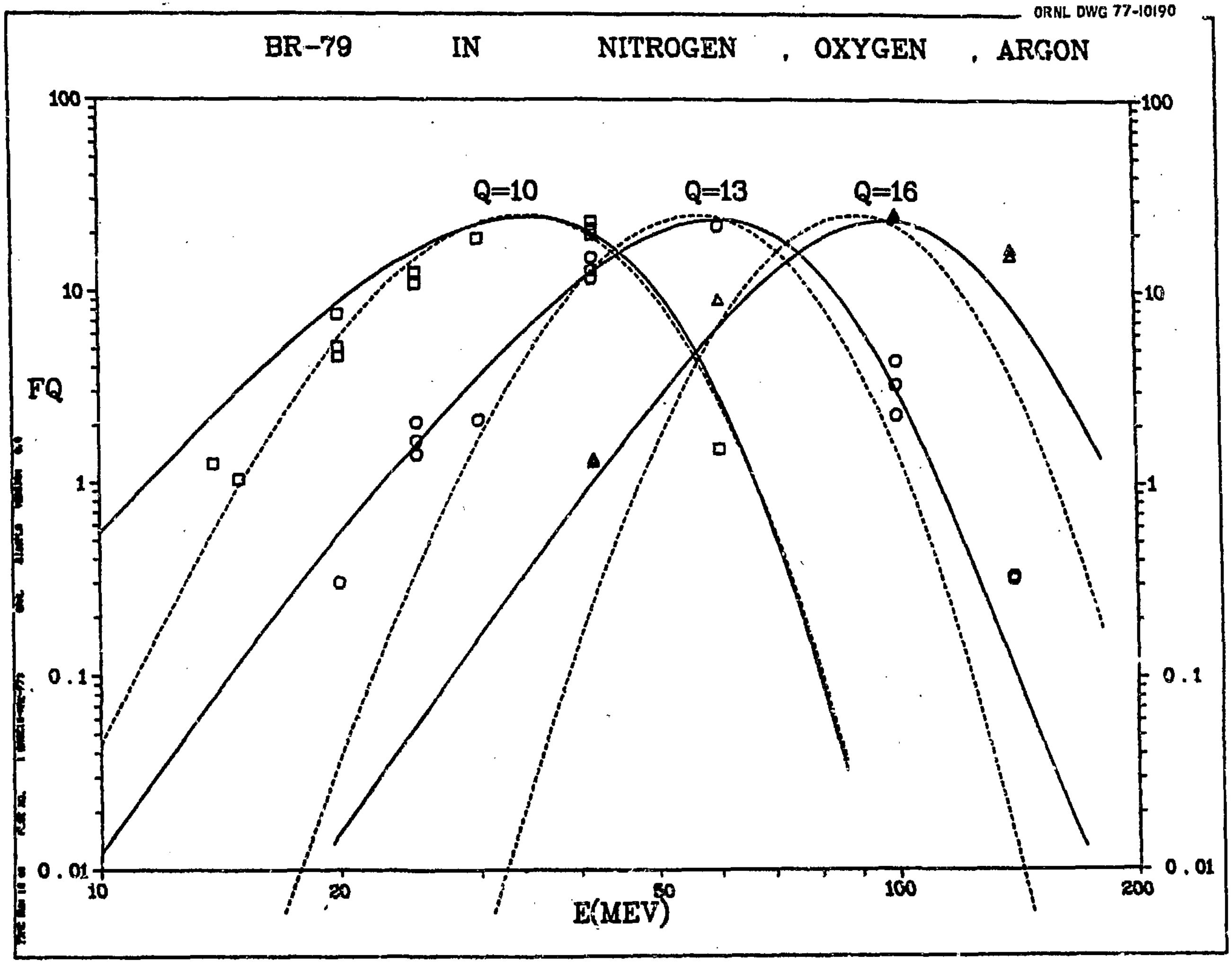
ORNL DWG 77-10190
BR-79 IN NITROGEN , OXYGEN , ARGON
Q=10
Q=13
Q=16
FQ
100
10
1
0.1
0.01
100
10
1
0.1
0.01
10
20
50
100
200
E(MEV)

FIGURE 7