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GENERALIZED SHAPED CHARGE DESIGN. ^{NTIS}

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CONTENTS

Symbols	iii
Abstract	1
Introduction	1
Constant Velocity Charge	4
Design	4
Experimental Setup and Results	10
Penetration	16
Calculation of Jet Characteristics	24
Convergence Effects	29
Introduction	29
Analysis of Convergence Effects	34
Example	37
Application to Nonsteady-State Jet Formation	39
Cone Angle and End Effects	40
Purpose and Design Characteristics	40
Experimental	43
Summary	51
References	52
Appendix A. Compression of Air in a Tandem Shaped Charge Stage	53
Appendix B. Calculation of Jet Properties	55

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SYMBOLS

V_0	liner velocity
α	cone half-angle
β	cone half-angle at time of collapse
V_2	liner velocity relative to a coordinate system moving with the cone collapse point
V_j	jet velocity
ℓ_j	jet length
ρ	density
r_0	outer liner radius
r_i	inner liner radius
d	liner thickness
R_0	outer explosive radius
r_t	target hole radius
r_j	jet radius
BHN	Brinell hardness
P	penetration
p	pressure
V'	velocity derivative $\partial V/\partial x$
δ	liner turning angle
ℓ	subscript ℓ refers to liner
$\dot{x}, \dot{y}$	velocity components in x and y directions
D	detonation velocity

GENERALIZED SHAPED CHARGE DESIGN

ABSTRACT

We have designed and tested a series of shaped charge devices. The tests have shown that by combining detailed Lagrange calculations of liner collapse with simplified incompressible flow models, we can tailor the explosive and liner contours of the charge to yield either constant velocity jets, stretching jets, or converging jets. Measured liner velocity was within 10% of theory, except for details near the apex and close to slide planes. We found that the effects of liner convergence and viscosity had to be included in the incompressible flow models to accurately correlate liner velocity, collapse angles, and jet velocity.

INTRODUCTION

We have examined, by calculation and experiment, two hypotheses of shaped charge warhead design:

- That the velocity gradient in a jet can be controlled
- That a second jet with a matched velocity profile can be inserted between the tail of the first jet and the slug

To test these hypotheses we have developed a method of calculating the design parameters of shaped charges. The method is based on

- A graphical summary — obtained from computed cylindrical implosions — of liner velocity as a function of explosive and liner thickness
- Hydrodynamic calculations of detailed liner velocity and collapse angle profiles
- Jet formation and penetration equations that relate the liner velocities and collapse angles to the resulting jet properties

Using this method, we can calculate jet properties, compare them to experimental properties, and then choose those properties best suited for target penetration.

In previous analyses of jet formation¹ it was assumed that:

- Liners were incompressible

- All parts of the liner had a constant velocity (V_0). (This implies that the liner is flat and that the liner's velocity does not change over time or distance.)
- The liner's velocity changed instantaneously from 0 to V_0
- The flow was completely fluid and nonviscous

A few years later, the analysis was corrected² to allow for a variable value of V_0 along the liner from apex to base. This correction made the calculation of liner collapse velocities, $V_0(x)$,^{3,4} consistent with the observed slug mass distribution in recovered specimens and with the corresponding jet velocity distribution.⁵ The experimental results in Ref. 5 also indicated that the first 4 cm of cone did not produce a noticeable amount of jet material.

Recent hydrodynamic calculations in Eulerian coordinates⁶ have determined all the parameters necessary to check the liner velocities obtained in Ref. 5. However, before we can make such a check, we need to correct the velocity of the liner for variations in thickness, because the computer study⁶ was done with a 1.9-mm-thick liner and the experiments⁵ with a 2.7-mm-thick liner. The correction — 0.4 km/s — was obtained by calculating the velocity vs charge to liner mass ratio of cylindrical charges.⁷ Figure 1 shows the raw data of Ref. 5 as well as the corrected experimental curve and the two theoretical curves. Note that the corrected experimental curve — obtained using jet-slug data — yields a higher value of V_0 than the Eulerian BRLSC (Ballistics Research Laboratory Shaped Charge) code calculations, even though the latter included some confinement.

No definite conclusion can be drawn from this comparison as to the exact value of V_0 because of inaccuracies in the calculations of Refs. 5 and 6. Possible errors in the experimentally inferred data have already been discussed.⁵ The disadvantage of the Eulerian method is that it often suffers from mass diffusion, leading to kinetic energy errors. Recent hydrodynamic calculations (HEMP code) — in Lagrangian rather than Eulerian coordinates — of the velocity of an unconfined charge also indicate somewhat lower velocities (Fig. 1) than inferred experimentally. This suggests that perhaps another reason exists for the discrepancy between the results derived from experimental data and those obtained theoretically. A probable cause is explained below as part of our experimental analysis of the jet formed by a constant velocity liner.

Fig. 1. Collapse velocity of an 86-mm standard precision shaped charge (COMP-B explosive).

CONSTANT VELOCITY CHARGE

Design

We have designed two conical shaped charges (hereafter designated designs I and II) to test the accuracy of the computational approach. We designed the charges in such a way that the collapse velocity (V_0) and the collapse angle (β) were nearly constant. To achieve this desired velocity profile, the 21° precision shaped charge was modified by

- Increasing the explosive diameter as axial distance from the cone apex increased
- Decreasing the liner thickness as the cone diameter increased

The design is shown in Fig. 2. Under ordinary circumstances the end of the copper liner would have a lower velocity than the rest of the liner, which would introduce undesirable gradients in V_0 and β . A simple way to eliminate such gradients is shown in the figure. The end of the liner is first cut with a tantalum wedge and then eliminated from the jet formation process by blocking its motion with a heavy steel ring. This procedure of eliminating end effects simplifies the jet profile analysis. In an attempt to increase the velocity of the tail of the jet further, we used a ring of lead powder to increase the liner velocity close to the cutter (see experimental shot 5).

The liner dimensions for the first shaped charge are summarized by the following equations. The outer surface of the liner is defined by

$$r_o = 13.35 + (x - 52.18) \cdot \tan 21^\circ,$$

where $52.18 < x < 180$ mm and r is in mm.

The outer surface of the spherical tip is defined by

$$r_o = [(14.30)^2 - (57.3 - x)^2]^{1/2}$$

for $52.18 > x > 43.0$ mm.

The inner surface is defined by the three equations:

$$r_i = 19.2 + (x - 75) \cdot \tan 22^\circ$$

for $180 > x > 75$ mm

$$r_i = 19.2 - (75 - x) \cdot \tan 21^\circ$$

for $75 > x > 53.14$ mm

Fig. 2. Constant velocity shaped charge design.

$$r_i = [(11.6)^2 - (57.3 - x)^2]^{1/2}$$

for $53.14 > x > 45.7$ mm.

Note that the cutter is positioned at $x = 150$ mm.

The explosive contour depends on the dimensions of the liner: The inner radius should match that of the copper liner but the outer radius depends on the desired liner velocity profile. For the standard unconfined shaped charge the outer radius of the explosive is 43 mm. Given that radius, the collapse velocity of the liner will gradually increase from the cone tip to a point near $x = 75$ mm. This point was estimated using a graphical summary of implosion velocities⁷ (Fig. 3). The summary was then used to calculate the increase in explosive diameter required to maintain a constant implosion velocity.

The resulting contour is given by

$$R_o = 43.15 \text{ mm}$$

for $0 < x < 99.84$ mm

$$R_o = 43.15 + (x - 94.84) \cdot \tan 17.5^\circ$$

for $94.84 < x < 180$ mm

Since the graphical summary used in the above calculation is based on cylindrically symmetric implosions, the effect of the axial degree of freedom needs to be checked on a two-dimensional (2D) Lagrange code. Figure 4 shows the grid (top) used for the calculation and a partly collapsed configuration (bottom). The calculation yields the implosion velocity and the collapse angle of the liner (Fig. 5). Note that Fig. 5 shows a maximum implosion velocity at $x_0 = 85$ mm. At smaller values of x the small radius of the liner limits the acceleration time and therefore the final kinetic energy. The rest of the profile is more easily controlled by the explosive thickness. For values of x_0 above 85 mm the velocity was constant within $\pm 5.5\%$.

The graphical summary in Fig. 3 was now used to calculate a correction term, which, when added to the first explosive contour, improved the velocity uniformity of the liner. The resulting second contour (design II) is defined by

$$R_o = 43.15 \text{ mm}$$

for $0 < x < 75$ mm

Fig. 3. Liner velocities as a function of explosive thickness, liner mass, and liner radius (obtained from cylinder implosion calculations).⁷

Fig. 4. Two frames of the HEMP grid used in the design calculations.

Fig. 5. Velocity (V_0) and collapse angle (β) profile for the liner of design I.

$$R_0 = 43.15 + (x - 75) \cdot \tan 13^\circ \text{ mm}$$

for $75 < x < 180 \text{ mm}$.

The Lagrange calculations of this second design resulted in the velocity (V_0) and collapse angle (β) profiles of Fig. 6. As can be seen in Fig. 6, both the V_0 and β profiles are more constant in this design than in the first shaped charge design. Note, however, that the velocity profile need not be completely uniform to yield a constant velocity jet because, if the copper jet is solid, it would have some material strength, thereby providing a mechanism to resist variations in the jet axial velocity. Furthermore, since the copper can be expected to work-harden during jet formation, the jet may have a fairly high strength. Unfortunately, the rate at which the hot copper jet is annealed may be so high that an estimate of its strength would be difficult. Comparison of the results obtained from the two designs, however, should yield some insight concerning the effect of material strength, and, therefore, some insight into how much care is required in the design.

Experimental Setup and Results

The experimental configuration is shown in Fig. 7. Both top and side views are shown. The jet was fired into a stack of 215 BHN (Brinell hardness) 1-in. steel plates. The jet shape and velocity were diagnosed with three separate x-ray shadowgraphs. Two charges of the first design and three charges of the second design were fired, with minor modifications in the setup. The results from different tests of the same design are identical, except for the minor differences shown by the shadowgraphs in Figs. 8 and 9. Table 1 summarizes numerical values from the shadowgraphs.

The shadowgraphs in Fig. 8 were obtained with design II, which yielded the more uniform implosion velocity profile. Four features should be noted:

- The tip of the penetrator is moving slightly faster than the main body of the penetrator. This part of the penetrator is produced by the part of the cone closest to the apex ($x_0 < 85 \text{ mm}$; see Fig. 2), where large collapse velocity and angle gradients produce fluctuations in jet velocity. The apex must therefore be designed with greater care to control such gradients.

Fig. 6. Velocity (V_0) and collapse angle (β) profile for the liner of design II.

Fig. 7. Top and side view of the experimental configuration used for design I and II shots.

Fig. 8. Uniform velocity jets of design II. The top figure (a) does not have a lead shot ring around the cone base (the other three do).

Fig. 9. Features of the shaped charge jet and jet-slug materials: (a) Jet slug interfacial material; (b) Slug material; (c and d) Jet and jet slug material.

Table 1. Shot data summary.

Test No.	Design No.	Charge No.	Explosive weight, kg	Target thickness, m	Distance from face of explosive to target (x), mm	X-ray times ^a measured on three shadowgraphs, μ s			Jet velocities	
						1	2	3	tip, km/s	rear, km/s
1	II	2	—	0.30	410	53.6	73.4	153.7	7.586	7.268
2 ^b	II	3	1.5	0.33	384	53.5	73.4	153.8	7.512	7.332
3 ^c	I	5	1.4	0.33	383	53.5	73.3	153.7	7.515	6.566
4	I	4	1.4	0.33	377	53.6	73.4	401.2	7.611	6.505
5 ^d	II	1	1.45	0.33	377	53.7	73.3	401.0	7.658	7.403

^aZero-time to x-ray burst includes about 4.2 μ s of system time and detonator function time.

^bLiner flushed with helium.

^cVacuum in liner.

^dAddition of 1000 g of lead tamper.

- The apex design also causes the halo of particles surrounding the penetrator. The diameter of the halo, if plotted against x, extrapolates to a point close to the cone tip, indicating that it is indeed the source of the halo. Such a halo could interfere with the cone collapse in staged systems.
- The main body of the penetrator is not stretching or contracting and therefore represents the uniform velocity jet that was the goal of this design.
- The interface between the penetrator and the slug does not break until it is stretched to a considerable length. One attempt (Shot No. 5) at increasing the rate of necking (narrowing) at the rear of the penetrator was only marginally successful. Note that at about 73 μ s the rear end of the penetrator necks slightly faster in shot 5 than shot 1. The effect is much too late, however, to be helpful in staged designs where a space between jet and slug would facilitate inserting a second liner without interfering with the jet of the first stage.

In Fig. 9, note that

- The interfacial material does not break up until the diameter is reduced to about 3 mm. Larger fragments are formed near the slug, where the strain rate is slower.

- Shot 4, at 401.2 μs , shows the front end of the slug separated from the rest. This is presumably due to the tension produced by the slug-jet velocity gradient. The rest of the slug is partly hidden among the fragments of the steel ring that had held the liner cutter. This ring broke up in shots 3 and 4, but not in shots 1, 2, and 5, which had the thinner explosive of design II. (The rings of shots 1, 2, and 5 were recovered after the shots.)
- The umbrella-like feature in shot 3 (it also appeared in shot 4 although it cannot be seen in Fig. 9) is produced by the comparatively large increase in liner velocity near the liner cutter. The increase in velocity of the last part of the jet caused it to penetrate itself. The breakup of the metal at the fringe of the umbrella suggests that the fragments will ultimately look like those in the halo that is attached to the tip. Therefore, the formation mechanism of the halo is probably similar.
- The effect of air, which is compressed by the liner, is apparently very small. Calculations described in Appendix A indicate only 1 to 2 kbars of air pressure at the contact point between jet and liner. Experiments showed no noticeable effect on jet diameter or jet velocity from changes in the atmosphere in the cones (see Table 1). However, the effect of air drag on the shape of the halo is noticeable. Compare vacuum shot 3 at 53 and 73 μs with shot 1 (or 5) at 54 and 73 μs (Figs. 9 and 8 respectively).
- The breakup of the halo fragments (Fig. 9) in shot 3 at 73 μs is due to collision with the lucite wall of the vacuum chamber.

Penetration

Table 2 summarizes the data on jet penetration of a series of 240 BHN steel plates (Figs. 10-14). The calculations needed for the comparisons in Table 2 follow. The jet from shot 1 has a length of 77 mm (ℓ_{T1}) when its center (x_1) is 106 mm from the cone base at $T_1 = 53.6 \mu\text{s}$. Half the jet has penetrated at

$$T = 53.6 + \frac{x \text{ (explosive to target)} - x_1 \text{ (jet to explosive)} + \text{penetration at } T}{\bar{v}_j}$$

$$= 53.6 + \frac{(410 - 106 + 37) \text{ mm}}{7.27 \text{ mm}/\mu\text{s}} = 100.5 \mu\text{s}.$$

Fig. 10. Target hole profile, shot 1.

Fig. 11. Target hole profile, shot 2.

Fig. 12. Target hole profile, shot 3.

Fig. 13. Target hole profile, shot 4.

Fig. 14. Target hole profile, shot 5.

Table 2. Calculated and measured penetration parameters.

Shot No.	T_1 , μs	x_1 , mm	ℓ_{T_1} , mm	ℓ_T , mm	P_c/P_m ($P_c/P_{m,tot}$)	Diameter (d_T)			$(d_t/d_j)_m$	$(d_t/d_j)_c$
						Jet mm	Target hole mm	Jet parts		
1	53.6	106	77	91	~0.97 (0.37)	5.7	~40	Tip	~7	8.7
						3.1	~27	Neck	~9	8.5
						7.5	~56	Tail	~7.5	8.3
2	53.5	106	81	89	~0.95 (0.33)	5.8	~38	Tip	~6.5	8.3
						3.7	~29	Neck	~8	8.5
						8.0	~52	Tail	~6.5	8.4
3	53.5	101	88	138	~0.83	5.2	~32	Tip	~6	8.6
						3.0	~20	Center	~7	8.0
4	53.6	104	87	142	~0.93	5.3	~33	Tip	~6	8.7
						2.9	~15	Center	~5	8.1
5	53.7	106	80	91	~0.97 (0.35)	~5.5	~38	Tip	~7	8.8
						~3.3	~25	Neck	~7.5	8.6
						7.1	~57	Tail	~8	8.5

Legend

- T penetration time
- T_1 x-ray time
- P penetration length (subscript c: calculated; m: measured)
- d diameter (subscript t: target; j: jet)
- ℓ jet length
- x_1 jet to explosive distance at T_1

The jet length (ℓ) at that time is

$$\ell_T = 77 + (7.586 - 7.268) (100.5 - 53.6) = 91 \text{ mm}$$

and the expected penetration (P_c) — assuming incompressible flow — is

$$P_c = \ell \sqrt{\rho_j / \rho_t} = (91) (1.07) = 97 \text{ mm},$$

where ρ_j is the density of the jet and ρ_t is the density of the target. This number is essentially the observed length of that portion of the hole in Fig. 10 that has a large diameter. The penetration length cannot be accurately established because a considerable amount of additional penetration is produced by the stretched jet-slug interfacial material. The additional penetration, as shown below, was absent in targets penetrated by jets of design I.

The hole diameter may also be calculated from the energy of the jet⁸ as long as the coupling efficiency is velocity independent. The calculation is made assuming an energy balance of

$$\frac{1}{2} (\rho_j \cdot \pi \cdot r_j^2 \cdot d\ell) v_j^2 = C (\pi \cdot r_t^2 \cdot dP)$$

where subscript j refers to the jet and t to the target; the value of $C = 3200 \text{ J/cm}^3$ for 240 BHN steel,⁸ and $dP = (\rho_j/\rho_t)^{1/2} d\ell$. Therefore,

$$r_t/r_j = \left(\frac{1/2 \rho_j v_j^2}{C (\rho_j/\rho_t)^{1/2}} \right)^{1/2} = 8.7 \text{ for the tip and } 8.3 \text{ for the rear.}$$

Figures 10 and 8 indicate experimental values (r_t/r_j) of 7, 7.5, and 9 for the jet tip, the maximum diameter near the tail, and the narrow necked area, respectively. This seems to indicate a somewhat lower efficiency for this jet, if indeed it is not hollow.

The results of the above calculations and those for shots 2 through 5 are summarized in Table 2. Two values for the ratio of calculated penetration (P_c) to measured penetration (P_m) are listed. The denominator is either the length of the large part of the hole (P_m) or the total length $P_{m,tot}$. The former value is only an approximation of how far targets are penetrated by uniform velocity jets, but it does demonstrate the validity of the computational procedure used in the design process. The additional penetration caused by the jet-slug interfacial material means that the value of $P_c/P_{m,tot}$ is small. As stated earlier, additional penetration was not seen in holes produced with the first design. The lateral expansion near the tail of the jet apparently destroyed the penetration capability of the thin stretching material between the jet and the slug. A ring of damage is noticeable in Figs. 12 and 13. The fragments ejected by or reflected from that area could have damaged the coherence of the interfacial material enough to destroy its ability to penetrate. The uncertainty in determining the exact length of the jet that effectively penetrates the target is the principal cause of error when we calculate the theoretical penetration (P_c) term for targets penetrated by jets of design I.

The variation of hole diameter follows the shape of the jet qualitatively. The quantitative correlation in Table 2 shows that the measured hole diameters tend to be smaller than those expected theoretically for all except the section penetrated by the necked portion near the tip of the jet (shots 1, 2, and 5). The discrepancy may be caused by

- Some of the jet energy being lost between the bent plates (Figs. 10-14)
- The jets being hollow or porous. The analysis given below indicates that conservation of momentum requires a smaller jet radius than observed.

Calculation of Jet Characteristics

Some of the basic assumptions in the usual calculations of jet formation can now be checked with the Lagrange calculations of liner collapse. These calculations include the exact pressure history that accelerates the liner and a full material description of the equation of state and strength of the liner material.

The liner configurations (Fig. 15) show that the criterion of rapid acceleration of the liner is satisfied approximately. This figure shows the liner of design II at time = 0 and its center line at 22 μ s after initiation of the octol explosive. The collapse angle of 33.3° is close to, but not exactly equal to, the value obtained from the standard liner deflection equation:

$$\beta = \alpha + 2 \sin^{-1} \frac{V_0}{2D} = \alpha + 2 \delta ,$$

where V_0 and D are the liner and detonation velocity. The insert on top shows that the particle direction and the collapse angle are not related as simply as the above equation suggests, although the approximation is reasonably good. The value of δ may also be calculated from either the collapse angle β or the direction of particle motion (as shown in the insert) if the acceleration is rapid and the flow nonviscous. The difference is actually finite and the above liner deflection equation yields an intermediate value of δ . The apparent deflection angle calculated from the particle motion direction increases as the liner approaches the axis. This effect is due to the stresses transmitted from the stagnation point. The variation in the magnitude of δ is about 0.5°. However, near the base of the liner — where velocity gradients are somewhat larger — it increases to about 0.7°. This deviation from the standard liner acceleration equations has only a minor effect (less than 1%) on the jet characteristics calculated with the standard equations. The values of δ (from particle motion), β , and V_0 may be taken from the Lagrange calculations to determine the jet characteristics. The value of δ is set equal to

$$\delta = \sin^{-1} V_0/2D + 0.5^\circ$$

in agreement with the observation in Fig. 15. The values of V_0 and β are summarized in Figs. 5 and 6. The equations used and the results are presented in Appendix B.

Fig. 15. Configuration of the liner before acceleration ($T = 0$); its center line after acceleration ($T > 0$); and various measures of the turn angle (δ) vs position along the liner (insert).

The calculated jet velocities may now be compared to the measured results. Figure 16 shows a comparison for design I. We did not use the common origin approximation here because the liner collapse locus (dashed line) was explicitly calculated. The solid lines radiating from this collapse locus show the initial particle velocities of the jet. The slug particle tracks, which have a slope of about 0.5 km/s, were not included. The velocity variation in the jet at time of formation is clearly evident — and so is the effect of the design change if we compare Fig. 17 to 16. Two noteworthy features of this presentation are

- The rear of the jet in Fig. 16 has a relatively high convergence rate (also suggested by Fig. 5), which results in a laterally expanding tail. Clearly, such a convergence effect is to be avoided in an efficient shaped charge device, so it was eliminated in design II (Fig. 17). The penetration depths observed showed that this flared section destroyed the penetration capability of the following jet-slug interfacial material.
- The jets are moving faster than indicated by the calculated tracks. The discrepancy appears to be outside the combined uncertainty of the calculations and the experimental measurements. Thus, the question of liner convergence effects on jet formation is discussed below.

Liner convergence effects may also change the calculated jet mass. The x-ray shadowgraphs of a jet from design I (shot 3) and one from design II (shot 1) were integrated graphically with the assumption that the jets were perfectly cylindrical. The jet mass obtained from the measured volume of design II (26 ± 2 g) is in reasonably good agreement with the calculated mass (27 g) of jet properties (Appendix B). The uncertainty in the comparison is due both to the uncertainty in jet radius and to uncertainties in the behavior of the liner near the apex between 7 and 5 cm (see Figs. 2, 5, and 6). The shadowgraph of design I was integrated between jet tip and the flaring section because of the difficulty in determining the mass in the flared section. In this case, therefore, the shadowgraph mass (11 g) was considerably less than that obtained from the calculations (24 g), indicating that about half of the mass was lost in the flared section. This result is qualitatively consistent with the particle trajectories in Fig. 16. Within these rather large uncertainties the jet mass observed is therefore consistent with the calculated values. To determine any effects of convergence and jet mass, more accurate

Fig. 16. Jet configuration and calculated particle tracks for design I.

Fig. 17. Jet configuration and particle tracks calculated for design II.

measurements must be made on a constant velocity jet. The smaller than expected hole diameters produced by these jets (Table 2) and the calculations presented in the next section, however, suggest that it is incorrect to use normal copper density, or jet volumes determined from jet radius, in the mass calculations.

CONVERGENCE EFFECTS

Introduction

The higher jet velocity observed in our experiments is a plausible effect of convergence. Since the leading portion of the liner, which forms the jet, speeds up during convergence and the trailing portion slows down, an increase in jet velocity is quite possible, even though the local β of the leading portion is increased. One might first conceive that the jet would have more energy because the slug would be moving more slowly and would have somewhat less energy. The slug's total energy, however, is only about 3% of that of the liner while the jet's is roughly 97%. A small change in this 3%, therefore, would have almost no effect on the energy of the jet. By conservation of energy, the jet energy cannot increase. The calculations that follow show that the slug velocity actually increases slightly from convergence effects.

The above information indicates that the effects of convergence must be addressed in the proposed program. Accurate tests of uniform jets provide the most effective way of determining these effects. An analytical approach to the problem is now presented. These experiments gave a jet velocity which was almost uniform: A jet about 6 cm long was moving at a constant velocity. This implies that one can go back and use the steady-state equations for the jet formation and determine what the jet velocity might be. In this case the implosion calculations showed an almost uniform implosion velocity of 0.20 cm/ μ s. Using the analytical equations, we calculated jet velocity to be 0.66 cm/ μ s. The average observed velocity, however, was 0.74 cm/ μ s.

A rough analysis was made by applying the three conservation laws: mass, energy, and momentum. The liner was split in two parts. Namely, that part which became the jet and that part which became the slug. To conserve energy, the two parts of the liner would have the same total kinetic energy as the initial liner. Because the inward radial momentum did not have to be conserved and the axial momentum did, the delta angles of the two parts of the liner could be different but the sum of the two axial momentums had to be the same as

the initial axial momentum of the liner. Two-dimensional calculations of implosions indicated that the δ angle, although it did increase slightly during the implosion, was essentially the same for the leading and trailing portion of the liner. This will be assumed in the model. Obviously, the sum of the masses of these two components must equal the mass of the original liner. We can now apply the conservation laws to the formation of the jet by the inner part of the liner and the formation of the slug by the outer part of the liner (see insert, Fig. 22). This gives five equations and seven unknowns. By assuming that the jet and liner formation is incompressible and that the velocity of the point of collapse is not changed appreciably by convergence effects, we can derive two more equations. It turns out that the two are redundant with respect to each other, so only one of them can contribute to the solution. This means there are six equations and seven unknowns. One more equation is required to provide a solution.

A study of a one-dimensional implosion of cylindrical liners was made to provide an additional equation. Let us now look at some 1D calculations of an imploding cylinder. Figure 18 shows plots of the calculated velocity profile of four copper liners just before the leading edge reaches the axis. They each have a different implosion velocity. Although some of their kinetic energy has been converted to internal energy by this time, (i.e., close to the time of collapse), the ratios of the velocity of the leading portion to the average are roughly the same. Figure 19 shows the cumulative average velocity per increment of the liner. For example, the average velocity of the first 7% of the liner is found at the intersection of an ordinate through the abscissa at 0.07 and the appropriate velocity curve. Let us assume that some fraction of the mass of the liner (m_ℓ) forms a jet with mass (m'_j). This fraction would be m'_j/m_ℓ . Let us also assume that the maximum implosion velocity of this fraction of the liner is called V_0 . Based on the data from Fig. 19, we can determine the ratio of V_{0j} to the average implosion velocity of the liner (V_0) vs the ratio m'_j/m_ℓ (Fig. 20). For comparison, Fig. 20 also shows some data from other implosion calculations having higher values of V_0 . Note that the data are not highly dependent on the value of V_0 . These curves were calculated for copper liners. If the liner material had a much higher compressibility, the data would probably depend more on V_0 . For metal liners, however, a single curve is probably adequate. The fact that the convergence effects are not highly dependent upon V_0 also indicates that they are not highly dependent on the

Fig. 18. Velocity profile of liner thickness at the time of collapse.

Fig. 19. Average velocity per liner increment at the time of collapse.

Fig. 20. Data from calculation of implosion of metal cylinders.

collapse angle β , as long as β does not get too large. The mathematical fit to this curve is

$$\frac{v_{0i}}{v_0} = \frac{1}{\left(0.12 + \frac{m'_j}{m_\ell}\right)^{2/5}}.$$

With this formula, we can now solve the equations that determine the convergence effects. The equations are presented below.

Analysis of Convergence Effects

Figure 21 shows the known parameters for a steady-state implosion and the predicted jet and slug parameters when convergence effects are not included. Figure 22 shows how this implosion is to be broken into the part that forms the jet and the part that forms the slug. The following equations would apply:

$$m_j + m_s = m_\ell \quad (\text{conservation of mass}) \quad (1)$$

$$\frac{m_\ell}{2} v_0^2 = \frac{m'_j}{2} v_{0j}^2 + \frac{m'_s}{2} v_{0s}^2 \quad (\text{conservation of energy}) \quad (2)$$

$$m_\ell v_0 \sin(\alpha + \delta_0) = (m'_j v_{0j} + m'_s v_{0s}) \sin(\alpha + \delta') \quad (3)$$

(conservation of axial momentum)

$$\frac{m_\ell}{2} v_0^2 = \frac{m'_j}{2} v_j'^2 + \frac{m'_s}{2} v_s'^2 \quad (\text{conservation of energy}) \quad (4)$$

$$m_\ell v_0 \sin(\alpha + \delta_0) = m'_j v_j' + m'_s v_s' \quad (\text{conservation of axial momentum}) \quad (5)$$

$$v_j' = v_1 + \sqrt{v_{0j}^2 + v_1^2 - 2 v_{0j} v_1 \sin(\alpha + \delta')} \quad (6)$$

(incompressibility of the liner)

$$v_s' = v_1 - \sqrt{v_{0s}^2 + v_1^2 - 2 v_{0s} v_1 \sin(\alpha + \delta')} \quad (7)$$

(incompressibility of the liner)

$$\beta = \alpha + 2\delta_0$$

$$V_l = V_0 \frac{\cos \delta_0}{\sin (\alpha + 2\delta_0)}$$

$$V_j = V_0 \frac{\cos \frac{\alpha}{2}}{\sin \left(\frac{\alpha}{2} + \delta_0 \right)}$$

$$V_s = V_0 \frac{\sin \frac{\alpha}{2}}{\cos \left(\frac{\alpha}{2} + \delta_0 \right)}$$

$$m_j = m_l \sin^2 \left(\frac{\alpha}{2} + \delta_0 \right)$$

$$m_s = m_l \cos^2 \left(\frac{\alpha}{2} + \delta_0 \right)$$

Fig. 21. Steady state implosion of a cone and the predicted jet parameters. The assumption is made that there are no convergence effects.

Fig. 22. Steady state implosion of a cone with the liner considered as two elements.

$$\frac{V_{0j}}{V_0} = \frac{1}{\left(0.12 + \frac{m_j}{m_\ell}\right)^{2/5}} \quad (8)$$

Equations (6) and (7) come from the solutions of the triangles shown in Fig. 23. There are seven unknowns (V_{0j} , V_{0s} , δ' , m'_j , m'_s , V'_j , and V'_s) and eight equations. As mentioned above, either Eq. (6) or (7) must be considered redundant. Equation (7), therefore, will be eliminated.

Example

Let us now test this approach against some experimental data. Design II gave a uniform implosion velocity of $V_0 \approx 0.20$ cm/μs. The liner mass per unit length — m_ℓ — was almost constant at 42.5 g/cm. Given this data, the steady-state equations of jet formation can then be used to predict the jet formation parameters. The known parameters and those predicted for nonconvergent jet formation are:

D (octol)	= 0.84 cm/μs
α	= 21°
V_0	= 0.20 cm/μs
m_ℓ	= 42.5 g/cm
δ_0	= 6.8°
β	= 34.6°
V_1	= 0.350 cm/μs
V_j	= 0.661 cm/μs
V_s	= 0.038 cm/μs
m_j	= 3.76 g/cm
m_s	= 38.74 g/cm
jet energy/cm	= $\frac{1}{2} (3.76 (0.661)^2 = 0.821$ energy units (e.u.)/cm

Now, using the same values of V_0 , m_ℓ , δ_0 , and V_1 and the seven equations given above, we can calculate the jet parameters, including convergence:

V_{0j}	= 0.386 cm/μs
V_{0s}	= 0.177 cm/μs
	= 7.99°
V'_j	= 0.725 cm/μs (observed value = 0.74 cm/s)
V'_s	= 0.044 cm/μs
m'_j	= 3.09 g/cm (observed value = 2.9 to 3.89 g/cm)

$$v_j = \sqrt{v_{0j}^2 + v_1^2 - 2v_{0j}v_1 \sin(\alpha + \delta')}$$

$$v_{j'} = v_j + v_1$$

$$\beta = \sin^{-1} \left(\frac{v_{0j} \cos(\alpha + \delta')}{v_j} \right)$$

$$v_s' = v_s - v_1 \quad v_s = \sqrt{v_{0s}^2 + v_1^2 - 2v_{0s}v_1 \sin(\alpha + \delta')}$$

$$\beta_s = \sin^{-1} \left(\frac{v_{0s} \cos(\alpha + \delta')}{v_s} \right)$$

Fig. 23. Derivation of Eqs. (6) and (7).

$$\begin{aligned} m'_s &= 39.41 \text{ g/cm} \\ \text{jet energy/cm} &= \frac{1}{2} (3.09) (0.725)^2 = 0.812 \text{ e.u./cm} \end{aligned}$$

The model predicts that jet velocity is increased but that its mass is decreased. The jet energy, however, should remain almost unchanged (it does, theoretically, drop by ~1%). It should be emphasized that the observed jet mass is fairly uncertain (see below). The jet velocity measurement was accurate; however, it was higher than the predicted value. It could be that the curve in Fig. 20 is higher than the calculations indicated. It is also possible that the experimental value of V_0 for the liner was actually higher than assumed. It is also possible that the splitting up of the liner from one mass having one velocity to two masses having two average velocities is still a crude approach that can only approximate reality. Another possibility is that the assumption of incompressibility of the liner, particularly for the portion which forms the jet, can lead to inexact results. At any rate the approach presented above; i.e., one that includes a first-order correction for convergence, does give predictions that approach experimental results. Work on the refinement of this model is in progress.

Application to Nonsteady-State Jet Formation

The equations developed above were applied to an experiment in which a steady-state jet was formed. We can also apply them to nonsteady-state jets. The equations themselves are still valid. It is the parameters that are fed into them that are dependent on whether or not the jet formation process is steady state. As can be seen from the equations, the input data are as follows: α , m_ℓ , δ_0 , V_0 , and V_1 .

The values of α , m_ℓ , δ_0 and V_0 for a given element of the liner are straightforward. However, the value of V_1 — the rate at which the collapse point of a given element moves along the axis — is a complicated function of the way in which parameters such as α , δ_0 , and V_0 may be varying with the axial dimension x . The nonsteady-state value of V_1 is given by

$$V_1 = V_0 \frac{(\cos(\beta - \alpha - \delta_0))}{\sin \beta}.$$

It is necessary, therefore, to calculate the true collapse angle, β , in order to calculate V_1 . The formulas for calculating the collapse angle β for the most general cases of nonsteady-state jet formation are given in Ref. 7.

Having calculated β for the liner element of interest (i.e., the collapse angle if the collapse was that of two plates), we can then go ahead with the procedures outlined above to determine the jet and slug parameters for a convergent liner.

CONE ANGLE AND END EFFECTS

Purpose and Design Characteristics

Further experiments have been done to investigate

- The effect of decreasing the cone angle on jet stability and breakup
- The effect on jet shape of joining two liner sections with a small separation into one jet stream

The first effect will lead to criteria for optimizing shaped charges at specific standoffs (distances from target). While jet stability and elongation to breakup should be maximized for long standoffs, this is not essential for short standoffs. In fact, it may be undesirable to maximize stability since smaller cone angles imply more total jet energy.

The second effect relates to the perturbations produced by free liner edges in the termination and start-up of jet formation. These effects change β so as to reduce the jet energy density at the end of the jet. They may also increase the energy at the start-up point, or jet tip. Unless compensated for in the design, these effects could cause undesirable interactions.

The design method was the same as described above. Four significant changes were made in the cone design (Fig. 24):

- The cone angle was changed from $\alpha = 21^\circ$ to $\alpha = 16.5^\circ$.
- The apex design was changed to eliminate the spray observed in Figs. 8 and 9.
- The explosive was changed from octol to a 90% HMX:10% Viton explosive called LX-07.
- The base of the cone was modified to change the angle of implosion discontinuously at about 160 mm from the apex. On selected shots liner cutters of a tungsten alloy ($W:N_i:Fe = 94/4.2/1.8$) were used.

The velocities and angles from the HEMP output were used to calculate the jet velocity (V_j) and the associated quantities β , V_0 , and V_2 . Figure 25 shows the calculated jet characteristics for an assembly that is designed to yield a constant velocity jet from the major portion of the cone. The jet velocity

Fig. 24. Design of 16.5° cone with clipper and alignment fixture.

Fig. 25. Summary plot of constant velocity jet design calculations.

near the cone apex oscillates slightly because of the details in the apex design. Refined apex designs should be considered in future work. Note that β is quite small near the apex, at the point where explosive and liner form a V-shaped void. The calculations showed this to involve a small amount of liner that should not affect the major portion of the jet formed by the rest of the liner.

At the point where the liner angle changes from 16.5° to 20.50° , the calculation shows a dip in the β curve and a spike in V_j . This is due to stretching of the liner caused by the sharp change in the collapse angle at that point. Our experiments indicate that this effect can produce a physical separation between the jets produced by the two sections of liner on both sides of the angle change.

The calculated jet parameters in Fig. 26 are for a design that allows the jet to stretch. The only difference between this design and the previous one is that the explosive is thinner near the cone base. The dashed line in Fig. 26 forms the outer explosive contour for this test. Aside from the gradients in V_j and β between $x_0 = -9$ and -2.5 the features in Fig. 26 are similar to those in Fig. 25.

Experimental

The experimental configuration shown in Fig. 27 allows enough of a flight path for a detailed analysis of the jet breakup pattern and the resulting particle tracks. A number of x-ray cassettes are placed in sequence down the flight path. The first is exposed by a 1-MeV generator, which produces a shadowgraph of the cone as it collapses. The other cassettes are exposed by the same low-energy x-rays used in the above experiments.

The target consists of a series of 25.4-mm 280-400 BHN steel plates, which are placed at a near grazing angle of 15° to the jet flight trajectory.

Two shots were fired. The x-ray shadowgraphs (Fig. 28) depict the constant velocity jet. The jet shows up as the large coherent particle near the center of the fragmented jet ($V_j = 8.7$ km/s). The cone apex produced a relatively large amount of fast material ($V_j = 10.1$ km/s), which appears to be unstable. The rear of the jet from the 16.5° cone section is somewhat slower ($V_j = 7.8$ km/s), while the jet from the 20.5° section is quite elongated and exhibits a particle shape close to that of a standard 21° charge. The fragmentation of the rest of the jet generates more randomly shaped particles.

Fig. 26. Stretching jet design calculations: variables as in Fig. 25.

Fig. 27. Experimental layout for the liner angle and edge effects tests.

Fig. 28. Jet configuration obtained from the constant velocity 16.5° cone design.

This comparatively rapid fragmentation process also produces thin streams of fine particles between the larger particles, with practically no particles of intermediate size.

The calculated jet mass may again be compared to the mass obtained from the shadowgraph. If a density of 8.93 g/cm^3 is assumed for the particles, the principal constant velocity particle has a mass of 17.5 g, based on a volume of 1.96 cm^3 . The rest of the jet — excluding the bifurcated apex particles — has a volume of 1.80 cm^3 and an implied mass of 16.0 g. The total calculated mass between $x_0 = -13$ and -2 cm is 29 g, which compares reasonably well with radiographic data, but again is somewhat smaller.

Two factors, however, modify this comparison:

- Analysis of the particle tracks shows that the slowest of the bifurcated particles intersect the cone collapse trajectory at a point corresponding to $X_0 = -10.7$. This shows that the jet velocity profile is slightly different than that shown by Fig. 25 and is approximately represented by the dashed line below $X = -10$. The theoretical jet mass should, therefore, be summed between $x_0 = -10.7$ and -2 cm . The result of the summation is 24.4 g.
- The above-mentioned convergence effects will further lower this value by an estimated 20%. Note that the experimental jet velocity is only slightly higher than predicted, apparently contradicting the approximately 10% velocity differential observed before. It was noted before that conservation of mass and momentum in the incompressible flow model predicted that the ratio of jet kinetic energy/liner kinetic energy is $\cos^2(\alpha/2) \approx 0.97$ for an $\alpha = 21^\circ$ half-angle cone. For a real material this energy ratio will hold except when thermal energy (e.g., temperature) takes up part of the liner kinetic energy. Hysteresis in the loading-unloading curves of the liner material causes the internal energy to increase as the jet formation rate and jet total energy density increases. A detailed analysis of a viscous material⁹ showed a loss in jet kinetic energy due to viscosity. The loss increased as the liner collapse angle decreased.

We still conclude, therefore, that convergence effects will increase jet velocity and decrease jet mass while viscosity and strength effects decrease jet velocity more at low cone angles than at large cone angles. The simple theory used above should calculate a high jet mass. The discussion of convergence effects showed that the proper theoretical mass should be

$\approx 24.4/1.2 = 20.3$. This suggests that the particle diameters from the radiographs are about $(1 - \sqrt{33.5/20.3}) 100\% = 28\%$ larger than that of a uniform density particle.

At the junction of the 16.5 and 20.5° cones (near $X_0 = -2$ cm) some unstable fragmenting particles were produced by the fluctuation in jet velocity which was described above. This fluctuation could be a means of separating jets, but appears to be too slow for the purposes of this program.

The results of the second test in Fig. 29 are for a jet that stretches over its whole length. The liner is cut by the above-mentioned conical cutter. The first shadowgraph in Fig. 29 shows the liner after it has converged to within the diameter of the hole in the center of the cutter. A small gap between the two sides of the cut liner still remains at the time of convergence ($t = 120 \mu s$) onto the jet stream. This liner-jet junction appears to have produced some instability in the jet stream as evidenced by particles moving laterally at 0 to over 0.4 km/s and longitudinally at a (V_j) of 5.3 km/s. This value of V_j is somewhat less than predicted by Fig. 26 and implies a reduction of velocity, which may have been caused by the cutting of the liner. This reduction is indicated by the dashed line in Fig. 26.

The target plates for both tests were cut through the middle of the hole, laid on edge, and photographed. Figure 30 shows both sides of the hole in a symmetric layout of the cut edges of the target. This layout clearly shows the hole shape. The hole is approximately round in a plane perpendicular to the flight path. Near the plate surface, however, the hole is enlarged by the deformation of surface material and therefore takes an approximately oval shape perpendicular to the jet trajectory.

The size of the hole may be obtained from Fig. 30. Note that because the targets were tilted 75° to the jet flight path, the major axis is $(\sec 75^\circ) \cdot d_t$, where d_t is the hole diameter perpendicular to the flight path. The major axis of the hole produced by the constant velocity jet (see V_j profile of Fig. 24) varies from $68 (\sec 75^\circ) = 260$ to $40 (\sec 75^\circ) = 150$ mm. This constant velocity jet, including its faster tip and stretching tail, perforated nearly three 25.4-mm, 380-BHN armor plates (Fig. 30, shot Cu 1).

The major axis of the stretching jet (see V_j profile, Fig. 25) varied from 97 to 77 mm over the greater length of the hole, but tapered to 50 mm in the fourth plate. This jet penetrated three and one-half armor plates. The distance between the charge and the targets was 17 cone diameters in both tests.

Fig. 29. Jet and slug configurations for the stretching jet from a 16.5° cone design.

Fig. 30. Targets penetrated by the constant velocity and stretching jets.

SUMMARY

The previous discussion has shown that:

- A combination of Lagrange computer calculations of liner collapse and comparatively simple jet formation models may be used to
 - Determine device configurations
 - Design jets with arbitrary velocity gradients
- The effects of liner convergence, strength of materials, and viscosity cause significant deviations from the simple theory
- The correlation between calculated and experimental jet mass for 21° half-angle cones is reasonably good and would require accurate measurements to show deviations conclusively. The agreement may be fortuitous, however, in that the effective particle density is lower. This is suggested by
 - A smaller hole than predicted from radiographed particle sizes.
 - The requirement that the jet kinetic energy of these converging systems is close to that calculated for plates. For 16.5° cones, however, the deviation is significant, suggesting hollow or porous particles. On the other hand, the jet penetration of targets at 16 cone diameters standoff was as expected from the effective jet length observed in the shadowgraphs.
- The velocity of the jet formed from the portion of the liner close (within 1 cm) to a cutter was lower than predicted in the calculations performed so far.

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APPENDIX A COMPRESSION OF AIR IN A TANDEM SHAPED CHARGE STAGE

The air in the metal cone of a shaped charge is compressed by the collapsing liner. It first flows towards the axis under the action of the liner and subsequently along the axis as it approaches the stagnation point. The air compression problem was investigated with a coupled Eulerian-Lagrange code. To facilitate the calculation it was considered from a coordinate system moving with the collapse point of the liner. This allows the code to neglect the metal jet formation processes and restrict the calculation to the air flow. Figure A1 shows the air flow pattern. Air is streaming into the Lagrange cone at 4.3 km/s under 1 bar pressure. The stagnation process results in a vortex 40 mm long with a 13 mm radius ($\sim 21 \text{ cm}^3$). The pressure-density profile is roughly represented in Table A1.

If the jet is included in this picture, the high-pressure gas volume would correspond to an annular layer 6 to 10 mm thick around the jet.

To get an order of magnitude effect of the gas pressure on the liner, we note that at a liner velocity of 2 km/s the liner passes through this zone in $\Delta t = 3$ to $5 \text{ } \mu\text{s}$.

Using $p = \rho \cdot d \cdot a$, we get a velocity perturbation

$$\Delta V = a \Delta t = \frac{P}{\rho d} \Delta t ,$$

where ρ is liner density and d liner thickness. Since d is 4 to 8 mm between 20 and 10 cm from the axis, we get an approximate value for the velocity perturbation

$$\Delta V = \frac{0.001 \text{ Mbar}}{(8.9) (0.6) \text{ g/cm}^2} 4 \text{ } \mu\text{s} = 7 \times 10^{-4} \text{ cm/}\mu\text{s}$$

or less than 0.4%. While presumably this effect would increase in the second stage it should still be small.

Table A1. Pressure-density profile.

Pressure, GPa	Density, Mg/cm ³	x, mm
0.22	0.09	2.5
0.21	0.10	10.0
0.15	0.08	15.0
0.09	0.05	20.0
0.02	0.01	30.0

Fig. A1. Gas flow produced by a collapsing liner in the coordinate system moving with the collapse point.

APPENDIX B
CALCULATION OF JET PROPERTIES

We have used the standard equations derived by Birkhoff et al.¹ to calculate jet characteristics:

$$V_j = V_0 \cdot \cos(\alpha + \delta - \beta/2)/\sin\beta/2 \quad (\text{Jet velocity}) \quad (\text{B1})$$

$$dm_j/dm_\ell = \sin^2 \beta/2 \quad (\text{ratio of jet to liner mass}) \quad (\text{B2})$$

Here β and α have the usual definition and

$$\delta = \sin^{-1} (V_0/2D) + 0.5, \quad (\text{B3})$$

as was observed in the text. We also get

$$dm_j/dx_0 = (dm_j/dm_\ell) (dm_\ell/dx_0) \quad (\text{B4})$$

where

$$dm_\ell/dx_0 = [r_0^2(x_0) - r_i^2(x_0)] \cdot \Pi \cdot \rho. \quad (\text{B5})$$

Here $r_0(x_0)$ and $r_i(x_0)$ are the outer and inner radii of the cone at x_0 (the axial distance at $t = 0$).

We can now calculate the time of jet formation by adding two terms:

- The time at which liner collapse starts:

$$\Delta t_1 = [x_0^2 + r_0^2(x_0)]^{1/2}/D, \quad (\text{B6})$$

where D is detonation velocity.

- The collapse time of the center of mass measured from the center to a point (r) near the axis. The axis is determined by the difference in the cross sectional areas, as defined by the average of the initial liner radius and the inner radius:

$$r = [0.25\{r_0(x_0) + r_i(x_0)\}^2 - r_i^2(x_0)]^{1/2}. \quad (\text{B7})$$

Therefore, we have the collapse time

$$\Delta t_2 = \frac{r_o(x_0) + r_i(x_0) - 2r}{2[V_0 \cos(\alpha + \delta)]} . \quad (B8)$$

This collapse point is further defined by

$$x = x_0 + V_0 [\sin(\alpha + \delta)] \Delta t_2 . \quad (B9)$$

The value of $\partial m_j / \partial x$ may be obtained by taking the derivative of Eq. (B9) and multiplying it by Eq. (B4). The resulting jet parameters are given in Tables B1 and B2 for designs I and II, respectively.

Table B3 compares the integrated jet mass (m_c) with that determined from the x-ray shadowgraphs (m_m). The table also lists the average jet velocity as calculated and measured. Note the reasonable agreement between the calculated mass and the jet mass obtained from the measured volume for design II (constant velocity jet). The calculated velocity is about 12% low because it does not include liner convergence effects. Note also that V_{jc} is the mass weighted average, $\sum_i m_{ji} V_{ji} / \sum_i m_{ji}$, while V_{jm} is a simple average of the tip and rear velocities (see Table 1).

Table B1. Computed jet parameters, design I.^a

x_0 , cm	x_t , cm	V_0 , km/s	β_t	V'_t , km/s·cm	δ_t , deg/cm	δ'_t , deg/cm	t , μ s	dm_x/dx_0 , g/cm	dm_j/dx_0 , g/cm	X'_t	dm_j/dx_j , g/cm	V_j , km/s	$r_j(x)$, mm
7	7.607	1.73	31.3°	0.014	6.35°	0.0083	16.11	30.59	2.225	1.214	1.833	6.268	2.55
8	8.796	1.87	32.2°	0.014	6.83°	0.0083	18.83	35.66	2.742	1.220	2.247	6.59	2.83
8.5	9.398	1.916	33.6°	0.0020	6.99°	0.0012	20.30	37.37	3.122	1.215	2.570	6.49	3.03
9	9.995	1.91	36.0°	-0.0040	6.97°	-0.002	22.03	38.86	3.711	1.211	3.065	6.08	3.30
9.5	10.585	1.858	37.8°	-0.011	6.79°	-0.0065	24.07	40.13	4.210	1.204	3.498	5.66	3.53
10.5	11.766	1.75	37.0°	-0.0030	6.42°	-0.0018	28.46	42.00	4.228	1.206	3.505	5.44	3.53
11	12.364	1.734	35.4°	-0.0030	6.37°	-0.0018	30.46	42.60	3.937	1.206	3.266	5.61	3.41
12	13.366	1.720	30.8°	+0.0022	6.32°	0.0013	34.40	43.12	3.041	1.210	2.512	6.32	2.99
13	14.785	1.760	27.6°	+0.0050	6.45°	0.0030	37.79	42.76	2.433	1.215	2.002	7.15	2.67
14	16.018	1.821	25.9°	+0.0070	6.66°	0.0041	40.90	41.51	2.085	1.220	1.708	7.84	2.47
15	17.267	1.903	24.5°	+0.0120	6.94	0.0071	43.69	39.36	1.772	1.232	1.438	8.61	2.26

^aSubscript t refers to time of jet formation; primes are derivative with respect to x.

Table B2. Computed jet parameters, design II.

x_0 , cm	x_t , cm	V_0 , km/s	β_t	V'_t , km/s·cm	δ_t , deg/cm	δ'_t , deg/cm	t , μ s	dm_x/dx_0 , g/cm	dm_j/dx_0 , g/cm	X'_t	dm_j/dx_j , g/cm	V_j , km/s	$r_j(x)$, mm
7	7.62	1.87	33.5°	0.00976	6.83°	0.0058	15.58	30.588	2.54	1.216	2.09	6.36	2.728
8	8.804	1.937	33.5°	0.0060	7.06°	0.00356	18.53	35.66	2.96	1.218	2.43	6.57	2.94
9	10.00	1.97	33.5°	0.000	7.17°	0.000	21.72	38.86	3.22	1.215	2.65	6.69	3.08
10.5	11.81	1.97	33.4°	0.0012	7.17°	0.00071	26.82	41.99	3.47	1.216	2.85	6.71	3.19
12	13.63	1.993	33.55°	0.0020	7.25°	0.00119	31.87	43.12	3.59	1.218	2.95	6.75	3.24
13	14.86	2.05	33.8°	0.000	7.44°	0.000	34.91	42.76	3.61	1.217	2.97	6.90	3.25
14.5	16.67	1.98	34.5°	-0.0012	7.21°	-0.00071	40.92	40.55	3.56	1.2139	2.94	6.55	3.23
15	17.29	1.99	36.0°	0.0020	7.24°	0.00119	42.66	39.37	3.76	1.219	3.08	6.33	3.31

Table B3. Computed vs measured jet mass and jet velocity.

Design	$\frac{m}{g}_c$	$\frac{m}{g}_m$	V_c , km/s	V_c , m/s	V_m/V_c
I ^a	24		6.42	7.05	(1.10)
	10.9	11	6.15	7.05	1.15
II	27	26 ± 2	6.66	7.46	1.12

^aThe first value corresponds to the total calculated jet and the second to the portion not destroyed by the convergence of the rear of the jet.

The comparison between calculated and measured results is more qualitative for design I. The interaction between the rear and the front part of the jet results in

- Some energy and momentum transfer to the front
- Some kinetic energy loss in the forward motion due to lateral expansion on the tail.

These details are not specifically included in the comparison. However, the first effect should result in an increased jet velocity and a higher V_j/V_{jc} ratio for the 11-g leading section of the jet. Furthermore, the average measured jet velocity is lowered because of the kinetic energy lost in the lateral direction. This causes a lowering in $\bar{V}_{jm}/\bar{V}_{jc}$. These effects are consistent with the last column in Table B3.

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