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**STATISTICAL TESTS FOR LOCATING POWER IN A
FOURIER LINE SPECTRUM**

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SUBJECT DESCRIPTORS

Welding
Electron Beam Welding
Welding Machines
Weld Properties
X-Radiation
Energy Dispersive X-Ray Analysis
X-Ray Spectroscopy
X-Ray Detection
Noise
Mathematics
Statistics
Analysis

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STATISTICAL TESTS FOR LOCATING POWER IN A FOURIER LINE SPECTRUM

Raymond D. Dixon and Joseph R. Geoffrey

Abstract. Fourier line spectra are used to detect noise on the electron beam of an electron beam welder. These spectra are calculated from data in the form of X-ray intensity as a function of time. Statistical tests are described that provide a means of determining the presence of random and periodic signals. The tests remove the uncertainty in deciding if power is present and if so, at which frequencies. The tests have proven to be consistent and reliable. They are of suitable generality to be useful when it is essential to quantitatively assess a signal for power.

INTRODUCTION

Several statistical tests, which are described later, were used to locate power in a Fourier line spectrum. The statistical approach to locating power was chosen because initial work yielded line spectra that were random in nature and that were typical of spectra obtained from a white noise source (see Figure 1b). These spectra are difficult to visually inspect for power, and the question as to whether any peaks in the spectra have a high probability of indicating power is not easily answered. Also, the time series was composed of 2^{11} points; thus, the line spectrum contained $2^{10} + 1$ terms. It would have been extremely time consuming to plot line spectra containing this many terms. This, in addition to the difficulty in interpreting the spectrum after it was plotted, stimulated the need for an analytical means of inspecting the line spectra. Thus, tests were used that would determine if a spectrum was from a white noise process and that would examine the terms in the spectrum to determine if power was indicated. The latter test was used to determine the significance level for which acceptance or rejection is given to the hypothesis that power at a harmonic frequency is greater than the white noise. Although the particular tests used are not new, application of these tests to the time series described later are noteworthy, as are the results obtained.

X-rays, generated by the electron beam in an electron beam welder striking a suitable target, were used to obtain the time series. The origin of the time series from which the line spectrum was obtained was the number of X-rays having energies between E_1 and E_2 per unit time. X-ray energies E_1 and E_2 were chosen so that the energy band (E_1 to E_2) did not include any characteristic X-ray lines and that E_2 (the high energy) was near the high-energy cutoff of the Bremsstrahlung spectrum.

The work is being done to characterize noise on the electron beam and to determine if this noise could cause some of the problems encountered in electron beam welding. Typical of the problems encountered are (1) geometrical variations in welds made using identical conditions and (2) the inability to produce identical welds, using the same welding conditions, but different welding machines.

The weld parameters considered to induce noise onto the electron beam are beam voltage, beam current, and filament current. The noise that would be induced onto the beam would be some fraction of that present on the source of each of these parameters.

Monitoring X-rays was chosen rather than measuring noise at the parameter monitoring points. This choice was made because X-rays indicate which frequencies are on the beam and how the amplitudes at these frequencies vary with the welding parameters. Measurements at the parameter monitoring points will not provide information about noise on the beam.

This report describes the analysis used to inspect the line spectra and thus to characterize the electron beam. The analysis is not limited to this particular application; actually, it is useful for inspecting any Fourier line spectrum.

EXPERIMENTAL

Electron beam welding is accomplished in a vacuum with a dense beam of energetic electrons. When the electrons strike a work piece, kinetic energy is converted to thermal energy, thereby producing a weld by melting and fusing the metal. A large number of X-rays also are produced in this process.

The electron beam welder used for this work was manufactured by Sciaky Brothers* and had a power capability of 30 kilowatts (kW), 60 kilovolts (kV), and 500 milliamperes (mA). The electron beam gun was of the Pierce type. In that design, high-energy electrons are produced in the gun, then drift in a field-free region where they strike the work piece.

For the work presented here, the beam was incident normal to a tungsten target. The electron gun was operated at powers less than 1.62 kW (60 kV and 27 mA) to prevent the beam from creating a cavity on the target surface. This resulted in all of the X-rays originating at or near the surface of the target, thereby simplifying the experiment. A 0.152-centimeter (cm)-thick beryllium window was placed in the side of the vacuum chamber, on the line of sight with the target surface, for an X-ray exit port.

The X-ray spectrometer consisted of a silicon (lithium) X-ray detector, a 2048 channel pulse-height analyzer, plus associated amplifiers and power supplies. In the multichannel scaling mode of operation, the dwell time per channel could be set from 0.001 to 0.9 seconds. For this work, a dwell time of 1×10^{-3} seconds was used. The reset time per channel was 400 nanoseconds (400×10^{-9} seconds). The data could be read out from the pulse-height analyzer memory through a plotter or onto punch paper tape. The detector was placed 70 cm from the impingement point of the electron beam. Various materials could be placed in the X-ray path to attenuate the X-ray intensity and to tailor the shape of the X-ray spectrum. All data were taken with a 90-degree X-ray takeoff angle.

The spectrometer was calibrated using an americium-241 gamma ray source, which allowed determination

of the energies E_1 and E_2 in addition to the beam voltage¹, to within ± 100 volts.

PROBLEM ANALYSIS

To a first approximation, the spectral intensity (I_E) of the X-ray Bremsstrahlung spectrum is calculated as follows:

$$I_E = K'J(E_0 - E) \quad (1)$$

where K' = a proportionality constant
 J = electron beam current
 E_0 = beam voltage
 E = X-ray energy

Integrating Equation 1 between energies E_1 and E_2 where $0 \leq E_1 < E_2 \leq E_0$ yields the following:

$$I = KJ(E_0 - D) \quad (2)$$

where $K = K'(E_2 - E_1)$
 $D = (E_1 + E_2)/2$

If J and/or E_0 are time dependent, then for small time (t) intervals, the following expression would apply:

$$I(t) = KJ(t)(E_0(t) - D) \quad (3)$$

The usual techniques of signal analysis are invoked whereby it is assumed that $I(t)$ can be represented as a finite Fourier series. Thus $I(t)$ can be written, using the discrete form for $2N$ even and equally spaced points, as follows:

$$I(r\Delta) = I(t) = \frac{I_0}{2} + \sum_{m=1}^{N-1} \left(h_m \cos \frac{\pi m r}{N} + g_m \sin \frac{\pi m r}{N} \right) + \frac{h_N}{2} \cos \pi r \quad (4)$$

where $t = r\Delta$
 $r = 0, 1, 2, \dots, 2N-1$
 Δ = the sampling time interval

The coefficients for $I(r\Delta)$ are defined in the usual manner and are found using a fast Fourier transform algorithm. The statistical analysis is done on

*Sciaky Brothers, Inc., 4918 W. 67th St., Chicago 38, Illinois.

the Fourier line spectrum of $I(r\Delta)$, which will be called the spectrum in this report, and which is expressed as follows:

$$\sum_{r=0}^{2N-1} \frac{I(r\Delta)^2}{N} = 2\left(\frac{I_0}{2}\right)^2 + \sum_{m=1}^{N-1} (h_m^2 + g_m^2) + 2\left(\frac{hN}{2}\right)^2 \quad (5)$$

A procedure outlined by Jenkins and Watts² is used to test whether the spectrum originates from a white noise source. This test begins with the sample spectrum estimate $h_m^2 + g_m^2$ computed at the harmonic frequencies as follows:

$$f_m = \frac{m}{2N\Delta} \text{ m, } = 0, 1, 2, \dots, N$$

The integrated spectrum, $I(f_m)$, is then estimated from the sample integrated spectrum as indicating by the following:

$$I(f_m) = \frac{1}{2} \sum_{p=1}^m (h_p^2 + g_p^2) \quad (6)$$

In practice, the final form of the estimates is

$$\frac{I(f_m)}{S^2} = \frac{1}{2S^2} \sum_{p=1}^m (h_p^2 + g_p^2) \quad (7)$$

where S^2 is the estimator of the variance (σ^2) of the number of X-rays per unit time, and is expressed as follows:

$$S^2 = \frac{1}{2} \sum_{m=1}^{N-1} (h_m^2 + g_m^2) + \frac{h^2 N}{4} \quad (8)$$

Note that $I(f_0) = 0$ if the mean is subtracted out and that $I(f_N)/S^2 = 1$. Thus, if $I(f_m)/S^2$ is plotted versus $2\Delta f_m$, the points should fall in approximately a straight line through (0,0) and (1,1). To judge the deviations of the points about the straight line, bands of $\pm\lambda/(N-1)^{1/2}$ are constructed about the theoretical line. If the points fall within the band, the source is white noise. The values of λ for 95

and 75 percent confidence levels are 1.36 and 1.02, respectively.*

Thus, this test will show whether or not the spectrum comes from a white noise source. If it does not, then tests are used to locate the power in the spectrum.

To test the largest term in the spectrum for power, a new variable (g) is defined³ such that

$$g = \frac{\max(h_m^2 + g_m^2)}{\sum_{m=1}^{N-1} (h_m^2 + g_m^2) + \frac{h^2 N}{4}} \quad (9)$$

where $\max(h_m^2 + g_m^2)$ = the largest term in the spectrum

The probability that g is greater than an arbitrary g^1 is shown³ by

$$P(g > g^1) = \sum_{P=0}^r \frac{(-1)^P (N-1)! [1-(P-1)g^1]^{N-2}}{(N-P-2)!(P+1)!} \quad (10)$$

where r = the largest integer such that $r \leq N-2$ and $1-(r+1)g^1 \geq 0$

Ideally one would like to know the value of g^1 such that if $g > g^1$, the hypothesis of white noise (or no power) at the 5(1) percent significance level can be accepted or rejected. It is better, however, to substitute g for g^1 and to determine what significance level results. Thus, for the 5(1) percent significance levels, significance is achieved when $P(g > g^1) \leq 0.05(0.01)$.

The above test determines only whether the largest concentration of power at a certain frequency is or is not from a white noise source. When it is desired to test at an arbitrary frequency, the terms are ranked from the largest to the smallest and then ratioed as in Equation 9. Thus, to test the q th largest, g is defined as follows:

*Jenkins and Watts² outlines this significance test and attribute the test to Kolmogoroff and Smirnov.

$$g = \frac{q\text{th largest of } (h_m^2 + g_m^2)}{\sum_{m=1}^{N-1} (h_m^2 + g_m^2) + \frac{h_N^2}{4}} \quad (11)$$

For this case,³ the probability that g is greater than g^1 is as follows:

$$P(g > g^1) = \frac{(N-1)!}{(q-1)!} \sum_{P=q}^r \frac{(-1)^{P-q} (1-P_g^1)^{N-2}}{P(N-1-P)! (P-q)!} \quad (12)$$

where r = the largest integer such that $1-rg \geq 0$

As discussed earlier, g is substituted for g^1 and the significance level is obtained. Again, for the 5(1) percent significance level, significance is achieved when $P(g > g^1) \leq 0.05(0.01)$. It should be noted that this test does not test the direct current (DC) term.

The DC and the high frequency terms are tested for significance using a Student's t test. For this test, a T value is calculated from

$$T = \frac{A_0}{2 \left(\frac{S^2}{2N} \right)^{1/2}} \quad (13)$$

where $A_0/2 = I_0/2$ to test the DC term

$A_0/2 = h_N/2$ to test the high frequency term

If the calculated T is greater than $T_{1-\alpha, N-1}$, where α represents the significance level, then $A_0/2$ is significant at that level. For this test, S^2 is defined by Equation 8.

The approach used for this work was to calculate the line spectrum, then test the DC and high frequency terms. Next, the spectrum was tested to determine if it was from a white noise source. After the white noise test, the spectrum was examined for power at the harmonic frequencies. The latter test started with the largest term and continued until three successive terms were rejected.

This served as a check on the white noise test in that, if a white noise source was detected, the first three terms tested were also insignificant. These findings would result in the testing being terminated.

A minicomputer manufactured by Modular Computer Systems* was used to complete all of the calculations. The integer range, using this machine, was -32767 to $+32767$ ($-2^{15} + 1$ to $2^{15} - 1$). Real numbers have a precision to six decimal places with magnitudes between 10^{-77} to 10^{76} (2^{-256} to 2^{255}). The integer range presented a slight problem when writing the program to accomplish the fast Fourier transform; however, the real number range must be considered when doing the calculations for determining power at the harmonic frequencies. This is especially true when the line spectrum contains a large number of terms or if g is near unity, as can be seen from Equations 10 and 12. For the work described here, Equations 10 and 12 were limited to approximately 20 terms in the series. It is unknown whether the series had converged. Several runs were completed on four spectra using different numbers of terms, however, and all yielded the same significant value. Thus, 20 terms in the series were considered adequate.

RESULTS AND DISCUSSION

As previously mentioned, the reason that statistical tests were developed was due partially to the random nature of the first few spectra obtained. Also, several hundred time spectra were anticipated, and no convenient means was available to plot the line spectra for visual inspection. The statistical tests also would provide a quantitative means of assessing the existence of power where there might be some question when using visual inspection. The spectra shown in this section were chosen for plotting because they are typical of those obtained. In addition, the chosen spectra indicate the usefulness of the tests when plotting cannot be readily accomplished

Figure 1 shows a time spectrum (a) and its resulting line spectrum (b). Statistical tests show the power spectrum to be from a white noise source. In all

*Modular Computer System, Inc., 1650 W. McNab Road, Ft. Lauderdale, Florida 33309.

FIGURE 1. Time series obtained with the electron beam welder operating at 20 kV and 14.5 μ A. A tungsten target was used, and no absorber was in the X-ray path. The time series is shown in (a); the resulting Fourier line spectrum in (b).

FIGURE 2. Time series obtained with the electron beam welder operating at 60 kV and 5.3 mA. A tungsten target was used, and 0.008 inches of tantalum was in the X-ray path. The time series is shown in (a), and the resulting Fourier line spectrum in (b). Amplitudes of the significant peaks in the power spectrum also are given.

cases for which the white noise test indicated a white noise source, the test for power at the harmonic frequencies indicated no power at the 5 percent significance level. In fact, the significance level was typically 60 percent (i.e., not significant). The experimental conditions used to obtain the data for Figure 1 were 20 kV beam voltage and 14.5 microamperes (μA) beam current. The low beam current was obtained by adjusting the current through the electron gun filament until 14.5 μA of beam current was obtained. The beam current required to obtain optimum operating conditions for this electron gun at 20 kV is approximately 5 mA. Thus, the white noise spectrum is consistent with the fact that the low filament current yields low beam current, which originates from random

electron emission, and that this is not characteristic of thermal emission.

Figures 2 and 3 are typical time spectra (2a and 3a) and their associated line spectra (2b and 3b). These figures show that some of the peaks could be unambiguously determined from plots of the line spectrum (notably the peaks at 120 and 360 hertz (Hz) in Figure 2, and at 120 and 360 Hz in Figure 3). Note that Figure 2(b) contains a large number of peaks, the significance of which cannot be evaluated by inspection. The test for power, however, showed that none of these peaks were significant. In Figure 3(b), the peaks at 234 and 240 Hz would probably be questionable upon inspection. The test for

FIGURE 3. Time series obtained with the electron beam welder operating at 40 kV and 4.85 mA. A tungsten target was used, and 0.008 inches of tantalum was in the X-ray path. The time series is shown in (a) and the resulting Fourier line spectrum in (b). Amplitudes of the significant peaks in the power spectrum also are given.

power showed the peak at 240 Hz to be significant while the peak at 234 Hz was not.

In all of the work done to date, the t test, used to test the DC term, has shown the DC term to be significant. In fact, the DC power is proportional to the power in the electron beam as predicted from X-ray theory. In general, the three tests described have proven to be an excellent way of testing line spectra for power and for locating the power.

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