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OBSERVATIONS ON THE OPERATION OF THE SEMPEX CODE

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September 22, 1976

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Prepared for U.S. Energy Research & Development Administration under contract No. W-7405-Eng-48.

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OBSERVATIONS ON THE OPERATION OF THE SEMPEX CODE

ABSTRACT

The authors' observations on the use of the SEMPEX computer code are given. The issues of proper sampling rate and total sampling interval are addressed. The effect that noise in the data has on the results and the ability of autocorrelation, truncation filtering, and the sliding-window technique to reduce these effects are also discussed.

INTRODUCTION

The purpose of this report is to put into some logical order the authors' observations in using the SEMPEX (Singularity Expansion-Method Pole Extraction) code. The authors have used this code extensively for about 18 months and hope that the experience gained in that time can be preserved. The code has been successfully applied to data calculated by transient electromagnetic computer codes for the scattered field from wire structures and to data from the LLL transient measurement range.

Because the SEMPEX program in essence does a simple curve fit to the data presented to it, the user must take care to sample the data at appropriate intervals and for a sufficient duration for the results to be physically meaningful. Also, because data is always corrupted by a certain amount of noise, steps must be taken to determine how the noise perturbs the results and to reduce its effects. Our observations on each of these issues follow. We will not attempt to prove our observations rigorously; but we will demonstrate various problems a user will encounter and give some guidelines on how to avoid them.

For the sake of simplicity, most of the examples presented will be for data generated from a known set consisting of a few poles rather than for data pertaining to a physical, realizable structure.

SEMPEX CODE OPERATION

A complete user's guide for the SEMPEX code is available as Air Force Weapons Laboratory Technical Report TR76-200.¹ We assume the reader has read this report and is familiar with the various input arguments and their definitions.

SENSITIVE PARAMETERS

The purpose of the SEMPEX code is to solve the equation

$$F(t) = \sum_{i=1}^N A_i e^{\alpha_i t} \quad (1)$$

for the A_i 's and the α_i 's, where A and α are complex. The A_i terms in this report are referred to as "residues"; the α terms are referred to as "poles".

Using the SEMPEX code to obtain significant (structurally dependent) poles and residues from a transient waveform is difficult unless one is familiar with the pitfalls of the code. There are many ways to obtain invalid results, therefore the following guidelines and examples should be useful.

Effects of NPTS/NPOLES Ratio

The ratio between the number of requested poles (NPOLES) and the number of sampled data points (NPTS) affects the accuracy of the calculated poles and residues. Figure 1 is a plot of the difference between the original data and data generated from the calculated poles and residues when $NPTS = 2 * NPOLES$. Figure 2, the same plot but when $NPTS > 2 * NPOLES$, shows this condition increases the percent rms error. The percent rms error E is defined as:

$$E \equiv \left\{ \frac{\sum_{I=1}^N [R(I) - O(I)]^2}{\sum_{I=1}^N [O(I)]^2} \right\}^{1/2}, \quad (2)$$

¹Darrel L. Lager, H. Gerald Hudson, and Andrew J. Poggio, "Users Manual for SEMPEX, A Computer Code for Extrapolating Complex Exponentials from a Time Waveform," Air Force Weapons Laboratory, Kirtland A.F.B., New Mexico, Technical Report No. TR76-200 (1977).

Fig. 1. Difference between reconstructed and original data (error equals reconstructed minus original) when $NPTS = 2 * NPOLES$ ($NPOLES = 50$, $NPTS = 100$).

Fig. 2. Difference (error) between reconstructed and original data when $NPTS > 2 * NPOLES$ ($NPOLES = 50$, $NPTS = 105$).

where $R(I)$ is the reconstructed data array,

$O(I)$ is the original data array,

N is the number of data points.

As NPTS is further increased, the error tends to reach some maximum, then converge back to the error of Fig. 1. Therefore, for the sake of consistency, the relation

$$NPTS = 2 * NPOLES \quad (3)$$

will be maintained in all the calculations presented in this report.

Effects of the Number of Requested Poles (NPOLES)

How is the pole set affected when more and less poles are requested than actually exist in the data? As an example, a data set generated from 12 poles (in six conjugate pairs) and a dc term is plotted in Fig. 3. SEMPEX was requested to fit the data using 16 poles, and the resulting pole plot is shown in Fig. 4. Table 1 is a listing of the poles and shows that the correct poles were accurately extracted.

Table 2 is a partial listing of the original and reconstructed data. Here, the data points used for extracting the poles, as well as the points between them, fit precisely, demonstrating the correct pole set was extracted. Table 1 shows that the three extra (curve-fitting) poles requested had very small residues. This is what one might expect, since there was no additional information in the data for these poles to represent. A further increase in NPOLES will randomly scatter the excess poles in the complex S-plane. Figure 5 is an isometric pole plot showing how the excess poles in such a case are scattered. The vertical lines represent the magnitude of the residues. Residues less than 10^{-3} times the largest residue are shown by circles.

When NPOLES is less than the number of poles that exist in the data, the results will be incorrect. Figure 6 shows the pole locations when the same data as in Fig. 3 are used but only 10 poles are requested. This pole plot obviously bears no resemblance to Fig. 4. Table 3 lists the calculated poles and shows that all poles and residues calculated were totally incorrect. The error plot of Fig. 7 clearly shows that the calculated pole set is wrong. Table 4 is a partial listing of the data points for the original and reconstructed data. Note in this case all of the data points used to calculate the poles, as before, fit exactly. However, the data points between these points are incorrect.

Fig. 3. Data generated from 12 poles and a dc term.

Fig. 4. Pole locations when NPOLES is greater than the number of poles in the data. Extra, "curve-fitting" poles are circled; data used was that of Fig. 3.

Table 1. Pole list for Fig. 4 showing accurate calculation of the poles in data of Fig. 3.

Residue			Pole		
Mag	Real	Imag	Real	Imag	
1.81062E-01	-1.81062E-01	3.00264E-15	-7.06124E+00	0.	dc term
1.00000E+00	1.41406E-07	1.00000E+00	-5.00000E+08	-2.00000E+08	Calculated pole set
1.00000E+00	1.41406E-07	-1.00000E+00	-5.00000E+08	2.00000E+08	
1.00000E+00	2.16309E-07	1.00000E+00	-5.00000E+08	-4.00000E+08	
1.00000E+00	2.16309E-07	-1.00000E+00	-5.00000E+08	4.00000E+08	
1.00000E+00	-4.34039E-08	1.00000E+00	-5.00000E+08	-6.00000E+08	
1.00000E+00	-4.34044E-08	-1.00000E+00	-5.00000E+08	6.00000E+08	
1.00000E+00	9.30876E-08	1.00000E+00	-5.00000E+08	-8.00000E+08	
1.00000E+00	9.30873E-08	-1.00000E+00	-5.00000E+08	8.00000E+08	
1.00000E+00	-3.37026E-07	1.00000E+00	-5.00000E+08	-1.00000E+09	
1.00000E+00	-3.37026E-07	-1.00000E+00	-5.00000E+08	1.00000E+09	
2.23137E-06	-4.19927E-07	-2.19150E-06	-6.81175E+08	1.18719E+09	Curve-fitting poles
2.23137E-06	-4.19926E-07	2.19150E-06	-6.81175E+08	-1.18719E+09	
9.99998E-01	3.51948E-07	-9.99998E-01	-5.00000E+08	1.20000E+09	
9.99998E-01	3.51948E-07	9.99998E-01	-5.00000E+08	-1.20000E+09	
3.00562E-09	3.00562E-09	-3.33606E-14	-2.03482E+08	1.59688E+09	

Table 2. NPOLES greater than the number of poles in the data. Data points with asterisks were used to calculate pole set. Here, data between them has essentially no error, indicating proper pole set was calculated.

Reconstructed data	Original data	Difference
-1.22693E-01	-1.22693E-01*	-3.56051E-09
1.92158E+00	1.92158E+00	-4.18135E-08
3.85506E+00	3.85506E+00	4.01684E-08
5.58261E+00	5.58261E+00	-3.24930E-08
7.02291E+00	7.02291E+00	-5.29556E-09
8.11365E+00	8.11365E+00	-1.19628E-08
8.81534E+00	8.81534E+00	4.22745E-08
9.11335E+00	9.11335E+00	2.08960E-08
9.01824E+00	9.01824E+00*	-3.26281E-11
8.56424E+00	8.56424E+00	6.75215E-09
7.80612E+00	7.80612E+00	4.51493E-08
6.81460E+00	6.81460E+00	4.74149E-08
5.67085E+00	5.67085E+00	2.20606E-08
4.46037E+00	4.46037E+00	8.76760E-08
3.26680E+00	3.26680E+00	8.78993E-08
2.16606E+00	2.16606E+00	5.98517E-08
1.22136E+00	1.22136E+00*	1.58528E-09
4.79385E-01	4.79385E-01	3.50486E-09
-3.21649E-02	-3.21649E-02	-1.28878E-08
-3.05412E-01	-3.05412E-01	2.74612E-08
-3.51601E-01	-3.51601E-01	-4.43452E-08
-1.98940E-01	-1.98940E-01	-3.50661E-08
1.10776E-01	1.10776E-01	-9.17127E-08
5.26445E-01	5.26445E-01	-1.79653E-08
9.92459E-01	9.92459E-01*	-2.47405E-09
1.45361E+00	1.45361E+00	-8.30203E-08
1.85969E+00	1.85969E+00	9.25662E-09
2.16948E+00	2.16948E+00	-7.77389E-09
2.35366E+00	2.35366E+00	3.83224E-08
2.39671E+00	2.39671E+00	2.69933E-08
2.29739E+00	2.29739E+00	4.16500E-08

Fig. 5. Three-dimensional pole plot in the upper half of the S plane showing condition when NPOLES is much greater than the number of poles in the data. Vertical lines indicate the log of residue magnitude; circled poles have small residues and are randomly scattered.

Fig. 6. Pole locations for data of Fig. 3 when NPOLES is less than the number of poles in the data. (Compare to Fig. 4.)

Fig. 7. Error in reconstructed data when NPOLES is less than the number of poles in the data (Fig. 6 and Table 3).

Table 3. Pole list for Fig. 4 when NPOLES is less than the number of poles in the original data.

Mag	Residues		Poles	
	Real	Imag	Real	Imag
9.85924E-02	-9.85924E-02	-1.86930E-09	3.39449E+08	0.
2.60896E+01	2.60896E+01	-4.41691E-06	-1.41032E+10	5.52930E-13
5.11522E+00	5.11522E+00	4.71793E-07	-1.83580E+09	-3.62572E-06
1.02956E-06	1.76068E-07	1.01439E-06	2.47608E+09	-5.92870E+08
1.02956E-06	1.76068E-07	-1.01439E-06	2.47608E+09	5.92870E+08
8.06838E-05	8.06708E-05	1.44503E-06	1.61715E+09	-1.17058E+09
8.06838E-05	8.06708E-05	-1.44503E-06	1.61715E+09	1.17058E+09
6.86469E-01	-6.54855E-01	2.05922E-01	-5.98835E+08	1.31691E+09
6.86469E-01	-6.54856E-01	-2.05922E-01	-5.98835E+08	-1.31691E+09
2.99777E+01	-2.99777E+01	4.19357E-06	-6.49101E+09	1.59688E+09

Table 4. Original and reconstructed data when NPOLES is less than the number of poles in the data. Data points with asterisks were used to calculate pole set; reconstructed data between these are in error.

Reconstructed data	Original data	Difference
-1.22688E-01	-1.22693E-01*	5.30256E-06
3.20779E-01	1.92158E+00	-1.60080E+00
2.40712E+00	3.85506E+00	-1.44794E+00
5.00554E+00	5.58261E+00	-5.77072E-01
7.33709E+00	7.02291E+00	3.14183E-01
8.96362E+00	8.11365E+00	8.49969E-01
9.73139E+00	8.81534E+00	9.16054E-01
9.69178E+00	9.11335E+00	5.78431E-01
9.01825E+00	9.01824E+00*	8.58614E-06
7.93347E+00	8.56424E+00	-6.30777E-01
6.65448E+00	7.80612E+00	-1.15164E+00
5.35853E+00	6.81460E+00	-1.45607E+00
4.16800E+00	5.67085E+00	-1.50285E+00
3.15046E+00	4.46037E+00	-1.30992E+00
2.32836E+00	3.26680E+00	-9.38445E-01
1.69343E+00	2.16606E+00	-4.72630E-01
1.22135E+00	1.22136E+00*	-9.97477E-06
8.83833E-01	4.79385E-01	4.04449E-01
6.56526E-01	-3.21649E-02	6.88691E-01
5.22545E-01	-3.05412E-01	8.27957E-01
4.72247E-01	-3.51601E-01	8.23848E-01
5.00478E-01	-1.98940E-01	6.99418E-01
6.02683E-01	1.10776E-01	4.91907E-01
7.71104E-01	5.26445E-01	2.44718E-01
9.92450E-01	9.92459E-01*	-8.45671E-06
1.24633E+00	1.45361E+00	-2.07281E-01
1.50664E+00	1.85969E+00	-3.53048E-01
1.74370E+00	2.16948E+00	-4.25778E-01
1.92762E+00	2.35366E+00	-4.26038E-01
2.03224E+00	2.39671E+00	-3.64473E-01
2.03862E+00	2.29739E+00	-2.58768E-01

Given a situation when NPOLES is less than the actual number of poles in the data, the error in the data points that are between those used to calculate the poles (compare Tables 2 and 4) is a good indication whether or not NPOLES is large enough. Another way of testing for NPOLES being too small is to increase NPOLES and compare the resulting pole set. If NPOLES was too small, the pole set will change dramatically and the percent error will decrease. As NPOLES is further increased, a point will be reached where the significant poles will not move. At this point NPOLES is equal to the number of poles in the data and meaningful results can be obtained.

Aliasing Error

A major problem in locating valid poles is aliasing error. This type of error is caused when the Nyquist sampling criterion has not been satisfied. The input data for the SEMPEX code consists of 512 equally-spaced data points. Since this is sampled data, it has an upper frequency limit of $1/2DT$, where DT is the time between samples. To completely define 512 data points would take 128 pole pairs. This would require a considerable amount of computer memory and running time. Under most circumstances meaningful data can be obtained using fewer data points, so the SEMPEX code uses only a subset of the original data. The parameter that controls this sampling is NDECI (for decimate) and Fig. 8 shows its effect. When $NDECI = 1$, all data is retained and the highest resolvable frequency remains $1/2DT$. When $NDECI = 2$, every other data point is sampled and the highest resolvable frequency is now $1/4DT$.

As $NDECI$ is increased, the highest resolvable frequency decreases, following the relationship

$$F_{HN} = \frac{1}{2 * NDECI * DT} , \quad (4)$$

where F_{HN} is the highest resolvable frequency.

When $NDECI$ is chosen such that F_{HN} is above the highest frequency content in the data, F_H , no aliasing problems will occur. If F_{HN} is less than F_H , all information above F_{HN} will be folded back below F_{HN} . Figure 9 graphically demonstrates the aliasing problem. When F_{HN} is ΔF below F_H , F_H will appear at a point $F_{HN} - \Delta F$ in the spectrum.

Figure 10 is a pole plot that was calculated in exactly the same way as the pole plot of Fig. 4 except $NDECI$ was increased to 12 in order to lower F_{HN} to 1.0646 GHz. Since ΔF is 0.1354 GHz, this lowered the 1.2-GHz pole to

Fig. 8. Effect of NDECI on sampling. Total number of sampled data points equals NPTS.

Fig. 9. Illustration of aliasing.

Fig. 10. Pole plot showing aliasing effects.

$F_{HN} = \Delta F$ or 0.9292 GHz. Table 5 shows that the 1.2-GHz pole is missing, but an 0.929-GHz pole has appeared. Figure 11 clearly shows something is wrong, for the rms error has jumped to 48.9%. In this case the aliased pole's residue was equal in magnitude to the other poles. Note that in Table 5 all poles below F_{HN} are calculated correctly, and note the residue of the aliased pole is unchanged from that for the true pole. Therefore, if the aliased pole or poles have small residues compared to the significant data, their effect on the rms error will be proportionally less. To demonstrate this, new data was generated, and the results are plotted in Fig. 12. Here again the magnitudes of the residues are plotted as vertical lines and in this case the aliased pole's residue is only 1/10 that of the aliased pole in Fig. 10. Figure 13 shows that the rms error introduced is 5.5%, or roughly 1/10 that of the previous case.

Therefore, to avoid aliasing problems, the user must choose the sampling frequency such that F_{HN} is above F_H . If F_H is not exactly known (which is generally the case) F_{HN} should be in a region of the data's frequency spectrum where the magnitudes of the higher frequency components are all about two orders of magnitude down from the significant portion of the spectrum. This situation would alias small-residue poles, which would have little effect on the overall error. They could be then eliminated because of their small residues.

Effect of Starting Point

The SEMPEX code, as mentioned previously, uses data files with 512 data points. With the parameter NBEGIN, the n^{th} point of the input array can be chosen as the starting point for processing. As far as the SEMPEX code is concerned, this starting point is defined to be $t = 0$ s.

For data that represents the impulse response of a structure, circuit, etc., the starting point can be chosen at NBEGIN = 1. This is necessary if the original impulse-response residues are to be accurately calculated. This assumes that the first point of the data represents the impulse response at $t = 0$.

A more difficult and nebulous problem is data that is pulse excited. This type of data contains a "source driven" region and a "natural response" or source free region. The poles of the source free region are the desired output. When the exciting field is a Gaussian pulse, NBEGIN must be set late

Table 5. Pole list showing aliased pole; $F_{HN} = 1.0646 \times 10^9$ Hz.

Mag	Residues		Poles	
	Real	Imag	Real	Imag
1.81062E-01	-1.81062E-01	-1.93179E-14	-4.24039E+01	0.
1.30319E-06	1.30319E-06	7.67386E-13	-6.73500E+08	4.71419E-12
1.00000E+00	1.36450E-06	1.00000E+00	-5.00000E+08	-2.00000E+08
1.00000E+00	1.36450E-06	-1.00000E+00	-5.00000E+08	2.00000E+08
5.04271E-08	-2.96942E-08	-4.07572E-08	-2.00289E+08	-2.62155E+08
5.04273E-08	-2.96946E-08	4.07572E-08	-2.00289E+08	2.62155E+08
9.99999E-01	1.28560E-06	9.99999E-01	-5.00000E+08	-4.00000E+08
9.99999E-01	1.28560E-06	-9.99999E-01	-5.00000E+08	4.00000E+08
9.99999E-01	1.22611E-06	9.99999E-01	-5.00000E+08	-6.00000E+08
9.99999E-01	1.22611E-06	-9.99999E-01	-5.00000E+08	6.00000E+08
1.00000E+00	4.39212E-06	1.00000E+00	-5.00001E+08	-8.00000E+08
1.00000E+00	4.39212E-06	-1.00000E+00	-5.00001E+08	8.00000E+08
1.00001E+00	3.62365E-05	-1.00001E+00	-4.99999E+08	-9.29167E+08
1.00001E+00	3.62365E-05	1.00001E+00	-4.99999E+08	9.29167E+08
9.99975E-01	-4.50848E-05	9.99975E-01	-4.99998E+08	-1.00000E+09
9.99975E-01	-4.50848E-05	-9.99975E-01	-4.99998E+08	1.00000E+09

} Aliased pole

Fig. 11. Error produced in reconstructed data by one aliased pole.

Fig. 12. Three-dimensional pole plot showing location of an aliased pole.

Fig. 13. Error introduced by aliased pole with only 1/10 the residue of the other poles.

enough in the time history to avoid the source driven area. This must be done because a Gaussian cannot be represented by a finite sum of complex exponentials. If the Gaussian-source-driven region is included in the processing, the results will be in error. Figure 14 shows the correct pole locations for a 60-m dipole. In calculating the results of Fig. 14, the processing started at B in Fig. 15, avoiding the source driven region. Moving the starting point back to A in Fig. 15 causes the pole location to be in error as shown in Fig. 16. A more dramatic difference between starting at points A and B can be shown by comparing the errors between original and reconstructed data. Figure 17 plots the error when the processing begins at point B in the source free region.

Figure 18 is the error when the processing starts at point A. As can be seen, the error went from 0.0023% to 118.6%. The code cannot obtain the correct results when in the source driven region because the source is a Gaussian pulse.

However, if the source can be represented by complex exponentials, the SEMPEX code can calculate the correct poles even when the processing starts in the source driven region. Figure 19 shows a time waveform consisting of a double exponential convolved with three complex exponentials as defined by the following expression:

$$F(t) = [\exp(-3.33 \times 10^7)t - \exp(-6.66 \times 10^7)t] \otimes \sum_{i=1}^3 \exp(\alpha_i t) \quad (5)$$

where:

$\otimes$ indicates convolution,

$$\alpha_1 = -4.96 \times 10^6 \pm j 7.67 \times 10^5,$$

$$\alpha_2 = -6.5 \times 10^6 \pm j 5.317 \times 10^6,$$

$$\alpha_3 = -8.37 \times 10^7 \pm j 2.185 \times 10^7.$$

Table 6 shows SEMPEX accurately located both the source and structure poles. This particular case deals with a simple source that takes only two poles to define. The more complicated the source with respect to its pole structure, the more poles will be required to define it. If the source should require 50 poles and the structure requires 50 poles, 100 poles would be required to define the waveform. If the processing is started in the source free region, the pole requirements drop to 50. Therefore, as a general rule avoid the source driven region if possible.

Fig. 14. Correct pole locations for a 60-m dipole; NBEGIN = 150.

Fig. 15. Time waveform used for studying effects of including the Gaussian-source driven region in the data samples used for extracting poles.

Fig. 16. Calculated pole locations for example of Fig. 14 when processing starts at A of Fig. 15, in the source driven region. (Source is Gaussian.)

Fig. 17. Error when processing starts in source free region. (Source is Gaussian.)

Fig. 18. Error when processing starts in source driven region. (Source is Gaussian.)

Fig. 19. Temporal data of double-exponential source convolved with three complex exponentials.

Table 6. Pole list showing source and structure poles accurately calculated when processing started in the source driven region with the source waveform of Fig. 19.

Residue			Pole			Error, $10^{-4}\%$	
Mag	Real	Imag	Real	Imag		Real	Imag
1.75228E-01	-1.75228E-01	-5.31701E-14	3.73955E+01	0.	dc	5	(close)
7.25672E+00	7.25672E+00	2.36044E-11	-3.33335E+07	-5.14586E-16	Source	27.5	(close)
4.98630E-00	-4.98630E+00	-5.52660E-11	-6.66685E+07	-3.35707E-15			
2.29783E-00	-9.79282E-01	-2.07870E+00	-4.96238E+06	7.67274E+05	α_1	3.025	138
2.29783E-00	-9.79282E-01	2.07870E+00	-4.96238E+06	-7.67274E+05			
2.90189E-01	-2.89557E-01	1.91370E-02	-6.64972E+06	5.31721E+06	α_2	1.03	15.3
2.90189E-01	-2.89557E-01	-1.91370E-02	-6.64972E+06	-5.31721E+06			
3.89822E-01	1.33627E-01	-3.66203E-01	-8.37008E+07	-2.18402E+07	α_3	17.1	0.045
3.89822E-01	1.33627E-01	3.66203E-01	-8.37008E+07	2.18402E+07			

Time Window (T')

Another parameter that, if not correct, will produce incorrect poles is the time window. The actual time window (T') is defined by the parameters DT, NDECI, and NPTS by the following relationship:

$$T' \equiv NDECI * DT * NPTS. \quad (6)$$

Sampling theory states the minimum time window (T) required to observe the waveform is:

$$T \equiv \frac{NPOLES}{F_H}, \quad (7)$$

where NPOLES is the number of poles in the data and F_H is the highest frequency in the data.

Unfortunately, this does not guarantee that the time window will be wide enough to accurately calculate the pole set. For example, T for data with nine poles and a high-frequency pole of 21.85 MHz, by Eq. (7) is:

$$T = 4.119 \times 10^{-7} \text{ s.}$$

Figure 20 is a plot of this data showing the actual time window T' used to calculate the pole set, and, by Eq. (6):

$$T' = 4.696 \times 10^{-7} \text{ s.}$$

Here NDECI = 4 sets F_{HN} to 31.93 MHz, which is greater than the highest-frequency pole of 21.8 MHz. Therefore, no aliasing can occur. NPOLES was chosen to be 15, well above the actual number of 9 poles, eliminating any problems associated with requesting too few poles. The results are tabulated in Table 7. The valid poles indicated by α_1 , α_2 , and α_3 have the percent error from the true value for their real and imaginary parts listed. As can be seen, the real and imaginary parts of the low-frequency pole α_1 are off by 2.97% and 5.32%, respectively. Another SEMPEX run was set up such that T' was increased to 6.57×10^{-7} s. This is slightly greater than half the period of the low-frequency pole. Table 8 lists the poles calculated. The percentage errors from the true pole locations are listed and indicate the low-frequency pole has now been accurately calculated.

Fig. 20. Nine-pole data with time window $T' = 4.69 \times 10^{-7}$ s.

Table 7. Pole list for data of Fig. 20 calculated with a time window less than 1/2 the period of the lowest-frequency pole.

Residue			Pole		Error, %		
Mag	Real	Imag	Real	Imag	Real	Imag	
4.61699E-01	-4.61699E-01	-2.92940E-12	-1.25829E+06	0.	dc		
4.93874E+00	-4.93874E+00	-2.98458E-11	-6.65483E+07	-2.63320E-13	source		
7.22545E+00	7.22545E+00	2.15934E-11	-3.32338E+07	-2.88240E-07			
2.51464E+00	-8.48248E-01	2.36726E+00	-4.81479E+06	-7.26415E+05	α_1	2.97 5.32	
2.51464E+00	-8.48248E-01	-2.36726E+00	-4.81479E+06	7.26415E+05			
2.85077E-01	-2.84689E-01	-1.48757E-02	-6.49597E+06	-5.31675E+06	α_2	0.0172 0.0085	
2.85077E-01	-2.84689E-01	1.48757E-02	-6.49597E+06	5.31675E+06			
2.61634E-08	-1.44633E-10	-2.61630E-08	3.47769E+06	-1.57573E+07	α_3	0.015 0.0636	
2.61635E-08	-1.44619E-10	2.61631E-08	3.47769E+06	1.57573E+07			
3.87732E-01	1.33050E-01	3.64189E-01	-8.36957E+07	2.18361E+07			
3.87732E-01	1.33050E-01	-3.64189E-01	-8.36957E+07	-2.18361E+07			
1.04625E-07	8.30058E-09	1.04295E-07	2.13405E+05	2.27022E+07			
1.04625E-07	8.30058E-09	-1.04295E-07	2.13405E+05	-2.27022E+07			
1.57941E-07	1.54160E-07	-3.43554E-08	-1.56835E+06	2.76800E+07			
1.57941E-07	1.54160E-07	3.43554E-08	-1.56835E+06	-2.76800E+07			

Table 8. Pole list calculated with a time window greater than 1/2 the period of the lowest-frequency pole.

Residue			Pole		Error, 10 ⁻³ %	
Mag	Real	Imag	Real	Imag	Real	Imag
4.99703E+00	-4.99703E+00	-4.96702E-11	-6.66033E+07	-7.23379E-12		
7.26603E+00	7.26603E+00	4.24656E-11	-3.33619E+07	-1.30319E-06		
2.29267E+00	-9.79150E-01	-2.07307E+00	-4.96180E+06	7.68141E+05	α_1 11.4	114
2.29267E+00	-9.79150E-01	2.07307E+00	-4.96180E+06	-7.68141E+05		
2.90189E-01	-2.89558E-01	1.91201E-02	-6.64965E+06	5.31718E+06	α_2 0.949	2.06
2.90189E-01	-2.89558E-01	-1.91201E-02	-6.64965E+06	-5.31718E+06		
3.89963E-01	1.33094E-01	3.66547E-01	-8.37037E+07	2.18347E+07	α_3 5.18	25
3.89963E-01	1.33094E-01	-3.66547E-01	-8.37037E+07	-2.18347E+07		

Figure 21 is an error plot of the difference between the original and reconstructed data calculated using the narrow time window of Fig. 20. Note the percent rms error is only 0.0045%. This would normally indicate the calculated pole set was correct, yet the low-frequency pole had errors as high as 5.32%. How then, when dealing with data containing unknown poles, can one determine whether or not the pole set is really correct? Figure 22 shows a comparison of the original data and data reconstructed from the pole set calculated with too narrow a time window. When the reconstructed data is extrapolated out to later time, the error becomes obvious. Therefore, if the extrapolation agrees favorably with the complete original data record, one can be reasonably confident that the correct pole set has been calculated. This comparison is the only method currently known that can detect improper poles due to too narrow a time window.

Observations indicate that the following rules may be useful in determining the proper time window: If Eq. (7) indicates T' to be greater than $1/2$ the period T_L of the lowest frequency in the data, then that is the proper minimum time window. If Eq. (7) indicates T' to be less than $0.5T_L$, then T' should be adjusted appropriately.

At present, no rational explanation is available to explain the time window requirements. Therefore, this is one area that needs further study and investigation.

Effects of Noise

For the previous discussion, all the input data was computer generated and, therefore, essentially noise free. When noise is added to the data, the results are no longer easy to interpret. Under certain conditions the SEMPEX code cannot find the proper pole locus even when only small amounts of noise are added. As an example Fig. 23 is a plot of a simple noise-free damped sine wave where

$$V_{1,2} = -5 \times 10^5 \pm j2\pi(5 \times 10^5),$$

$$A_{1,2} = 0 \pm j1.$$

Fig. 21. Difference between original and reconstructed data with the time window too narrow. (Error equals reconstructed minus original.)

Fig. 22. Comparison of original and reconstructed data when time window is too narrow.

Fig. 23. Generated data: one conjugate pole pair with $\alpha_{1,2} = -5 \times 10^5 \pm j2\pi(5 \times 10^6)$. Data points indicated were used in calculating pole lists of Tables 9 and 10.

The data points in the figure indicate the data used for the calculations, and Table 9 shows the correct pole was calculated. Then ± 0.005 volts of uniformly distributed random noise was added to the data, and the code was rerun using the same parameters used to calculate the poles in Table 9. Since each pole was generated with a residue of $0 \pm j1.0$, it was desired to add noise equal to $\pm 0.5\%$ of the value of the residue. As Table 10 shows, that noise with a magnitude of $\pm 0.5\%$ of the magnitude of the residue caused a 2.77% error in the residue, a 1.6% error in the imaginary part of the pole, and a 110.6% error in the real part of the pole. On the surface it would appear the SEMPEX code cannot find the proper poles in the presence of noise. However, Table 10 shows only three poles were calculated and there were three poles in the original data. When noise was added to the data, effectively, additional information was added to the original three-pole data.

In coping with this problem, the next step would be to request more poles so the extra ones would fit the noise. Another run was made with $NPOLES = 10$ and all other input parameters unchanged. Figure 24 shows the data points chosen to calculate the poles. The results of this run are listed in Table 11. Note there is a considerable improvement in the pole locations. The error in the real part decreased from 110.6% to 0.515%, the imaginary part from 1.6% to 0.025%. The error in the magnitude of the residue changed from 2.7% to 0.033%. This shows that, given the proper conditions, meaningful results can be expected.

This example was extremely simple, yet required some varying of the parameters. With more complicated data the results are more confusing. Figure 25 shows the noise-free, upper-left half-plane pole locus for a 60-m dipole. Figure 26 shows the time waveform used to give Fig. 25 but with $\pm 1\%$ of peak noise added. The SEMPEX code was rerun on this noisy waveform using the same parameters as were used to calculate the poles of Figure 25. Figure 27 is a plot of the resulting pole locations. As expected, the pole locations are no longer correct, other than for five exceptions.

At first glance the poles in Fig. 27 would seem to be totally wrong. But a careful examination shows that the poles enclosed by boxes didn't move much. An examination of the residues calculated from the noise-free data indicates these boxed poles had the largest residues. The higher-frequency poles have residues of less magnitude, and, since the noise level is constant with frequency, it affects higher-frequency poles more.

Table 9. Calculated pole list for noise-free damped sine wave of Fig. 23.

Mag	Residue		Pole		Error, %	
	Real	Imag	Real	Imag	Real	Imag
2.00513E-02	-2.00513E-02	-3.30846E-14	1.00825E+03	0.		
1.00000E+00	-2.98553E-08	-1.00000E+00	-5.00005E+05	5.00000E+06	α_1	1×10^{-3}
1.00000E+00	-2.98553E-08	1.00000E+00	-5.00005E+05	-5.00000E+06		

Table 10. Pole list calculated for damped sine wave of Fig. 23 with $\pm 0.5\%$ -residue-magnitude, uniformly distributed noise added.

Residue error, %	Residues			Poles		Pole error, %	
	Mag	Real	Imag	Real	Imag	Real	Imag
2.77	4.70000E-03	-4.70000E-03	-1.18575E-14	3.28617E+07	1.60413E-17	110.6	1.6
	1.02774E+00	-5.91867E-03	-1.02772E+00	-1.05320E+06	4.91977E+06		
	1.02774E+00	-5.91867E-03	1.02772E+00	-1.05320E+06	-4.91977E+06		

Table 11. Ten-pole list calculated from data of Fig. 23 with added uniformly distributed noise equal to $\pm 0.5\%$ of residue magnitude.

Residue error, %	Residue			Pole		Pole error, %	
	Mag	Real	Imag	Real	Imag	Real	Imag
0.0334	9.99666E-01	1.56536E-03	-9.99664E-01	-4.97421E+05	4.99871E+06	0.515	0.025
	9.99666E-01	1.56536E-03	9.99664E-01	-4.97421E+05	-4.99871E+06		
	8.54356E-04	-4.86578E-04	-7.02257E-04	1.83016E+06	-7.58211E+06		
	8.54356E-04	-4.86578E-04	7.02257E-04	1.83016E+06	7.58211E+06		
	2.50735E-03	1.57510E-03	1.95086E-03	-2.67559E+06	-1.30850E+07		
	2.50735E-03	1.57510E-03	-1.95086E-03	-2.67559E+06	1.30850E+07		
	1.10828E-05	-1.08030E-05	2.47439E-06	1.48233E+07	-1.71338E+07		
	1.10828E-05	-1.08030E-05	-2.47439E-06	1.48233E+07	1.71338E+07		
	1.02522E-03	-7.93953E-04	6.48623E-04	-4.07987E+06	2.14975E+07		
	1.02522E-03	-7.93953E-04	-6.48623E-04	-4.07987E+06	-2.14975E+07

Fig. 24. Generated data of Fig. 23, showing data points used to calculate a ten-pole set from Fig. 23 input data with ± 0.005 V uniformly distributed random noise added.

Fig. 25. Upper-left half-plane pole locus for 60-m dipole.

Fig. 26. Computed scattered field of 60-m dipole with $\pm 1\%$ -peak-amplitude, uniformly distributed noise added.

Fig. 27. Upper-left half-plane pole locus for 60-m dipole with $\pm 1\%$ -peak-amplitude uniformly distributed noise added.

Therefore, if a given pole's residue is well above the noise level, that pole will exhibit little degradation due to the noise. Other poles in the data with small residues will be dominated by the noise and their locations will be greatly affected.

NOISE REDUCTION

Autocorrelation, Integration, and Filtering

Because of this sensitivity to noise, techniques to lessen the effects of the noise were investigated. Methods like autocorrelation, integration, and filtering were evaluated but proved to be unsuccessful. Autocorrelation reduced the noise content significantly but changed the relative magnitudes of the residues. Dominant poles became more dominant while poles with small residues were minimized along with the noise. Therefore, no real improvement in the pole locus could be obtained. Filtering the data proved to be unsuccessful since the filter could remove the out-of-passband noise but couldn't do anything about the in-band noise.

The filter used was a truncation type filter. The Fast Fourier Transform of the data was taken, and those frequency terms not desired were set to zero. The inverse transform was taken and the resulting waveform used for the analysis. The utility of the filter was not for in-band noise reduction, but for eliminating all unwanted information above a given cutoff frequency F_c . This insured that if the Nyquist sampling rate was adjusted such that F_{HN} was slightly lower than F_c , no aliasing of high-frequency noise would occur. This technique simulates an ideal filter in that at one frequency step in the spectrum the function has some value and at the next frequency step the function value is zero. The inverse transform of a discontinuity such as this is $\text{SIN}(X)/X$. To minimize this distortion, F_c must be chosen to fall in a natural notch of the frequency spectrum. Figure 28 shows some acceptable cutoff points.

To demonstrate how the filter operates, a run was made using the data whose frequency spectrum is plotted in Fig. 28. The cutoff point F_c chosen was 0.9978×10^7 Hz, which corresponds to the first notch in the spectrum and also defines F_H . The input parameters were adjusted to reflect the band limited nature of the filtered data with $TMAX = 1.7037 \times 10^{-6}$ s; $NDECI = 15$; by Eq. (4), $F_{HN} = 0.99978 \times 10^7$ Hz; and $NPOLES$ set to 12. The original unfiltered data had approximately 50 poles. If the high-frequency information

Fig. 28. Plot showing acceptable filter-cutoff points.

had not been removed, requesting only 12 poles would have caused errors in the pole locations. Table 12 shows the calculated pole listing indicating the percent error from the true, unfiltered pole locations. As can be seen, there was essentially no change in pole location for the poles in the passband of the filter. The remaining poles are either above the cutoff frequency of the filter or have relatively small residues compared to the dominant poles. Therefore, this filter is very useful in eliminating unwanted high-frequency information, thus avoiding any aliasing problems and removing ambiguous poles from the pole set.

Noise Reduction by Clustering

Figure 29 is a three-dimensional plot of the 60-m dipole pole locus, showing the magnitudes of the residues. Figure 30 is the same plot for the pole locus of the noisy example described above. Note in Figure 30 there are no small residues as found in Figure 29. Therefore, if one is dealing with relatively unknown data where the pole locations and noise levels are not well defined, how can one tell which poles are valid structural poles and which poles are noise dominated? Given the condition where all input parameters are correctly defined, incrementing one parameter, NBEGIN, will expose noise-dominated poles. If a pole set has been properly calculated with NBEGIN equal to 150, then a second pole set calculated with NBEGIN equal to 151 will cause a migration of noise-dominated poles. Figure 31 shows this effect.

The pole locations for NBEGIN = 150 are plotted as A's and pole locations for NBEGIN = 151 are plotted as B's. Those poles with the largest residues (circled) are relatively stable, in fact the first three are superimposed. Those poles whose residues are small show definite migrations which are due to the influence of the noise. To obtain as accurate an estimate of the real pole location as possible, NBEGIN is incremented many times to get a family of poles. Poles that are structure-dependent and have a good signal-to-noise ratio will form a cluster. These clusters then make up the valid identifiable pole set for that structure, and all other poles are discarded. The real and imaginary parts of the poles of each cluster are then averaged to obtain the final pole set. Then the residues are calculated by using the averaged pole set and fitting it to the original data by least-squares techniques.

An example problem will demonstrate the above procedure. Table 13 lists the residues and poles used to generate the test data, and Fig. 32 is the

Table 12. Pole list calculated from data of Fig. 28 after filtering; $F_c = 9.978 \times 10^6$ Hz.

Residue			Pole		Error, %	
Mag	Real	Imag	Real	Imag	Real	Imag
4.24325E-01	-1.69840E-01	-3.88852E-01	-1.51608E+06	4.80983E+06	0	0
4.24325E-01	-1.69840E-01	3.88852E-01	-1.51608E+06	-4.80983E+06		
4.15382E-01	4.12089E-01	-5.21985E-02	-1.06133E+06	2.34745E+06		
4.15382E-01	4.12089E-01	5.21985E-02	-1.06133E+06	-2.34745E+06	0	0
1.31787E-01	-1.03818E-01	-8.11765E-02	-1.82365E+06	-7.28019E+06		
1.31787E-01	-1.03818E-01	8.11765E-02	-1.82365E+06	7.28019E+06		
1.16712E-01	-1.16712E-01	1.33582E-12	-5.86320E+07	-9.99785E+06	0.00495	0
4.47096E-03	-4.47096E-03	-9.25835E-13	-2.65076E+07	-2.05472E-09		
3.08035E-03	-3.08035E-03	-3.48166E-13	-1.21610E+07	9.99785E+06		
2.63935E-03	-2.63935E-03	7.26992E-15	9.98323E+01	0.	All remaining poles either are in the right half-plane above the filter cutoff or have a low residue.	
2.79824E-04	-2.79824E-04	3.27516E-15	6.08861E+05	-9.99785E+06		
2.49492E-08	-2.49492E-08	-2.12471E-19	8.71701E+06	9.99785E+06

Fig. 29. Three-dimensional plot of noise-free pole locus for 60-m dipole.

Fig. 30. Three-dimensional plot of noisy pole locus for 60-m dipole.

Fig. 31. Multiple plot showing migration of noise-dominated poles.

Fig. 32. Amplitude vs time response for six-pole-pair example of clustering.

Table 13. Listing of poles and residues for generating example of noise reduction by clustering.

Residue	Pole
$0 \pm j1.0$	$5 \times 10^5 \pm j20 \times 10^5$
$0 \pm j1.0$	$5 \times 10^5 \pm j40 \times 10^5$
$0 \pm j1.0$	$5 \times 10^5 \pm j60 \times 10^5$
$0 \pm j1.0$	$5 \times 10^5 \pm j80 \times 10^5$
$0 \pm j1.0$	$5 \times 10^5 \pm j100 \times 10^5$
$0 \pm j1.0$	$5 \times 10^5 \pm j120 \times 10^5$

amplitude vs time response. Then 14 mV of uniformly distributed noise was added to the data. This represents a worst case signal-to-noise ratio of -31 dB where 0 dB is 1 V. Since all the poles in this test case have the same residues, the noise will affect each of them about the same. This data was then processed by incrementing NBEGIN from 1 to 10. This produced ten pole sets as listed in Tables 14-23. Each table lists the residues and poles calculated plus how far off the poles were with respect to the actual poles. At the bottom of each list is the average absolute error for the pole set. For example, in Table 14 the real part of a given pole was off by an average of 1.6%. The upper-left half-plane pole plot of the ten runs is shown in Fig. 33, and the clustering effect is quite obvious. Each cluster is then averaged, and the results are listed in Table 24. The average error for the ten runs is also listed. Note that the error for the real part of a given pole decreased from an average of 1.32% per individual run to 0.235% for the composite pole set. The imaginary part of the ten-run pole is relatively insensitive to the noise, thus the improvement was not as great (0.0192% to 0.0114%). As the noise in the data increases, the clusters become more and more spread out (Fig. 34), until the noise dominates and the clusters are no longer discernable (Fig. 35).

This iterative technique is the best method found so far for obtaining valid pole sets from noisy data. The technique solves the biggest problem area of the SEMPEX code and enables the code to be used as a valid research tool. With the noise problem at least under some control, the major problem is now dealing with electromagnetic data with source driven regions. As discussed earlier, to obtain valid results when the computation starts in the source drive region, enough poles must be requested to describe both the source and the structure. Otherwise the results obtained will be invalid. In cases where the source region is short (i.e. the exciting pulse is narrow) the processing can be started at some later time. However, when the source persists throughout the time record, difficulties become more likely.

Table 14, Pole set extracted from noisy waveform (Fig. 32) using NBEGIN = 1.

Residue			Pole		Error, %	
Mag	Real	Imag	Real	Imag	Real	Imag
1.22290E-01	-1.22290E-01	-1.45051E-13	-3.87566E+03	0.	0.52	-0.0085
1.00228E+00	2.15928E-03	-1.00227E+00	-5.02613E+05	1.99983E+06		
1.00228E+00	2.15928E-03	1.00227E+00	-5.02613E+05	-1.99983E+06	+2.2	-0.044
1.00423E+00	8.47084E-03	-1.00420E+00	-5.11044E+05	3.99822E+06		
1.00423E+00	8.47084E-03	1.00420E+00	-5.11044E+05	-3.99822E+06	-1.78	-0.028
9.95728E-01	9.05910E-03	-9.95686E-01	-4.91132E+05	5.99830E+06		
9.95728E-01	9.05910E-03	9.95686E-01	-4.91132E+05	-5.99830E+06	-1.02	-0.0237
9.97471E-01	8.64599E-03	9.97433E-01	-4.94940E+05	-7.99810E+06		
9.97471E-01	8.64599E-03	-9.97433E-01	-4.94940E+05	7.99810E+06		
5.08188E-02	-1.64663E-02	4.80772E-02	-8.05047E+06	9.22480E+06		
5.08188E-02	-1.64663E-02	-4.80772E-02	-8.05047E+06	-9.22480E+06		
1.01141E+00	-2.76266E-03	1.01141E+00	-5.17010E+05	-9.99996E+06	+3.4	-0.0004
1.01141E+00	-2.76266E-03	-1.01141E+00	-5.17010E+05	9.99996E+06		
9.94436E-01	-4.55892E-03	9.94426E-01	-4.96374E+05	-1.20012E+07	-0.726	0.01
9.94436E-01	-4.55892E-03	-9.94426E-01	-4.96374E+05	1.20012E+07		
Average absolute error					1.60	0.019

Table 15. Pole set extracted from noisy waveform (Fig. 32) using NBEGIN = 2.

Residue			Pole		Error, %	
Mag.	Real	Imag	Real	Imag	Real	Imag
1.20004E-01	-1.20004E-01	6.83605E-15	-1.43960E+04	0.	-2.95	-0.12
9.89105E-01	5.73032E-02	-9.87444E-01	-4.85266E+05	1.99764E+06		
9.89105E-01	5.73032E-02	9.87444E-01	-4.85266E+05	-1.99764E+06		
1.00189E+00	9.85823E-02	9.97032E-01	-5.10121E+05	-4.00041E+06	2.02	0.010
1.00189E+00	9.85823E-02	-9.97032E-01	-5.10121E+05	4.00041E+06		
9.98113E-01	1.49694E-01	-9.86824E-01	-4.99371E+05	5.99943E+06		
9.98113E-01	1.49694E-01	9.86824E-01	-4.99371E+05	-5.99943E+06	-0.126	-0.0095
9.98116E-01	1.97202E-01	-9.78441E-01	-4.95577E+05	7.99962E+06		
9.98116E-01	1.97202E-01	9.78441E-01	-4.95577E+05	-7.99962E+06		
9.94175E-01	2.45430E-01	-9.63405E-01	-4.32574E+05	9.99867E+06	-1.48	-0.0133
9.94175E-01	2.45430E-01	9.63405E-01	-4.32574E+05	-9.99867E+06		
1.00182E+00	2.93531E-01	-9.57853E-01	-5.11320E+05	1.19989E+07		
1.00182E+00	2.93531E-01	9.57853E-01	-5.11320E+05	-1.19989E+07	2.26	0.0092
3.28452E-02	-3.28452E-02	-8.52651E-14	-9.85168E+07	-1.27750E+07		
2.66093E-10	2.66093E-10	3.92284E-21	1.66643E+07	-1.27750E+07		
Average absolute error					1.62	0.0278

Table 16. Pole set extracted from noisy waveform (Fig. 32) using NBEGIN = 3.

Residue			Pole		Error, %	
Mag	Real	Imag	Real	Imag	Real	Imag
1.26968E-01	-1.26968E-01	6.75016E-14	-4.58617E+04	0.	-0.02	0.007
9.97687E-01	1.00951E-01	9.92567E-01	-4.99905E+05	-2.00014E+06		
9.97687E-01	1.00951E-01	-9.92567E-01	-4.99905E+05	2.00014E+06		
5.60268E-09	2.70927E-09	-4.90407E-09	1.41143E+07	-3.19323E+06	-1.27	-0.009
5.60268E-09	2.70927E-09	4.90407E-09	1.41143E+07	3.19323E+06		
9.93733E-01	1.96876E-01	-9.74035E-01	-4.93651E+05	3.99964E+06		
9.93733E-01	1.96876E-01	9.74035E-01	-4.93651E+05	-3.99964E+06	-3.8	-0.0106
9.88371E-01	2.87814E-01	9.45537E-01	-4.81003E+05	-5.99936E+06		
9.88371E-01	2.87814E-01	-9.45537E-01	-4.81003E+05	5.99936E+06		
1.00207E+00	3.84538E-01	-9.25351E-01	-5.11989E+05	7.99979E+06	2.4	-0.0026
1.00207E+00	3.84538E-01	9.25351E-01	-5.11989E+05	-7.99979E+06		
9.95745E-01	4.68359E-01	-8.78719E-01	-4.95577E+05	1.00013E+07		
9.95745E-01	4.68359E-01	8.78719E-01	-4.95577E+05	-1.00013E+07	-0.885	0.013
1.00023E+00	5.56144E-01	-8.31361E-01	-5.04279E+05	1.20002E+07		
1.00023E+00	5.56144E-01	8.31361E-01	-5.04279E+05	-1.20002E+07		
Average absolute error					1.53	0.00732

Table 17. Pole set extracted from noisy waveform (Fig. 32) using NBEGIN = 4.

Mag	Residue		Pole		Error, %	
	Real	Imag	Real	Imag	Real	Imag
1.24169E-01	-1.24169E-01	-1.10672E-15	-1.29383E+04	0.		
3.48828E-28	-3.48828E-28	8.95689E-40	5.29551E+07	3.72322E-13		
9.97557E-01	1.45821E-01	-9.86841E-01	-5.04905E+05	2.00022E+06	0.981	0.011
9.97557E-01	1.45821E-01	9.86841E-01	-5.04905E+05	-2.00022E+06		
9.92164E-01	2.88598E-01	-9.49263E-01	-4.93993E+05	3.99954E+06	1.2	0.115
9.92164E-01	2.88598E-01	9.49263E-01	-4.93993E+05	-3.99954E+06		
9.96201E-01	4.23182E-01	-9.01850E-01	-5.02281E+05	6.00040E+06	0.456	0.0067
9.96201E-01	4.23182E-01	9.01850E-01	-5.02281E+05	-6.00040E+06		
9.95853E-01	5.54581E-01	8.27142E-01	-5.07320E+05	-8.00040E+06	1.46	0.005
9.95853E-01	5.54581E-01	-8.27142E-01	-5.07320E+05	8.00040E+06		
9.98713E-01	6.73164E-01	-7.37752E-01	-5.05850E+05	1.00002E+07	1.17	0.002
9.98713E-01	6.73164E-01	7.37752E-01	-5.05850E+05	-1.00002E+07		
9.95507E-01	7.74522E-01	6.25420E-01	-5.05395E+05	-1.19983E+07	1.08	0.0142
9.95507E-01	7.74522E-01	-6.25420E-01	-5.05395E+05	1.19983E+07		
9.26631E-24	-9.26631E-24	1.70729E-34	4.41236E+07	1.27750E+07		
Average absolute error					1.05	0.0084

Table 18. Pole set extracted from noisy waveform (Fig. 32) using NBEGIN = 5.

Residue			Pole		Error, %	
Mag	Real	Imag	Real	Imag	Real	Imag
1.25545E-01	-1.25545E-01	2.00735E-13	-5.85415E+04	0.		
1.87555E-02	-1.87555E-02	-4.12527E-12	-1.32176E+07	-8.48109E-15		
9.96303E-01	1.88360E-01	-9.78335E-01	-5.03980E+05	2.00137E+06	0.796	0.0685
9.96303E-01	1.88360E-01	9.78335E-01	-5.03980E+05	-2.00137E+06		
9.96758E-01	3.83280E-01	-9.20121E-01	-5.06193E+05	3.99915E+06	1.24	0.0213
9.96758E-01	3.83280E-01	9.20121E-01	-5.06193E+05	-3.99915E+06		
9.93832E-01	5.52775E-01	8.25919E-01	-5.03071E+05	-6.00042E+06	0.614	0.007
9.93832E-01	5.52775E-01	-8.25919E-01	-5.03071E+05	6.00042E+06		
9.92272E-01	7.03574E-01	6.99706E-01	-4.96918E+05	-7.99933E+06	0.616	0.0084
9.92272E-01	7.03574E-01	-6.99706E-01	-4.96918E+05	7.99933E+06		
9.99646E-01	8.29843E-01	5.57361E-01	-5.11907E+05	-1.00016E+07	2.38	0.016
9.99646E-01	8.29843E-01	-5.57361E-01	-5.11907E+05	1.00016E+07		
9.98342E-01	9.22278E-01	3.82218E-01	-5.13287E+05	-1.20018E+07	2.66	0.015
9.98342E-01	9.22278E-01	-3.82218E-01	-5.13287E+05	1.20018E+07		
3.50872E-05	3.50872E-05	9.85203E-16	5.85623E+06	-1.27750E+07		
Average absolute error					1.38	0.0227

Table 19. Pole set extracted from noisy waveform (Fig. 32) using NBEGIN = 6.

Residue			Pole		Error, %	
Mag	Real	Imag	Real	Imag	Real	Imag
1.17317E-01	-1.17317E-01	-9.64857E-14	1.58088E+04	0.		
2.87820E-21	-2.87820E-21	-2.22477E-32	3.89002E+07	2.96418E-13		
9.85022E-01	2.43406E-01	-9.54475E-01	-4.89460E+05	1.99949E+06	2.2	0.0255
9.85022E-01	2.43406E-01	9.54475E-01	-4.89460E+05	-1.99949E+06		
9.85496E-01	4.67529E-01	-8.67536E-01	-4.91572E+05	3.99908E+06	1.68	0.023
9.85496E-01	4.67529E-01	8.67536E-01	-4.91572E+05	-3.99908E+06		
9.91682E-01	6.66805E-01	-7.34033E-01	-5.01543E+05	6.00042E+06	0.308	0.007
9.91682E-01	6.66805E-01	7.34033E-01	-5.01543E+05	-6.00042E+06		
9.86910E-01	8.22349E-01	5.45649E-01	-4.94409E+05	-8.00036E+06	1.12	0.0045
9.86910E-01	8.22349E-01	-5.45649E-01	-4.94409E+05	8.00036E+06		
9.91973E-01	9.35290E-01	3.30519E-01	-5.03785E+05	-9.99979E+06	0.756	0.0021
9.91973E-01	9.35290E-01	-3.30519E-01	-5.03785E+05	9.99979E+06		
9.89853E-01	9.85933E-01	-8.80095E-02	-4.94830E+05	1.19979E+07	1.03	0.0175
9.89853E-01	9.85933E-01	8.80095E-02	-4.94830E+05	-1.19979E+07		
9.77927E-09	-9.77927E-09	-4.20605E-20	1.31452E+07	1.27750E+07		
Average absolute error					1.18	0.0133

Table 20. Pole set extracted from noisy waveform (Fig. 32) using NBEGIN = 7.

Residue			Pole		Error, %	
Mag	Real	Imag	Real	Imag	Real	Imag
1.20421E-01	-1.20421E-01	2.84217E-14	3.06565E+04	0.		
9.90446E-01	2.92722E-01	-9.46201E-01	-5.00610E+05	1.99854E+06	0.122	0.075
9.90446E-01	2.92722E-01	9.46201E-01	-5.00610E+05	-1.99854E+06		
9.85474E-01	5.51584E-01	-8.16648E-01	-4.95379E+05	3.99914E+06		
9.85474E-01	5.51584E-01	8.16648E-01	-4.95379E+05	-3.99914E+06	0.926	0.0215
9.86225E-01	7.62778E-01	6.25148E-01	-4.97483E+05	-6.00001E+06		
9.86225E-01	7.62778E-01	-6.25148E-01	-4.97483E+05	6.00001E+06		
3.36919E-12	3.01381E-12	1.50611E-12	1.93676E+07	-6.40250E+06	0.504	0.00017
3.36919E-12	3.01381E-12	-1.50611E-12	1.93676E+07	6.40250E+06		
9.84760E-01	9.09775E-01	3.76911E-01	-4.97673E+05	-7.99963E+06		
9.84760E-01	9.09775E-01	-3.76911E-01	-4.97673E+05	7.99963E+06	0.466	0.00463
9.92317E-01	9.87685E-01	-9.57695E-02	-5.08564E+05	1.00003E+07		
9.92317E-01	9.87685E-01	9.57695E-02	-5.08564E+05	-1.00003E+07		
9.86808E-01	9.67441E-01	-1.94544E-01	-4.95929E+05	-1.20003E+07	1.71	0.008
9.86808E-01	9.67441E-01	1.94544E-01	-4.95929E+05	1.20003E+07		
Average absolute error					0.757	0.0178

Table 21. Pole set extracted from noisy waveform (Fig. 32) using NBEGIN = 8.

Mag	Residue		Pole		Error, %	
	Real	Imag	Real	Imag	Real	Imag
1.14722E-01	-1.14722E-01	-5.46228E-14	1.86480E+05	0.		
2.49693E-02	-2.49693E-02	1.79411E-13	-7.19973E+06	0.		
8.10794E-05	8.10794E-05	5.34295E-16	5.58496E+06	6.34328E-13		
9.89754E-01	3.34656E-01	-9.31461E-01	-5.09090E+05	1.99943E+06	1.82	0.0285
9.89754E-01	3.34656E-01	9.31461E-01	-5.09090E+05	-1.99943E+06		
9.89873E-01	6.29348E-01	-7.64049E-01	-5.11217E+05	4.00051E+06	2.24	0.0128
9.89873E-01	6.29348E-01	7.64049E-01	-5.11217E+05	-4.00051E+06		
9.85819E-01	8.49303E-01	5.00523E-01	-4.95679E+05	-5.99806E+06	0.864	0.0323
9.85819E-01	8.49303E-01	-5.00523E-01	-4.95679E+05	5.99806E+06		
9.87786E-01	9.68827E-01	1.92600E-01	-5.04216E+05	-8.00072E+06	0.843	0.009
9.87786E-01	9.68827E-01	-1.92600E-01	-5.04216E+05	8.00072E+06		
9.84149E-01	9.72893E-01	-1.48417E-01	-4.96473E+05	-1.00007E+07	0.705	0.007
9.84149E-01	9.72893E-01	1.48417E-01	-4.96473E+05	1.00007E+07		
9.84742E-01	8.67086E-01	-4.66774E-01	-5.00051E+05	-1.20003E+07	0.0102	0.0025
9.84742E-01	8.67086E-01	4.66774E-01	-5.00051E+05	1.20003E+07		
Average absolute error					1.08	0.0154

Table 22. Pole set extracted from noisy waveform (Fig. 32) using NBEGIN = 9.

Residue			Pole		Error, %	
Mag	Real	Imag	Real	Imag	Real	Imag
1.21023E-01	-1.21023E-01	2.13163E-14	1.31982E+04	0.		
9.90644E-01	3.79782E-01	-9.14954E-01	-5.11380E+05	2.00047E+06	2.28	0.0235
9.90644E-01	3.79782E-01	9.14954E-01	-5.11380E+05	-2.00047E+06		
4.79911E-14	4.79906E-14	2.20862E-16	2.35681E+07	-3.39028E+06		
4.79911E-14	4.79906E-14	-2.20862E-16	2.35681E+07	3.39028E+06	1.86	0.0265
9.82183E-01	6.98696E-01	-6.90296E-01	-4.90709E+05	3.99894E+06		
9.82183E-01	6.98696E-01	6.90296E-01	-4.90709E+05	-3.99894E+06		
9.89174E-01	9.13550E-01	-3.79330E-01	-5.17017E+05	6.00004E+06	3.40	0.00067
9.89174E-01	9.13550E-01	3.79330E-01	-5.17017E+05	-6.00004E+06		
9.89340E-01	9.89324E-01	5.67735E-03	-5.10009E+05	7.99930E+06		
9.89340E-01	9.89324E-01	-5.67735E-03	-5.10009E+05	-7.99930E+06	2.0	0.00875
9.86849E-01	9.08581E-01	-3.85163E-01	-5.03721E+05	-9.99920E+06		
9.86849E-01	9.08581E-01	3.85163E-01	-5.03721E+05	9.99920E+06		
9.81633E-01	6.85377E-01	-7.02754E-01	-4.92729E+05	-1.19984E+07	1.45	0.0133
9.81633E-01	6.85377E-01	7.02754E-01	-4.92729E+05	1.19984E+07		
Average absolute error					1.95	0.0134

Table 23. Pole set extracted from noisy waveform (Fig. 32) using NBEGIN = 10.

Residue			Pole		Error, %	
Mag	Rea	Imag	Real	Imag	Real	Imag
1.24178E-01	-1.24178E-01	6.66134E-15	2.24869E+04	0.		
9.82043E-01	4.15879E-01	-8.89637E-01	-4.92410E+05	2.00127E+06	1.51	0.0635
9.82043E-01	4.15879E-01	8.89637E-01	-4.92410E+05	-2.00127E+06		
9.85926E-01	7.58212E-01	-6.30211E-01	-5.01670E+05	4.00148E+06	0.334	0.037
9.85926E-01	7.58212E-01	6.30211E-01	-5.01670E+05	-4.00148E+06		
1.79742E-02	9.66703E-03	-1.51532E-02	-3.09077E+06	5.50091E+06		
1.79742E-02	9.66703E-03	1.51532E-02	-3.09077E+06	-5.50091E+06		
9.81875E-01	9.56021E-01	2.23835E-01	-4.99221E+05	-5.99725E+06	0.156	0.0458
9.81875E-01	9.56021E-01	-2.23835E-01	-4.99221E+05	5.99725E+06		
9.79956E-01	9.58942E-01	2.01852E-01	-4.94636E+05	7.99816E+06	1.07	0.023
9.79956E-01	9.58942E-01	-2.01852E-01	-4.94636E+05	-7.99816E+06		
9.81914E-01	7.85927E-01	-5.88620E-01	-4.99424E+05	-1.00000E+07	0.116	0
9.81914E-01	7.85927E-01	5.88620E-01	-4.99424E+05	1.00000E+07		
9.81054E-01	4.56084E-01	-8.68593E-01	-4.95652E+05	-1.19994E+07	0.87	0.005
9.81054E-01	4.56084E-01	8.68593E-01	-4.95652E+05	1.19994E+07		
Average absolute error					1.14	0.029

Table 24. Averaged pole-set data from Tables 14-23 (10 runs), showing effectiveness of clustering for lowering error.

Residues (magnitude)			Poles					
			Real part			Imaginary part		
Actual	Computed	Error, %	Actual	Computed	Error, %	Actual	Computed	Error, %
1.0	1.00056	0.056	5 E5 ^a	4.99962 E5	0.0076	2 E6	1.99924 E6	0.038
1.0	1.00058	0.058	5 E5	5.00555 E5	0.111	4 E6	3.99961 E6	0.0098
1.0	0.99886	0.114	5 E5	4.9878 E5	0.244	6 E6	5.99937 E6	0.0105
1.0	0.99958	0.042	5 E5	5.00769 E5	0.154	8 E6	7.99954 E6	0.0058
1.0	1.0023	0.23	5 E5	5.03488 E5	0.698	10 E6	10.0002 E6	0.002
1.0	1.00094	0.004	5 E5	5.00985 E5	0.197	12 E6	12.9997 E6	0.0025
Average absolute error		0.084			0.235			0.0114
Average absolute error per run					1.32			0.0192

^aE5 means 5×10^5 .

Fig. 33. Pole clusters from ten iterations with ± 14 mV added noise.

Fig. 34. Example of pole-cluster spreading with increasing noise; ± 0.28 V noise added.

Fig. 35. Multiple pole plot demonstrating noise-dominated pole behavior.

CONCLUSIONS AND RECOMMENDATIONS

The above discussion and the authors' experience lead to the following conclusions and recommendations:

- The iterative technique described enables the user to differentiate between noise-dominated poles and structurally significant poles, with the mean of the pole clusters converging on the true value of the pole.
- The time-window requirements described give a user some guidelines on how long a temporal waveform must be observed to give an accurately calculated pole set. However, investigations on a theoretical level are needed to uncover precisely how the time window affects the accuracy of the pole locations.
- Presently the multiple-run technique uses two data points per pole in the calculations. Using three, four, or more data points per pole might improve the accuracy and should be investigated.
- Laboratory pulse generators may require many poles to characterize their response. If the response of such a generator persists throughout the data record, the existing code's present capacity of fifty poles will have to be increased in order to calculate the correct pole set for the excited body. The absolute maximum will probably be governed by the capability of the polynomial-root-finding procedure.
- The current version of the SEMPEX code solves the matrix equation

$$AA^T x = A^T B.$$

If the condition of $2 * NPOLES = NPTS$ is observed, this equation reduces to

$$Ax = B.$$

The advantage here is that the transpose of the A matrix need not be computed. Avoiding this computation will obviously save computation time and may decrease the ill-conditioning of the operation.

ACKNOWLEDGMENT

Computer time used for the examples in this report was funded by the Air Force Weapons Laboratory, Kirtland A.F.B., New Mexico, under Project Order 76-090.

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Printed in the United States of America

Available from

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