NUMERICAL SIMULATION OF LASER FUSION PLASMAS USING A REDUCED DESCRIPTION FORMALISM

Rickey James Faehl

(Ph.D. Thesis)

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Reduced Description Formalism

Abstract

A computational study of laser fusion plasmas with a new numerical model is undertaken. The plasma is described by fluid equations and the fields by the time dependent Maxwell's equations. Problems of stimulated Brillouin scattering, self-steepening of the critical surface, wave trapping near the critical surface, and competition of inverse bremsstrahlung absorption with Brillouin scattering are treated to demonstrate the flexibility of the model. New results are obtained for intensity-dependent inverse bremsstrahlung absorption. Absorption of laser energy via density fluctuations in the underdense plasma is discussed and studied numerically for assumed fluctuation distributions.

Acknowledgments

"I am in blood
Stepp'd in so far that, should I wade no more,
Returning were as tedious as go o'er"

Macbeth Act III, Scene IV

It is not false modesty to state that this study could not have been completed without the aid and support of many people. I wish to acknowledge my deepest appreciation here to the Tull's, Estabrook's, Valeo's, Krueer's, and Thomson's, without whose friendship and hospitality I should have had a much more trying time commuting between Livermore and Albuquerque. Kent Estabrook, Ernest Valeo, Jeff Thomson, Bruce Langdon, and Barbara Lasinski have also provided, aside from a spare bed on occasion, valuable discussions, criticism, and/or support for this project.

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The difficult job of preparing this manuscript has been superbly accomplished by Mrs. Esther Caster.

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John DeGroot is my first mentor in plasma physics. If I have

learned anything of the simulation of plasmas, I must credit John for redirecting my experimentalist inclinations toward that end. He taught me not simply a collection of sundry techniques, but the intuition for modeling. Thus, I regard as much more subtle and difficult to transmit.

Finally, Dr. William Krueger has taken me from a callow graduate student to a physicist. My understanding of the physical world and appreciation of its mystery and excitement have been immeasurably enhanced by him. He has taught me to be surprised, to expect the unexpected. I freely acknowledge my profound gratitude for this.

And throughout, Allyson has not contributed in any direct way to this study, but if not for her, I should not have maintained a semblance of sanity as long as I have.

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CHAPTER 1

Introduction

Laser fusion offers the promise of a vast untapped energy source. To achieve this, laser energy must be used to efficiently compress a small mass of fusible material. Plasma processes are able to significantly alter the propagation and absorption of this energy. These alterations are inexplicable for the most part within the context of atomic processes, but are of utmost importance for the efficient implosion of laser fusion pellets. Although much understanding of these processes has already been derived from numerical and analytical studies, continuing anomalies between theory and experiment indicate the need for further study.

This study concerns the use of a reduced description to numerically describe reflective, absorptive, and modulational processes in laser plasmas. The reflective instabilities in particular pose large problems to laser fusion. They add nothing useful to the implosion process, but are potentially capable of scattering out large fractions of the incident energy before it can be used for compressional work. Despite extensive numerical modeling of these instabilities, the initial calculations were unable to predict the observed levels of scattering or its spectra. It is clear in retrospect that the scattering was modified by long-time phenomena (compared to the simulation duration), and competition with higher dimensional effects.

Investigation of these areas would be beyond the scope of this thesis,

but the mere fact that such features make an observable contribution suggests that the simulations may have missed other interesting phenomena.

One reason particle simulations may not have detected such effects is that, while they represent a close approximation to actual collisionless plasmas, they have certain profound limitations. Among these are the high level of numerical noise and the slow speed. These lead to a tendency to follow systems which are too small for too short a time. In consequence, an effect seen in a careful simulation is probably real, but it is not necessarily the one which will persist long enough to register on an experimentalist's meter.

A flexible numerical tool, involving a reduced description of the plasma, was developed to complement the simulation technique and to circumvent the above restrictions. The two-fluid plasma equations together with Maxwell's equations were found to provide a powerful tool for the description of laser-illuminated plasmas, including wave propagation, hydrodynamics, laser absorption, and scattering instability evolution. The dramatically reduced calculation speed facilitated long duration, quiet studies of the self consistent one-dimensional laser/plasma interaction problem.

A reduced description of the plasma is discussed in Chapter 2. Stimulated reflection coefficients are calculated with this method for a linear density gradient plasma, and these are compared with corresponding simulation values. The good agreement, up to levels when trapping dominates, confirmed the validity of this approach.

The inverse bremsstrahlung absorption coefficient is a crucial parameter in laser fusion schemes, yet it is not contained within the particle simulation technique. It was a trivial factor to include in the reduced description. Comparisons with theory verified the calibration. Another linear problem concerns the magnitude of the laser field in the vicinity of the critical surface. Analytic theory gives a standing Airy pattern in a linear density gradient. The exact steady pattern calculated here was compared with that solution, and was also done for nonlinear density gradients, various magnitudes of collisional absorption, and finite laser bandwidth. The resulting intensity patterns were related to the Airy value through approximate analytic models or simple heuristic expressions. Finally, Kidder predicted that the zeroth order ponderomotive force from the standing laser wave near the critical density would steepen the density profile. This was numerically confirmed.

In Chapter 3, the reduced plasma description was used to study the post-saturation evolution of the scattering instabilities. A common feature of many laser plasma scattering experiments is the enhanced width of the scattered spectrum. The stimulated scattering should, however, narrow the spectrum, at least within the linear theory framework. Homogeneous, periodic calculations were conducted first, to eliminate the complication of end-effects. It was found convenient to spatially Fourier analyze the reduced equations first, and to follow the evolution of the spectrum directly. Two distinct features were found, harmonic generation driven by large amplitude

acoustic waves, and repeated scattering. This latter has been observed experimentally in gases and liquids, so there are grounds for expecting it in long scale length plasmas. The implied long propagation paths are inconsistent with backscattering, but may be plausible for sidescattering. Finite system calculations verified the existence of these scattering mechanisms in locally homogeneous systems, but they were severely weakened in inhomogeneous ones. The absence of spectral broadening in the backscatter direction, for scale lengths typical of recent experiments, while broadening is still occurring in a sidescatter mode, is consistent with the measurements reported by an NRL group.

Chapter 4 deals with the trapping of intense electromagnetic signals in self-consistent longitudinal density fluctuations. This phenomenon was apparently first suggested by Gurevich and Karpman, though not verified. Particle simulations and reduced description calculations show the existence of such structures near, but below, the critical density when the field is above a threshold value. The spectral evolution of the field and the density perturbations were followed in the fluid calculations. The final k -spectrum has a nearly exponential form, reminiscent of a solitary wave. Periodic calculations show that mode coupling leads toward this result as well. End effects play a role in the establishment of the stable structures, but the effect depends primarily on the local value of the field. Formation is not greatly inhibited by gradient effects, as long as the density scale length is significantly greater than the width of the hole. Bandwidth also has little effect on either

formation or stability of the structures. The one-dimensional calculations are finally discussed with respect to other, higher dimensional calculations performed by Valeo and Estabrook and by others.

In Chapter 5, various absorption related topics are studied with the reduced description. The first section deals with reductions in classical absorption induced by density modifications. Stimulated Brillouin scattering as expected is found to have a highly deleterious effect on collisional absorption. Self-steepening tends to be dominated by SBS as an absorption reducer, except in very steep gradient profiles. In that case, steepening is found to reduce absorption efficiency by factors on the order of fifty percent. In the next section, the intensity dependence of the inverse bremsstrahlung cross-section, which is usually neglected, is treated. For intensities comparable to those being considered for stimulated processes, a standing wave pattern in moderate length system is found to induce reduction factors of 2-3 in total absorption. The effect depends primarily on the local magnitude of the field, so a standing wave is more effective in inhibiting absorption than a traveling wave. Finally, in the last section, a new absorption mechanism, based on scattering oscillating electrons from finite density fluctuations, is discussed. Calculations show that this mechanism has the desirable properties that it is relatively insensitive to density modulations near the critical density, extends over a wide density range, and produces reasonably soft heated electron distributions. Various sources for the density fluctuations are discussed.

The conclusions of this study are summarized in the last chapter. The relevance of the results to laser fusion experiments is briefly discussed, and extensions of this work to future studies are presented.

CHAPTER 2

The Reduced Description Model of Laser Plasma

There are many ways to describe a plasma system. The complete classical description consists of specifying the position and momentum of each particle and the fields at each position. A plasma is not meaningful, however, unless the physical dimensions are greater than several Debye lengths. Such a volume contains too many particles to permit practical applications of a complete description. Plasma simulation techniques^{1,2} follow the evolution of a system by moving quasiparticles in their self-consistent fields. The fields are self-consistent because their sources, density and current density are just the local averages over the positions and velocities of the particles. This procedure is virtually equivalent to solving Liouville's equation for a given set of initial conditions. The technique closely models the behavior of actual collisionless plasmas, but it seems improbable that all problems of interest require such detailed information. In particular, the high noise levels and expense involved with such techniques indicate that an accurate, reduced description of the plasma would have many applications. One such reduced description, which couples the high frequency field equations with the two-fluid plasma equations, has been derived here. The body of this work is devoted to determining the extent to which this description can be applied to laser plasma

systems. In this chapter, the formalism is first described, and then tested against a variety of conditions for which known solutions, or approximate solutions, exist.

SECTION I

Description of the Reduced Description Formalism (RDF)

The collective processes which characterize the fast time scale behavior of a plasma require a wave description. The reduced equations must therefore contain at least enough information to follow waves. The first two fluid equations of a species with charge, q , and mass, m , i.e., for particle density, n , and flux, $\bar{\Gamma} = n\bar{v}$,

$$\frac{\partial n}{\partial t} = -\nabla \cdot \bar{\Gamma} \quad (2-1)$$

$$\frac{\partial \bar{\Gamma}}{\partial t} = -\nabla \left(\frac{\Gamma^2}{n} \right) + \frac{q}{m} n \bar{E} + \frac{q \bar{\Gamma}}{mc} \times (\nabla \times \bar{A}) - \frac{1}{m} \nabla \cdot \bar{p} - \nu \bar{\Gamma} \quad (2-2)$$

can easily be shown to yield such a wave equation. In these equations $\bar{E} = -\nabla \phi - \frac{1}{c} \frac{\partial \bar{A}}{\partial t}$ is the longitudinal electric field, $\bar{A}$ is the vector potential and ν a heuristic damping term, which can be regarded as playing the role of either Landau and/or collisional damping. Viscosity terms, whether real or artificial, were not included initially. These two equations are only the first in an infinite sequence of moment equations. The sequence is truncated here by assuming a scalar pressure, either isothermal $P_j = n_j \theta_j$ or adiabatic, $P_j \propto n_j$. Two equations such as (2-1) and (2-2) are included for each plasma species. The fields which appear in (2-1) and (2-2) are the macroscopic ones given by Maxwell's equations. In the transverse gauge, these can be derived from the potential equations

$$\nabla^2 \phi = -4\pi e(n_i - n_e) \quad (2-3)$$

$$\frac{\partial^2 \bar{A}}{\partial t^2} - 2\nu_{em} \frac{\partial \bar{A}}{\partial t} - c^2 \nabla^2 \bar{A} = -4\pi c \bar{J}_{tr} \quad (2-4)$$

where $\bar{J}_{tr}$ is the component of the current density transverse to the direction of propagation, and ν_{em} the electromagnetic wave damping rate. The macroscopic quantities which are the sources of (2-3) and (2-4) are just the ones calculated in (2-1) and (2-2), so the set (2-1) - (2-4) comprise a closed description. These are the equations which will be hereafter referred to as the "reduced description formalism," or RDF for short. The derivation and differencing of the equations is discussed in Appendix A.

The above model was tested against the simulation SBS coefficients of Kruer, Estabrook, and Sinz³. This was felt to provide a stringent test of the technique, for the correct steady state reflection coefficient required proper treatment of a variety of diverse phenomena, such as electromagnetic wave propagation and damping in a plasma, stimulated scattering from self-consistent plasma waves, saturation of the instability, and the hydrodynamic response of the plasma. Figure (2-1) shows the simulation values as solid circles and the reduced description values fit with the solid line, for an ion damping rate of about one-quarter the acoustic frequency. The reflection coefficients are remarkably consistent up to values on the order of fifty percent. Since the ion trapping observed in the

simulations for the strong scattering cases enhanced the ion damping rate, the discrepancies do not seem at all unreasonable. The field strength above which the reduced description model predicts too much scattering serves to delineate the regime of validity of the technique, for a given linear damping rate. Inclusion of reflection dependent damping coefficients is an intriguing but untested idea. Lacking such physics, the code is probably accurate so long as trapping has not begun.

The collisional absorption appears as a damping term (2-4). For a fixed, linear density gradient the total absorption coefficient in the geometrical optics limit can be evaluated in closed form, as was done by Dawson, Kaw, and Green⁴. Their expression is

$$A_{\text{Absorb}} = (1 - \exp(-\frac{32}{15} \frac{L}{c\tau})) \quad (2-5)$$

where L is the density scale length in centimeters, c the vacuum speed of light and $\tau = 7.59 \times 10^8 \theta_e^2 / n_{cr}$, θ_e the electron temperature in KeV, and n_{cr} the critical density in cm^{-3} . The absorption coefficients calculated with the reduced model equations agreed with (2-5) within a few percent over absorption between one to ninety percent (c.f. Table 5-1). This is excellent agreement, especially since the analytic theory neglected the absorption of the evanescent wave.

Simulations have discovered several regions in which collective processes can efficiently absorb the laser energy. Strong plasma turbulence can be excited near the critical density by either the

electrostatic instabilities or resonance absorption, and near one-quarter critical density by either Raman scattering or the two plasmon ($2\omega_p$) instability. This absorption is driven by waves transverse to the propagation direction by trapping in large amplitude Langmuir waves, or by nonnormal incidence, none of which can be expected to be represented by the model equations in a slab geometry. Such absorption may be able to be included by adding locally large coefficients to the damping terms, but it is not purported that the calculations give them self-consistently.

SECTION II

Factors Affecting the Swelling of Electromagnetic Waves Waves Near the Critical Density

The propagation of electromagnetic waves in inhomogeneous plasmas has been previously studied, especially in connection with ionospheric radio wave propagation. A large number of analytic solutions have been found for steady, unperturbed conditions (c.f. Ginzburg⁵). In laser fusion plasmas the radiation intensity is often great enough to drive perturbations unstable, so the unperturbed state has not elicited interest comparable to the "turbulent" plasma state. The latter requires knowledge of the local field energy density, however, and near threshold or near the transition point for different nonlinearities, this quantity must be known with considerable precision. The complication then is that the local energy density near the critical density can be an order of magnitude greater than the vacuum traveling wave value even in steady, unperturbed plasmas. When effects such as collisional absorption, nonlinear density profiles, and laser incoherency are considered, the situation becomes more realistic but the uncertainties about the local field density increase proportionately.

The reduced description technique proved an inexpensive, simple tool for quantitative study of unperturbed intensity patterns under a variety of conditions. The Airy equation is derivable from (2-4) by neglecting damping, assuming the potential varies sinusoidally in

time with frequency ω_0 , and the density linearly with position,

$$\frac{\partial^2 A}{\partial p^2} + p A = 0 \quad (2-6)$$

where $n(x) = (1 - x/L)n_{cr}$, and $p = (\omega_0 L/c)^{2/3} x/L$. The solutions for (2-6) are Airy functions, and the intensity pattern for these functions is tabulated in many mathematical references. The RDF calculations in a fixed linear density profile confirm this Airy pattern. Figures (2-2) and (2-3) show a typical gradient and intensity pattern.

Under realistic conditions the density gradient cannot always be expected to be linear, though one hopes that it can be approximated as such in certain regions. The above analytic solutions do not carry over to gradients of the form $n=n_{cr}(1 - x/L)^p$, $p \neq 1$. (This is an unproved assertion, but the fact that the power series solutions are in terms of three point recurrence relations implies that closed form solutions are impossible.) An approximate analytic expression is suggested, however. Since $(1 - x/L)^p$ approaches $1 - px/L$ near the critical density (i.e., as x goes to zero) it is plausible that the pattern can be described by an Airy function with an effective scale length, L/p . This renormalization is reasonable only near the critical density, but this is the region of most of the swelling.

The steady intensity patterns were calculated in fixed profiles with $1 \leq p \leq 6$ using the RDF equations. The parameter most pertinent here was felt to be the peak intensity of the standing wave pattern normalized to the vacuum standing wave peaks. The calculated values

of this ratio are shown in Figure (2-4) plotted as circles, with the solid line representing the peak values predicted on the basis of the effective scale length Airy pattern. The simple theory can be seen to be consistently within ten percent of the observed values. This agreement is felt to be quite satisfactory. (It should be noted that the ratio shown in the figure is not necessarily the most natural one from an experimental viewpoint, where one would normally measure the RMS traveling intensity of the vacuum wave. This latter is a factor of $\sqrt{4}$ smaller than the peak standing wave value, so comparison with the traveling value gives a swelling ratio on the order ten to twenty instead of three to four.)

Dissipation was neglected in deriving the Airy equation, but for laser fusion to be effective, absorption must be nonnegligible. This is not a serious oversight, however, for Ginsburg shows that dissipation with a linear position dependence can be included in such a manner that (2-6) again results, although ρ is then a complex variable. For large absorption coefficients, the complex nature of (2-6) should differ qualitatively from the real analogous equation. Weak absorption, however, can be expected to give Airy-like patterns, with reduced amplitudes.

The collisional damping rate is quadratic in density, so that it is not linear in the spatial coordinate even for a linear density gradient. It is no longer possible to transform (2-4) into an Airy equation via a simple change of variables. The conclusions of the previous paragraph, therefore, need no longer be true. Physically,

one might conjecture that the quadratic dependence of the damping would lead to most dissipation in the region most critical to the swelling, i e., near the critical density, so that again it is not obvious what the steady pattern would be.

The effect of damping was studied through the use of a damping term derived from Dawson, et.al⁴.

$$2\nu_e = - \frac{7.63 \times 10^{-4} \left(\frac{n}{n_{cr}} \right)^2 Z}{\theta_e^{3/2}} \quad (2-7)$$

where $n_{cr} = 10^{21} \text{ cm}^{-3}$ has been assumed, Z is the average ionization state, taken as unity here. For a given scale length L and gradient power-law, the only available "free" parameter is the electron temperature. The electron temperature is not actually free, since it is determined by the balance between energy gained from the laser and energy lost through conduction, radiation, and expansion. Nevertheless, if it is kept in mind that the actual temperature must be calculated self-consistently, it seems reasonable to use this parameter to vary the total amount of absorption.

A series of calculations were performed in fixed linear profiles which had a scale length of 10 vacuum wavelengths. The swelling factor was again measured as the ratio of the first peak below critical density to the vacuum peaks. This swelling factor was found to be very closely the same as the Airy factor, although the field was reduced in overall magnitude. The percentage reduction in the intensity at the first peak is plotted versus the percentage absorbed in

Figure (2-5). The calculated values have been least squares fitted to give an empirical expression for the peak,

$$I_{\text{eff}} = (.44 - .56e^{-K_T})I_0 \quad (2-8)$$

where K_T is the argument of the exponential in (2-5). The peak intensity depends roughly linearly on the reflection coefficient for absorption less than fifty percent. For larger absorption the peaks fall progressively below (2-8), because the reduced back signal is no longer large enough to form a complete standing wave. The pattern is clearly not a standing wave when the absorption has reached ninety percent. Even with near total absorption, however, the peak is reduced by only twenty percent from its vacuum standing wave value. Swelling near the critical density partially compensated for the attenuation. These are also Ginzburg's conclusions for the highly absorbing case, in which he proposes a mirror surface to estimate the structure of the field in the plasma. The expression derived there agrees moderately well with the calculated values when the absorption is high. The shape invariance of the intensity pattern has also been confirmed by Goldman, Dubois, and McKinnis⁶, using a different numerical technique.

Because actual density profiles are formed by the interaction of the expanding plasma with the laser, the profile need not be linear over any significant distance. To model this effect, calculations were also performed in nonlinear power law profiles ($p = 2,4$). Those

results are also shown in Figure (2-5) ($p = 2$, Δ , and $p = 4$, $\square$), where the reduction factor is plotted as a function of total absorption. It is found that the factor is within about ten percent of the curve in the figures for a wide variety of scale lengths and profile shapes.

A more intrinsic factor which should affect the swelling of electromagnetic waves is the lack of coherence of the incident laser beam. This partial coherence should "smear" the standing wave pattern to some extent. Since the average energy from the laser is not altered by finite bandwidth, the smearing should decrease the peaks and increase the nodes on the average. Figure (2-6) shows a calculated average bandwidth which exhibits just these features [c.f. Figure (2-2)]. The uncertainty about the pattern then is the quantitative dependence of the peaks on the bandwidth.

This problem can be modeled by averaging an ensemble of Airy functions. The distribution of Airy functions can be roughly related to the frequency spectrum as follows. Define the random variable $\tilde{\omega}$ such that $\omega = \omega_0 + \tilde{\omega}$. For convenience suppose that the incident signal has a gaussian line shape, so the frequency distribution is

$$\rho(\tilde{\omega}) = \frac{1}{\sqrt{2\pi} \Delta\omega} e^{-\omega^2/2(\Delta\omega)^2}$$

normalized to unity. Now the Airy function scales according to the normalized length

$$a(\omega) = \left(\frac{c^2 L}{\omega^2} \right)^{\frac{1}{3}} \quad (2-9)$$

If this parameter is separated into mean and fluctuating components, one can easily identify

$$a = a_0 + \tilde{a},$$

$$a_0 = \left(\frac{c^2 L}{\omega_0^2} \right)^{\frac{1}{3}} \quad (2-10)$$

$$\tilde{a} = \frac{2}{3} \left(\frac{c^2 L}{\omega_0^2} \right)^{\frac{1}{3}} \frac{\tilde{\omega}}{\omega_0} \quad (2-11)$$

provided $\sqrt{\langle \tilde{a}^2 \rangle} \ll a_0$. The ensemble average of Airy functions, $\langle \text{Ai}^2(x_0) \rangle$, with differing scale lengths, evaluated at x_0 is therefore

$$\langle \text{Ai}^2(x_0, a_0) \rangle = \int_{-\infty}^{\infty} d\tilde{a} f(\tilde{a}; \Delta a) \text{Ai}^2(x_0, a_0 + \tilde{a}) \quad (2-12)$$

$$\text{where } f(a; \Delta a) = \frac{1}{\sqrt{2\pi} \Delta a} e^{-a^2/2(\Delta a)^2} \quad \text{and } (\Delta a)^2 = \frac{4}{9} \left(\frac{c^2 L}{\omega_0^2} \right)^{\frac{2}{3}} \left(\frac{\Delta \omega}{\omega_0} \right)^2.$$

The time averaged peak amplitude squared for $L = 10\lambda_0$ and $p = 1$ is shown in Figure (2-7) as a function of relative bandwidth $\Delta\omega/\omega_0$. The bars are due to the fact that temporal fluctuations in the stochastic model for the laser bandwidth lead to dispersion around the

mean value. They are included for convenience and do not indicate any uncertainty in measurement. The solid line in the figure is the result obtained by numerically evaluating Eq (2-12). The agreement is qualitatively good out to beyond $\frac{\lambda_w}{\lambda_0} = 10^2$, and more significantly, it is in good quantitative agreement in the regime $\frac{\lambda_w}{\lambda_0} \lesssim 5^2$. High power lasers needed in laser fusion may not be able to operate with too large a bandwidth. Consequently, five percent bandwidth is probably close to the upper limit for practical applications.

The agreement between the model and calculation encouraged us to study the scaling of the swelling factor with system length. The results are shown in Figure (2-8), for different bandwidths. The model predicts significant reduction in the peak amplitude at modest bandwidths for the longer systems. The reduced description calculations agreed moderately well. As an example, at $L = 42\lambda_0$, the model predicts a swelling in the peak intensity of 4.5 when $\frac{\lambda_w}{\lambda_0} = .02$. The measured swelling factor was $4.8 \pm .3$. Agreement to within ten percent certainly constitutes satisfactory accuracy for a model as simple as this. The fact that it gives roughly the correct scaling with both bandwidth and density scale length indicates that the results shown in Figure (2-8) are probably a useful measure of the reduction in swelling with bandwidth in realistic systems.

It should be noted that the magnitude reduction here amounted only to a factor of about fifty percent. The effect of bandwidth on parametric instability thresholds, however, tends to be order of

magnitude⁷. Nevertheless, when threshold knowledge is sensitive, bandwidth reduction in the intensity may need to be considered.

SECTION III

Self-Steepening of Density Profiles Near the Critical Density

Kidder⁸ has proposed that the plasma profile will steepen at the critical density because of the zero-order radiation pressure. This problem is especially amenable to study through the reduced description, since "unwanted" physics can be omitted to isolate a specific effect. In fact, the reduced description methodology is so attractive for this case that Lee, et al.⁹ have used similar techniques already to investigate steepening. Kidder derived a simple analytic expression for the density jump. This, along with Lee's results, indicates that there is ample data to which these calculations can be compared.

Kidder balanced radiation pressure against the plasma pressure at the classical turning point of the electromagnetic field. He assumed that the plasma flows through this point at the ion acoustic speed, c_s . When a laser impinges on this surface, however, it is reflected. Conservation of momentum suggests that the plasma must take up the excess momentum, so that the net flow is inhibited.* The resulting density jump is then given by

$$\frac{\Delta n}{n_{cr}} = \frac{\alpha + \sqrt{\alpha(4 + \alpha)}}{2}, \quad \alpha = \left(\frac{v_{os}}{v_{th}} \right)^2 \quad (2-13)$$

* Conservation of flux indicates that the lowered density below the critical density should be supersonic. Such a velocity profile will lead to a larger density jump than is given by (2-13).

Dissipation near the critical density, of course, will reduce the amount of reflection, but this effect can amount to no more than a factor of 2 in the radiation momentum exchange.

The RDF calculations showed definite steepening at the critical density, but also revealed certain numerical problems. Figure (2-9) shows the upper and lower density "shelves" as a function of vacuum field strength, (V_{os}/V_{th}) . The upper levels are entirely consistent with those reported by Lee. For sufficiently low ion damping, indicated by ϕ , the lower density was also in adequate agreement with Lee's published data. At larger ion damping magnitudes, however, the lower density was consistently too high. The reason for this is related to the differencing of the ion flux equation, (2-2), and the form used for damping.

The simple theory derived by Kidder assumed initially that the plasma flowed sonically. This is only true for free expansion, but in the calculations, it was often subsonic by an early point, and decelerating. This is clearly nonphysical but readily understandable if one examines the leading terms of the differencing used in (2-2),

$$\Gamma_j^{t+\Delta t} = \Gamma_j^t \left\{ \frac{1 - .5\Delta tv}{1 + .5\Delta tv} \right\} - \frac{\Delta t}{(1 + .5\Delta tv)} \left\{ \frac{\Gamma^2}{n} \right\}_{j+\frac{1}{2}} - \frac{\Gamma^2}{n} \left\{ \right\}_{j-\frac{1}{2}} \Bigg\}^{t+\frac{\Delta t}{2}} \quad (2-14)$$

where only the convective term is shown. At low enough density, the second term on the RHS of (2-14) can be arbitrarily small. This is primarily the term that drives the low density expansion. Then since

the time step is fixed and finite, it is possible for the fixed decrease in the flux, i.e., the first term on the RHS, to exceed the convective increase. The net result is that the flux at that position suffers a net decrease. It is decelerated. Inhibition of the low density flow eventually propagates throughout the entire expansion, slowing the expansion to nonphysically low speeds. For subsonic speeds, Kidder's derivation would predict a smaller density jump. Hence, the lower density shelf is too high when $v = .0025\omega_0$, while at $v = .0002\omega_0$, the expansion is always at least sonic, and the predicted density jump is observed.

Figure (2-10) shows a typical steepened density for $(V_{os}/V_{th}) = 0.5$ and $v = .0002\omega_0$. The resolution of the density jump is degraded by the modest amount of expansion which has occurred at the time of this "snapshot". Unfortunately, the greatly reduced ion damping allowed stimulated Brillouin scattering at even this low pump level. The time averaged field pattern shows this unambiguously at later times, though not at the time shown in the figure. Whenever Brillouin scattering did not interfere with steepening, however, the density jump at the critical density was given approximately by (2-13). Figure (2-11) shows Eq (2-13) compared with the measured density jump. The agreement is good, despite the fact that (2-13) should be no better than a lower bound on the density jump.

The reduced description technique overall performed adequately in studying profile steepening at the critical density. Certain model dependent characteristics were found which could seriously alter the

results. Although these were identified and avoided, they illustrated that this technique, flexible though it is, could not be applied naively to any problem. As with any numerical method, reliable results were obtainable only after the technique itself was understood enough that improper solutions could be recognized and discarded.

Figure 2-1 Stimulated reflection coefficients as a function of normalized field strength. Points are taken from Ref 3, solid line shows RDF coefficients for similar parameters.

FIGURE 2-2 Typical density profile used in swelling calculations, $m_e/M_1=0.0$.

FIGURE 2-3 Time-averaged field pattern as a function of position, $(v_{os}/v_{th})^2=0.5$.

FIGURE 2-4 Swelling factor($\circ$) as a function of power-law density profile, $n \propto (x/L)^p$, $L=10\lambda_D$. Solid line indicates Airy-swelling for effective scalelength L/p .

FIGURE 2-5 Normalized standing wave intensity as a function of collisional absorption (inverse bremsstrahlung), $p=1(\bullet)$, $p=2(\triangle)$, $p=4(\circ)$.

FIGURE 2-6 Typical intensity pattern for a slightly incoherent pump (time-averaged), as a function of position, $\Delta\omega/\omega_0 = 0.02$.

FIGURE 2-7 Swelling factor as a function of laser bandwidth, $(V_{os}/V_{th})^2=0.5$, $L=10\lambda_0$.

FIGURE 2-3 Swelling factor as a function of linear density profile length, for $\Delta\omega/\omega_0=0.00, 0.01, 0.02, 0.03, 0.05$.

Figure 2-9 Density jump as a function of field strength, $v_i = 0.0002$ ($\circ$), $v_i = 0.0025$ (Δ).

Figure 2-10 Density profile at $\omega t = 2000$,
 $(V_{os}/V_{th}) = 0.7$, $v_i = 0.002 u_o$, $m_e/m_i = 0.01$.

Figure 2-11 Calculated density jump as a function of field strength, $m_e/M_i=0.01$. Solid line is Eq(2-13). The error bars reflect the considerable uncertainty in determining upper and lower shelves from computer generated graphics.

CHAPTER 3

Stimulated Brillouin Scattering and Mode Coupling:

Nonlinear Evolution of a Parametric Instability Process

Stimulated Brillouin mode coupling effects will be studied in this chapter. This does not refer to the linear instability phase of the process^{10,11}, which is certainly a mode coupling effect, but instead to the interaction of the large amplitude waves created by the scattering. Since the exponential growth is not of primary interest, attention will be focused on the post-saturation, nonlinear evolution of the waves. The reduced plasma formalism is felt to possess several distinct advantages over simulations for such a study. Among these are much shorter calculation times, much lower noise levels, and the ability to isolate particular physical processes.

The rationale for such an investigation are (1) there is ample evidence from laser plasma scattering experiments that some mechanism(s) broaden the electromagnetic wave spectrum^{12,13,14,15}, (2) to the best of our knowledge, SBS mode coupling has not been previously investigated, and (3) related to the above, understanding the source of the broadening may provide insights into the microdynamics inside the plasma, which are otherwise inaccessible. Clearly, simulations and analytic theory have shed considerable light* on the Brillouin scattering process, usually with regard to the level of reflection. Nevertheless, mode coupling studies of other parametric instabilities

*Please note pun.

(i.e. the parametric decay instability) have shown that they are rich in wave-wave interactions. There is every intuitive reason to believe that conditions can be found in which they are also propitious in Brillouin scattering. Stimulated Raman and Brillouin studies in solid state media moreover have long been accompanied by complex spectra (c.f. Bloembergen¹⁶). The fact that the characteristic spectra, asymmetrically broadened toward longer wave lengths, are reported in virtually all laser plasma experiments (in which the spectrum was measured) suggests similar mechanisms may be occurring.

The chapter is ordered as follows: In Section I, a wave coupling version of the reduced description formalism (RDF) is described and used to identify the qualitative characteristics of Brillouin wave coupling. Some of the quantitative wave field features are examined near threshold and the extrapolation to higher intensities is inferred. This section involves considerable detail since it is the basis for evaluating more realistic, inhomogeneous calculations later. Still, the topic is somewhat esoteric, and the applications-minded reader may wish to pass over this section lightly. In Section II, the infinite medium assumption is relaxed to study the effect of finite length on the mode-coupling. Inhomogeneous density profiles are also examined. Finally, in Section III, the possible applications of these calculations are illustrated by concentrating on a series of scattering experiments conducted by an NRL group, Ripin, et al.¹⁵.

SECTION I

Homogeneous Brillouin Scattering Studies

Before proceeding, a historical footnote seems in order. Solid medium laser experiments were the first to see the pronounced asymmetrical broadening of scattered spectra^{16,17,18}. Investigations of SBS in solids, liquids, and gases have all observed such spectra for sufficiently high intensities. The high intensity is apparently crucial*, for the spectrum changes from a series of discrete Stokes lines at moderate intensities into a diffuse, broadened spectrum at the greater intensities. Although such phenomenon have been studied for nearly a decade within the framework of nonlinear optics (i.e., cubic field nonlinearities), it is felt that the properties of a dense plasma are sufficiently unique that the earlier results are not directly applicable. They are still useful insofar as they indicate that spectral broadening can be the nonlinear result of a known linear process, that is stimulated process.

The first RDF calculations were conducted in homogeneous, periodic systems. Although end effects were omitted, the improved spectral resolution was an enormous advantage in following the mode-coupling. It was convenient in this situation to Fourier analyze the RDF model equations, and so observe the evolution of the spectrum

*Early experimental observations of the spectral broadening occurred during inadvertent self-focusing and filamentation.

directly. The equations (2-1)-(2-4) were thus reduced to two sets of coupled wave equations, though each mode was intrinsically coupled to all other modes. The algebra in deriving the sets is straightforward but tedious and so has been deferred to an appendix. The final equations for the electromagnetic and ion density modes are

$$\ddot{A}_i + 2v_{ek} \dot{A}_k + \omega_k^2 A_k = -\omega_0^2 \sum_k n_k A_k - k, \quad (3-1a)$$

$$\ddot{n}_k + 2v_{ik} \dot{n}_k + \omega_{ik}^2 n_k = -\frac{1}{2} n_0 \left(\frac{v_0}{v_{th}} \right)^2 \omega_{ik}^2 \sum_k A_k A_k - k, \quad (3-1b)$$

where $\omega_k^2 = \omega_{pe}^2 + k^2 c^2$, $\omega_{ik}^2 = \frac{6}{M_i} k^2$. It should be noted that this technique retains phase, as well as amplitude information, and so should be quite general¹⁹.

To isolate the dominant wave-coupling mechanisms, a series of calculations were done in which the intermode k-spacing, the number of modes, the ambient density, and the pump field strength, $\frac{v_{os}}{v_{th}}$ were varied. Changing system length due to plasma expansion or too large an intermode spacing (i.e., too short a system length) may mask such interactions in finite simulations. It is possible that particle simulations are suitable for this type of study, but the point is moot until such an attempt is made. Using the set (3-1a) and (3-1b), two distinct types of mode-mode interactions were observed to be particularly strong in the saturated state of the SBS instability.

The first mechanism is simply harmonic generation, driven by the large amplitude waves. For sufficiently strong pump, and density close to critical, the coupled wave calculations show strong spatial harmonic generation at $3K_0$, $5K_0$, ... for the high frequency waves, and $4K_0$, $6K_0$, $8K_0$, ... for the ion waves. This process can be explained as a nonresonant, forced oscillation. Viewed strictly in k -space the SRS instability transfers energy from a large amplitude electromagnetic wave at K_0 to another electromagnetic one at $-K_0$ and an ion acoustic wave at $2K_0$ (i.e., forward traveling with respect to the pump), with the latter also carrying off the bulk of the momentum. This process is resonant, in that it not only satisfies $\vec{K}_0 = \vec{K}_{em} + \vec{K}_{ia}$, but also $\omega_0 = \omega_{em} + \omega_{ia}$. Once the decay products have reached large amplitude, however, one should consider the possibility of nonresonant coupling to new waves. For the present case, quasiresonant is a more descriptive term, since we will still seek triplets such that $\sum_{i=1}^3 \vec{K}_i = 0$, though frequency mismatches may be allowed.

Let us examine the $3K_0$ electromagnetic wave as a typical example. Since it is the density fluctuations which couple the electromagnetic waves, and since a large amplitude ion wave does exist at $2K_0$ (by assumption), the most obvious triplet is the coupling of the forward traveling pump and the forward traveling ion acoustic wave to a $3K_0$ EM wave. Except in a physically insignificant region near the critical density, we find $\omega_{3K_0} \neq \omega_0 + \omega_{ia}$. The process involves the pumping of a wave whose natural frequency, is $\omega_{3K_0} = (\omega_p^2 + (3K_0)^2 c^2)^{1/2}$, with a pump whose effective frequency is $\omega_0 + \omega_{ia}$, and effective amplitude is

$\omega_{pi}^2 n_i A_{K_0}$. A forced harmonic oscillation such as this will eventually drive the $3K_0$ wave to a steady amplitude, which is easily calculable. Before we insert observed values to actually test this model, we should consider one other possible triplet that could lead to a $3K_0$ wave.

It is well known that a large amplitude ion acoustic wave can effectively decay into its higher harmonics, as long as $k\lambda_D \ll 1$. As $k\lambda_D \approx \left(\frac{v_{th}}{c}\right) \left(\frac{kc}{\omega_0}\right) \left(\frac{n_{cr}}{n_0}\right)^{1/2}$, the inequality is very well satisfied near the critical density. The ion acoustic wave will, therefore, expend its energy at a given rate, depending on its amplitude, in generating higher harmonic ion waves, independent of any coupling with the high frequency wave. Once an appreciable amplitude is attained by the second harmonic wave, that is $4K_0$, it can in principle couple a $-K_0$ electromagnetic wave to a $3K_0$ electromagnetic wave.

A typical spectrum showing harmonic generation is shown in Figure 3-1. The initial density in this calculation was $.935 n_{cr}$, the initial pump strength was $(V_{os}/V_{th})^2 = .25$, with pump depletion allowed, and to minimize ion Landau damping $\theta_e/\theta_i \approx 100$. A small signal in this case was even observable at $5K_0$, though reduced in intensity by roughly 5 orders of magnitude. The relative importance of the two coupling mechanisms can be judged by comparisons with the forced oscillation model. A straightforward derivation yields the steady RMS level of such an oscillation.

$$A_k^2 = \frac{\omega_{pe}^4 n_i (K \pm K_0) A_{K_0}^2}{2(\omega_k - \omega)^2 + 4\nu_e \omega^2} \quad (3-2)$$

TABLE 3-1

High Frequency Harmonic Generation In Periodic Simulations

$$\left(\frac{V_{os}}{V_{th}}\right)^2 = 0.25, \quad n_o = .926 \, n_{cr}$$

$$n_{2K_o}^2 = 3.5 \cdot 5 \times 10^{-3}$$

$$n_{4K_o}^2 = 4.5 \cdot 5 \times 10^{-5}$$

K	A_K^2 (measured)	A_K^2 (coupling with n_{2K_o})	A_K^2 (coupling with n_{4K_o})
.27	3.5×10^{-1}	-	-
.81	$3. \times 10^{-3}$	3.85×10^{-3}	-
1.35	1.2×10^{-5}	3.12×10^{-6}	4.05×10^{-6}

where $\omega_k = (\omega_p^2 + (Kc)^2)^{1/2}$, $\omega = \omega_{k_0} + \omega_{ik} \pm \omega_{k_0}$, and ν_e is the damping increment on A_{ik} . Using the observed values for n_{ik}^2 , Table 3-1 shows the agreement between the observed and predicted levels for these harmonics, assuming that the coupling is between the K_0 pump wave and $2K_0$ ion wave. The largest discrepancy in this limited sample occurred for a case in which the magnitude of the second harmonic ion wave was within a factor of four of the primary. This suggests that the second coupling scheme can be quite effective if the density wave harmonics are powerfully driven, though the lower order scheme would be the more commonly encountered one.

The second mechanism encountered in the calculations might be called repeated scattering¹⁷. Once the pump has driven a back-scattered wave to a large amplitude, the secondary wave can itself act as an SBS pump. The new electromagnetic wave (i.e., secondary decay product) is propagating along with the original pump wave, but downshifted by approximately twice the frequency of the acoustic wave*. This secondary backscattering will, of course, also generate backward traveling ion waves, and once ion waves of comparable magnitude have been created in both directions, the possibility of quasis resonant couplings should be considered. If one could neglect

*It should be noted that the frequency shift is not necessarily that of a normal ion acoustic wave. The deviation from the normal mode frequency may be either positive or negative depending on the exact wavenumber. The measured frequency shift is consequently an unreliable indicator of the density or wavenumber of the original scattering event.

all but the last backscattering event, however, it is plausible that this wave, too, could drive SBS if it exceeded the appropriate threshold. There is no a priori reason why the other waves can be neglected, but as will be cited below, calculations indicate that the system evolution is consistent with this hypothesis. The procedure then is that a series of consecutive backscatters, i.e., in alternating directions, leads to a sequence of lines separated by twice the acoustic wave frequency. The process can be terminated either by competing mode-mode interactions or when the dissipation rate integrated over the entire spectrum equals the rate at which energy is fed into the spectrum from an external source.

Stimulated rescattering is clearly evident in Figure 3-2, the late time spectrum of a launched wave calculation for which $n_0 = .5 n_{cr}$, $(v_{cs}/v_{th})^2 = .25$, $\theta_e = 2$ KeV, and the system length, $L = 40\lambda_0$. The spectrum is observed to broaden in time in exactly the manner described above. Although additional waves have also reached significant levels by this time, the contrast level between the rescattered waves and the pump has been sufficiently depleted so that no further broadening occurs, but the spectral width has already attained roughly eight percent of the original frequency ($\Delta\omega/\omega_0 \sim 1\%$ for $m_e/M_1 = 1/1836$). Furthermore, somewhat later in the calculation, off-resonant coupling, facilitated by the broad spectrum, filled in the spectral "valleys." Thus, the occurrence of repeated scattering would not necessarily lead to measurement of a discrete line spectrum.

Figure 3-3 shows both harmonic generation and repeated scattering in a calculation in which $n_o = .935$, $(V_{os}/V_{th})^2 = .50$, and $L = 40\lambda_o$. Harmonic generation occurred initially, but the slow broadening due to repeated scattering continued after the former had ceased. During the repeated scattering, the original pump wave was gradually depleted, as expected. The three wave coupling manifested itself in the fact that the newly driven waves only induced transient depletions in the waves directly involved in pumping. Since the pump waves were strongly coupled to their respective sources, they were quickly restored to their previous level. Consequently, the evolving spectrum was immune to large fluctuations. As Figure 3-4, a later spectrum from the same calculation, shows though, large and dramatic alterations could be induced. The spectral changes are correlated with the appearance of a band of up-shifted, higher-k waves, and these waves are clearly drawing energy only from a narrow region of the spectrum, centered around the wavenumber of the original pump wave. The spectral broadening, moreover, assumes an entirely different character after this, diffusing at a rapid rate to the lowest wavenumbers in the calculation. Upshifted waves, up to three times the original wavenumber, are also evident. The role of the blue shifted waves in the broadening was not obvious within the context of simple mode-coupling discussions, but can be clarified if they are identified with a purely growing branch of the SBS instability. Comparison of the spectra in Figure 3-4 with the growth curve for a standing wave pumped ion/electromagnetic instability (see Appendix Figure B1) shows marked similarities, for

instance. The full discussion of this problem is better served in Chapter 4, however, in which the formation of localized field/density modulations is treated. Suffice it to say here, the spectrum broadens in precisely the fashion envisioned earlier in the section until the blue shifted waves reach large amplitude, and thereafter, it changes explosively.

The energy content in the broadened spectrum is also interesting. To study this, however, an alternate calculational strategy was used. Instead of launching a wave with an initial, known energy density, i e., $\langle v_{os}^2 / v_{th}^2 \rangle$ and allowing it to deplete, the pump was initialized at a given level, and then maintained. Saturation at a steady level could only be accomplished through dissipation or scattering energy from unstable regions of k -space to stable ones, at a rate equal to that at which the instability coupled energy into the system. This certainly bears a resemblance to the weak turbulence picture of saturation of the parametric decay instability, though the analogy cannot be pushed too far.

Figure 3-5 shows the average energy density of electromagnetic waves in the saturated state as a function of pump intensity divided by threshold pump intensity*. This total field energy scaled linearly with the pump ratio, up to ratios of 8-10. Even in this regime, it was qualitatively evident that the RMS fluctuations increased with

*Since the intensity is merely the energy density in the mode times its group velocity, the pump strength ratio at a given density is the same for intensity and energy density.

increasing pump strength. This is clear in Figures 3-6 and 3-7 which show the time history of the total energy density at $(V_{os}/V_{os_{thr}})^2 = 2$ and 7 respectively. Neglecting the transient overshoot, both go to a steady state in which the total energy varies around a constant value. At the lower level, neither the fluctuations nor the mean have any temporal structure. Nevertheless, even with the temporal evolutions at $(V_{os}/V_{os_{thr}})^2 = 7$, a recurring pattern is discernible over several thousand $\omega_0 t$. The mean level of the fluctuations, as shown in Figure 3-5 as a function of the pump ratio, increases roughly linearly with increasing pump strength. For pump strengths stronger than shown in Figure 3-5, the saturation level either increases faster than linearly or a steady level is not reached at all, within the limitations of the computational model.

To obtain a clearer picture of the temporal evolution, we have plotted the time histories of the modes surrounding the pump in Figure 3-8 for the case $(V_{os}/V_{os_{thr}})^2 = 7^*$. The traveling wave pump was maintained at mode 28, that is $K_0 = 28 \frac{2\pi}{L_0}$, fixed at a level corresponding to the mean energy density of a standing wave pump at the same $(V_{os}/V_{th})^2$. After rapid initial growth, the linearly most unstable wave in the system, $K = -27$, saturated at a level several times that of the pump. It in turn immediately drove another back-scattered wave, $K = +27$, to high levels. This alternating direction cascade continued until the spectrum had settled down to a quasisteady

*To be precise, we plot the wavenumbers in both directions around the pump, i.e., the intervals $(K_0 - 3, K_0 + 2)$ and $(-K_0 - 2, -K_0 + 3)$.

state. That state had been reached by about $\omega_0 t \approx 1500$ and continued with only minor changes until $\omega_0 t \approx 4000$. The exact timing of these events, of course, is not particularly relevant, since the calculations employed an unphysically low electron to ion mass ratio ($m_e/M_i = .01$). The timing of the significant phases of the spectral evolution are mentioned only to illustrate the relative stability of the spectrum over periods long compared with other time scales. For pumps nearer threshold, the stable phase persisted throughout the duration of the calculation ($\omega_0 t = 7500$). For the present case, however, qualitative changes occurred after $\omega_0 t = 4000$. These again were associated with the appearance of blue shifted modes. The shorter wavelength waves saturated at amplitudes greater than that of the pump. Thereafter, the intensity of the previously created red shifted lines are also altered, and additional red lines such as $\pm (K_0 - 4)$ begin to be observable. The direct role played by the blue lines is not obvious, but it is clear in stronger pump cases that their appearance signals the rapid diffusive spread of the spectrum to the longest wavelength modes allowed in the system. Again it is tempting to identify these lines with the purely growing branch of the ion scattering instability found from linear theory. By $\omega_0 t = 4000$, though, the system is certainly nonlinear. It should be noted that the growth of the blue lines shown in Figure 3-8 is only approximately exponential, and any growth rate is ill-defined. Nevertheless, this is only further proof that the nonlinear state of the SBS instability is rich in wave-wave interactions, which had not been previously discussed.

Summarizing the conclusions of this section, a series of calculations were conducted in a k-space version of the reduced description model, for the express purpose of determining whether nonlinear wave interactions associated with the SBS process could influence the spectral characteristics. Two such mechanisms were clearly isolated, harmonic electromagnetic wave generation and repeated scattering. The latter was able to broaden the electromagnetic wave spectrum extensively at moderately low pump strengths $(V_{os}/V_{th})^2 = .2 - .4$. Furthermore, a powerful diffusive broadening mechanism was observed for sufficiently strong pumps, which was capable of driving waves at arbitrarily small wavenumbers. Unfortunately, it was not possible to determine what "sufficiently strong" meant in terms of real system parameters, since the system periodicity was so strongly built into the computational model. The effect of waves escaping from "the box", or any end effects in fact, could not be treated with this method.

SECTION II

Inhomogeneous Brillouin Scattering Studies

A very suitable method for treating large scale inhomogeneities, finite system length, and end effects on the Brillouin scattering instability was found to be, simply integrating the RDF equations, (2-1)-(2-6), directly. Particle simulations might also have been useful, but the freedom from particle noise was a distinct advantage in resolving weak mode-coupling effects. Further, the reduction in computer time needed for a given case facilitated wide parameter studies that would have been prohibitively expensive with particle codes.

The first calculations treated a locally flat density profile, with the ends smoothed to reduce classical reflection. Several wavelengths of vacuum were retained on either end of the profile to facilitate the introduction or removal of electromagnetic waves. (Transmitting boundary conditions on electromagnetic waves are exact in vacuum, but only approximate in dispersive media.) The source of the waves, or "laser", was an antenna cell just within one boundary. If the pump was initiated smoothly, a wave could be grown, which propagated into the plasma. The propagation was an aspect of the scattering problem totally neglected in the homogeneous, periodic calculations. Strong scattering, however, at the "front" of the plasma could significantly alter the instability evolution of the "interior".

For the problem of SBS in a homogeneous, finite length system, analytic expressions for the steady reflection coefficient can be derived in two limits. These are when the product of the ion damping rate $\frac{\nu_i}{\omega_i}$ and the interaction length, $k_0 L$, is either much greater or much less than unity. In the former case, the process is backward amplification while in the latter, the instability is absolute, growing exponentially at all points in space.

The expression for reflectivity when $\frac{\nu_i}{\omega_i} \ll \frac{1}{k_0 L_0}$ is²⁰

$$R = \text{sn}(\theta L_0, C_b(0)/C_f(0)) \quad (3-3)$$

where sn is a Jacobian elliptic function, $C_b(0) = k_b^{1/2} A_b(0)$ is the normalized backtraveling amplitude at the incident boundary, $x = 0$, $C_f(0)$ the normalized forward amplitude, i.e., the laser amplitude, and $\theta \approx \sqrt{4k_{\text{vac}} (v_{os}/v_{th}) (2n_o/n_{cr})^{1/2}}$. Since the reflection coefficient for SBS is essentially $(C_b(0)/C_f(0))^2$, the conditions for appreciable stimulated reflection is $\theta L \gtrsim \frac{\pi}{2}$. At $n_o = .5n_{cr}$, this corresponds to $(v_{os}/v_{th})^2 \gtrsim (L/\lambda_o)^{-2}$. Since this expression is valid only in the short system limit, we may say that $L = 10\lambda_o$ is a typical length, giving a threshold of $(v_{cs}/v_{th})^2 = .01$. This is orders of magnitude greater than the homogeneous threshold. Once the threshold is exceeded, however, the reflection very rapidly rises to unity.

The time required to establish the steady state is not given by (3-3) - but is related to the growth rate. Near the inhomogeneous threshold this may be longer than hydrodynamic expansion times. As

an example, an intensity twice the inhomogeneous threshold is predicted to give sixty-five percent reflection. Even with a reduced ion/electron ratio ($m_e/M_i = .01$), though, the periodic calculations showed that at this pump strength the time to exponentiate two orders of magnitude was roughly $\omega_0 t = 10^4$. Since the expression was valid only for fairly short systems, it is clear that hydrodynamic expansion will extensively alter the density profile before the steady state is established. The effects of profile modification may in general be expected to limit the applicability of (3-3) to physically unrealizable conditions.

A series of cases were made to examine SBS in the weak dissipation/short system limit. For intensities above the inhomogeneous threshold by a significant factor (such as 10), the ion density fluctuations were observed to grow. Correlated with the increasing density fluctuations was the rise in reflectivity. Regardless of the particular intensity, the growth of the ion waves and of the reflectivity did not subside until virtually one hundred percent of the laser was being reflected. As the intensity was reduced, only the rate of growth was decreased (see Fig 3-9). Growth still continued until total reflection was established. Despite the qualitative agreement with the analytic theory, it was not possible to verify the quantitative aspects, especially near threshold where the theory needed to be tested. Even at ten times threshold, the growth had not saturated in typical calculational times (2000-4000 $\omega_0 t$). The near threshold regime, though, has little applicability in plasmas because

the expansion time is comparable to the growth time. In the stronger pump cases, the calculations were consistent with the theory.

The other limit, $\frac{v_i}{\omega_i} \gg \frac{1}{k_o L_o}$, is easily obtained for $k_o L \geq 10$ when θ_e is not too much greater than θ_i . An analytic expression for the steady state reflection is also derivable in this case, (c.f. Krue²¹)

$$C_b(x)^2 = C_b(0)^2 (1 - R/(\exp(\emptyset x) - R)) \quad (3-4)$$

where $\emptyset = 1/4(m_e M_i)(V_{os}/c)^2 \omega_p^2 (\omega_i v_i (k_2^2/(k_2 - k_o))) (1 - R)$. This expression predicts exponential amplification of a backscattered signal until it reaches the front edge of the plasma. Given an initial intensity, $C_b(x_o)^2$, the steady state reflection can be computed without excessive difficulty. Numerical calculations were conducted in a finite slab plasma to test the theory. For $v_i/\omega_i \doteq \frac{1}{4}$ the backward traveling signal does indeed increase exponentially as it propagates through the slab as shown in Figure 3-10. The observed gain coefficient is a function of time, though. It is smaller than $\emptyset$ during a transient phase of the instability, but rises toward it as a limiting value.

In the numerical model, the initial backscattered signal is due to classical reflection from the back edge of the plasma. For a given profile, the fraction reflected is fixed, so that $C_b(L)^2/C_f(0)^2$ is a constant, depending only on the profile shape. Equation (3-4) evaluated at $x = L$ divided by $C_f(0)^2$ should also be a constant then.

Within the accuracy with which the "steady" reflection level may be measured on the plots ($< 5\%$)*, this is found to be the case. It should be mentioned that this "steady" level has only limited significance, since hydrodynamic expansion of the slab had created a shallow ramp in front of the homogeneous region after about $\omega_0 t = 10^3$. Although the expansion was enhanced by the reduced mass ratio, cases lasting between $2-4 \times 10^3 \omega_0 t$ are still very short compared to other laser fusion time scales. The pre-ramp could be expected to alter the "steady" state reflection, since strong SBS was evident in the ramp after a few thousand $\omega_0 t$.

The early analytic theories, from which (3-3) and (3-4) were derived, were motivated by scattering in crystalline media. Unfortunately, the static conditions of solid-state materials are not obtainable in short-pulse laser plasmas. So long as the finite profile did not change much from its initial configuration, the calculations of heavily ion wave damped SBS were nevertheless in excellent agreement with the analytic results. Plasma expansion and profile modification introduced factors not included in the theory, however, after which the computational results could diverge significantly from that predicted.

Harmonic satellite generation and repeated scattering were strongly modified in the finite slab geometry. For pump powers only

*To avoid ambiguity, note that the five percent uncertainty applies to the measurement of any reflection level, not that a noise level of five percent was present.

slightly above the inhomogeneous threshold ($R < 20\%$) only the narrow SRS line is observed in the reflected spectrum, and only the pump line in the transmitted. When the pump is increased to the level of strong scattering ($R \geq 50\%$), much temporal evolution is observed. As an example, for $(V_{os}/V_{th})^2 = .81$, $n_0 = .85 n_{cr}$, and $L = 10-14\lambda_0$, the average reflection coefficient reached seventy percent after $\omega_0 t = 2 \times 10^3$. Thereafter successive downshifted spectral lines appeared alternatively in the forward and backward directions, clearly to 3rd order in the backward direction and 2nd in the forward. Figure 3-11 shows these spectra at roughly their maximum width. Both the relative frequency shifts and the relative intensities are consistent with a repeated scattering model. Somewhat later, the primary scattering had reached sufficient magnitude for pump depletion to occur. The reason this terminates re-scattering is that the backward wave is exponentially amplified under these conditions. Although exponentiation also persists throughout the transient phase of the instability, the rate is small enough that an intense backscattered signal has a long interaction path in the plasma. The interaction length continually decreases as the process approaches the steady state, however. The net result is that the strong scattering near the front "starves" the rescattering process, i.e., there is little field over the full interaction path in the forward direction. Since the analytic theory predicts strong SRS when $\phi_{L_0} \simeq 5-10$, we expect this inhibition of repeated scattering to occur at even lower

intensities in actual laser plasma experiments than is seen in the relatively short computational systems.

Under strong SBS conditions, harmonic generation was also easily observable. Spatial Fourier analysis of the field and density within the plasma (k -spectrum) showed $3K_0$ and $5K_0$ harmonic content in the field and harmonics up to 5th order in the density fluctuations. None of these lines appeared outside the plasma, though, as is expected from a forced oscillation. The harmonics of the high frequency wave are nonpropagating modes. They may contribute to the internal mode-coupling dynamics, or they may contain a significant fraction of the electromagnetic energy in the plasma, but they are not part of the measured spectra in laser plasma experiments.

Calculations were also performed in inhomogeneous, linear density gradient plasmas. In light of the fact that rescattering is an inherently front/back symmetric process, it seems even less propitious in a density gradient than was seen before. Nevertheless, a marginal amount of rescattering was observed for strong scattering cases. This was only transient, and so seems an unlikely candidate for experimental measurement. Higher harmonics of the primary density wave were also seen, but the electromagnetic harmonics were relatively weaker, and again even the small $3K_0$ content was not manifested in any frequency components.

On the basis of these one-dimensional slab studies, certain conclusions can be drawn. The analytic expressions for steady state SBS reflection, in the highly damped and highly undamped limits, were

consistent with the calculations. Only indirect evidence was available in the undamped case, but the general features of the highly damped calculations agreed very reasonably with the analytic expression. These provide confidence in extrapolating a limited number of numerical results to a more general parameter space. Fourier harmonics of the field were generated in the plasma interior but were found to be nonpropagating. Re-scattering, on the other hand, clearly broadened the scattered (and transmitted) spectra during early phases of strong SBS. Pump depletion eventually destroyed this process. This late time behavior was precisely the information we sought with the reduced plasma description. A potentially important process such as stimulated rescattering was shown to be throttled in the self-consistent evolution of Brillouin backscattering.

These results do not rule out mode-coupling as a spectral broadening mechanism, but they do mitigate against its importance in stimulated Brillouin backscattering. The stimulated scattering is not intrinsically one-dimensional, as studied above, and it is clear that scattering in other directions can behave qualitatively differently. Sidescattering, for instance, depletes the laser strongly in its original trajectory, while pumping uniformly along that of the scattered photons. Furthermore, in an inhomogeneous plasma, refractory losses in the sidescattered direction tend to filter out all but a narrow cone of angles for effective gain. Without pump depletion, rescattering can prosper. With a natural directional selector, the

process, assumes a pseudo-one dimensionality, and again the physical situation is not too far from the numerical model. This suggests that the homogeneous, periodic calculations can be applicable to side-scattering.

SECTION III

Application of Inhomogeneous Calculations to a
Scattering Experiment Performed at NRL

The homogeneous and inhomogeneous simulations of stimulated Brillouin scattering performed in the previous sections will now be applied to a typical scattering experiment to illustrate their utility. The experiment at NRL¹⁵ is especially interesting since measurements at different angles of laser incidence led to different backscattered spectra. The published measurements are therefore reproduced in Figure 3-12. Although the total energy collected at each angular position was about the same, the spectral characteristics varied considerably. Application of the results of the previous sections yields a qualitative explanation for this variation.

The spectra measured for normal incidence was remarkably narrow. This was tentatively identified as stimulated Brillouin backscatter, and that hypothesis seems reasonable. The density scale length was estimated to be about 20 microns in this Nd-laser experiment. This is comparable with scales used in the inhomogeneous RDF calculations. In those, strong SBS was observed for field energy commensurate with the thermal energy of the plasma. The laser power reported for the experiments seems sufficient to easily satisfy these intensity requirements. Little spectral broadening was observed in the calculations, however, despite even strongly occurring SBS. This is consistent with the backscatter measurements.

At the intermediate angular position ($\theta = 24^\circ$), some broadening of the spectrum was measured. It was an irregular spectrum, though, consisting of many distinguishable lines. Analysis of this measurement would involve too many details, which would tend to distract from the main points. We will thus proceed to the measurement at the largest angle.

At the 45° position, the spectrum possesses a nearly continuous spectrum, albeit with structure, asymmetrically broadened toward longer wavelengths. The total spectral width at this angle is an order of magnitude wider than the one measured at normal incidence. This type of spectra is similar to the calculated ones in which repeated scattering dominated. Although the focal lens could have collected some directly backscattered SBS, this source is no more likely to lead to broadening than it did for normal incidence. At the laser power of the experiment, however, Brillouin sidescatter is apparently above the inhomogeneous threshold²². These waves would presumably be generated parallel to the density surface, but would then refract out of the plasma at some other angle. Since sidescattering is inherently multidimensional, the calculations in this chapter do not apply to the process directly. The magnitude of sidescatter can be estimated, though, from knowledge of experimental parameters and inhomogeneous theory. The trajectory of a wave can, furthermore, be calculated as a function of initial density and angular orientation. From this, one can infer coherence lengths for subsequent SBS backscatter²³.

The likelihood that repeated scattering is responsible for broadening at a given angle relative to the target normal can be evaluated. The details of this analysis unfortunately are tedious, and so have been postponed until Appendix E. The key points, nevertheless, can be briefly summarized. At the intensity reported in the NRL experiment, both Brillouin back- and sidescatter are far above theoretical thresholds. For typical scale lengths, the coherence lengths were long enough for sidescattered waves of reasonable magnitude to repeatedly backscatter. The absence of broadening in the normal direction, only sporadic broadening at 24° , and pronounced broadening at 45° is consistent with the occurrence of strong sidescatter, but only below about $n_0 = .6 n_{cr}$. Although discrete lines were not evident in the last spectra, there is apparently some structure at $10 \text{ \AA}$ spacing. Assuming linear dispersion relations and a hydrogenic plasma, however, a discrete spacing of $20\text{--}25 \text{ \AA}$ is expected. There are several possible explanations for the measured spacing which would make them consistent with repeated SBS backscatter. Nonlinear frequency shifts could cause the frequency decreases to be lower than linear theory predicts. Alternatively, the Brillouin shift in the polyethylene targets could be more typical of carbonic rather than hydrogenic plasmas. This alone would account for the observed spacing. Finally, although polyethylene, aluminum, and copper target materials were used in the experiments, the paper does not specify which type the broadened spectra example was from. The discrepancy, moreover, is only on order 2, so that repeated scattering is not strongly argued against on that basis.

Figure 3-1 Electromagnetic field spectrum at $\omega_0 t = 1400$, pump depletion allowed, $n_0 = .935 n_{cr}$, $L = 40 \lambda_0$, $\theta_e/\theta_1 = 100$, $m_e/M_1 = 0.01$. $(V_{0E}/V_{th})^2 = 0.25$.

Figure 3-2 Electromagnetic field spectrum at $\omega_0 t = 2800.$,
 pump depletion allowed, $n_0 = 0.5 n_{cr}$, $L = 40 \lambda_0$,
 $\theta_e / \theta_i = 100$, $m_e / M_i = 0.01$, $(V_{os} / V_{th})^2 = 0.25$.

Figure 3-3 Electromagnetic field spectrum at $\omega_0 t = 1800.$, same calculation as Figure 3-1.

Figure 3-4 Electromagnetic field spectrum at $\omega t = 2400$, same calculation as in Figure 3-1.

FIGURE 3-5 Averaged field energy density in the saturated state as a function of pump strength divided by threshold pump strength, $(V_{os}/V_{os, \text{threshold}})^2$.

FIGURE 3-6 Time history of the field energy density, $(V_{os}/V_{os_threshold})^2 = 2.$, $m_e/M_i = 0.01$, $L = 40\lambda_o$, $n_o/n_{cr} = .75$, fixed magnitude traveling wave pump.

Figure 3-7 Time history of the total field energy density, fixed, traveling wave pump, $n_0 = 0.75 n_{cr}$, $L = 40 \lambda_c$.
 $(V_{os}/V_{os\text{thresh}})^2 = 7$.

Figure 3-8 Time histories of modes near the wavenumber of the pump, $K=+28$, $n_0 = .75 n_{cr}$, $L = 40 \lambda_0$,

$(V_{os}/V_{s_{thresh}})^2 = 7$. Waves in the + direction are labeled with a "B", while - direction waves are labeled with "R".

Figure 3-9 Reflection coefficient from a finite slab as a function of time, for various field strengths, $n_0 = .5 n_{cr}$, $L = 11 \lambda_0$, $\theta_e = 5 \text{ KeV}$, $v_i = 0$.

Figure 3-10. Field profile in a finite slab, averaged over 250 p.p.t., reflection coefficient is 69%, $n_0 = .75 n_{cr}$, $E_0 = 8.0 \times 10^6$ (V/cm), $\omega = 866 \omega_{pe}$, $\epsilon = 0.25$, $\mu = 1.0$.

Figure 3-11. Frequency spectra of (a) reflected and (b) transmitted field between $\omega = 0.9\omega_0$ and $\omega = 1.1\omega_0$, $(V_{os}/V_{th}) = 0.5$, $\beta = 10^{-3}$, $\omega_0 = 10^8$ rad/sec, $\omega_0/\omega_{ce} = 1.0$, $\omega_0/M_1 = 0.01$.

FIG. 2. (a) Shift of backscatter time-integrated spectra towards the red for increasing angle θ between the target normal and laser direction.

Figure 3-12 Time-integrated spectra measured at different angular orientations of the laser with respect to the target (after Ref. 15).

CHAPTER 4

ELECTROMAGNETIC WAVE TRAPPING AND HOLE FORMATION

Formation of stable electromagnetic wave structures near the critical density of laser plasmas has aroused considerable interest. Simulations have shown that the density wells associated with these structures can extend to much lower density than the surrounding plasma. The two dimensional surfaces near the critical density thus exhibit strong modulations on a laser wavelength scale. Since most collisional absorption occurs in this vicinity, such large perturbations in the density may have profound ramifications on absorption efficiency. Moreover, the collective absorption processes near the critical density will experience large inhomogeneities, which may severely limit their effectiveness, as well. The existence of these "tebble"-like states and the conditions under which they may be expected to occur is, therefore, a problem with direct applications to laser fusion. In this chapter, one dimensional studies will be conducted to better identify the threshold conditions.

Previous numerical and/or analytic studies of trapped electromagnetic wave states have been pursued by Eaw, Schmidt, and Wilcox²⁴, Eaw, Nishikawa, and Yoshida²⁵, Gorovitch and Karpman^{26, 27, 28}, Vukobratovic and Estabrook²⁹, and by Longden and Lasinski³⁰. As a result of these studies, a set of characteristics of the state has become apparent.

* The author has recently been advised of the numerical work of Perslund, Firdel, Linman and Lee³¹.

The bubbles are only observed reasonably near the critical surface. A slab, or "sandwich" state is found to be unstable to the growth of transverse perturbations, of a sausage type. The bounded ellipsoidal states that result from the break-up suggest that the three dimensional structures would approximate an electromagnetic bubble imbedded in the plasma. Once formed these are observed to be relatively stable over thousands of laser periods. This does not imply that they do not affect other processes, only that they do not partake directly.

Although the work cited above has gone far toward understanding the qualitative nature of the trapped wave state, little work has been directed toward defining threshold conditions. It is true that the phenomena occurs only near the critical density, but how near is not clear. Also, there are several reasons to believe that the field intensity must be of order $(V_{cs}/V_{th})^2 \sim 1$, but it would be useful to know if there are conditions under which the state could form under less strongly driven conditions. In this chapter, the improved understanding of the phenomena derived from numerical studies has permitted a more accurate formulation of the formation criteria.

All the work in this chapter will be limited to one dimension. Even though the slab state is unstable, the one-dimensional calculations are sufficient to follow the formative phase, which is the critical one for studying the threshold and onset of wave trapping. Limitation of the analysis and calculations to the slab geometry therefore poses no drawbacks. This chapter is basically divided into

three parts (as is all gall). In the first part, numerical and analytic studies of wave trapping are performed in homogeneous density profiles. These calculations are described first. In the process of analyzing these, several key concepts relative to the wave trapping phenomena emerged. Among these are that the magnitude of the standing wave component is the significant parameter and that parametric instabilities play no more than a secondary role. A simple model based on a cubic nonlinearity is very helpful in understanding the conditions. The understanding gained from the homogeneous studies is then applied to more realistic inhomogeneous plasmas in the second section. Wave trapping still occurred in linear gradient density profiles, but only close to the critical surface. The models derived for homogeneous plasmas were found to be easily modified for use in inhomogeneous plasmas. In particular, the models provide a clear understanding of the effect of laser bandwidth on the formation threshold. Finally, the results of the first two sections are discussed in light of previous work, and in the effect they indicate "bubbles" have in actual laser fusion schemes.

Before proceeding, it is worth noting that both particle and RDF simulations were employed in this work. The two techniques agree remarkably well on threshold and early dynamics of the wave-trapping process. In fact, they were very complementary. The RDF calculations were free from particle noise and fast enough to conduct parametric studies. They showed a tendency to become unstable, however, when large gradients were present. The particle simulations on the other

band required a much larger allocation of resources, but their insensitivity to large gradients and kinetic nonlinearities permitted calculation into the fully nonlinear state. The RFF inability to follow the nonlinear state was not viewed as a great liability, since the late time behavior is intrinsically multidimensional, and since the formative phase was the primary concern here.

SECTION I

WAVE TRAPPING/HOLE FORMATION IN A HOMOGENEOUS PLASMA

A typical homogeneous plasma calculation was conducted on a density profile such as may be seen in Figure 4-1. A vacuum space on either side of the profile facilitated both expansion of the profile and propagation of the electromagnetic waves. The pump was introduced as a traveling wave on the left boundary of the simulation. When the wave was incident on the plasma with sufficient intensity, density fluctuations could be coupled with the electromagnetic field.

Stimulated Brillouin scattering is a familiar example of such coupling. Brillouin scattering was in fact observed over a wide range of densities. Within roughly ten percent of the critical density, however, a qualitatively different type of coupling could also be observed. Instead of backscattering the energy, the electromagnetic field coalesced into highly localized structures, accompanied by deep density wells. The well could extend to virtually vacuum at the bottom, with the expelled plasma accumulating as overdense walls. The field was initially seen to possess only a local maximum; that is, the localized field had no nodes. Although such a strong vacuum state lasted with few qualitative changes for thousands of cycles, it did evolve slowly, broadening the hole width. This is clearly seen in

Figures 4-2 and 4-3. In Figure 4-2 the density profile at $\omega_0 t = 500$ shows several density cavities just after the hole formation; after $\omega_0 t = 2000$, however, Figure 4-3 shows that the width has expanded

dramatically. The field at the later time has at least one node within the cavity. The evolution from symmetric to antisymmetric state suggests a picture of atomic ground and excited state wave functions. The simulation, though, showed a continuous evolution, as opposed to an abrupt jump, so the analogy should perhaps not be over-labored. One of the more fascinating characteristics of this late time evolution was that the frequency of the trapped field decreased from ω_0 to about $.5-.6 \omega_0$. This is not germane to the present purposes, unfortunately, and so will not be further discussed. The interested reader may find more information on this phenomena in Largdon and Lasinski³⁰.

The density regime in which the hole-forming dominated was not precisely specified above, but a fairly sharp density separation between SBS and this process nevertheless was observed. As an example, two simulations, each 10 vacuum wavelengths long, were conducted. Both were irradiated by a laser of strength $(v_{os}/v_{th})^2 = .81$ and the dissipation was the same in both cases. Figure 4-4 shows, however, that the calculation at $n_0 = .95 n_{cr}$ had developed multiple vacuum holes by $\omega_0 t = 1200$. The other simulation, at $n_0 = .85 n_{cr}$ is shown at the same time in Figure 4-5. Strong SBS is observed, but there is no tendency either at this time or later for the fields to collapse into nonpropagating density wells. This is confirmed in both NDF and particle simulations, even when the intensity was strong enough to backscatter seventy to eighty percent of the laser energy. In the particle simulations, such strong scattering led to density waves in

which $\left(\frac{\Delta n}{n_0}\right)^2 \sim .1$. This resulted in particle trapping and, thus, enhanced damping of that wave. The RDF calculation could not treat particle trapping, so the density fluctuations grew to physically unrealistic levels. It is significant, then, that even when the ion acoustic fluctuations were far in excess of the critical density, no wave trapping occurred at $n_0 = .85 n_{cr}$. Thus, it can be inferred that the simple presence of overdense density fluctuations is not sufficient in itself to guarantee hole formation.

Between $n_0 = .95 n_{cr}$ and $.85 n_{cr}$ there is admittedly a transitional regime, so the boundary between scattering and trapping is not mathematically sharp. The intermediate density, however, is very useful for illustrating the effect system length has on wave-trapping threshold. Because trapping is only marginal in this regime, one can see the weak effect due to system length. Two simulations were conducted at $n_0 = .90 n_{cr}$, to study system length effects. One was 10 vacuum wavelengths long, the other 20. The field strength was chosen to be just above threshold for hole formation. In both cases, holes formed initially. In the shorter system, however, the vacuum depression eventually lost its field and the surrounding plasma expanded in to fill the void. The longer system not only exhibited multiple holes, an index of having far exceeded the threshold, but persisted strongly throughout the simulation duration, to $t = 3000$. Although it can be inferred from this that system length tends to decrease the wave trapping threshold, later analysis of the phenomena will show that the

effect is actually due to the strength of the standing wave component present at a given density.

The simultaneous localization of the field and density fluctuations suggest that the two are coupled. Brillouin scattering is one example of just such an interaction. The first order ponderomotive potential couples with an ion acoustic wave there. For an ion acoustic wave one-half the wavelength of the pump wave, the potential and density perturbations move in phase. This resonance leads to exponential growth of both at the expense of the pump. Given this known interaction, it is seductive to look for another resonant instability which would result in the localization. As the American College Dictionary defines "seduce", however, i e. to lead astray, analysis of the coupled wave equations does not give any instability other than the SBS near the critical density for a traveling wave. The analysis itself is rather lengthy and since it is not original, it has been included as Appendix B. An example of the numerically derived real and imaginary frequency components for a traveling wave pump is shown in Figure 4-6. This is quite significant since RDF calculations which employed only the solution of those equations in real space produced wave trapping in agreement with the other numerical techniques. Although difficult to rigorously prove, the qualitative differences between wave trapping and scattering indicate that SBS is not the mechanism responsible for the localization. Since a traveling wave, furthermore, possesses no gradients in its time-averaged potential, there are no zero order forces which could give rise to the phenomenon.

A standing wave pump leads to qualitatively different results.

First, zero order forces exist in this mode. These are periodic with twice the wave number of the electromagnetic wave, and could be expected to produce a density ripple with the same wavelength. A coupled wave stability analysis around the homogeneous state moreover yields a different set of dispersion curves than a traveling wave, as may be seen by comparing Figures 4-6 and 4-7. The unstable waves are divided into a branch with $k < k_0$, which possesses the same properties as SBS, and one at $k > k_0$. This latter is a parametric coupling of a purely growing density fluctuation to an electromagnetic disturbance at the pump frequency. The branches are analogous to the parametric decay/oscillating two stream instabilities which occur transverse to the laser direction. In the presence of the standing wave, the plasma will also respond, to a zero-order force with periodicity of $2k$. The instability predicted in the homogeneous plasma must compete with and/or be modified by the density ripples produced by the zero order force. Since the instability occurs in first order, it seems likely that the zero order force will dominate the system initially. The unstable waves though may still play a significant role in the localization process.

To clarify the relative importance of these two autonomous density perturbations, coupled wave calculations were performed. The wave equations were Fourier transformed here, and a subset of the one-dimensional k -space was numerically solved in a periodic system of finite length. Figure 4-8 shows that the linear instability theory,

indicated by the solid line for these conditions is confirmed, if the $2k_0$ density ripple is suppressed. This calculation was conducted at $n_0 = .85 n_{cr}$, $(v_{os}/v_{th})^2 = .1$, $\theta_e = .75$ KeV, $\theta_e/\theta_i = 3.33$, and $L = 40 \lambda_0$. The standing wave was fixed at a given level, so that pump depletion was forbidden. The relative coarseness of the calculational k -space was an inevitable consequence of the system discreteness. A longer system would result in finer resolution. The length used is, however, representative of recent experiments. The one wave with $k < k_0$ which exhibited exponential growth also had a real frequency in excellent agreement with linear theory. All the waves with $k < k_0$ were purely growing, again in agreement with theory. If the background modifications were ignored, linear theory was well satisfied.

If the density was permitted to evolve without constraints the spectra showed that the $2k_0$ density ripple almost immediately dominated the ion waves. Unstable growth still occurred, but it was drastically modified from that predicted by linear theory. The measured growth rates in the presence of the zero order density ripple but with a fixed amplitude pump are shown as (Δ) in Figure 4-8. The linearly fastest growing waves can be seen to be severely weakened, while previously stable waves with $k < k_0$ have been destabilized. Furthermore, all the measured growth rates possessed either no oscillatory component or one which was so small as to be unresolvable over $1000 \omega_0 t$. In

neither this case nor the above, though, did the fields coalesce into self-consistent density wells.

Wave trapping was observed only in the coupled wave simulations when both pump depletion and the $2k_0$ density fluctuations were allowed. In the above runs, the pump wave had been fixed. When it was allowed to deplete, the spectrum spread rapidly and simultaneous wave trapping/hole formation occurred. Figure 4-9 shows a typical profile of the time averaged field and density, for $n_0 = .95 n_{cr}$, $(V_{os}/V_{th})^2 = .125$, and $L = 40 \lambda_0$. Comparison with other simulations at the same density clearly shows the distinctive density wells containing intense localized field packets. The field magnitude has been normalized to (V_{os}/V_{th}) , so that the pump components initially had amplitude of unity, and the time averaged peaks were at 2. The single most intense peak in the Figure 4-9 has a magnitude $\langle A^2 \rangle = 25 A_{k_0}^2$, despite the presence of dissipation and pump depletion. The field spectrum in all trapping cases evolved relatively early into a continuous exponential shape as shown in Figure 4-10. This was correlated with the field localization first, while the depth of the density hole increased continuously toward a steady value. This is consistent with observations in RDF calculations that intense field localization would relax if the hole had not formed in approximately an ion acoustic time scale. As useful as these calculations were in interpreting the formative phase of wave trapping, they could not follow the evolution into the stable vacuum hole state. As Figure

4-9 shows, the hole collapsed indefinitely and when the reconstructed density became negative, the formulation became numerically unstable.

Analysis of the fluid equations used in the RDF calculations indicate that density deformation may be pressure stabilized. To be precise, steady driftless conditions require $d(V_i)/dt = 0$. Neglecting dissipative terms, this is possible when the plasma pressure gradients balance the gradients in the ponderomotive pressure. Though formally the condition for steady density is the vanishing of the divergence of the flux $n\bar{v}$, we expect the cancellation of these two largest force contributors to be roughly equivalent for practical purposes.

The steady density distribution can be derived from the assumption of isothermal pressure balance, regardless of the mechanism responsible for the deformation. Pressure balance can be expressed mathematically as

$$0 = - (n_e/M_i) v_{os}^2 \nabla A^2/2 - ((e_e + e_i)/M_i) \nabla n_i/n_i \quad (4-1)$$

where the potential has been normalized with respect to the vacuum amplitude. This is easily solved for the physically reasonable boundary conditions

$$A^2(x_0) = A_{\max}^2, \quad n_i(x_0) = n_{\min} \quad (4-2)$$

The solution in terms of the unspecified constants $n_{\min}$ and $A_{\max}^2$ is

$$n_1(x) = n_{\min} \exp(-n_0^2(A^2 - A_{\max}^2)/2) \quad (4-3)$$

where $n_0^2 = (V_{os}^2/V_p^2)$ and V_0 contains both electron and ion energy, i.e. $V_0^2 = (\theta_e + \theta_i)/m_e$. The amplitude, though normalized to the vacuum pump strength, contains any swelling factors. We can further define $n_{\min}$ in terms of $A_{\min}^2$ and $A_{\max}^2$ by noting that the density maxima, which is approximately $2n_0 - n_{\min}$, occurs at the field minima. This redefinition simplifies the definition in case of a standing wave, for which $A_{\min}^2 = 0$. The new definition then is

$$n_{\min} = 2n_0 / (1 + \exp(n_0^2(A_{\max}^2 - A_{\min}^2)/2)) \quad (4-4)$$

We can now insert the spatially modulated density function into the wave equation for the electromagnetic wave, and seek self-consistent solutions.

The electromagnetic wave equation has the new form,

$$\frac{\partial^2 A}{\partial t^2} - c^2 \nabla^2 A + \omega_p^2 \frac{n_1(x)}{n_0} A = 0 \quad (4-5)$$

which is the equation derived by Kaw, et al.²⁴. For small field strength, or at least small standing wave component, the exponential may be expanded to give a cubic nonlinearity. This nonlinear wave equation is commonly used in nonlinear optics. It has approximate solutions of the form $A \propto \text{cn}(x, m)$, where cn is a jacobean elliptic

$$\text{function, } m = \frac{\alpha}{2 \sqrt{1 - \frac{n_o}{n_{cr}}}} \left(\frac{v_o}{v} \right)^2, \quad \alpha = \frac{2 \frac{n_o}{n_{cr}}}{1 + \exp(-\eta_o^2 A_{\max}^2 / 2)}, \quad \text{for a}$$

standing wave, and boundary conditions have been applied such that $A(\pm \infty) \rightarrow 0$, as $m \rightarrow 1$. The derivation is given in detail in Appendix F.

The jacobian solution results from a weak field approximation, yet we find that it possesses many of the same qualitative features as observed in simulations of wave trapping. It is therefore worth examining more closely.

In the limit that the wave amplitude goes to zero, the ϵn -function becomes simply the cosine function, with wavelength given by the linear dielectric function. The wavelength, given by $4K(m)$ (K a complete elliptic integral), and the peak amplitude, $\text{cn}(K, m)$ increase as the field strength increases, however. The m -parameter governs the characteristics of each, and it possesses a complicated dependence on density and field strength. Figure 4-11 shows the vacuum field strength dependence of the wavelength, for various densities between .25 n_{cr} and .95 n_{cr} . For moderate intensity, the field strength dependence is clearly weak below .5 n_{cr} , but pronounced above about .75 n_{cr} . As $m \rightarrow 1$ in fact, the wavelength goes to infinity and the functional form to a hyperbolic secant (sech). Although this approximate solution is expected to breakdown for strong fields, it exhibits the same qualitative characteristics as hole formation. Because the onset of localization, i.e. $\lambda \rightarrow \infty$, is sudden and the structure which results is similar to the holes, we will define hole forming threshold as the

field strength which gives $m = 1$. This is meaningful only as an upper bound on the threshold, but it still represents a considerable improvement over previous trapping criteria. Figure 4-12 shows the dependence of this threshold on density. Note that the field strength, (V_{os}/V_{th}) is the value in the plasma, not the vacuum one. These two values are easily related by considering conservation of energy flux. Since $v_{gr} A^2 \Big|_1 = v_{gr} A^2 \Big|_2$, we find $(V_{os}/V_{th})^2 = (V_{os}/V_{th})_{vac}^2 (1 - n_o/n_{cr})^{-1/2}$. This is confirmed in RDF calculations.

The above discussion concerned a linear swelling factor. As the wave pattern goes from a periodic wave train to an isolated electromagnetic pulse, the peak amplitude is expected to be amplified. To estimate this nonlinear swelling factor, note that the above model is equivalent to $A^2/\lambda \Big|_1 = A^2/\lambda \Big|_2$, for linear dispersion. It is not the purpose here to derive a nonlinear dielectric function; but it is plausible to extrapolate the same form to nonlinear conditions. This is unproved, so its validity is judgeable by its success in predicting calculated field magnitudes. The nonlinear wavelength is given by $4K(m)$, a sample of which was displayed in Figure 4-11. Although this prescription is undefined at $m = 1$, the observed swelling factors are in reasonable agreement for fields such that m is slightly less than unity.

Additional simulation characteristics can be understood as well through this model. The observed hole width at half-depth was initially one-half vacuum wavelength, or $k_o \Delta x = 3.14$. The width of the secant squared function at half-height is $k_o \Delta x = 2.7$. The difference

is of order ten percent, which is about the uncertainty in reading computer generated plots. Also, multiple holes were sometimes observed initially. The system, however, continued to evolve from that point, with the holes eventually coalescing*. This is certainly consistent with a sech^2 -type asymptotic state. Let us now return to the simulation results to see if the relatively simple theory aids in quantitative understanding.

A series of RFL calculations were conducted at $r_c = .95 n_{cr}$ in which the laser amplitude was varied to find the hole forming threshold in a finite sized system. Ion damping was included but for these purposes there was no ion temperature. The horizontal axis in Figure 4-11 therefore directly represents the field strength. A typical homogeneous system length was $11\lambda_0$. Density holes were unambiguously observed for $\eta_0^2 \geq .64$ under these conditions. A field strength of $\eta_0^2 = .50$ led to initial field localization at the front edge of the plasma, but this convected away as the front edge freely expanded. Well defined holes never formed. When the system length was increased to $14.3\lambda_0$, however, all other conditions the same, a stable cavity of width $.51\lambda_0$ and minimum density $.1 n_{cr}$ formed at the front edge. Clearly, a field strength of $\eta_0^2 = .50$ was in the transitional regime in which length can play a role. At $\eta_0^2 = .36$ certain characteristics of the wave trapping process were observed, but no holes ever developed.

*A discussion of the role of initial conditions on hole evolution is given in Section II.

In all these calculations, the pump wave was transmitted in from the left boundary, and so it was purely traveling initially. If the simple theory is correct, there should be no density rippling in a traveling wave. When the pump wave reaches the back edge of the slab, though, classical reflection scatters a fraction of the beam back. The edges were sheathed to reduce this effect, but a small percentage (< 5%) was nevertheless reflected. This is a physical, not numerical, effect, governed by the geometry of the interface. It is thus present in both particle and RBF simulations. This small standing wave component is not sufficient to drive self-trapping, but it will induce density ripples of magnitude

$$\frac{\delta n}{n_{cr}} = .25 \frac{n_c}{n_{cr}} (V_{cs}/V_{th,local})^2 R \quad (4-6)$$

For $R = 3\%$, the magnitude of the fluctuations can be seen in Table 4-1 to increase steadily as the critical density is approached. More important than the magnitude of the fluctuations though is the total density of the peaks, $n_c + \delta n$, which affects the group velocity more strongly than the average density. Below $n_c = .9 n_{cr}$, an increase of a few percent does not seriously inhibit the propagation of a traveling wave. A density ripple of three percent at $.95 n_{cr}$, however, reduces the effective group velocity by over twenty percent. This results in additional reflection, so that further upstream* the standing wave

* We define upstream to be the direction opposite to that of the laser, or rather points in space in that direction. Downstream is hence in the direction further along the laser path.

Table 4-1. Magnitude of Density Fluctuations as a Function of Field Strength and Ambient Density.

n_0/τ_{cr}	r_e^2				
	.2	.4	.6	.8	1.0
.75	.0023	.0045	.0068	.0090	.0113
.80	.0027	.0054	.0081	.0107	.0134
.85	.0033	.0066	.0099	.0132	.0165
.90	.0043	.0085	.0128	.0171	.0213
.95	.0064	.0127	.0191	.0255	.0319

component is enhanced. In the field of the stronger standing wave component, deeper density ripples are formed. One needs only a slight enhancement in this manner to have one hundred percent reflection upstream. Once total reflection is achieved, Figure 4-11 may be used to estimate the swelled field strength and wavelength.

The total reflection results from a bootstrap mechanism. It depends on the amount of reflection at the back edge and on the magnitude of the density ripple which that reflection induces. The total system length needed to establish total reflection thus increases as the pump strength increases. At $n_0^2 = .50$ the above analysis indicates that a standing wave at full pump strength easily exceeds the hole formation threshold. The standing wave component had not reached this threshold level by the front edge of shorter slab calculations. When the system length was increased by only twenty-five percent, though, the standing wave had been increased to the point that hole threshold strength had been attained. The simulation at $n_0^2 = .36$ did not show self-trapping, but this intensity was apparently close to the threshold anyway. A much longer simulation might have resulted in trapping. The exact threshold for these unrealistic profiles, however, is secondary to the fact a model has been derived which qualitatively and semi-quantitatively agrees with simulations.

The analytic model for self-trapping, combined with the mechanics of standing wave formation, describes the hole threshold near the critical density, but does it also explain the simulation results at lower density? In short, yes. The homogeneous simulations showed

only SBS below about $.85 n_{cr}$, even when the vacuum strength exceeded the analytic threshold levels for hole formation. At the lower density, though, the standing wave could not bootstrap efficiently. Consequently, only a small fraction of the field was in a standing wave mode. The configuration of the simulations favored the traveling wave mode; observation of wave-trapping resulted only when a significant percentage of the wave could be converted into standing waves, i.e. near the critical density. The analytic model indicates that threshold for trapping does decrease dramatically as the critical surface is approached, but the total nonobservation of wave trapping below $.90 n_{cr}$ was due to the traveling wave character of the laser. Understanding the phenomena better has therefore permitted interpretation of what were otherwise baffling simulation results.

Before proceeding to the more realistic inhomogeneous studies the results of this section should be summarized. The key conclusion is that wave trapping, and hole formation, is a phenomena which must be driven by a standing wave. Unstable wave growth apparently plays a role during the formative phase, but this has not been identified with any linear instability. Such a linear parametric instability was discovered, but the self-consistent growth of the waves in the presence of a linearly driven density ripple drastically modified the growth spectrum. The free response of the plasma to the electromagnetic field of sufficient intensity was required to observe the localized state. The field spectrum of this localized state was nearly exponential, leading to the supposition that the stable state was akin to a

solitary wave. Such solitary waves may be regarded as nonlinear normal modes of the plasma, in the presence of a standing wave. This is consistent with the steady-state analysis of the field-density distribution. The field distribution which was reconstructed from a weak-field approximation was found to possess the trapped state characteristics seen in the simulations. Finally, synthesis of these results led to the conclusion that the absence of holes below $.90 n_{cr}$ was due to inability of the homogeneous slab to establish a standing wave of sufficient magnitude. Numerical threshold values were calculated analytically, but it is doubtful that the homogeneous threshold is of much practical value. If holes are to be significant in laser fusion it is necessary that their threshold in inhomogeneous plasmas be known. Nevertheless, the cubic nonlinearity model can be expected to provide an upper bound for homogeneous systems.

SECTION II

Wave Trapping/Hole Formation in Inhomogeneous Plasma

Whenever a density profile extends far above critical density, some fraction of the energy will generally be reflected, giving a standing wave component. While the standing wave component may be at a significant level in the inhomogeneous plasma, the inhomogeneities themselves may spoil the trapping. Numerical studies were therefore performed to understand inhomogeneity effects on wave trapping.

The homogeneous model for self-trapping was based on an exponential nonlinearity in the field, albeit truncated to first-order terms. This model can be easily extended to a linear density profile plasma. The wave equation in a linear profile is the Airy equation discussed in Chapter 2. Earlier in this chapter, we derived an expression for the steady density distribution as a function of intensity, Eq (4-3). If the ambient density in that expression is replaced with a linear gradient density function, $n_c(x) = (1 - \frac{x}{L})n_{cr}$, a type of nonlinear Airy equation is produced

$$\frac{\partial^2 A}{\partial x^2} + \frac{\omega_0^2}{c^2} \left(1 - \frac{2n_{cr} \left(1 - \frac{x}{L} \right)}{\left(1 + e^{-1/2 \frac{2\omega_0^2}{n_{o \max}^2}} \right)} \right) e^{-1/2 \frac{2\omega_0^2}{n_{o \max}^2} (A^2 - A_{\max}^2)} A = 0 \quad (4-7)$$

Analytic solutions of this equation are unknown, so it was solved numerically. The solution characteristics were found to be similar to those of the jacobian function. As the pump power increased, the field experienced both a swelling in amplitude and a dilation in wavelength.

This latter strongly affected the inhomogeneous solution, since the swelling depends on the number of wavelengths which fit on the profile. In consequence, the nonlinear swelling of the field was attenuated by the effective reduction in the profile length. This simple model, naively interpreted, does not represent the physics accurately at all.

Since the nonlinear evolution of the linear profile is initiated by an Airy-like intensity pattern, one would expect the strongest and earliest effects to occur near the critical density. Once the perturbations reach significant levels, however, they will tend to lock the additional swelling in the field into the density troughs. The actual nonlinear state is then determined by the competition between the tendency of the field to increase in wavelength and inertia of the perturbed plasma which remembers its initial spatial distribution and attempts to resist wavelength dilation. These intuitive ideas were used to refine the density modulation model in Eq (4-7). The exponential nonlinearity was retained, but only the linear Airy function appeared in the nonlinearity. Despite the possibility that this could underestimate the nonlinear field effects, the computed intensity pattern exhibited huge swelling factors in modest profile lengths at moderate field strengths. Figure 4-13, for instance, shows the model results for an $L = 10 \lambda_0$ density profile when $(V_{os}/V_0)^2 = .3$. The linear swelling amounts to a factor of 3.6 ($\times 4$ for comparison with traveling waves) for such a scale length. The inclusion of density modulations has amplified the peak intensity to over 18. The minimum density in such a field moreover is down to $.1 n_{cr}$ as shown in

Figure 4-14. The model calculations thus show a hole forming threshold for this scale length of $(V_{os}/V_{\theta})^2 = .2-.3$.

The linear swelling in the intensity pattern increases with scale length. Since this model uses the linear pattern to evaluate the density perturbations, the hole threshold should decrease with increasing scale length. The definition of threshold for a continuous process is somewhat arbitrary, but a plausible one is: that vacuum field strength for which the minimum density first falls below $.01 n_{cr}$. Then the model calculations confirm the decrease in threshold. Figure 4-15 gives the calculated hole threshold as a function of linear scale length.

Particle simulations were also performed to study self-trapping in inhomogeneous plasmas. These yielded consistently higher thresholds than did the model calculations discussed above. If allowance is made for the energy SBS is removing from the beam under these conditions, however, this result is not surprising. The rule of thumb for SBS is that fifty percent backscatter occurs whenever

$(V_{os}/V_{th})^2 \frac{\lambda_o}{L_o} = 5-10$, depending on the ion damping rates. As an example of the effect of such scattering, consider a simulation conducted on an 8 wavelength linear density profile, with electron to ion temperature ratio of 10. Ion Landau damping is thus weak, and the lower limit on the SBS prescription is probably applicable. One should thus expect strong SES above $(V_{os}/V_{th})^2 \approx .6$. The adjusted field strength threshold for holes on this profile is $(V_{os}/V_{th})^2 = .5$

$((v_{os}/v_e)^2 = .45)$. Holes did not form, however, in a simulation for which $(v_{os}/v_{th})^2 = .75$. The minimum density near the critical density reached only $.3 n_{cr}$. This minimum near the critical density is consistent with the model calculations, if the energy reaching the critical density were reduced by fifty to sixty percent. Under the same conditions, a field strength of 1.1 resulted in stable vacuum holes, as Figure 4-16 indicates. Assuming that particle trapping limits the average scattering factor, to say fifty to seventy percent, this calculation is also roughly consistent with the model derived thresholds. RDF simulations in profiles up to 20 wavelengths long behave in a similar fashion.

The above examples indicate that beam modification in the underdense plasma must be included in evaluating self-trapping thresholds in inhomogeneous plasmas. The only stimulated effect allowed in these one dimensional simulations was Brillouin backscatter, and knowledge of its nonlinear behavior permitted fairly accurate estimation of its effects. Since SBS has inhomogeneous thresholds close to the wave trapping ones, it must probably be considered whenever the latter threshold is being evaluated. There are other effects, however, which are also efficient at reducing the energy flux to the critical density. Since wavetrapping has its lowest threshold in that region, the local density near the critical surface is the parameter which determines whether holes will form or not. Raman back- and sidescatter and Brillouin sidescatter are examples of flux reducing mechanisms which can not be readily addressed in this study, but which pose considerable

problems to hole formation. Likewise, any strong dissipation mechanism would absorb the energy and hence inhibit hole formation.

Although it is not practical to categorically list all the effects which could influence wave trapping, any effort aimed at providing more realistic threshold conditions must consider some. Brillouin scattering has already been discussed. As further illustrations, classical absorption and finite bandwidth will be briefly discussed below.

As the linear swelling studies in Chapter 2 showed, the primary effect collisional damping exerts on the intensity pattern is a uniform decrease in its magnitude. Calculations over a wide range of scale lengths indicate that the curve in Figure (2-5) is useful for estimating this magnitude, within roughly ten percent, as a function of total absorption. For moderately strong absorption the decrease in $\langle A^2 \rangle$, mean squared amplitude, can be nearly a factor of 2. This reduced effective intensity is then the parameter to be used in evaluating hole thresholds.

Finite bandwidth effects on the intensity pattern were also studied in Chapter 2. For realistic bandwidths, the incoherence related reductions tended to be small, i.e. reductions of twenty to forty percent for scale lengths between $20-100 \lambda_0$. This factor must be cautiously interpreted. Despite relatively minor attenuation in the intensity pattern, bandwidth also severely weakens resonant parametric instabilities, such as SBS. Inhibition of scattering instabilities in the underdense plasma will allow a greater fraction of the original

beam to reach the critical surface. The net result of these two opposing tendencies on wave trapping is not generally clear. To help resolve uncertainties, several simulations were conducted in an S wavelength long density profile, with varying amounts of bandwidth. The incident field strength was $(V_{os}/V_{th})^2 = 1.1$, sufficient to create vacuum holes after $\omega_0 t = 500-800$ with a monochromatic wave, when $\frac{m}{M_i} = .01$. The results for bandwidths of $\Delta\omega/\omega_0 \approx .03, .05$, or $.10$ were that virtually no difference was observed in either hole formation or subsequent evolution. In fact, if there was any difference in the calculations it was that the hole seemed to form slightly earlier in the large bandwidth cases. These calculations do not form a definitive study but they indicate that the net effect of bandwidth under non-linear conditions may be to increase the energy which actually reaches the critical surface.

It is of considerable interest to study the competition of the anomalous absorption mechanisms near the critical surface with wave trapping. Development of strong holes will conceivably disrupt the critical surface to the extent that collective processes will be gradient stabilized. On the other hand, if microturbulence is established first, the enhanced dissipation could lower the local field level below hole-forming threshold. To further complicate matters, resonance absorption may find the "lumpy" surface a hospitable environment, while the other processes are spoiled. Unfortunately, only the self-consistent two-dimensional study of these interacting

processes will be able to provide answers to these questions. The one dimensional calculations employed herein are not adequate for this problem.

Inhomogeneous thresholds have been studied in this section, and several useful guides have been derived. Model calculations were used to find thresholds as a function of density scale length. This threshold decreased with increasing scale length, due to the length dependence of the linear swelling factor. The model calculations relied on this swelling factor, which may have tended to overestimate the thresholds. As an upper bound, though, the calculations, as represented in Figure 4-15, were consistent with simulations. Uncertainties were introduced when stimulated Brillouin scattering was allowed in the calculations, for they reduced the energy flux to the critical surface. Brillouin scattering could be estimated from other nonlinear studies, and allowance for stimulated reflection brought simulations into closer agreement with the model predictions. SBS is representative of a body of phenomena which can occur in the underdense plasma, but which do not effect the critical surface region directly. This led to the general rule, that only the local energy density near the critical density was needed to evaluate hole thresholds. The question of competition between holes and anomalous absorption in the same region, unfortunately, must await higher dimensional simulations to be resolved. It is finally appropriate to discuss these studies within the overall efforts which have been made to understand wave trapping.

SECTION III

Discussion of Wave Trapping/Hole Formation Results

Self-focusing and self-trapping of intense laser beams in dispersive media were first investigated within the context of nonlinear optics (c.f. Ref 16). The saturable cubic nonlinearity used in those studies is also useful in plasmas, but the saturation physics is quite different than in dispersive liquids or solids. The nonlinear optics work thus does not possess direct applicability to laser fusion plasmas, and will hence not be considered further. Perhaps the first investigation of condensed electromagnetic states in plasmas was conducted by Volkov³¹. Gurovich and Karpman²⁷, however, were the first to phrase the problem of trapped electromagnetic energy in density wells properly. They discovered a number of solitary wave states depending on the ratio of the electromagnetic to ion acoustic group velocities, and on whether the dielectric function was positive or negative. In particular, when the former ratio was less than unity, they found a traveling wave broke up into a solitary wave with the characteristics found for holes, except that the coupled density/field structure propagated at the field group velocity. For $\epsilon < 0$, their analysis uncovered stationary solitons, similar to the one found in Section I. In the negative- ϵ case, however, they limited themselves to small amplitude pumps. In subsequent papers they studied the stability and effect of dissipation on this negative- ϵ soliton.

The analysis of negative- ϵ electromagnetic wave states has been further treated by Marburger and Tooper³², who recognized the standing wave nature of such a solution, but kept the ions immobile, and by Lai³³ who relaxed the ion immobility assumption and found one dimensional, hole-like solutions in the overdense plasma.

Kaw, Schmidt, and Wilcox²⁴ performed an elegant analysis of the exponential saturating nonlinearity used in Eq (4-5). They derived both filamentation, or self-focusing, and self-trapping from this equation. The filamentation admitted of a legitimate stability analysis. Their analysis also pointed to an asymptotic waveguide-like solutions. Although they did not explicitly state the importance of standing waves for the self-trapped state, their form for the exponential nonlinearity implied it. A stability analysis for such conditions was inappropriate, as demonstrated in Section I, but they did find eigenstates which possessed cavity-like properties. Threshold for this eigenstate was vague but the explicit examples given were for $(v_{os}/v_e)^2 \sim 1$ and $\epsilon < 0$.

The condition of negative- ϵ for self-trapped states has been generally translated into occurrence only in overdense plasmas. The homogeneous calculations in Section I show that this is overly restrictive, if not actually misleading. Localized wave states can easily evolve from profiles which are initially everywhere underdense. Even the suggestion by Kaw, et al. that growing ion waves could trap the field did not differentiate between normal ion acoustic modes, which did not trap the field, and zero frequency density fluctuations driven by

standing waves. This is significant primarily in that it shows that the localized state is not the usual end product of SBS.

The soliton state which Gurovich and Karpman²⁷ derived for positive- ϵ is limited to density close enough to the critical density that the ion acoustic speed is greater than the field group velocity. This occurs only when $1 - \frac{\epsilon}{m_e c^2} \frac{m_e}{M_i} \leq \frac{n_0}{n_{cr}} \leq 1$, which occupies an exceedingly small volume. Although this state could evolve from SES, it is much less likely than the one driven by a standing wave.

A growing experimental and theoretical literature is occurring about localized electrostatic cavities, or "cavitons", near the critical density. Since the form of the coupled density/electrostatic field equations is similar to that for electromagnetic waves, this is hardly surprising. If the electromagnetic field localizes as a soliton, then the cavitons may be regarded as Langmuir solitons. It is not necessary here to discuss the theoretical developments in detail; it should be noted that the studies of Zakharov³⁴, Valeo and Krue³⁵, Schmidt³⁶, and Denavit, Pereira, and Sudan³⁷ tend to overlap with these wave trapping studies, except that the former admits the possibility of direct wave-particle interactions.

The close relation between the two phenomena is illustrated by the two and three dimensional studies of Denavit, Pereira, and Sudan³⁷, and those of Valeo and Estabrook²⁹. In the former case, the initially slab electrostatic state was found to be unstable to sausage-type instabilities. The final state was found to be localized, electrostatic "bubbles". Valeo and Estabrook found almost precisely the same

behavior for the electromagnetic waves. The latter, however, exhibited no tendency to dissipate late in time. Both analyses were confirmed with computer simulations. Langdon and Lasinski³⁰ have also confirmed the two dimensional behavior of the electromagnetic state, and have uncovered interesting characteristics of the trapped field.

Experimental studies of cavitons have been conducted with micro-waves by Kim, Stenzel, and Wong³⁸. Experiments in laser plasmas by Fidmann and Sigel³⁹ for Nd lasers and Donaldson and Spalding⁴⁰ for CO₂ lasers have reported curious structure near the critical surface. The CO₂ experiments actually mapped an extensive cavity in the density profile, but the dimensions, 50 μm by 200 μm , are much larger than any cavity seen in simulations to date. This, however, may be due to the limited space or time covered in the simulations. Other experiments have tentatively observed extraordinary roughness of the critical surface, which could be a signature of electromagnetic "bubbles". None of the experiments reported to date, though, can claim unambiguous detection of the localized state.

The conclusions from this study may be briefly summarized. One dimensional thresholds for wave trapping/hole formation near the critical density were found to be competitive with those of strong Brillouin backscatter. Although extensive study of the competition of this effect with other collective processes was not feasible within the one dimensional framework, those effects may be estimated roughly by evaluating the extent by which they reduce the energy density in the region of the critical surface. Finite bandwidth in particular

seemed more effective in spoiling resonant collective processes, i e. SES, than in inhibiting hole formation.

FIGURE 4-1 Typical slab density profile, $L=10\lambda_0$, $n_0=.95 n_{cr}$.

FIGURE 4-2 Density profile at $\omega_0 t = 500$; initial profile length, $L = 10\lambda_0$, $n = .95 n_{cr}$. Well defined holes are centered at $x/\lambda_0 = 18.0, 19.2, 20.8$.

FIGURE 4-3 Density profile at $\omega t=2000$, same calculation as shown in Figure 4-2. Hole at $x/\lambda_0=19.2$. Note underdense plateau at $n \approx 0.75 n_{cr}$ on laser side of hole.

Figure 4-4 (a) Time-averaged field profile at $\omega_0 t = 1200$. Peaks are swelled by a factor of 2 above linear value. $(V_{os}/V_{th}) = .9$, $L = 16 \lambda$, $n_0 = .95 n_{cr}$. (b) Density profile at same time as (a). The beginnings of field coalescence can be seen at $\omega_0 x/c = 56, 70, 82, 96, 109$.

Figure 4-5 (a) Time-averaged field profile at $\omega_0 t = 1200$. An exponentially back-amplified signal is evident, though peaks are smaller than linearly swelled standing wave. $(V_{os}/V_{th}) = .9$, $L = 16 \lambda_D$, $n_0 = .85 n_{cr}$. (b) Density profile at the same time.

Figure 4-6 Numerical solution to dispersion relation for parametric coupling of electromagnetic waves and ion density fluctuations, traveling wave pump, $n=.8 n_c$, $(v/v_{th})=.548$, $\Gamma_i=.001 \omega_0$, $\Gamma_{em}=.00087 \omega_0$. Frequencies plotted versus $k=k_{\text{pump}}$.

Figure 4-7 Numerical solution to dispersion relation for parametric coupling of electromagnetic waves to ion density fluctuations, standing wave pump, $n=.8 n_{cr}$, $(V_{os}/V_{th})=.447$, $\Gamma_i=.001 \omega_0$, $\Gamma_{em}=.00087 \omega_0$. Plotting convention the same as in Figure 4-6.

Figure 4-8 Growth rates measured in a coupled wave simulation, standing wave pump, $(V_{os}/V_{th})=.316$, $n=.85 n_{cr}$, $\Gamma_c=.0025 \omega$, $\Gamma_m=.001 \omega$. The solid line is theoretical prediction for these conditions; solid circles(●) are measured growth rates when $2k$ density ripple is artificially suppressed; triangles(Δ) show growth rate when density ripple allowed to grow.

FIGURE 4-9(a) Reconstructed density profile from a coupled wave simulation, $(V_{os}/V_{th})^2 = .125$, $m/M_i = .01$, $n = .95 n_{cr}$, $L = 40 \lambda_0$. Note negative density trough at $\omega_0 x/c = 20$. Fourier codes do not prohibit such behavior but become numerically unstable when it occurs.

FIGURE 4-9(b) Reconstructed intensity profile at same time as (a). Peak at $\omega_0 x / c = 160$ is $A^2 = 25$, compared with $A_0^2 = 2$.

FIGURE 4-10 Time averaged field spectrum; initial spectrum is shown in red while final spectrum to the same scale is in black.

FIGURE 4-11 Wavelength of cn-function as a function of vacuum field strength, for various densities $n_0 = .25, .50, .75, .85, .95 n_{cr}$.

FIGURE 4-12 Threshold field strength for hole formation as a function of density. Threshold criterion is taken to be $m=1$, where $A \propto cn(x;m)$.

Figure 4-13. Field profile derived from model calculations, $L=10^3$, $(V_{os}/V_\theta)=548$.

Figure 4-14 Density profile derived from model calculations for same parameters as Figure 4-13.

Figure 4-15 Inhomogeneous threshold for vacuum hole formation as a function of system length, derived from model calculations.

Figure 4-16 Density profile from particle simulation showing late time hole characteristics. $L_0 = 8\lambda_0$, $\eta_0^2 = 1.0$, $m_e/M_1 = 0.01$.

CHAPTER 5

RDF Study of Absorptive Processes in Laser Plasmas

In the preceding chapters, the reduced description formalism has been applied to stimulated scattering, steepening of the plasma near the critical density, wave trapping, and linear swelling factors under a variety of conditions. Discussion of dissipative processes has been conspicuously absent. Extensive numerical investigations have already been conducted to understand collisionless dissipation. Moreover, the particle trapping which characterizes the saturated state of many of these mechanisms is not directly describable within the RDF context. There are aspects of the absorption problem to which this method can be profitably applied. Some of these will be presented in this chapter.

The effect of density modifications on collisional absorption is studied in the first section. The motivation for considering this topic stems from the fairly sensitive density dependence of the collisional dissipation rate, i.e. as density squared. Cross density modifications near the critical density, such as self-steepening, can significantly reduce the total absorption efficiency. Brillouin scattering can also induce large density modulations, but these tend to reduce absorption by scattering the energy out of the plasma before it can be absorbed. Self-consistent evaluation of the reduction in efficiency must be approached numerically, and an RDF type code is almost ideal for this purpose. Large lagrangian MHD codes, for

instance, are zoned too coarsely to resolve collective effects, and particle simulations do not treat collisional phenomena accurately.

In the second section, a nonlinear characteristic of inverse bremsstrahlung itself is examined. Quantum mechanical field-dependent corrections (i.e. nonlinear) occur in the free-free transition rate. These correspond to multiphoton processes and they reduce the overall absorption rate. The magnitude of the effect is studied by substituting a field dependent damping rate for the usual collisional one in the EDF calculations. The absorption coefficient is found to be significantly reduced for moderate field strength ($V_{os}/V_{th} \sim 1$), when swelling factors are included.

A new absorption mechanism is discussed in the third section. The Dawson-Oberman theory, developed in the early 1960's, indicates that dissipation occurs when electrons are scattered from finite ion density fluctuations, as well as from individual ions. Much study has been centered on the resonance which occurs for long wavelength ion fluctuations. Short wavelengths, on the order of several Debye lengths, however, can also give a significant enhancement over collisional absorption, especially if the fluctuations occur between about one-half critical density and the critical density itself. Although the strength of the mechanism depends on the spatial distribution of the ion waves, and their orientation relative to the field of the laser, the potentially greater volume available for absorption can lead to anomalously efficient dissipation. Conditions for the

generation of such waves are discussed. Finally, RDF calculations for various ion wave spectra and magnitudes are presented.

SECTION I

Reduction in Collisional Absorption Due to
Self-Consistent Density Modifications

In Chapter 2, the RDF calculations were used to confirm analytic expressions for collisional absorption of laser energy in a linear density profile. Though the theory was derived in the geometrical optics limit, the exact wave treatment in the calculations agreed with the theory over a wide range of conditions. Modeling the plasma with a uniform density gradient is a reasonable approximation if the field strength is much less than the thermal energy. This corresponds to $(v_{os}/v_{th})^2 \ll 1$. At higher power, however, the results of the previous chapter indicate that self-consistent density modulations must be taken into account. In this section we will examine some of the ways self-consistent density modifications can reduce the classical absorption.

There are several mechanisms by which density modifications reduce absorption. First, whenever $(v_{os}/v_{th})^2 L/\lambda_0 \gtrsim 5-10$, significant backscattering due to SBS occurs in the underdense region²¹. The energy in this case is simply scattered out of the plasma. It never reaches high enough density to experience efficient collisional dissipation. This can be very effective in reducing absorption, since the classical damping rate scales as the square of the density. The group velocity of the light also decreases with increasing density, so the light spends more time in the regions of higher dissipation. A second

mechanism which can lower the absorption efficiency is self-steepening of the density profile at the critical density^{8,9}. The ponderomotive pressure from the laser inhibits plasma flow at the critical density, the classical turning point, causing an accumulation above the critical density and a shelf below it. A density jump therefore forms. Though this mechanism does not inhibit energy from reaching the high density region, it finds less plasma there, and hence reduced absorption. Finally, propagation of the laser to the critical density and reflection from it leads to a standing wave structure. At low intensity, the pattern takes the form of an Airy function. As seen in the Chapter 4, however, the density ripples which accompany the standing wave can lead to wave trapping and orders of magnitude enhancement over the Airy swelling factor. The peaks of the standing wave pattern occur at the density troughs, though, and this complicates evaluation of the net effect on inverse bremsstrahlung absorption.

The reduced description formalism is almost uniquely suited for such studies. Two dimensional MHD modeling of laser fusion schemes, for instance, depend critically on energy deposition information. These rely on absorption cross-sections similar to the one derived by Dawson et al.⁴. It is usually prohibitively expensive to resolve any fine structure in the underdense plasma in these calculations, but even a finely zoned problem would miss the first and third mechanisms just discussed. The second, self-steepening, could be observed, since only intensity magnitude is needed there. Even then, steepening

due to electrostatic wave pressure would not be observable. On the plasma time scale, particle simulations can resolve self-consistent plasma effects, but collisional phenomena is not accurately represented in such codes. The RDF technique can incorporate realistic collisional processes, while still following the plasma evolution on its own time scale. For the present study, then, it is the logical numerical tool.

A series of calculations performed on a 7.3 wavelength linear profile illustrate the trends. The electron temperature was not calculated self-consistently in these runs. The plasma was assumed to be isothermal, at .5 KeV. For such short scale lengths, this temperature is in reasonable agreement with hydrodynamic results. An ion damping rate was included at a level corresponding to a 3:1 electron to ion temperature ratio. The electron to ion mass ratio was further set to .01, to reduce calculational time. Expansion, steepening, and stimulated Brillouin scattering were all intrinsically included in the calculations. The reduction in classical absorption as a function of intensity can be seen in Figure 5-1. At low intensity, the absorption was well described by Eq (2-5). With increasing field strength, however, the steady absorption levels decreased monotonically. To show that the points on Figure 5-1, represent true steady absorption levels, the time resolved absorption curves for various intensities are plotted in Figure 5-2. Since the total absorption in short scale length calculations increased significantly

as the plasma expanded, the constant levels are evidence that SBS balances collisional absorption to a high degree.

The above calculations show substantial reduction in absorption as a function of intensity. Since multiple effects contributed toward this end, however, further simulations were necessary to evaluate the relative importance of each.

Steepening was evident in certain of the above calculations, though not to the extent predicted, c.f. Ch 2, Sec III. Competing effects were responsible for at least part of that. As an example, in the cases in which $(V_{os}/V_{th})^2 \sim .5$, steepening should be strongly evident but relatively strong SBS under these conditions greatly reduced the electromagnetic flux which reached the critical density. Thus, both steepening and collisional absorption are affected by SBS in the same qualitative manner. These two processes can fortunately be separated on the basis of scale length. Brillouin scattering can be stabilized by sufficiently large inhomogeneities, while the steepening, being independent of scale length, should reach the same asymptotic state even in large density gradients. This led to a series of very short scale length calculations, to evaluate the effect of self-steepening on collisional absorption efficiency.

The density profile was initialized by allowing an overdense slab to freely expand for several thousand $\omega_0 t$. The resulting effective scale length was between $1 - 3 \lambda_c$. Although the density jump was measured directly, knowledge of the slope is important for determining the exact intensity structure in the vicinity of the critical

density. This is clearly important in evaluating collisional absorption. Knowledge of the swelling pattern in nonlinear profiles derived in Chapter 2 allowed highly accurate determination of this effective scale length. This in turn was found to be virtually identical to the slope of the profile at the critical density. The steepening of the profile could therefore be verified by measuring the decrease in the swelling factor.

Figure 5-3 shows the absorption history curves for various intensities. The profile was initially flattopped at $n = 3.0 n_{cr}$. The electron temperature was fixed at .5 KeV, and the ion fluid damping constant was set at $\nu_i = .0002 \omega_o$. As noted in Chapter 2 this form of ion dissipation can limit free expansion to anomalously low values. The value used here was low enough to permit realistic expansion, but the loss in smoothing allowed too much low density noise. The region around the critical density was less affected. Since the plasma is continuously expanding, the total absorption, as shown in the figure, monotonically increases. The steepening due to even modest fields, however, results in large fractional reductions in the absorption. With the low ion damping constant, SBS occurred, in even short scale lengths, above $(V_{os}/V_{th})^2 \approx .25$. Monitoring the time averaged intensity pattern verified this unambiguously after $\omega_o t = 2000$ at that intensity. Figure 2-10, at that time, though, shows that profile steepening had already occurred. At later times, the reduction was thus due to a combination of steepening and stimulated scattering. It is

interesting that a steady absorption level was established at higher intensities, despite the profile expansion.

To examine steepening effects at higher intensities, an order of magnitude larger damping constant was used. Although the expansion was nonphysically slow, the total absorption in the shorter system was lower, and the steepening more difficult to resolve, Figure 5-4 shows that the total absorption was still reduced by a significant factor by the density modifications.

The results of these calculations do not admit ready generalizations. It would be useful to derive expressions for the reduction in classical absorption due to steepening, as a function of intensity. In an expanding profile, however, steepening alone is not sufficient to give a steady absorption level. Even the relative reduction is found to be a function of time. Steepening is clearly capable of causing a significant reduction in the absorption over that of a fixed profile, especially in short scale length plasmas. The quantitative amount of reduction, though, depends on the rate of expansion, the rate of absorption, and the degree of steepening at the critical density. In previous studies of steepening, employing both particle and hybrid (particles ions and fluid electrons) simulations⁴¹, much steeper critical density gradients were seen with the particle calculations. The present fluid calculations are even more diffusive than the hybrid ones. The absorption coefficients measured here should therefore be regarded as upper bounds when steepening near the critical density is present.

Finally, we consider reductions due to the standing wave component of the field in an inhomogeneous density profile. As shown in the last chapter, a standing wave leads to both finite density ripples and nonlinear field swelling. Since the field is peaked in the density depressions, the density squared dependence of inverse bremsstrahlung could result in net absorption reductions. This must be partially balanced, however, by the enhanced field amplitude. To evaluate this mechanism, two sets of calculations were performed. The first set consisted of ordinary RDF calculations with $\left(\frac{v_{os}}{v_{th}}\right)^2 = .5$, $m_e/M_i = .01$, $L = 10 \lambda_o$, with the temperature varied formally to alter the amount of absorption. Stimulated scattering, self-steepening, and density rippling could therefore interact to lower the absorption coefficient. In the other set of calculations, the fluid equations were first reduced to a wave equation for the ion density fluctuations. The inhomogeneous density was introduced as a fixed background condition, to describe the medium. Stimulated Brillouin scattering in this configuration is almost exactly the same as in the full RDF calculations. The SBS reductions were thus included correctly; no self-consistent density modifications were allowed. The absorption was computed using the fixed initial profile. The results are summarized in Table 5-1. The discrepancy between the full RDF computed absorption and the wave equation approach amounted to no more than a few percent over the entire range of absorption. We conclude that density rippling is effectively cancelled out by the field swelling.

TABLE 5-1

ABSORPTION AS A FUNCTION OF ELECTRON TEMPERATURE

Linear Density Gradient

$$L = 10 \lambda_o, \quad (v_{os}/v_{th})^2 = .5$$

m_e/M_i	0. $\rightarrow$ (Theory)	0.01 (Fluid Eqs)	0.01 (Wave Eqs)
θ_e (KeV)			
0.10	92.52 (95.69)	89.44	90.00
0.25	54.00 (54.87)	27.78	29.20
0.50	24.40 (24.52)	9.62	10.32
1.00	9.46 (9.47)	4.11	4.79
2.00	3.42 (3.45)	1.57	---

SECTION II

Direct Intensity Dependence in Collisional Absorption

In this point, the classical absorption was computed with an intensity independent coefficient, v_{em} . This is tantamount to considering only single photon interactions. Several authors^{42,43,44}, however, have shown that the correct quantum mechanical result includes both the thermal and the oscillating components of the electron velocity. Silin⁴⁵ has furthermore derived the same result from purely classical arguments. Since the oscillating velocity is proportional to the field amplitude, $V_{os} = eE/m\omega$, the local value of the field itself plays a direct role in the classical absorptive process.

Let us examine the magnitude of this intensity dependence. In the limit that $(V_{os}/V_{th}) \ll 1$, the full expression reduces back to Eq (2-7). In the other limit, $(V_{os}/V_{th}) \gg 1$, Silin gives

$$v = v_o \frac{12}{\sqrt{2\pi} \left(\frac{V_{os}}{V_{th}} \right)^3} \left\{ \ln \left[\frac{1}{2} \left(\frac{V_{os}}{V_{th}} \right) \right] + 1 \right\} \quad (5-1)$$

For calculational purposes, we have fit the exact expression with

$$v = v_o \left(1 + \frac{1}{3} \langle A^2 \rangle \right)^{-\frac{3}{2}} \quad (5-2)$$

In the weak field limit, this of course gives the usual result, and in the strong field limit, it is within ten percent of the expression

given in Eq (5-1). Note that the time averaged field, calculated, is used to evaluate (5-2). For a traveling wave field, the average value of the field at a given density is approximately

$$\sqrt{E^2} = \frac{1}{\sqrt{2}} E_0 \left(1 - \frac{n}{n_{cr}} \right)^{-1/2} \quad (5-3)$$

The intensity dependence is therefore a strong effect only for fields such that $(v_{os}/v_{th}) > 1$. The picture is dramatically different for the more realistic standing wave mode. Then the time averaged pattern is an Airy function, normalized to $2^{1/2} v_{os}/v_{th}$. Field effects are localized, but at locations where the field itself is largest. The Airy swelling factor, moreover, leads to proportionally largest reductions in the collisional dissipation rate near the critical density. The reduction in collisional damping in a standing wave pattern can be significant at lower intensities than previously suspected.

A series of calculations done in a fixed linear density profile confirm the qualitative ideas advanced above. Figure 5-5 shows absorption history curves at various vacuum field strengths. The electron temperature was fixed at .5 KeV, which gives forty-four percent absorption in a $20 \lambda_0$ scale length, for $\lambda_0 = 10^{-4}$ cm. The actual absorption is reduced by a factor of 2 for $(v_{os}/v_{th})^2 = 1.0$. For $(v_{os}/v_{th})^2 \rightarrow 4$, the absorption falls below the classical figure by a factor of 4-5. Figure 5-6 shows the total absorption coefficient for these conditions as a function of field strength.

To evaluate the reduction in inverse bremsstrahlung due to intensity dependence of the cross-section, consider the simpler model problem of absorption of a standing wave in a homogeneous plasma. If the steady part of the absorption coefficient is denoted by G_0 , the absorption integrated over one wavelength is

$$ABS = \frac{1}{k} \int_0^{2\pi} G_0 \sin^2 y \, dy \quad (5-4)$$

where $y = kx$, and $ABS = -\frac{dA^2/dx}{A^2}$. The integral is easily evaluated to give

$$ABS = \pi G_0 / k \quad (5-5)$$

Now suppose G has the following spatial, i.e. intensity, dependence

$$G = G_0 / (1 + \frac{1}{3} A^2 \sin^2 y)^{3/2}$$

The integrated absorption then becomes

$$ABS_1 = \frac{4}{k} \int_0^{\pi/2} \sin^2 y \, dy / (1 + \frac{1}{3} A^2 \sin^2 y)^{3/2} \quad (5-6)$$

With a little help from standard integral tables (c.f. Gradshteyn and Ryzhik, Eqs. 2.564 and 2.598), this integral can be expressed in terms of elliptic integrals,

$$ABS_1 = 4 \left[\frac{A^2}{3} \left(1 + \frac{A^2}{3} \right)^{1/2} \right]^{-1} (K(a) - E(a)) G_c / k \quad (5-7)$$

where $a = \sin^{-1} \left\{ \frac{A/3^{1/2}}{(1 + A^2/3)^{1/2}} \right\}$. In the limit that $A \rightarrow 0$, it can be easily verified that (5-7) reduces to (5-5). On the other hand, when $A \gg 0$, $ABS_1 \propto A^{-3}$, which can be a significant reduction. A measure of the reduction can be gained by computing

$$f = ABS_1 / ABS = \frac{4}{\pi} \left[\frac{A^2}{3} \left(1 + \frac{A^2}{3} \right)^{1/2} \right]^{-1} (K(a) - E(a)) \quad (5-8)$$

For a standing wave we will take $A^2 = 2v_{os}^2/v_{th}^2$, although this should be multiplied by $(1 - \frac{n_o}{n_{cr}})^{-1/2}$ to account for swelling. These results are displayed in Table 5-2.

In an inhomogeneous plasma, both the field structure and the absorption integral are more complicated. We may obtain an estimate of the exact reduction, however, by considering the swelling in an Airy pattern. The low amplitude absorption integral in a linear density gradient was evaluated in Chapter 2, Eq (2-5). Exactly the same absorption would have occurred in a homogeneous plasma if $G_o = \frac{32}{15} (cr)^{-1}$. The difference between the homogeneous analysis and absorption in an Airy pattern is that the peak value of the field, A^2 , swells near the critical density. The effective absorption therefore will be given by (5-7), for an averaged value of the peak field. Though sophisticated averages could be performed, a reasonably accurate estimate results from simply averaging the largest and smallest lobes of the Airy pattern. These f-factors are compared with

TABLE 5-2

Percentage Reduction in Inverse Bremsstrahlung
Absorption as A Function of Field Strength

A^2	$\sin(a)$	f
0.0	0.0	1.00
0.0625	0.1429	0.968
0.2500	0.2774	0.910
0.5625	0.3974	0.825
1.0000	0.5000	0.722
2.2500	0.6550	0.530
4.0000	0.7560	0.381
9.0000	0.8666	0.200
16.0000	0.9180	0.15

simulation results in Figure 5-7, for $L = 4.8 \lambda_0$, $\epsilon_e = .5$ KeV, and $\frac{n_e}{M_i} = 0.$, the low amplitude absorption is 12.5% ($Z=1$). This averaging procedure gives astonishingly good agreement.

For comparison purposes, a traveling wave possesses no spatial structure, and a time averaged value of $A^2 = .5(1 - \frac{n_e}{n_{cr}})^{-1/2} (v_{os}/v_{th})^2$. Integrating the absorption over one wavelength gives

$$ABS_2 = \lambda_0 G_0 / (1 + \frac{A^2}{3})^{3/2} \quad (5-9)$$

and a reduction factor of

$$f = (1 + (1 - \frac{n_e}{n_{cr}})^{-1/2} (v_{os}/v_{th})^2 / 6)^{-3/2} \quad (5-10)$$

This f-factor is in very satisfactory agreement with simulations.

The fractional reduction in absorption was observed to be insensitive to density scale length. A series of calculations were performed with $(v_{os}/v_{th})^2 = 1.0$, $\epsilon_e = 0.5$ KeV, and $n_e/M_i = 0.0$. The following scale lengths were calculated: $L/\lambda_0 = 21.0, 16.4, 13.2, 10.3, 8.6, 6.5$, and 4.8 . Though the magnitude of absorption varied by almost a factor of four over these parameters, the f-factor changed by only three percent, i.e. from .43 to .40. Figure 5-8 shows the intensity dependence of absorption in two profiles, $L = 21. \lambda_0$ and $L = 4.8 \lambda_0$. There is clearly little difference in the fractional reductions.

On the basis of the above results, we conclude that the total absorption can be modeled satisfactorily by

$$A = 1 - e^{-K_T f} \quad (5-11)$$

where $K_T = \frac{32}{15} \frac{I_0}{c\tau}$, and f is given by Eq (5-8), with A^2 appropriately averaged over the intensity pattern in the plasma. The significance of this simple theory is that it relates the intensity dependent absorption coefficient to the linear Airy function, suitably modified by factors such as were treated in Chapter 2.

The results of these two sections should be briefly summarized. As expected SBS proved capable of reducing the collisional absorption rate significantly. Self-steepening was difficult to observe in moderate scale length plasmas due to the flux reduction caused by SBS, but in sufficiently steep gradients, absorption reductions on the order of 2 were measured in RDF calculations. There was enough numerical uncertainty in these results that general prescriptions for the effect were impossible to derive. Numerical diffusion associated with the Eulerian algorithm undoubtedly smoothed the profile more than it is realistically, however, so the calculated results probably overestimated the total absorption. There are indications from particle simulations that the real profile at the critical density has a steeper gradient than the RDF calculations.

Finally, inclusion of another effect in the calculations, intensity dependent inverse bremsstrahlung rate, yielded very significant reductions in the absorption at modest field strength. The factor is strong enough to swamp thermal effects for $(v_{os}/v_{th}) \gg 1$. The ramifications to the problem of cold plasma heating may be enormous. It is conventionally assumed that a cold dense plasma absorbs laser radiation so efficiently that it rapidly heats to several hundred eV, after which the plasma begins to thermally bleach. If the rise time of the laser is sufficiently fast, however, $(v_{os}/v_{th})^2$ can be very large number. Clearly, (5-2) then predicts that the plasma temperature is irrelevant, and that the collisional absorption is weak. Only if the laser intensity is increased slowly enough so that $(v_{os}/v_{th}) \leq 1$ will the conventional heating history be valid. Of course, this does not imply that the plasma will remain unheated. With such a large value of (v_{os}/v_{th}) and a relatively weak collisional absorption rate, parametric instabilities may well fulfill the role of heating the plasma up to a few hundred electron volts. Shocking up of the critical surface could then spoil these processes for the duration of the laser pulse, but the turbulent heating is rapid enough that they would have already served a purpose. In any case, self-consistent interaction of many processes could well depend on the initial mode of heating.

In these sections, various processes were discussed which tended to decrease the inverse bremsstrahlung efficiency. The next section deals with a new method for increasing the overall efficiency.

SECTION III

Enhancement of Laser Absorption with Short
Wavelength Ion Fluctuations

Most collective absorption processes are limited to the immediate vicinity of the critical surface. Self-steepening, cavitons⁴⁷, and bubbles, however, not only inhibit collisional absorption, they may gradient stabilize the collective processes. Uncertainty about the critical surface structure, therefore, has lead to renewed interest in mechanisms capable of absorbing the energy over a broad density range. Collisional or inverse bremsstrahlung is one such mechanism, but in the previous section, it was shown how its efficiency can be severely degraded by self-consistent density or intensity effects. In this section, short wavelength ion fluctuations will be examined, and shown to result in significant enhancements over collisional absorption.

Dawson, Oberman, and Ron⁴⁸⁻⁵⁰ showed that finite density correlations can enhance the absorptivity of a plasma. Collisional absorption consists of the scattering of oscillating electrons from massive ions. The Dawson-Oberman theory predicts enhanced absorption when electrons are scattered off ion waves. Consider a spectrum of ion fluctuations such that $n_s(x) = \sum_k n_k e^{ikx}$. Transposition of the electrons from their equilibrium positions by an external electric field will result in a finite electrostatic field in the plasma. Fourier transformation of the continuity equation and Poisson's equation, for a uniform external field, gives

$$n_s(\omega) = -i \frac{\bar{V}}{\omega} \cdot \nabla n \quad (5-12)$$

$$\bar{k} \cdot \epsilon_L(k, \omega) \bar{E}_{k, \omega} = -4\pi e n_s(k, \omega) \quad (5-13)$$

where the oscillating electron velocity is given by

$$V_{osc} = -eE_{laser}/m\omega \cos(\omega t) \quad (5-14)$$

where $\epsilon_L(k, \omega)$ is the dielectric constant function. The average power dissipated is just given by $\bar{J} \cdot \bar{E}$. By comparing this with the laser field energy, one can then define an effective collision frequency. The details are included in Appendix D. A given density fluctuation spectrum at an angle θ_k to the laser E-field,

$$v^* = \frac{\omega}{2} \left(\frac{\Delta n}{n_{cr}} \right)^2 \sum_k \frac{\text{Im}(\epsilon_L)}{\epsilon_L^2} \cos^2 \theta_k \quad (5-15)$$

where $\Delta n \equiv \left(\sum_k n_k^2 \right)^{1/2}$. Figure 5-10 shows the qualitative effect of spectral width for $k\lambda_D = .5$, when $\Delta n = .07$.

The above discussion tacitly assumed that the ion fluctuations would be distributed over the entire density profile. Since the actual distribution depends on the mechanism which creates the waves, some discussion of possible wave sources is needed. One of the most attractive, and simultaneously most damning, characteristics of short wavelength ion enhanced absorption is that the waves themselves are not a

* The fluctuation spectrum must be "flat-topped" for this form to be true.

consequence of the absorption. They are secondary laser plasma effects, created as a byproduct of more fundamental interactions. Within the category of "more fundamental" one should include free expansion and conduction of energy into the overdense plasma. As the following will show, both of these can become ion wave sources, though it is not clear that either will be useful in this context. High energy ions, cavitons, parametric instabilities, and streaming across self-generated magnetic fields are all more esoteric mechanisms, but under given conditions, these could be realistic sources.

First, conduction of heated electrons into an overdense plasma must be balanced by a cold return current. This can be unstable with respect to an ion-electron streaming instability. The associated ion acoustic waves can be broad in angle and at levels of $\left(\frac{\delta n}{n_0}\right)^2 \sim 10\text{--}30\%$, commensurate with trapping saturation. Unfortunately, simulations of this process indicate it occurs at considerably greater than critical density, i.e. $n \simeq 5 \times 10^{21} - 10^{22} \text{ cm}^{-3}$ for $n_{\text{cr}} = 10^{21} \text{ cm}^{-3}$. The density fluctuations must be in the underdense plasma before they can contribute to the absorption. Let us consider an example to test the plausibility of this.

Suppose $\lambda_0 = 1 \text{ }\mu\text{m}$, $L = 20 \text{ }\mu\text{m}$, $n \propto r^{-3}$, $\theta_e = 2 \text{ KeV}$, $\theta_e/\theta_i = 10$, and that the waves are generated at $n = 5 \times 10^{21} \text{ cm}^{-3}$. The wave source is therefore somewhat more than $10 \text{ }\mu\text{m}$ from the critical surface. If a hydrogenic plasma is assumed, the ion acoustic speed is $c_s \simeq .0015 c$, and the time for such a wave to propagate to the critical surface is $\Delta t = 2.10^{-3} c_s$. Even a 10:1 temperature ratio will result in

huge Landau damping in this time, however. For any vestige of the original wave to reach the critical surface, the wave source must be within about 5 μm of that point. Aside from casting doubt on the return current streaming instability as an appropriate ion wave source, this exercise also indicates that ion waves are unlikely to propagate far from their generation location.

The last point is crucial in evaluating plasma expansion as a fluctuation source. In an electron-ion plasma, the more mobile electrons set up an ambipolar potential which drags the ions along. If two ion species are present, each will receive an energy $Z_i e\phi$ from the potential. A velocity differential can thus be established

$$\Delta V = \sqrt{\frac{2Z_1 e\phi}{M_1}} \left\{ 1 - \sqrt{\frac{Z_2 M_1}{Z_1 M_2}} \right\} \quad (5-16)$$

where $M_1 \leq M_2$. These two "beams" will equilibrate in a time⁵¹

$$\tau_{ii} \approx \frac{M_1^2 (\Delta V)^3}{4\pi n_2 e n_1 Z_1^2 \left(2 + \frac{M_1}{n_2} \right)} \quad (5-17)$$

If $\tau_{ii} \ll \frac{V_1 + V_2}{2L}$, the beams will be collisionally tied together; that is, they will not expand at different speeds. If $\Delta V > V_{thi}$, however, an ion-ion streaming instability can occur. Although simulations show that the expansion potential can drive such an instability in a collisionless plasma, the process occurs primarily in the low density region of the blow-off. Unless the ion waves created in the

low density region can propagate closer to the critical surface, they are unlikely to be useful in laser absorption.

Although the potential accompanying the expansion may not be useful for these purposes, the one found near the critical surface due to either self-steepening or resonant absorption may be ideal. In the first place, this potential can be much stronger than the ambipolar one. Second, it is ideally located, for instabilities which can generate the necessary fluctuations are already at the desired densities. Finally, streaming instabilities which originate near the critical density, driven by an outward propagating beam, could conceivably generate short wavelength fluctuations throughout the profile. This mechanism has already been observed in the microwave experiments of Wong, et al.⁵², in which fluctuations of $\frac{\delta n}{n_0} = 5\%$ were seen. Since it was the fast ion bunch which generated the waves, and since such bunches are often measured in high power laser/plasma experiments, it is at least plausible that short wavelength ion waves are present in such experiments. Other mechanisms, such as streaming across self-generated magnetic fields, could also be discussed, but the above discussion is suggestive enough to show how ion waves may be created in laser fusion plasmas.

Once the ion spectra exists, it is proper to inquire how it affects absorption. The magnitude of v^* is 2-5 times greater than the collisional damping rate in the low intensity limit, depending on the electron temperature. As Figure 5-11 shows, it possesses a significant increment over a wide density range, down to roughly

$.5 n_{cr}$ for $k\lambda_D \approx .5$. The exact magnitude, however, can vary widely in a real experiment, due to the cosine squared factor in Eq (5-14) which depends on the angle between the ion waves and the laser electric field vector. For lack of accurate distribution information, we have averaged over a presumed isotropic fluctuation spectrum. This reduces the magnitude by a factor of between 1.5 and 3, depending on the laser polarization.

Using an angle averaged reduction factor of .5, Eq 5-14 was evaluated for a 20 wavelength linear density profile, and included in a series of RDF calculations. Collisional absorption was also included. The absorption from this classical effect alone was eighteen to nineteen percent for $\epsilon_e = 1$ KeV. With a uniform distribution of ten percent density fluctuations, the absorption rose to over ninety percent, for this scale length. The large enhancement is due to a modest increase in the local dissipation rate, but extended over many wavelengths.

Intensity dependence was included only through SBS. The stimulated scattering, however, reduced the classical absorption to seven to eight percent for $L = 20 \lambda_0$ and $(v_{os}/v_{th})^2 = .50$. Since Brillouin scattering is representative of scattering type instabilities and because its behavior is moderately well understood in one dimension, it is illustrative to consider its competition with the enhanced absorption. The extended character of the fluctuation induced mechanism is in contrast to other absorptive processes, such as those near the critical density. Brillouin scattering occurs at lower density

than those processes, and so evolves independently of them. For density fluctuation enhancements in the absorption, a large density overlap exists with SBS. The effect of the latter can therefore be evaluated by performing the self-consistent calculations. For a $20 \lambda_o$ scale length, the intensity dependence on absorption is indicated by Figure 5-12, for $\frac{\delta n}{n_{cr}} = 7\%$. The calculations were extended only up to $(v_{os}/v_{th})^2 = 1.0$, for which very large SBS would be expected in these conditions. Even at this field level, however, almost thirty-five percent absorption occurred.

Another series of calculations examined the effect of the magnitude of fluctuations. Figure 5-13 shows the absorption as a function of $\frac{\delta n}{n_{cr}}$ for a $20 \lambda_o$ fixed density profile. The absorption due to collisions alone was again eighteen percent. When the profile was allowed to relax, i.e. finite ion mass ($m_e/M_i = .01$), Brillouin scattering was again possible. In the classical case this reduced absorptivity to less than eight percent. In Figure 5-13, though, the absorption can be seen to remain well above this level, even for $\frac{\delta n}{n_{cr}} \div 3\%$. The fractional reduction is also reduced from the collisional case, as comparison with Figure 5-1 shows.

Finally, the fact that SBS scales roughly as $(v_{os}/v_{th})^2 L/\lambda_o$ indicates that scale length effects may cancel when the enhanced absorption mechanism is competing with stimulated scattering. Although conclusive results have not been obtained, the scale length dependence of absorption in a series of simulations with $\epsilon_e = 1$ KeV, $(v_{os}/v_{th})^2 = 0.5$, $m_e/M_i = .01$, and $\frac{\delta n}{n_{cr}} = .07$ at $k\lambda_D = .5$ indicate

relative insensitivity to the length. Figure 5-14 shows these results. Since actual length effects are expected to be more complicated than the one dimensional calculations, it is not clear that Figure 5-14 has practical applications. Two dimensional simulations, RDF or otherwise, are more appropriate for addressing such problems.

The RDF simulations overall show that modest short wavelength ion fluctuation levels can result in twenty to fifty percent absorption efficiency. Once the fluctuations are created, they give absorption which is relatively resistant to self-steepening or SBS depletions. One dimensional electrostatic particle simulations with a fixed density spectrum, moreover, confirm the theoretical dissipation magnitude, Eq (5-14), and yield a "soft" heated electron distribution. Suprathermal tails are conspicuously absent. There are still many uncertainties in applying this mechanism to actual experiments. Neither the angular nor spatial distribution of density fluctuations can be estimated accurately. Even the spectrum and magnitude are only crudely estimable. The insensitivity to density modifications, softness of the heated electron spectrum, and relatively robust level of absorption, however, make fluctuation induced absorption very attractive for laser fusion. The calculations discussed in this section were based on reasonable estimates of the fluctuation distribution. The promise of these calculations is sufficiently great that further studies are strongly suggested. Two- or three dimensional studies involving oblique and focused beams in arbitrary fluctuation

distributions, and two dimensional particles simulations to determine realistic distributions, seem logical extensions of the present work.

FIGURE 5-1 Collisional Absorption as a function of vacuum field strength. $m_e/M_1=.01$, $\theta_e=0.5$ KeV, $L=7.5 \lambda_0$, $\nu=.0025 \omega_c$.

Figure 5-2 Absorption as a function of time for various laser field strengths, same parameters as in Figure 5-1.

FIGURE 5-3. Absorption histories showing the effect of self-steepening for $\nu=0.0002$, ω_0 ; $m_e/M_i=0.01$, $\theta_e=0.5$ KeV, $I=7.5 \times 10^9$, $(V_{os}/V_{th})=.1, .3, .4, .5, .6$.

FIGURE 5-4 Absorption histories showing the effect of self-steepening for $\alpha = .0025$; $m/M_i = .01$, $\theta = 0.5$ KeV, $(V_0/V_{th}) = 0.0, 0.5, 0.7, 0.9$. The combination of steep gradients and relatively large V_0/ω_0 effectively suppressed SBS in these simulations.

Figure 5-5 Absorption as a function of time, $m_e/M_e = 0$, $\theta = 0.5$ KeV, $L = 20 \lambda_D$, $(V_{os}/V_{th})^2 = 0, .25, 1.00, 4.00$.

Figure 5-6 Absorption as a function of field strength, $m_e/m_i=0.0$, $\theta_e=0.5$ KeV, $L=20\lambda_0$, $Z=1.0$.

Figure 5-7 Absorption fraction, f , as a function of field strength, $m/M_i=0.0$, $\theta=0.5$ KeV, $L=4.8 \lambda$. For purposes of comparison, the solid line is Eq(5-8).

Figure 5-8 Absorption fraction, f , as a function of field strength for different scale lengths, $L = 21.0 \lambda$ (●) and $L = 4.8 \lambda$ (Δ), $m/M_i = 0.0$, $\theta_0 = 0.5$ KeV. Dashed line represents $f = (1 + 2(eV_{os}/V_{th})^2/3)^{-1/2}$.

Figure 5-9 Density fluctuation induced absorption as a function of wavelength of the fluctuation, $\Delta n/n_{cr} = .07$, $m_e/M_i = 0.0$, $\theta_e = 50$ KeV, $L = 21.0 \lambda_0$.

Figure 5-10 Density fluctuation induced absorption rate, v^* as a function of position, same density profile as in Figure 5-11(a), $k\lambda_D = 0.5$, $\Delta k\lambda_D = 0.5$, $\Delta n/n_{cr} = .07$.

Figure 5-11(a) Density profile used for computing the density fluctuation induced absorption rate, ν , in Figures 5-11(b)-(d). Fluctuations are assumed to be uniformly distributed over the entire profile, $L=21.0 \lambda_0$.

Figure 5-11(b) Density fluctuation induced absorption rate, γ^*
 $k\lambda_D = 0.3$, $\Delta n/n_{cr} = .07$.

Figure 5-11(c) Density fluctuation induced absorption rate, v^*
 $k\lambda_D = 0.5$, $\Delta n/n_{cr} = .07$.

Figure 5-11(d) Density fluctuation induced absorption rate, ν^*
 $k\lambda_D = 0.8$, $\Delta n/n_{cr} = .07$.

Figure 5-12 Absorption as a function of field strength, $\theta = 1.0$ KeV, $L = 21 \lambda_0$, $k\lambda_D = 0.5$, $\Delta n/n_{cr} = 0.07$, $m_e/M_i = 0.01$, $v_i = 0.0025 \omega_0$.

Figure 5-13. Absorption coefficient as a function of the magnitude of the density fluctuations; $\theta_e = 50^\circ$, $k_p = 0.5$, $l = 21$, $m/M_0 = 0.0$.

Figure 5-14. Absorption as a function of path length. $\theta \approx 1.0$ KeV, $n/n_c = 0.1$, $\gamma_{os}/V_{th} = 0.25$.

CHAPTER 6

Conclusions and Recommendations

The numerical method described in this work has been demonstrated to be a fast, flexible tool for examining a variety of laser plasma problems. The technique of solving the two fluid plasma equations on the plasma time scale, coupled self-consistently with Maxwell's equations, has proved useful to studying scattering, modulational, and absorptive phenomena. Although the conclusions reached in the course of this work have potential applications to laser fusion, particularly in the areas of understanding "bubble" formation and self-consistent inverse-bremsstrahlung absorption, the primary goal was to illustrate the utility of the reduced description formalism. This has hopefully been accomplished.

Several limitations and flaws in the RDT technique have been discovered. The marginal success in studying ponderomotive steepening near the critical density aptly showed the diffusiveness of the numerical algorithm. Although the equations were almost completely time and space centered, the effective cell size was large enough to give considerable Eulerian smoothing. This may have contributed to the general stability of the scheme, but a higher order algorithm, possibly Lax-Wendroff, coupled with more sophisticated dissipation expressions will probably lead to more accurate results. In this vein, the capability of following longitudinal electric fields was demonstrated but not exploited in this study, because of noise related problems. A

dissipation expression for the electron fluid which more closely models Landau damping should open completely new classes of problems for RDF investigation.

The RDE technique can also be extended by including higher order moment equations and transport equations appropriate to other quantities. A temperature equation has been implemented into the scheme already, without excessive difficulty. Although the thermal conduction was usually high enough to justify the isothermal assumption, problems involving self-consistent heating need to treat the energy balance properly. Also, dense, high temperature plasmas are well-known emitters of high frequency radiation, such as X-rays. This radiation requires a much shorter time scale than the laser, and so is not calculated in the field equations. Various techniques for transporting such radiation are known, however, and one of these might be profitably incorporated into the RDE equations.

Finally, higher dimensional effects clearly are important to understand. Extension of the technique from one to two dimensions is not trivial, but since Estabrook has already written such a code, it apparently is feasible. Although such a numerical tool would not be as inexpensive to run as the present formalism, it would still be a useful addition to the repertoire of techniques needed to study laser fusion problems.

In summary, a flexible one dimensional numerical tool has been written and used to study stimulated scattering, electromagnetic density profile modulation, self-consistent inverse bremsstrahlung

absorption, short wavelength ion density induced absorption, steepening near the critical density, and electromagnetic field structure of TE waves near the critical density. These were studied merely as representative types of problems for which the technique can be useful. Extensions of the code have furthermore been suggested which may increase the number of applications even more. The utility of the technique, and its potential for future use, have thus been amply demonstrated.

APPENDIX A

The Reduced Description Equations

Derivation of the reduced description for plasma is straightforward. The complete description is the Liouville equation for the N -particle distribution function,

$$\frac{df^N(\bar{x}_N, \bar{v}_N, t)}{dt} = 0 \quad (A1)$$

Integration over all but one of the particles leads to an appropriate one particle distribution,

$$\frac{\partial f}{\partial t} + \bar{v} \cdot \frac{\partial f}{\partial \bar{x}} + \frac{\bar{F}}{m} \cdot \frac{\partial f}{\partial \bar{v}} = \left(\frac{\partial f}{\partial t} \right)_{coll}$$

where $f=f(\bar{x}, \bar{v}, t)$, $\bar{F}$ is the force at the phase space point $(\bar{x}, \bar{v})$ at time t , and $\left(\frac{\partial f}{\partial t} \right)_{coll}$ represents the effect of multiple particle interactions on f . This level of description can be further reduced by integrating over all velocities, $\bar{v}$. Therefore, instead of solving (A2) at $\left(\frac{\bar{x}}{\Delta x} \right) \left(\frac{\bar{v}}{\Delta v} \right)$ points, one has a set of quantities, for which solutions at only $\left(\frac{\bar{x}}{\Delta x} \right)$ points must be found. The quantities are formed by taking velocity moments of (A2). All such moments are of the form $M = \int g(\bar{x}, \bar{v}, t) d^3\bar{v}$, where $g=1$ gives $M=n(\bar{x}, t)$, the number density, $g=v$ gives $M=n\bar{u}$, the number flux, $g=v^2$ leads to $M=n\theta$, the thermal energy density, and so forth. Explicitly,

$$\frac{\partial}{\partial t} \int \bar{v} g f d^3\bar{v} + \frac{\partial}{\partial \bar{x}} \cdot \int \bar{v} \bar{v} g f d^3\bar{v} + \int \bar{v} \frac{\bar{F}}{m} \cdot \frac{\partial f}{\partial \bar{v}} d^3\bar{v} = \int \bar{v} \left(\frac{\partial f}{\partial t} \right)_{coll} d^3\bar{v} \quad (A3)$$

where $\int \bar{v} g \frac{\partial f}{\partial \bar{v}} d^3\bar{v} = \frac{\partial}{\partial \bar{x}} \cdot \int \bar{v} \bar{v} g f d^3\bar{v}$ has been used. Unfortunately, the second integral in (A3) is a higher order moment than $\int \bar{v} g f d^3\bar{v}$. Thus, each moment equation contains the divergence of a higher order moment. Self-consistent solutions therefore involve an infinite sequence of equations. To be useful, this sequence must be truncated, on physical grounds.

The reduced description employed here consisted of the first two moment equations for ions and the first two for electrons. The first moment, $g=1$, is just the continuity equation, (specie notation has been suppressed)

$$\frac{\partial n}{\partial t} = -\frac{\partial}{\partial x} \cdot n\bar{u} + S \quad (A4)$$

where S indicates either a source or sink for the specie, i.e. ionization or recombination. Usually, we took $S=0$.

The second moment equation, $g=\bar{v}$, is a momentum transfer equation,

$$\frac{\partial n\langle\bar{v}\rangle}{\partial t} + \frac{\partial}{\partial x} \cdot n\langle\bar{v}\bar{v}\rangle = -\frac{n}{m}\langle F\rangle + \langle\bar{v}\frac{\partial f}{\partial t}|_{coll}\rangle \quad (A5)$$

If we separate $\bar{v}=u+\bar{v}$, where $u=\langle\bar{v}\rangle$ and $\langle\bar{v}\rangle=0$, the divergence term separates into a convective and a pressure expression

$$\frac{\partial n\bar{u}}{\partial t} = -\frac{\partial}{\partial x} \cdot n\bar{u}\bar{u} - \frac{1}{m}\frac{\partial}{\partial x} \cdot \bar{P} - \frac{n}{m}\langle F\rangle + \langle\bar{v}\frac{\partial f}{\partial t}|_{coll}\rangle \quad (A6)$$

The averaged force is derived from Maxwell's equations. It is convenient to write the forces in terms of potentials

$$\langle F\rangle = q\left(-\frac{\partial\phi}{\partial x} - \frac{1}{c}\frac{\partial\bar{A}}{\partial t} + \frac{\bar{u}}{c} \times \left(\frac{\partial}{\partial x} \times \bar{r}\right)\right) \quad (A7)$$

Observation that

$$\bar{u}_\perp \doteq -\frac{e\bar{A}_\perp}{m_e c}$$

leads to

$$\langle F_1\rangle = -q\frac{\partial\phi}{\partial x} + \frac{1}{2}\frac{qe}{m_e c^2}\frac{\partial A^2}{\partial x}$$

the important dissipation term is rigorously determined by the collisional physics, but we used a simple relaxation expression

$$\langle\bar{v}\frac{\partial f}{\partial t}|_{coll}\rangle = -\nu n\bar{u} \quad (A8)$$

the stability characteristics of the algorithm, however, can be strongly influenced by the form used, so a little discussion about dissipation expressions is required at this point.

The effect of a constant relaxation rate on an equation like (A6) can be analyzed simply. First, it is clear that flux perturbations are exponentially damped

$$\bar{n} \propto e^{-\nu t} \quad (\text{A9a})$$

Operating on (A4) with $\frac{\partial}{\partial t}$ and on (A6) with $\frac{\partial}{\partial t}$ leads to

$$\frac{\partial n}{\partial t} \propto -\nu n \quad (\text{A9b})$$

In other words, all density fluctuations are damped as

$$n \propto e^{-\nu t}$$

This is probably adequate for ion waves, since neither ion Landau damping nor ion collisions exhibit strong wavelength dependence. Electron fluctuations, though, exhibit a pronounced damping at short wavelengths. Eq (A8) is, therefore, inadequate for electron fluids. Marginal stability was achieved by letting $\frac{\nu}{\omega_p} \gg 1$, but short scalelength noise was still present. For electrons, two alternative forms may lead to improvement.

Viscosity is a well understood fluid dissipation mechanism.

By analogy with viscosity, it is plausible to take

$$\frac{\partial \bar{n}}{\partial t} \propto D \frac{\partial^2}{\partial x^2} \bar{n} \quad (\text{A10})$$

This also leads to

$$\frac{\partial n}{\partial t} \propto D \frac{\partial^2}{\partial x^2} n$$

Thus this type of damping yields a diffusive smoothing on both flux, $\bar{n}$, and on density fluctuations. It has the desirable property that short wavelengths are more damped than long wavelength ones

$$n \propto e^{-k^2 D t}$$

For realistic results, it is not clear what value of D should be used. The diffusion coefficient must therefore be chosen on

be used. The diffusion coefficient must therefore be chosen on physical grounds to give a desired damping magnitude at a particular wavelength.

A second commonly used form for dissipating gradients in the electron fluid is the "von Neumann artificial viscosity"

$$\frac{\partial n \bar{u}}{\partial t} \propto Q \frac{\partial u^2}{\partial x} \quad (A11)$$

where Q is a constant of order unity. This form is difficult to analyze because of its intrinsic nonlinearity. It has proved highly successful, however, in the calculation of shock phenomena. For treating waves, (A11) is not as useful as (A10). Other expressions besides these two could doubtless be devised, but these examples were felt to be illustrative of the problems in computing electron damping.

Returning now to the full momentum transfer equation, (A5), Eqs. (A6), (A7), and (A8), plus an equation of state, can be combined to yield

$$\frac{\partial n \bar{u}}{\partial t} = -\frac{\partial}{\partial x} n \bar{u} \bar{u} - \frac{\partial}{\partial x} \frac{n}{m} + \frac{q n}{m} \frac{\partial \phi}{\partial x} - \frac{1}{2} \frac{q e n}{m c^2} \frac{\partial A^2}{\partial x} - \nu n \bar{u} \quad (A12)$$

The equation of state used in (A12) and in the actual algorithm was an isothermal one, i.e. $\bar{P} \rightarrow P = n \theta$, $\theta = k_B T$. Eqs. (A4) and (A12) for each specie comprise the equation set used to describe the plasma.

The equations for the potentials, ϕ and A , are in one dimension simply

$$\frac{\partial^2 \phi}{\partial x^2} = -4\pi e (n_i - n_e) \quad (A13)$$

$$\frac{\partial^2 A}{\partial t^2} + c^2 \frac{\partial^2 A}{\partial x^2} - \omega_{ce}^2 \frac{n}{n_{cr}} A = 0 \quad (A14)$$

Poisson's equation, (A13), couples the ion and electron fluids together, while (A14) couples the electromagnetic field to

together, while (A14) couples the electromagnetic field to the electron fluid. Note that the gauge condition for (A13) and (A14), $\nabla \cdot \bar{A} = 0$, is trivially satisfied in slab geometry.

Finally, the number of equations could be even further reduced by assuming the the electrons were inertialess, that is $\frac{d\bar{u}}{dt} = 0$. Then it follows that

$$0 = -\frac{e_0}{m_e} \frac{\partial \bar{u}}{\partial x} - \frac{en}{m_e} \frac{\partial \bar{u}}{\partial x} - \frac{1}{2} \left(\frac{e}{mc} \right)^2 n \frac{\partial A^2}{\partial x}$$

or

$$en \frac{\partial \bar{u}}{\partial x} = -e_0 \frac{\partial n}{\partial x} - \frac{1}{2} \frac{e^2}{mc^2} n \frac{\partial A^2}{\partial x} \quad (A15)$$

(A15) can then be substituted into the ion momentum equation

$$\frac{\partial(nu)_i}{\partial t} = -\frac{\partial}{\partial x} \left(\frac{n u_i^2}{n_i} \right) - \frac{e_0 + e_i}{m_i} \frac{\partial n_i}{\partial x} - \frac{n_i n_i (e_i)^2}{m_i^2 (n_i c)^2} \frac{\partial A^2}{\partial x} - \nu(nu)_i \quad (A16)$$

where we have assumed that the electron and ion gradients are virtually identical. Now we proceed to the differencing of the equations, (A4), (A14), and (A16).

For convenience, the quantities are normalized so that $c=1$, $\omega_0=1$, and $n_{cr}=1$. The vector potential is normalized to its vacuum magnitude, A_0 , and the field strength parameter $V_{os}/V_{th} = eA_0/(mc^2 \theta_e)^{1/2}$, is represented as Y below. Also let $s_1 = \frac{\Delta t}{\Delta x}$, $s_2 = \frac{\Delta t}{(\Delta x)^2}$, $s_3 = (1 + 5\nu \Delta t)$, $s_4 = (1 - 5\nu \Delta t)$, and s_5 and s_6 like s_3 and s_4 except that ν_e appears in place of ν .

$$n_j^i = n_j^{i-1} - .5 s_1 (g_{j+1}^{i-k_2} - g_{j-1}^{i-k_2}) \quad (A17)$$

$$g_j^{i+k_2*} = g_j^{i-k_2} - .5 s_1 c_s^2 (n_{j+1}^i - n_{j-1}^i) - .25 s_2 \frac{m_e}{m_i} Y (A_{j+1}^{2i} - A_{j-1}^{2i}) \quad (A18)$$

$$g_j^{i+k_2} = g_j^{i+k_2*} - .5 s_1 ([g_{j+1}^{i+k_2*}]^2 / n_{j+1}^i - [g_{j-1}^{i+k_2*}]^2 / n_{j-1}^i) \quad (A19)$$

$$A_j^{i+k_2} = A_j^{i-k_2} \frac{s_3}{s_5} - \frac{\Delta t}{s_5} n_j^i A_j^i + \frac{s_4}{s_5} (A_{j+1}^i + A_{j-1}^i - 2A_j^i) \quad (A20)$$

APPENDIX B

Linear Theory of the Stimulated Brillouin/Zero-Frequency Instability

This appendix is devoted to a derivation and analysis of the linear theory of the instabilities discussed in Chapters 3 and 4. The derivation is not in itself original, and in fact follows the formalism of Nishikawa⁵⁴, i.e. the instability of two coupled harmonic oscillators driven by a large amplitude pump wave. To the reader familiar with Ref. 54, the notation may lead to confusion. The symbols used are similar enough to provoke recognition, but unfortunately they do not always have the same connotations. The high frequency waves, for instance, are denoted as wave 1, and the low frequency as wave 2. This is precisely opposite to the notation used by Nishikawa. The reader will hopefully keep this in mind while going through this section. An apology is perhaps also due for the detail included here. Without going through every step, it is hoped that the unfamiliar reader will be able to follow the development more easily because of this strategem.

These instabilities involve the coupling of an electromagnetic wave with a secondary electromagnetic wave and an ion density fluctuation. The wave equation for the electromagnetic wave is directly obtainable from Maxwell's equations written in terms of potentials, in the transverse gauge, $\nabla \cdot \bar{A} = 0$. The equation for $\bar{A}$ is the only one needed in this case;

$$\frac{\partial^2 \bar{A}}{\partial t^2} - c^2 \nabla^2 \bar{A} = 4\pi \bar{J} \quad (B1)$$

The source term $\bar{J}$, in the absence of transverse gradients can

be accurately expressed as $J = n_0 e^2 A / m_e c$ in a homogeneous, nonrelativistic plasma. (Vector notation will henceforth be suppressed.) The by Fourier analyzing (B1), one obtains the following linearized equation

$$\frac{\partial^2 A}{\partial t^2} k_1 + (\omega_p^2 + k_1^2 c^2) A_{k_1} + 2\Gamma_1 \frac{\partial A}{\partial t} k_1 = -\frac{\omega_p^2}{n_0} n_{k_1 - k_0}^* A_{k_0} \quad (B2)$$

where $\omega_p^2 = 4\pi e^2 n_0 / m_e$ is the local plasma frequency. We have taken the instantaneous density to be of the form $n(\vec{r}) = n_0 + n(\vec{r})$, and added a phenomenological damping term proportional to $2\Gamma_1$.

The equation for the density fluctuation is derived from the RDF equations in Appendix C, and so is not duplicated here. After Fourier analyzing the ion wave equation, the following results

$$\frac{\partial^2 n}{\partial t^2} k_2 + \frac{k_2 \Gamma}{M_i} e k_2 n_{k_2} + 2\Gamma_2 \frac{\partial n}{\partial t} k_2 = -\frac{1}{2} (e/m_e c)^2 (m_e/M_i) k_2^2 A_{k_2 - k_0}^* A_{k_0} \quad (B3)$$

Equations (B2) and (B3) describe the coupled harmonic oscillators.

In Nishikawa's analysis, a standing wave pump was explicitly assumed

$$A_{k_0}(\omega_0) = 2A_{k_0} \cos(\omega_0 t).$$

This is somewhat less general than it need be, however. We will instead assume

$$A_{k_0}(\omega_0) = \frac{1}{2} A_0 \{ e^{ik_0 x - i\omega_0 t} + e^{-ik_0 x + i\omega_0 t} + \alpha e^{ik_0 x + i\omega_0 t} + \alpha e^{-ik_0 x - i\omega_0 t} \} \quad (B4)$$

For $\alpha=1$, the pump reduces to a standing wave form, while for $\alpha=0$, a traveling wave pump results. The rationale for this form is that in a laser fusion plasma, the incoming laser signal will propagate into the critical density and be reflected. The fraction of incident intensity that arrives at a given

point will depend on the amount of dissipation which has occurred at higher densities. It can always be expressed as a mixture of traveling and standing waves, such that $0 \leq \alpha \leq 1$.

We now assume that the waves have the temporal dependence $A_{k_1} \sim e^{i\omega_1 t}$ and $n_{k_2} \sim e^{i\omega_2 t}$. Eqs(B2) and (B3) then become

$$(-\omega_1^2 - 2i\omega_1 \Gamma_1 + \omega_{k_1}^2) A_{k_1}(\omega_1) = -\frac{\omega_p^2}{n_0} \{ A_{k_0}(\omega_0) n_{k_1-k_0}^*(\omega_1 - \omega_0) + \alpha A_{k_0}(-\omega_0) n_{k_1-k_0}^*(\omega_1 + \omega_0) \} \quad (B5a)$$

$$(-\omega_2^2 - 2i\omega_2 \Gamma_2 + \omega_{k_2}^2) n_{k_2}(\omega_2) = -\frac{1}{2} n_0 \left(\frac{e}{m_e c} \right)^2 \left(\frac{m_e}{M_i} \right) k_2^2 \{ A_{k_0}(\omega_0) A_{k_2-k_0}^*(\omega_2 - \omega_0) + \alpha A_{k_0}(-\omega_0) A_{k_2-k_0}^*(\omega_2 + \omega_0) \} \quad (B5b)$$

On the right hand side of (B5a), the frequency dependence must be near ω_{k_2} if resonance with the normal mode is to be attained. This implies two separate matching conditions

$$\begin{aligned} \omega_0 &= -\omega_1 + \omega_2 \\ \omega_0 &= -\omega_1 + \omega_2 \end{aligned} \quad (B6)$$

The dispersion relation is obtained from (B5b) and the two equations arising from (B5a)

$$\begin{vmatrix} -(\omega_0 + \omega_2)^2 + \omega_{k_1}^2 - 2i\Gamma_1(-\omega_0 + \omega_2) & 0 & \lambda \\ 0 & -(\omega_0 + \omega_2)^2 + \omega_{k_1}^2 - 2i\Gamma_1(\omega_0 + \omega_2) & \alpha \lambda \\ \nu & \alpha \nu & -\omega_2^2 + \omega_{k_2}^2 - 2i\omega_2 \Gamma_2 \end{vmatrix} \begin{vmatrix} A_{k_1}(-\omega_0 + \omega_2) \\ A_{k_1}(\omega_0 + \omega_2) \\ n_{k_2}(\omega_2) \end{vmatrix} = 0 \quad (B7)$$

where $\lambda = \omega_p^2 A_{k_0} / n_0$ and $\nu = \frac{1}{2} n_0 (e/m_e c)^2 (m_e/M_i) k_2^2 A_{k_0}^2$. Since the electromagnetic wave frequency, ω_0 , is much greater than the ion frequency, we can use the approximations,

$$\begin{aligned} -(\omega_0 + \omega_2)^2 + \omega_{k_1}^2 - 2i(-\omega_0 + \omega_2)\Gamma_1 &\approx 2\omega_0(\omega_{k_1} - \omega_0 - \omega_2 - i\Gamma_1) \\ -(\omega_0 + \omega_2)^2 + \omega_{k_1}^2 - 2i(\omega_0 + \omega_2)\Gamma_1 &\approx 2\omega_0(\omega_{k_1} - \omega_0 - \omega_2 - i\Gamma_1) \end{aligned}$$

to arrive at the dispersion relation

$$4\omega_0^2(\omega_2 + i\Gamma_1 - \delta)(-\delta_2 - i\Gamma_1 - \delta)(-\omega_2^2 + \omega_{k_2}^2 - 2i\omega_2 \Gamma_2) - 2\omega_0 \lambda \nu (-\omega_2 - i\Gamma_1 - \delta) - 2\omega_0 \alpha^2 \lambda \nu (\omega_2 + i\Gamma_1 - \delta) = 0. \quad (B8)$$

In this equation $\delta = \omega_0 - \omega_{k_1}$. Eq (B8) can be rewritten in the more useful form

$$(\omega_2^2 - (\delta + \Gamma_1^2) + 2i\omega_2\Gamma_1)(\omega_2^2 - \omega_{k_2}^2 + 2i\omega_2\Gamma_2) + \frac{\lambda v}{2\omega_0} \{ (1 - \alpha^2)(\omega_2 + i\Gamma_1) + \delta(1 + \alpha^2) \} = 0 \quad (B9)$$

There are two cases of primary interest, the decay of the incident wave into normal modes of the system ($\omega_2 = x + iy$), and a forced oscillation in which the secondary electromagnetic wave has the same frequency as the pump and the density fluctuation is purely growing ($\omega_2 = iy$). The former case will be analyzed first.

Case $\omega_2 = x + iy$. First let us examine the instability near threshold. We expect the frequency, x , to be approximately the normal mode of oscillation, the ion acoustic wave. This suggests an expansion $x = \omega_{k_2} + \Delta$, such that $\Delta \ll \omega_{k_2}$. It is convenient to introduce the notation $2B = \omega_{k_2}^2 - (\delta^2 + \Gamma_1^2)$ and $K = \lambda v$. Then (B9) separates into the following real and imaginary components,

$$\Delta = \frac{2Ky - K(1 - \alpha^2)(\Gamma_1 + y)}{8\omega_0 \{ \omega_{k_2}^2 (\Gamma_1 + \Gamma_2) + \Gamma_2 (B - 2\Gamma_1 y) \}} \quad (B12)$$

$$\frac{2Ky - K(1 - \alpha^2)(\Gamma_1 + y)}{2\omega_0 \{ \omega_{k_2}^2 (\Gamma_1 + \Gamma_2) + \Gamma_2 (B - 2\Gamma_1 y) \}} \omega_{k_2} (B - (\Gamma_1 + \Gamma_2)y - 2\Gamma_1\Gamma_2) = \frac{4\Gamma_2 y (B - \Gamma_1 y) + 2\omega_0 K - K(\omega_{k_2}^2 (1 - \alpha^2) + \delta(1 + \alpha^2))}{2\omega_0} \quad (B13)$$

Working to first order in Δ , we may eliminate Δ for it as a function of y in (B11) and then substituting this value into (B10). We obtain

$$4(\omega_{k_2}^2 - \alpha^2 + \omega_{k_2}^2 \Delta (B - (\Gamma_1 + \Gamma_2)y) - \Gamma_2 y (B - \Gamma_1 y) - \Gamma_1 \Gamma_2 (\omega_{k_2}^2 + 2\Gamma_1\Gamma_2)) = -\frac{K}{2\omega_0} \omega_{k_2}^2 (1 - \alpha^2) + \delta(1 + \alpha^2) \quad (B14)$$

$$4\omega_{k_2}^2 \Delta^2 - 2(\omega_{k_2}^2 + 1) + 4(B - \Gamma_1 y) \Gamma_2 (\omega_{k_2}^2 + 1) + 4\Gamma_1 (\omega_{k_2}^2 + 1) (\omega_{k_2}^2 - \Gamma_2 y) + \frac{K}{2\omega_0} (1 - \alpha^2) \Gamma_2 = 0 \quad (B15)$$

Thresholds may be obtained by setting $y=0$ in (B13). For a standing wave pump ($\alpha=1$) a familiar expression results

$$K_{thr}(\alpha=1) = (k_2) K_0 \quad (B14)$$

where $K_0 = \frac{1}{\alpha} k_2 \frac{1}{1+\alpha}$. The corresponding condition for a traveling wave is more complex, however,

$$K_{thr}(\alpha=0) = K_0 \frac{k_2^2 + \delta^2 - \omega_{k_2}^2}{1 + (\frac{k_2}{k_0} (1+\alpha) + B1_2)} \quad (B15)$$

In the limit that the damping on the electromagnetic wave is much smaller than the ion damping or the acoustic frequency, (B15) is virtually the same condition as (B1). In both cases, however, it can be seen the threshold goes to zero if either damping rate goes to zero.

The growth rate near threshold is not a parameter of interest in most cases, since it is by definition close to zero. Instead we will next calculate the growth rate far above threshold. By this we mean that the damping rates are negligible compared to the growth rates or the mismatch terms. Under those conditions, (B9) reduces to

$$(\omega_2^2 - \delta^2)(\omega_2^2 - \omega_{k_2}^2) + \frac{K}{2\omega_0} \{ \omega_2^2 (1 - \alpha^2) + \delta (1 + \alpha^2) \} = 0 \quad (B16)$$

Again letting $\omega_2 = x + iy$, the real and imaginary components are

$$(x^2 - y^2)^2 - (x^2 - y^2)(\delta^2 + \omega_{k_2}^2) + \omega_{k_2}^2 (\delta^2 - 4x^2 y^2) + \frac{K}{2\omega_0} \{ x(1 - \alpha^2) + \delta(1 + \alpha^2) \} = 0 \quad (B17)$$

$$2xy(2(x^2 - y^2) - (\delta^2 + \omega_{k_2}^2)) + \frac{K}{2\omega_0} (1 - \alpha^2) y = 0 \quad (B18)$$

These two equations can be combined to eliminate $(x^2 - y^2)$,

$$(x^2 - y^2) = \frac{1}{2} (\delta^2 + \omega_{k_2}^2) - \frac{K}{8\omega_0 x} (1 - \alpha^2)$$

so that (B17) becomes

$$-\frac{1}{4} (\delta^2 + \omega_{k_2}^2)^2 + \left(\frac{K}{8\omega_0 x} (1 - \alpha^2) \right)^2 - 4x^2 y^2 + \frac{K}{2\omega_0} \{ x(1 - \alpha^2) + \delta(1 + \alpha^2) \} = 0 \quad (B19)$$

It is simple to solve (B19) for a standing wave, using the expression for $(x^2 - y^2)$,

$$y = \left(\frac{1}{2} \sqrt{\frac{K}{\omega_0}} - \frac{(\delta^2 + \omega_{k_2}^2)}{4k_2} \right)^{1/2} \quad (B20a)$$

$$x = \left(\frac{K}{2\sqrt{3}K\omega_o} - \frac{(1^2 + \frac{\omega^2}{k_2^2})^2}{(\frac{\omega}{k_2})^2} \right)^{1/3} \quad (\text{B20b})$$

The maximum growth rate is found from $\frac{dy}{dy} = 0$ in (B20a). The maximum occurs for $\omega_o = 2^{-2/3} (K/\omega_o)^{1/3}$, giving

$$y_{\max} = \left(\frac{3}{8} \left(\frac{K^2}{2\omega_o} \right)^{1/3} - \frac{\omega^2}{4k_2^2} \right)^{1/2}$$

For large pump powers, the growth rate far above threshold scales as $K^{1/3}$. These results agree with Nishikawa's equations, as they should under these conditions.

Nishikawa did not consider the growth rate for a traveling wave pump. Eq (B19) then assumes the approximate form

$$-\frac{1}{4}(\omega^2 + \frac{\omega^2}{k_2^2})^2 + \frac{1}{x^2} \left(\frac{K}{8\omega_o} \right)^2 - 4x^2 y^2 + \frac{K}{2\omega_o} (\omega k_2 + \delta) = 0 \quad (\text{B21})$$

This equation is quadratic in x^2 , with the solution

$$x^2 = \frac{K}{2\omega_o} \left(\frac{(\omega k_2 + \delta) + \sqrt{(\omega k_2 + \delta)^2 + y^2}}{8y^2 - k_2^2} \right) \quad (\text{B22})$$

There are two limits in which x and y are easily obtainable,

(1) $y \gg \omega k_2 + \delta$, and (2) $y \ll \omega k_2 + \delta$. The latter has only limited use since a large growth rate was assumed in deriving (B16). The strong growth analysis, (1), then proceeds as follows.

Assuming $y \gg \omega k_2 + \delta$, (B22) becomes

$$x^2 = \frac{K}{16\omega_o y} \quad (\text{B23})$$

which can then be substituted in the earlier expression for $x^2 - y^2$. The resulting solutions are

$$y = \left(\frac{K}{\omega_o} \left\{ \frac{2 + \sqrt{3}}{16} \right\} \right)^{1/3} \quad (\text{B24a})$$

$$x = \left(\frac{K}{\omega_o} \left\{ \frac{1}{16(2 + \sqrt{3})^2} \right\} \right)^{1/3} \quad (\text{B24b})$$

This solution possesses the same intensity dependence as the standing wave solutions, in the large growth limit, i.e. as

$K^{1/3}$. The difference in the numerical factors is also small,

so that we may conclude that the instability which results from

the coupling of ion density fluctuations with electromagnetic waves is insensitive to the standing or traveling wave nature of the pump, when the growth rates greatly exceed the ion acoustic frequency, ω_{k_2} , or the frequency mismatch, δ , for a normal mode decay type interaction.

Case $\omega_2 = i\nu$. For a purely growing density fluctuation, i.e. $x=0$, (B9) becomes

$$(y^2 + (\delta^2 + \Gamma_1^2) + 2\Gamma_1 y)(y^2 + \omega_{k_2}^2 + 2\Gamma_2 y) + \frac{K}{2\omega_0} i(1 - \epsilon^2)(\nu + i\Gamma_1) + (1 + \epsilon^2) = 0. \quad (B25)$$

The requirement that y is real means that this type of solution is possible only for $\alpha=1$. Thus a purely growing instability can only occur if the pump is purely standing. Any traveling components will lead to a very small, but finite, frequency component.

Threshold is obtained from (B25) by setting $y=0$.

$$K_{thr} = -\frac{\omega_0^2 k_2^2 (\delta^2 + \Gamma_1^2)}{\delta}. \quad (B26)$$

The threshold attains its minimum value at $\delta = -\Gamma_1$.

$$K_{min} = 2\omega_0^2 k_2^2 \Gamma_1. \quad (B27)$$

Note that (B26) is meaningful only for $\delta < 0$. Therefore, a necessary condition for the purely growing instability is that only waves such that $k > k_{pump}$ are unstable.

Far above threshold we expect the damping rates to have little effect on the instability. Neglecting Γ_1 , Γ_2 , and ω_{k_2} with respect to y in (B25) gives

$$y = \left(\frac{-\delta^2 + (\delta^4 - 4\delta K/\omega_0)^{1/2}}{2} \right)^{1/2} \quad (B28)$$

for a standing wave pump. The maximum growth rate occurs at

$$\delta_0 = (-K/2\omega_0)^{1/3},$$

$$V_{\text{max}} = \frac{F}{2} \frac{1}{\epsilon_0} \frac{1}{\omega_p^2}$$

Since the instability is not a decay into normal modes, it is not surprising that a growth increment exists over a large region of k -space. The region is bounded by the points at which $y=0$. These are easily obtained from (B23), for $\omega=0$. Instability is found over the following region of k -space:

$$\left(\frac{\omega}{K_0} - \frac{\omega_p^2}{K_0^2} \right) (1 - \frac{1}{2} \frac{\omega_p^2}{K_0^2} + \frac{1}{2} \frac{\omega_p^4}{K_0^4}) < \frac{1}{2} \frac{\omega_p^2}{K_0^2} + \frac{1}{2} \frac{\omega_p^4}{K_0^4} - \frac{1}{2} \frac{\omega_p^6}{K_0^6}$$

where $K_0 = K_{\text{min}}$ far above threshold. (B30) shows that the spectral width increases linearly with the ratio of intensity to threshold intensity. This broad spectrum may be a significant factor for pumping with a broad band pump.

Eq (B9) was solved numerically as well. A few representative samples are included to show the main features of the more exact numerical solution. For clarity, the figures are for cases without damping, i.e., $\gamma_1 = \gamma_2 = 0$. (Figures 4-6 and 4-7 are examples of cases with finite damping.)

Figures B-1 and B-2 show the real and imaginary roots when $n = 5 n_{cr}$ and $(V_{os}/V_{th})^2 = 1$, for standing wave and traveling wave pumps, respectively. Both modes give a positive frequency shift for $k < k_0$, and a negative shift for unstable waves with $k > k_0$. The traveling wave case always remains positive frequency, however, while for a purely standing wave, part of the unstable spectrum is at zero-frequency. The unstable spectrum in the latter case splits into two distinct lobes.

Figures B-3 and B-4 are analogous to the previous figures except that the field strength has been increased to $(V_{os}/V_{th})^2 = 1.5$.

Both positive and negative frequency shifts are enhanced, and the width of the unstable spectrum has virtually doubled, in excellent agreement with (B30).

Finally, in Figures B-5 and B-6, the numerical solutions at $n = 8 n_{cr}$ are displayed. Because of a desire to plot all solutions on the same axes, the standing wave pump is only $(V_{os}/V_{th})^2 = 2$, while the traveling wave one is at $(V_{os}/V_{th})^2 = 4$. The negative frequency shifts for the latter case can be seen to reduce the frequency to less than 10% of the usual ion acoustic frequency. It should be remembered that the frequency in all cases has been plotted as a function of the ion wavenumber minus the pump wavenumber. Overall, the analytic results derived earlier in the Appendix were in excellent agreement with the numerical solutions.

Figure B1. Numerical solution of Eq (B9), $(V_0/V_{th})^2=0.1$, $n_0=0.5 n_{cr}$, $m_e/M_i=0.01$, $\gamma_1=\gamma_2=0$, Standing Wave Pump.

Figure B2. Numerical solution of Eq (B9), same parameters as Figure B1 except Traveling Wave Pump.

Figure B3 Numerical solution of Eq (B9), $(V_{os}/V_{th})^2=0.3$, $n_o=0.5 n_{cr}$, $m_e/M_i=0.1$, $\Gamma_1=\Gamma_2=0$, Standing Wave Pump.

Figure B4 Numerical solution of Eq (B9), same parameters as Figure B3, except Traveling Wave Pump.

Figure B5 Numerical solution of Eq (B9), $(V_{os}/V_{th})^2=0.2$, $n_0=0.8 n_{cr}$, $m_e/M_1=0.01$, $\beta_1=\beta_2=0$, Standing Wave Pump.

Figure B6 Numerical solutions of Eq. (B9), $(V_{gs}/V_{th})^2=0.3$, $n_0=0.8 n_{cr}$, $m_e/M_i=0.01$, $\Gamma_1=\Gamma_2=0.$, Traveling Wave Pump.

APPENDIX C

Derivation of Coupled Wave Equations From The Reduced Description Equations

Numerical solution of the coupled wave equations, (3-1a) and (3-1b), is a powerful method of following spectral evolution in a periodic medium. The equations for coupling electromagnetic waves to ion density waves are easily derivable from the RDE equations. The high frequency field is already in the form of a wave equation

$$\frac{\partial^2 A}{\partial t^2} + \frac{\partial A}{\partial t} + \frac{A}{\tau} - \nabla^2 A = 0 \quad (C1)$$

Spatial analysis of (C1) gives

$$\frac{\partial^2 A_k}{\partial t^2} + 2\nu \frac{\partial A_k}{\partial t} + \omega_p^2 A_k - \omega_k^2 A_k = -\omega_k^2 A_k n_{k-k} \quad (C2)$$

where n_k is normalized with respect to n_{cr} . We will denote the linear frequency relation by

$$\omega_k^2 = (\omega_p^2 + k^2 c^2)$$

so that (C2) clearly takes the form of a linear harmonic oscillator for A_k , driven by the coupling terms, $A_k n_{k-k}$.

$$\frac{\partial^2 A_k}{\partial t^2} + 2\nu \frac{\partial A_k}{\partial t} + \omega_k^2 A_k = -\omega_k^2 A_k n_{k-k} \quad (C3)$$

In general, the coupling products are not in phase with A_k , but near resonance can still induce significant effects.

The equation for n_k , the density fluctuation, involves more algebra but is still straightforward. Operate on the continuity equation with $\frac{\partial}{\partial t}$

$$\frac{\partial^2 n}{\partial t^2} = -\frac{\partial^2}{\partial t \partial x} n u \quad (C4)$$

and on the flux equation with $\frac{\partial}{\partial x}$

$$\frac{\partial^2}{\partial x \partial t} \bar{n} u = - \frac{\partial^2}{\partial x^2} \bar{n} u u - c_s^2 \frac{\partial}{\partial x} \bar{n} - \frac{1}{2} \left(\frac{e}{mc} \right)^2 \frac{2m_e}{M_i} \frac{\partial A^2}{\partial x} - v \frac{\partial \bar{n} u}{\partial x} \quad (C5)$$

where $c_s^2 = (\theta_e + \theta_i) / M_i$. Now, note that

$$v \frac{\partial \bar{n} u}{\partial x} = -v \frac{\partial \bar{n}}{\partial t}$$

and neglect convection in periodic systems on the grounds that it is the product of two first order quantities. Then (C4)

and (C5) can be combined to give

$$\frac{\partial^2 \bar{n}}{\partial t^2} + v \frac{\partial \bar{n}}{\partial t} - c_s^2 \frac{\partial^2 \bar{n}}{\partial x^2} = \frac{1}{2} \left(\frac{e}{mc} \right)^2 \frac{m_e n_0}{M_i} \frac{\partial A^2}{\partial x} = 0 \quad (C6)$$

Fourier analysis of (C6) yields

$$\frac{\partial^2 \bar{n}_k}{\partial t^2} + v \frac{\partial \bar{n}_k}{\partial t} + c_s^2 k^2 \bar{n}_k = - \frac{1}{2} \left(\frac{e}{mc} \right)^2 \frac{m_e}{M_i} \frac{n_0}{c_F} k^2 \sum_{k'} A_{\vec{k}}^* A_{\vec{k}-\vec{k}'} \quad (C7)$$

For N modes, Equations (C3) and (C7) represent $2N$ coupled harmonic oscillators, where the coupling functions are derived naturally in the process of deriving the equations. Similar sets can be derived to describe the coupling of any two types of waves.

APPENDIX D

Derivation of the Ion Fluctuation Induced Dissipation Rate

A simple, heuristic derivation can be given for the dissipation rate which ion fluctuations induce in the presence of a high frequency wave. (This follows a derivation given by Krue.) Consider a density fluctuation of wavenumber, k , magnitude, δn , and a frequency is much less than that of the high frequency field, so that the latter sees a constant perturbation over many periods. A high frequency electric field causes the more mobile electrons to translate with respect to the ions. If the field (electromagnetic or electrostatic) has the form

$$E = E_0 \cos(\omega_0 t)$$

then the electrons will collectively exhibit the velocity

$$v_{os} = -\frac{eE_0}{m\omega_0} \sin(\omega_0 t).$$

This gives an oscillating current

$$J = -en v_{os}.$$

Since the electric field is presumed to be uniform in the field direction, the continuity equation for the electrons becomes

$$-\omega_0 \delta n(\vec{x}, t) = -\vec{\nabla}_{os} \cdot \nabla n(\vec{x}, t) \quad (D1)$$

The field in the plasma generated by this oscillation of charge is

$$\nabla \cdot \epsilon \vec{E} = 4\pi e \delta n. \quad (D2)$$

Harmonic analysis of (D2) gives

$$E(k, \omega_0) = -4\pi e i \delta n(k, \omega_0) / \epsilon(k, \omega_0). \quad (D3)$$

The rate at which these internal fields do work per unit volume

is just

$$\frac{dU}{dt} = -\mathbf{J} \cdot \mathbf{E} \quad (D4)$$

Now denote the Fourier transform of δn by δn_k ,

$$\delta n = \sum_k \delta n_k e^{i\mathbf{k} \cdot \mathbf{x}},$$

so that (D4) may be written

$$\frac{dU}{dt} = -\text{Re} \left(\sum_{\mathbf{k}, \mathbf{k}'} \frac{(-4\pi e i V)}{\epsilon |\mathbf{k}|^2} \delta n_{\mathbf{k}'} (-e V_{0s}) n_{\mathbf{k}} e^{i\mathbf{k} \cdot \mathbf{x}} e^{i\mathbf{k}' \cdot \mathbf{x}} \right) \quad (D5)$$

If the density fluctuations, n_k , are normalized with respect to n_{cr} , and it is noted that $\omega_o^2 = 4\pi e^2 n_{cr} / m_e$, (D5) can be greatly simplified,

$$\frac{dU}{dt} = -\omega_o^2 \frac{E^2}{4\pi} \cos^2(\omega_o t) \sum_{\mathbf{k}'} \left(\frac{n_{\mathbf{k}'}}{n_{cr}} \right)^2 \text{Im} \left(\frac{1}{\epsilon(\mathbf{k}', \omega_o)} \right) \quad (D6)$$

For an electrostatic wave, note that

$$\frac{dU}{dt} \approx -\nu^* \frac{E^2}{8\pi} \quad (D7)$$

Comparison of (D6) and (D7), therefore gives a time averaged dissipation rate of

$$\nu^* \approx -\omega_o \sum_{\mathbf{k}'} \left(\frac{n_{\mathbf{k}'}}{n_{cr}} \right)^2 \text{Im}(\epsilon(\mathbf{k}', \omega_o)) / \epsilon^2(\mathbf{k}', \omega_o) \quad (D8)$$

For a three dimensional distribution of density fluctuations, only the component aligned with the electric field contributes to (D8), so the expression within the summand should rigorously be multiplied by $\cos^2(\mathbf{k}', \mathbf{e}_E)$.

The dissipation rate derived above is for the induced plasma fields, but since their source is the electromagnetic field, (D8) also represents the electromagnetic damping rate so long as $\nu^* \ll \omega_p$. If the inequality is not well satisfied, depletion of the plasma fields must be accounted for, and (D8) will be only an upper bound.

APPENDIX E

Application of Inhomogeneous Calculations
to a Scattering Experiment Performed at NRL

The inherent three-dimensionality of SBS should not be obscured by the attention spent on the backscattering mode. This latter is interesting in itself, since it has the lowest threshold in a homogeneous plasma. For short density scale lengths, however, the threshold for sidescatter is lower than for backscatter. According to Liu and Rosenbluth²², the inhomogeneous threshold for Brillouin backscatter is given by

$$\left(\frac{v_{os}}{v_{th}}\right)^2 \frac{L_n}{\lambda_o} > \frac{1}{2\pi} \left(\frac{L_n}{L_u}\right) \left(\frac{n_o}{n_{cr}}\right) \quad (E1)$$

where L_n (L_u) is the density (velocity) scale length. Since the derivatives which define the scale length are in the direction of propagation, oblique incidence increases the scale lengths and decreases the threshold. The same source²² also derived a threshold condition for Brillouin sidescatter in a spherical plasma, radius of curvature, R ,

$$\left(\frac{v_{os}}{v_{th}}\right)^2 \frac{L_n}{\lambda_o} > \frac{1}{2\pi (k_o L_u)^{1/3} \left(1 + 2\left(\frac{L_n}{R}\right) \left(\frac{k_y}{k_y}\right)^2 \left(\frac{n_o}{n_{cr}}\right) \left(1 + \frac{L_u}{L_p}\right)^{2/3}\right)} \quad (E2)$$

Under common experimental conditions, the sidescatter threshold is much lower than the backscatter. To illustrate this, we will consider parameters typical of a series of NRL experiments¹⁵.

In the NRL experiments, a subnanosecond 1.06 μ laser was focused onto a slab target. Although the condensed account of the experiments suggested a variety of interesting pieces of information, the primary point of interest here was that they

measured the spectra of the light returning through the focusing lens. These measurements were conducted at three different orientations of the target with respect to the laser. The magnitude of the signal collected by the lens was virtually independent of the orientation, although a slight increase with increasing angle was evident in their published data. Since no statistics were given, it is not possible to infer trends from such small variations (15%). The spectral characteristics, however, changed qualitatively as the orientation increased from 0° to 24° to 45° . The spectra exhibited progressively more structure, and at 45° a well-defined asymmetrical red-shift. The purpose of publishing the data was to support a Brillouin backscatter hypothesis. Using the published data, however, and the results of the homogeneous periodic RDF calculations, it is the intention here to show that substantial sidescatter could also have been measured.

The density and velocity scale lengths were theoretically estimated to be 30 and 200 μ by Ripin. Evaluating (E1) and (E2) at $n_o = .5 n_{cr}$ for normal laser incidence gives $(v_{os}/v_{th})^2 L_n / \lambda = .012$ for backscatter threshold, but .002 for sidescatter threshold. For oblique incidence, both scale lengths are increased as $L_n = L_{n_o} / \cos \theta$. The backscatter threshold decreases as a simple cosine of the incidence angle. The velocity gradient is in the ion acoustic direction, which is determined by the wave-number matching condition. The sidescatter threshold thus

decreases somewhat faster than $\cos\theta$. At 24° and 45° , we find that $(V_{os}/V_{th})^2 L_{n_0}/\lambda = .0018$, and .0009, respectively, for sidescatter. Sidescatter apparently has an order of magnitude lower threshold than backscatter under these conditions.

These considerations were based on (1) and (E1). There is no particular reason to doubt these expressions, but they are only threshold conditions. What is usually more interesting is the pump strength at which a large fraction of the laser begins to be scattered. Particle simulations indicate that this onsets when $(V_{os}/V_{th})^2 L_{n_0}/\lambda = 5-10$. The difference between this expression and (E1) is two to three orders of magnitude for backscatter. If one can extrapolate the difference for sidescatter, then the threshold level provides an accurate estimate of the strength of the process. Confirmation of this for Brillouin sidescatter or for the extent of existence of the modes, however, probably must wait extensive two-dimensional numerical simulations. Sidescatter nevertheless seems highly competitive with backscatter in the XRL experiments.

Lacking the hard evidence a multi-dimensional simulation might yield, let us engage in a small gedanken experiment. Let us assume that strong sidescatter occurs in the plasma. Since there is no reason to think this would inhibit backscatter, we may also assume that the two modes are sufficiently separated in direction that they do not directly interact. Let the density profile be linear so that $n(x) = n_{cr}(1 - x/L)$, and let us consider what happens to an electromagnetic wave sidescattered

at $x=X_0$. It is easy to show that k_y is a constant of motion in slab geometry. The electromagnetic wave will therefore retrace out of the plasma along a parabolic trajectory. The time to retrace to any lower density (i.e. $x=X_1 < X_0$) is given by

$$t = \frac{2L}{c}(X_1 - X_0/L)^{1/2} \quad (E3)$$

If the electromagnetic wave suffers no further interactions, it will leave the profile after a time $T=2L(1-X_0/L)^{1/2}/c$, at an angle $\theta = \tan^{-1}(k_y/k_x)$, or

$$\theta = \tan^{-1}\left(\frac{X}{L-X_0}\right) \quad (E4)$$

A sufficiently intense sidescattered wave may undergo further interactions, however. There are two distinct factors which influence this. The first is the trajectory, which as noted above will be approximately parabolic regardless of intensity. Although this path is independent of the nature of the new interactions, it plays a vital role in determining the viability of wave-wave processes. For parametric processes especially, the trajectory determines the rate at which phase mismatch occurs. If we consider the stimulated backscatter instability for this wave, a coherence length, L , can therefore inferred,

$$\int_{X_0}^L (k_0 - k_1 - k_2) dx \approx 1 \quad (E5)$$

where k_1 is the wavenumber of a backscattered wave, k_0 that of the refracting sidescattered wave, and k_2 that of an ion acoustic wave. Interestingly, the small frequency shift in the secondary wave means that it will retrace the path of the

pump (sidescattered) wave. Also, the length defined by the above is actually a density range, within which the three wave are coherent.

The above integral can be explicitly evaluated for a linear density gradient. The y components are assumed to be perfectly resonant initially. In a slab geometry, therefore, they will always be resonant. The evaluated integral gives

$$L - X_0 = L \left(\frac{4c}{3\omega_0 L} \right)^{2/3} \quad (E6)$$

The time required for the wave to propagate to this lower density is then

$$\Delta t = \frac{32}{3\omega_0} \left(\frac{\omega_0 L}{c} \right)^{2/3} \quad (E7)$$

Finally, the distance the wave will have traveled in the y-direction in this time is

$$\Lambda = \frac{c}{\omega_0} \left(\frac{X_0}{L_0} \right)^{1/2} \frac{32}{3} \left(\frac{\omega_0 L}{c} \right)^{2/3} \quad (E8)$$

The interaction length for stimulated Brillouin backscatter is thus $L_I = (\Lambda^2 + (L - X_0)^2)^{1/2}$.

Effective backscatter can only occur if the sidescattered intensity along the above interaction path is sufficiently intense. Inhomogeneous simulations have shown that the product of $(v_{os}/v_{th})^2$ and L_n must exceed 5-10 for strong backscatter to occur. The scale length there was the full density profile length, and not the interaction one. Taking the lower bound gives a threshold (for strong scatter) $(v_{os}/v_{th})^2 = .25$ for the NRL experiments. The actual sidescattered intensity can be estimated only in the crudest fashion, but since its inhomogeneous threshold is an order of magnitude lower than

backscatter, we expect at least a comparable amount of energy to be side-scattered. In the NRL experiment, the focused power density was estimated to be $3 \times 10^{15} \text{ W/cm}^2$ and the electron temperature to be .7 KeV. This corresponds to $(V_{os}/V_{th})^2 = 4$. Stimulated reflection in the backward direction has been measured to be about 10% in many experiments, though only 5% in the NRL experiment. Even this amount, though, lends credence to the supposition that the sidescattered light in the NRL experiment was intense enough to stimulate strong Brillouin backscatter.

Under the above conditions, strong SBS would be similar to the model mode coupling calculations discussed in Chapter 3, Section I. Repeated scattering is therefore probable. In fact, if the intensity of the side-scattering could be determined accurately, those results could plausibly extrapolated to give the magnitude of the spectral broadening. As it is, calculations at $n_o = .5 n_{cr}$ with modest pump levels led to 8% relative bandwidth. When this is scaled down to realistic mass ratios, a bandwidth of .5% is still obtained. This compares well with the bandwidths reported in several scattering experiments. Unfortunately, quantitative comparisons with the repeated scattering calculations will be impractical until the magnitude of sidescattering can be more accurately gauged.

Finally, the sidescattering hypothesis can reasonably account for increasing spectral width with increasingly oblique target irradiation. If we suppose that the expansion plume dynamics are dominated by the slab surface, then the oblique incidence may actually correspond to the incidence angle relative to the plasma normal surface. The scattered spectra were measured from light collected by the focusing lens, either $f/1.9$ or $f/14$. (The $f/14$ lens subtends a half-angle of 4° , while $f/1.9$

subtends almost 30° .) For normal incidence, a narrow backscattered signal was measured, which is entirely consistent with stimulated Brillouin backscatter. Even with the $f/1.9$ lens, the original sidescattering event would have been forced to occur above about $n_0 = .8 n_{cr}$ (.995 n_{cr} with the $f/14$ lens.). At 24° incidence, the total back-signal was only slightly greater, some broadening was evident, but the signal was still dominated by the narrow central peak. Here, the $f/14$ lens could collect sidescatter originated between $.8-.9 n_{cr}$, which seems too high a density for strong sidescatter. If the other lens were used, however, densities as low as $.6 n_{cr}$ could have been sampled. This certainly includes part of the density regime in which sidescattering might be expected. The NRL report did not specify which lens was used for these measurements, unfortunately, so the intermediate angle must be regarded as ambiguous for these purposes.

The broadest, most diffuse spectra were measured from targets tilted 45° to the laser, but with total energy comparable to the others. Light refracted from between $n_0 = .4-.6 n_{cr}$ would have been collected by the $f/14$ lens here, and to paraphrase Goldilocks, this fits (sidescattering) just (about) right. The reasoning behind this is straightforward. Scattering processes occurring above $.8 n_{cr}$ are doubly weakened because the ion wavelength is increasing, and so decreasing the growth rate, and because the process must compete with a robust ensemble of absorptive mechanisms near the critical density. Below about $.25 n_{cr}$ on the other hand, Brillouin scattering must compete with the stronger Raman back- and sidescattering processes. The logical regime to look for Brillouin scattering is thus between $.3$ and $.7 n_{cr}$. The fact that the best defined examples of spectral

broadening occurred at 45° strongly suggests that Sbs was involved, while the inhomogeneous simulations in Ch 3, Section II showed that backscattering was too weak a broadening source to account for the measurements. For a variety of reasons then, Stimulated Brillouin sidescatter, followed by repeated backscattering, must be considered as consistent with the NRL experiments, and at least as attractive as any other broadening mechanism for these conditions.

APPENDIX F

Derivation of Jacobean Wave Function

Equation (4-5) was the nonlinear wave equation derived in Chapter 4. If a standing wave is assumed, and the exponential nonlinearity in (4-5) expanded for weak fields, i.e. $\eta_0^2 \ll 1$, the following equation results

$$\frac{\partial^2 A}{\partial (xk_0)^2} + (1 - \alpha + \frac{1}{2}\eta_0^2 A^2 \alpha) A = 0 \quad (F1)$$

where $\alpha = 2(n_0/n_{cr}) / (1 + \exp(-\frac{1}{2}\eta_0^2 A_{\max}^2))$. If we let $A = C_1 \text{cn}(xk_0, m)$,

(F1) becomes

$$C_1 \text{cn}(xk_0, m) (2m - 1 - 2m \text{cn}^2(xk_0, m)) + (1 - \alpha + \frac{1}{2}\eta_0^2 \alpha C_1^2 \text{cn}^2(xk_0, m)) C_1 \text{cn}(xk_0, m) = 0 \quad (F2)$$

Eq(F2) is satisfied for two choices for m

$$m = \alpha/2$$

$$m = \alpha \eta_0^2 C_1^2 / 4$$

If the first expression is chosen, identical satisfaction of (F2) occurs only at $\eta_0^2 = 2/C_1^2$, which is also the matching point for the second choice. Since a standing wave always gives $C_1^2 > 2$, the exact matching is at least in fair agreement with the assumption of weak fields, to expand the exponential nonlinearity. The jacobian function, cn, therefore is a solution to the cubic nonlinearity wave equation only at one field strength, but we expect it to be close to the exact solution for a spread of field strengths around that point.

CHAPTER 4

ELECTROMAGNETIC WAVE TRAPPING AND HOLE FORMATION

Formation of stable electromagnetic wave structures near the critical density of laser plasmas has aroused considerable interest. Simulations have shown that the density wells associated with these structures can extend to much lower density than the surrounding plasma. The two dimensional surfaces near the critical density thus exhibit strong modulations on a laser wavelength scale. Since most collisional absorption occurs in this vicinity, such large perturbations in the density may have profound ramifications on absorption efficiency. Moreover, the collective absorption processes near the critical density will experience large inhomogeneities, which may severely limit their effectiveness, as well. The existence of these "pebble"-like states and the conditions under which they may be expected to occur is, therefore, a problem with direct applications to laser fusion. In this chapter, one dimensional studies will be conducted to better identify the threshold conditions.

Previous numerical and/or analytic studies of trapped electromagnetic wave states have been pursued by Eaw, Schmidt, and Wilcox²⁵, Eaw, Nishikawa, and Yoshida²⁶, Gurevich and Karpman^{27,28,29}, Vukobratovic and Estabrook³⁰, and by Longen and Lasinski³¹. As a result of these studies, a set of characteristics of the state has become apparent.

* The author has recently been advised of the numerical work of Berslund, Firdel, Lineman and Lee³².

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