A BOUNCE-AVERAGED MONTE CARLO COLLISION OPERATOR
AND RIPPLE TRANSPORT IN A TOKAMAK

Jay M. Albert* and Allen H. Boozer

Plasma Physics Laboratory, Princeton University

P. O. Box 451

Princeton, New Jersey 08544

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ABSTRACT

A bounce-averaged Monte Carlo operator is presented that simulates bounce-averaged perturbative Lorentz pitch angle scattering of particles in toroidal plasmas, in particular a tokamak. In conjunction with bounce-averaged expressions for the deterministic motion, this operator allows a quick and inexpensive simulation on time scales long compared to a bounce time. An analytically tractable model of transport due to toroidal magnetic field ripple is described.

MASTER

*Present address: Laboratory of Plasma Studies, 369 Upson Hall,
Cornell University, Ithaca NY 14853

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I. INTRODUCTION

For many transport processes in toroidal magnetic confinement devices, the shortest time scale of interest is set by a mirror or bounce time. Just as averaging over the gyromotion allows a simplified treatment in terms of drift kinetics, it is often feasible to further average the motion over a bounce trajectory. In such cases it is often permissible to use a perturbative treatment, that is, to integrate the effect under consideration along an unperturbed bounce orbit. Such amenable effects include toroidal ripple in a tokamak,¹⁻⁴ transport in a stellarator,⁵ electron transport in ELMO Bumpy Torus,⁶ and resonant transport in an axially asymmetric mirror.⁷ In all of these cases, transport depends on pitch angle scattering, for which the Lorentz operator is taken to be a good representation. The main topic of this paper is the presentation of an accurate Monte Carlo⁸ implementation of the bounce-averaged Lorentz operator, in the case that the scattering is a perturbation to the collisionless motion. Tokamak bounce orbits are used, but the method could be simply applied to other types of devices with similar particle dynamics.

In section II, the Monte Carlo operator is derived from a bounce-averaged Fokker-Planck equation written by Cordey.⁹ The trapped/passing boundary is a troublesome region because scattering is not perturbative for particles right at the boundary. In section III, a previously proposed⁶ scheme for a bounce-averaged Monte Carlo operator is discussed. Section IV tests the performance of the present operator by applying it to an

ensemble of particles initially delta-distributed in pitch. The ensemble is seen to relax to pitch-space isotropy on the proper time scale. Section V illustrates the use of the operator in a fully bounce-averaged treatment of the banana-drift regime of ripple transport in a tokamak.

II. DERIVATION OF THE OPERATOR

The collisionless bounce motion follows from the adiabatic invariance of μ , with the energy decomposed as $1 = (v_{||}/v)^2 + \mu B/E$, and model magnetic field

$$B = B_0(1 - \epsilon \cos \theta) . \quad (1)$$

The effect of pitch angle scattering on the particle distribution function is given by

$$\frac{\partial f}{\partial t} = C(f) = 2\nu_{90} \lambda \frac{\partial}{\partial \mu} \mu \lambda \frac{\partial f}{\partial \mu} \quad (2)$$

where μ is nondimensionalized by B_0/E , $\lambda = v_{||}/v$, and ν_{90} is the Spitzer 90° collision frequency. For scattering to be a perturbation to the bounce motion, the bounce time τ_b must be short compared to the scattering time: $\tau_b \nu_{90} \ll 1$. Then, following Cordey,⁹ the version of equation (2) for the bounce-averaged distribution function can be obtained by dividing through by λ and applying

$$\langle \cdot \rangle = \frac{1}{2\pi} \int \langle \cdot \rangle d\theta \quad (3)$$

where the limits of integration are $-\theta_b$ to θ_b for trapped particles (TPs) and $-\pi$ to π for passing particles (PPs). This gives

$$\frac{\partial f}{\partial t} = \frac{2v_{90}}{\langle v/v_n \rangle} \frac{\partial}{\partial \mu} \mu \langle \frac{v_n}{v} \rangle \frac{\partial f}{\partial \mu} \quad (4)$$

where

$$\langle v_n/v \rangle = \begin{cases} \frac{2}{\pi} \sqrt{\frac{2\epsilon(1-\zeta^2)}{1-\epsilon}} [E(x) - (1-x^2)K(x)] , & \text{TPs} \\ \frac{2}{\pi\zeta} E(1/x) , & \text{PPs} \end{cases} \quad (5a)$$

$$\langle v/v_n \rangle = \begin{cases} \frac{2}{\pi} / \sqrt{\frac{2\epsilon(1-\zeta^2)}{1-\epsilon}} K(x) , & \text{TPs} \\ \frac{2}{\pi\zeta} K(1/x) , & \text{PPs} \end{cases} \quad (5b)$$

and where

$$\begin{aligned} \zeta &= \langle v_n/v \rangle (\theta=0) = [1-\mu(1-\epsilon)]^{1/2} \\ x^2 &= \frac{\zeta^2}{2\epsilon} \frac{1-\epsilon}{1-\zeta^2} , = \sin^2(\theta_b/2) \text{ for TPs.} \end{aligned} \quad (5c)$$

These integrals over the unperturbed orbit are exact, using (1). Usually $\epsilon \ll 1$, and then $1-\zeta^2 \approx 1$ for trapped particles. Using this last approximation even for passing particles recovers the expressions of Cordey.⁹

The next step is to convert equation (4) to the standard form

of the diffusion equation. Introducing the variable z :

$$z = \langle v_{||}/v \rangle \quad (6)$$

one verifies that, since $v_{||}/v = \{1 - \mu(1 - \varepsilon \cos \theta)\}^{1/2}$, to lowest order in ε , $\partial/\partial \mu (v_{||}/v) = -\frac{1}{2}(v/v_{||})$ so that

$$\frac{\partial z}{\partial \mu} = -\frac{1}{2} \left\langle \frac{v}{v_{||}} \right\rangle .$$

In terms of z , equation (4) becomes

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial z} \left[D(z) \frac{\partial f}{\partial z} \right] , \quad (7)$$

with

$$D(z) = \frac{v_{90}}{2} \mu \left\langle \frac{v_{||}}{v} \right\rangle \left\langle \frac{v}{v_{||}} \right\rangle \quad (8)$$

and with $\int f dz = 1$. This reflects the fact that the unaveraged phase space volume element is proportional to $d(v_{||}/v)d\theta$.

The Monte Carlo equivalent of (7) is well known⁸:

$$z' = z + \langle \Delta z \rangle \pm \sqrt{\langle (\Delta z)^2 \rangle} , \quad (9a)$$

with

$$\langle \Delta z \rangle = \frac{\partial D}{\partial z} \tau = \frac{\tau v_{90}}{2} \frac{\partial}{\partial z} \left(\mu \left\langle \frac{v_{||}}{v} \right\rangle \left\langle \frac{v}{v_{||}} \right\rangle \right) \quad (9b)$$

$$\langle (\Delta z)^2 \rangle = 2D\tau = \tau v_{90} \mu \left\langle \frac{v''}{v} \right\rangle \left\langle \frac{v}{v''} \right\rangle \quad (9c)$$

where τ is a time step much less than v_{90}^{-1} and where $\pm$ is a sign chosen at random with equal probability. It could be replaced by a Gaussian random variable with zero mean and unit variance, but because the number of iterations will be large, the central limit theorem makes this unnecessary. The terms are shown in figure 1; they are singular at the trapped/passing boundary.

Since $\langle \Delta z \rangle / \langle (\Delta z)^2 \rangle^{1/2} \sim (\tau v_{90})^{1/2} \ll 1$, the systematic piece is much smaller than the diffusive piece. Nevertheless, the effect of the systematic, or drift, piece is comparable after $\sim 1/\tau v_{90}$ iterations. Without it, particles would linger and eventually accumulate in regions of low D . The drift term always points in the direction of high D in just such a way as to balance this tendency. Furthermore, $D(z)$ goes to zero at the endpoints of the allowed range, $0 < \zeta < 1$, fast enough to prevent an unphysical step out of the range. This may be seen by linearizing the operator (9) about the endpoint $\zeta = 0$, and then minimizing ζ' with respect to ζ ; even with the random sign chosen as $-$, $\zeta > 0$ implies $\zeta' > 0$. The analogous valid behavior occurs at $\zeta = 1$. In general, for (9) to be an accurate approximation to (7) after sufficiently many iterations, the time step τ must be small enough that the stepping terms are small and nearly constant during an iteration:

$$\begin{aligned} \frac{\partial \langle \Delta z \rangle}{\partial z} \delta z &\ll \langle \Delta z \rangle \ll 1 \\ \frac{\partial \langle (\Delta z)^2 \rangle}{\partial z} \delta z &\ll \langle (\Delta z)^2 \rangle \ll 1 \end{aligned} \quad (10)$$

or, as long as the particle is not very close to the singularity at the trapped/passing boundary (where $\langle v/v_n \rangle \rightarrow \infty$ as $x \rightarrow 1$),

$$\tau v_{90}/2\varepsilon \ll 1. \quad (11)$$

Very near the passing/trapped boundary, $x \approx x_* = 1 - \tau v_{90}/2\varepsilon$, condition (10) will always be violated because of the singularity of $\langle v/v_n \rangle$. In particular, the slope of the systematic term gets intolerably large. To prevent the Monte Carlo operator from generating unrealistically large flux out of this singular region, one technique is the following. Denoting the diffusion coefficient evaluated at x_* by D_* , the terms are made smoother by redefining $D(z)$ by

$$\frac{1}{D} \rightarrow \frac{1}{D} + \frac{1}{D_*}$$

so that $D(z)$ is almost unchanged away from the singular region but is bounded within it. Then the Monte Carlo coefficients become modified according to:

$$\langle (\Delta z)^2 \rangle \rightarrow \langle (\Delta z)^2 \rangle / (1 + \langle (\Delta z)^2 \rangle / \langle (\Delta z)^2 \rangle_*) \quad (12a)$$

$$\langle \Delta z \rangle \rightarrow \langle \Delta z \rangle / (1 + \langle (\Delta z)^2 \rangle / \langle (\Delta z)^2 \rangle_*)^2. \quad (12b)$$

III. A DIFFERENT SCHEME

Tolliver⁶ took a different approach to generate bounce-averaged Monte Carlo coefficients. In his procedure, the mean and variance of the change in μ per bounce are calculated by considering a delta-distributed ensemble of particles starting at $-\theta_p$ ($-\pi$ for passing particles) and executing one unperturbed bounce transit, integrating the Lorentz operator along the way:

$$\begin{aligned}\langle \delta\mu \rangle &= \oint dt \int (\mu - \mu_0) C(f) d^3v \\ \langle (\delta\mu)^2 \rangle &= \oint dt \int (\mu - \mu_0)^2 C(f) d^3v\end{aligned}\quad (13)$$

where $f = \delta(\mu - \mu_0)/\langle v/v_n \rangle$. (Tolliver's λ is our μ , his ζ is our λ , and his Q corresponds to our $\langle v_n/v \rangle$.) These integrals easily evaluate to

$$\begin{aligned}\langle \delta\mu \rangle &= 2\tau_b v_{90} \frac{\partial}{\partial \mu} \left(\mu \left\langle \frac{v_n}{v} \right\rangle \right) \\ \langle (\delta\mu)^2 \rangle &= 4\tau_b v_{90} \mu \left\langle \frac{v_n}{v} \right\rangle.\end{aligned}$$

The prescription is then to select $\mu' - \mu$ from a Gaussian distribution with mean and variance:

$$\mu' = \mu + \langle \Delta\mu \rangle + \varepsilon \sqrt{\langle (\Delta\mu)^2 \rangle}, \quad (14a)$$

$$\langle \Delta\mu \rangle = \langle \delta\mu \rangle \frac{\tau}{\tau_b} = 2\tau v_{90} \frac{1}{\langle v/v_n \rangle} \frac{\partial}{\partial \mu} \left(\mu \langle v_n/v \rangle \right) \quad (14b)$$

$$\langle (\Delta\mu)^2 \rangle = \langle (\delta\mu)^2 \rangle \frac{\tau}{\tau_b} = 4\tau v_{90} \frac{\mu \langle v_n/v \rangle}{\langle v/v_n \rangle} \quad (14c)$$

where ξ is a Gaussian random variable with zero mean and unit variance. Taking $\langle \delta z \rangle \approx (\partial z / \partial \mu) \langle \Delta \mu \rangle$ and $\langle (\delta z)^2 \rangle \approx (\partial z / \partial \mu)^2 \langle (\Delta \mu)^2 \rangle$, the diffusive term (14c) is equivalent to that of (9c) but (14b) gives

$$\langle v/v_{||} \rangle \frac{\partial}{\partial z} \mu \langle v_{||}/v \rangle \quad (15)$$

for the systematic piece, which is not the same as (9b). Expression (15) fails to capture the collisional change of the bounce time, $2\pi qR/v \langle v/v_{||} \rangle$, during a bounce. Furthermore, this approach would sanction rewriting equation (7) as

$$\frac{\partial f}{\partial t} = \frac{1}{D} \frac{\partial^2 f}{\partial y^2} \quad , \quad \int f D \, dy = 1$$

where y is defined by

$$\frac{\partial z}{\partial y} = D(z) \, ,$$

and calculating, as in (13)-(14),

$$\langle \Delta y \rangle = 0 \quad , \quad \langle (\Delta y)^2 \rangle = \frac{2\tau}{D}$$

which also gives $\langle (\Delta z)^2 \rangle \approx (\partial z / \partial y)^2 \langle (\Delta y)^2 \rangle = 2D\tau$, but misses the systematic piece entirely.

It so happens that the variability of $\langle v/v_{||} \rangle$ is mostly confined to the singular region of the trapped/passing boundary, so that elsewhere the numerical difference between our operator

(9) and that of Tolliver (14) is usually small. A more pressing problem is that Tolliver's operator, like ours, suffers from the divergence of τ_b/τ_{90} at the singular region. A modification to prevent $\langle v/v_{90} \rangle$ from diverging, similar to the modifications in (12), would be required if (14) is to be used.

IV. NUMERICAL VERIFICATION

It is relatively simple to numerically implement the above algorithms. As a test, an ensemble of particles was chosen to be initially distributed as a delta function in z . Our Monte Carlo operator (9), as modified by (12), was then iterated with the stepping terms recalculated after every step with the new value of the diffusing variable. To find the updated ξ corresponding to a new z , a straightforward Newton's method was used on relation (5) for z in terms of ξ .

Since the number of particles $n(z)\Delta z$ in a small range Δz about z is given in terms of the f of equation (7) by simply $f(z)\Delta z$, the actual distribution function obtained is just given by the histogram of $n(z)$. According to (7), f should relax to an isotropic, or Maxwellian, distribution in a time on the order of ν_{90}^{-1} , or about $1/\tau\nu_{90}$ iterations of the operator. This means $n(z)$ should become a constant function of z , allowing for statistical fluctuations (which scale like the reciprocal square root of the number of particles). Figure 2 shows the results for typical parameters. The modified algorithm (9) is seen to relax the distribution to isotropy on the time scale of τ_{90} , as it should.

V. RIPPLE TRANSPORT

Use of the bounce-averaged Monte Carlo operator in calculating test particle transport will be illustrated by considering the effect of small toroidal magnetic field ripple. Not only is the bounce-averaged description of the particle motion very suitable for computation, but with the assumption of radial locality the particle diffusion coefficient can be calculated analytically.

To model the deviation from axisymmetry caused by the discreteness of the magnet coils, the expression (1) for B is modified to read:

$$B = B_0(1 - \epsilon \cos\theta - \delta \cos N\varphi) \quad (16)$$

where N is the number of coils and δ is a small inhomogeneity parameter. To avoid trapping in the local ripple wells, we take δ small enough that

$$\left| \frac{Nq\delta}{\epsilon \sin\theta_b} \right| < 1 \quad (17)$$

for most of the trapped particle population; again, the exceptional particles, with turning points on the midplane, are problematic (see, e.g., reference 3).

In describing particle dynamics in terms of bounce-averaged drifts, the following expressions¹⁰ are extremely useful:

$$\langle \dot{\alpha} \rangle = \frac{1}{e} \frac{\partial J / \partial \beta}{\partial J / \partial E}, \quad \langle \dot{\beta} \rangle = - \frac{1}{e} \frac{\partial J / \partial \alpha}{\partial J / \partial E} \quad (18)$$

$$J = \int v_{||} ds$$

for Clebsch coordinates α and β such that

$$\vec{B} = \nabla \alpha \times \nabla \beta.$$

With¹¹ the poloidal flux $\alpha \approx r R B_\theta$ and $\beta = \varphi - q\theta$, these yield the toroidal precession per bounce

$$\varphi_p = \langle v_r \rangle \tau_b / R \propto \rho q^2 \tau_b / \varepsilon r \quad (19a)$$

and the ripple-induced radial step per bounce

$$\Delta = \langle v_r \rangle \tau_b = (N\pi / \sin \theta_p)^{1/2} \rho \delta \left(\frac{q}{\varepsilon} \right)^{3/2} \sin(N\varphi_p - \frac{\pi}{4}) \quad (19b)$$

where τ_b is the particle bounce time. For $\tau \geq \tau_b$, the collisionless particle motion can be written in slightly simplified form as a 2-D area-preserving map,⁴ with time step τ_b :

$$\begin{aligned} r' &= r + \Delta \cos N\varphi \\ \varphi' &= \varphi + \varphi_p \end{aligned} \quad (20)$$

For $N\Delta \varphi_p / dr \geq 1$, the motion is stochastic;⁴ we assume instead that this quantity is small. Transport is then driven by pitch angle scattering, which couples to φ by

$$\begin{aligned} \delta\varphi(\tau) &= q\delta\theta_p = q \left[\frac{\partial\theta_p}{\partial z} \right] \left[\langle \Delta z \rangle \pm \sqrt{\langle (\Delta z)^2 \rangle} \right] \\ &= \phi_{c0}(\tau) \pm \phi_c(\tau) \end{aligned} \quad (21)$$

The first equality relies on collisionality being only a perturbation to the bounce motion.

With the notation

$$\Lambda = \Delta \frac{\tau}{\tau_b}, \quad \Phi_p = (\varphi_p + \Phi_{c0}) \frac{\tau}{\tau_b} \quad (22)$$

the full map is

$$\begin{aligned} r' &= r + \Lambda \cos N\varphi \\ \varphi' &= \varphi + \Phi_p \pm \Phi_c \end{aligned} \quad (23)$$

with time step τ . For this discretization to be valid, τ must be small enough that i) the parameters Λ , Φ_p , and Φ_c remain nearly constant, and ii) the change in $N\varphi$ is small. This second condition is more stringent because $N \gg 1$. It requires τ small enough that

$$\cos N\Phi_p \approx 1. \quad (24)$$

The analogous requirement $\cos N\Phi_c \approx 1$ is guaranteed by $\tau v_{g0} \ll 1$, which is separately required for good statistical behavior of the Monte Carlo operator.

Equations (23) are in very convenient form for computation. Furthermore, the map can be treated analytically² if radial dependences of the parameters are ignored. One can then directly compute the local radial diffusion coefficient:

$$\begin{aligned}
 D_r &= \lim_{m \rightarrow \infty} \frac{(r_m - r_0)^2}{2(m\tau)} = \frac{1/K(\tau)}{1/K^2(\tau) + 1/Z^2(\tau)} \frac{\Delta^2}{2\tau} \\
 &= \frac{1/K(\tau_b)}{1/K^2(\tau_b) + 1/Z^2(\tau_b)} \frac{\Delta^2}{2\tau_b}
 \end{aligned}
 \tag{25}$$

where

$$\kappa = \frac{2}{(N\Phi_c)^2}, \quad z = \frac{1}{N\Phi_p}
 \tag{26}$$

are the collisional and precessional decorrelation times of φ in units of τ . Here $N\Phi_p$ is taken to lie in $(-\pi, \pi)$, and then because of (24), lies well inside $(-\pi/2, \pi/2)$. Thus $z \gg 1$, and the statistical requirement of the Monte Carlo operator, $\tau\nu_{90} \ll 1$, imposes $\kappa \gg 1$. The diffusion coefficient (25) still requires averaging over velocity space. These results agree with the results of Yushmanov,² and reduce to the special cases of the collision-dominated and precession-dominated banana drift regimes treated by Linsker and Boozer.¹

VI. SUMMARY

A Monte Carlo operator has been derived for pitch-angle scattering of single particle orbits in a tokamak, assuming the scattering to be a perturbation to the collisionless motion. This formulation is easy to implement numerically. Each iteration of the operator corresponds to on the order of one period of the collisionless motion: a bounce time for trapped particles, and a poloidal rotation of 2π for passing particles. Where the dynamics under consideration are amenable to a bounce-averaged description, this avoids integration over uninteresting details of the particle

orbits. This operator was compared to a previously proposed operator and was found to have more satisfactory relaxation to velocity space isotropy for a wide range of orbit parameter values. The formulation was also shown to be useful analytically, by rederiving a general expression for the diffusion coefficient for tokamak ripple transport in a clear and unified way.

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REFERENCES

- ¹R. Linsker and A. H. Boozer, Phys. Fluids 25, 143 (1982).
- ²P. N. Yushmanov, Nucl. Fusion 23, 1599 (1983).
- ³R. J. Goldston and H. Towner, J. Plasma Phys. 26, 283 (1981).
- ⁴R. J. Goldston, R. B. White, and A. H. Boozer, Phys. Rev. Lett. 47, 647 (1981).
- ⁵J. D. Beidler, W. N. G. Hitchon, H. E. Mynick, and J. L. Shoet, Proc. Sherwood Conf., 1985.
- ⁶J. S. Tolliver, Phys. Fluids 28, 1083 (1985).
- ⁷B. V. Chirikov, Sov. J. Plasma Phys. 5, 492 (1979).
- ⁸A. H. Boozer and G. Kuo-Petravic, Phys. Fluids 25, 143 (1981).
- ⁹J. G. Cordey, Nucl. Fusion 16, 499 (1976).
- ¹⁰T. G. Northrop and E. Teller, Phys. Rev. 117, 215, (1960).
- ¹¹A. H. Boozer, Phys. Fluids 23, 2283 (1980).

FIGURE CAPTIONS

Figure 1. Monte Carlo terms a) $\langle \Delta z \rangle / \tau v_{90}$ and b) $\langle (\Delta z)^2 \rangle / \tau v_{90}$ versus z , for $\varepsilon = 0.2$

Figure 2. Time development of $f(z)$ versus z , for an ensemble of 5120 particles according to the operator of section II, with $\varepsilon = 0.2$, $\tau v_{90} = 10^{-3}$. All particles were at $z = 0.5$ initially.

a) $t/\tau_{90} = 0.02$

b) $t/\tau_{90} = 0.2$

c) $t/\tau_{90} = 1$

d) $t/\tau_{90} = 2$

Figure 1.

Figure 2)a, b

Figure 2)c, d

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