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EQUILIBRIUM CALCULATIONS FOR PLASMA CONTROL IN CIT

D. J. Strickler
Y-K. M. Peng
N. Pomphrey
S. C. Jardin

FUSION ENGINEERING DESIGN CENTER

OAK RIDGE NATIONAL LABORATORY

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**D. J. Strickler
Y-K. M. Peng**

**N. Pomphrey
S. C. Jardin**

Princeton Plasma Physics Laboratory

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ABSTRACT

The free-boundary equilibrium code VEQ provides equilibrium data that are used by the Tokamak Simulation Code (TSC) in design and analysis of the poloidal field (PF) system for the Compact Ignition Tokamak (CIT). VEQ serves as an important design tool for locating the PF coils and defining coil current trajectories and control systems for TSC. In this report, VEQ and its role in the TSC analysis of the CIT PF system are described. Equilibrium and coil current calculations are discussed, an overview of the CIT PF system is presented, a set of reference equilibria for modeling a complete discharge in CIT is described, and the concept of a plasma shape control matrix is introduced.

1. INTRODUCTION

The poloidal field (PF) system for the Compact Ignition Tokamak (CIT) is determined through a design and analysis process using the free-boundary equilibrium code VEQ1 and the Tokamak Simulation Code (TSC).² TSC is a plasma transport and electromagnetics code that tests the consistency of the PF system with design physics and engineering guidelines. It uses preprogrammed PF coil currents and control matrices to characterize feedback systems for controlling plasma position and shape. VEQ provides these equilibrium data for TSC fiducial discharge calculations³ and plasma disruption modeling. The first step in the design process is to generate a detailed table of plasma geometry (e.g., elongation κ , triangularity δ), profile parameters (e.g., plasma internal inductance $l_i/2$, poloidal beta β_p), and volt-second requirements at N representative points in time where preprogrammed currents are needed for a simulation. This table is often based on a preliminary TSC analysis. The next step is to calculate N reference equilibria and the associated coil current distributions, constraining the parameters in the table. Here VEQ computes PF coil current distributions of minimum stored energy while satisfying constraints on, for example, plasma shape, profiles, coil current magnitude, and volt-seconds. The PF currents are then varied about their reference values in a set of free-boundary, unconstrained equilibria in order to compute plasma shape control matrices relating shape perturbations to correction currents, $d\mathbf{I} = \mathbf{A} \epsilon$, where the elements of ϵ are typically errors in major radius R_0 , minor radius a , κ and δ at the 95% flux surface, and flux linkage. This analysis is repeated until a solution is found that is consistent with plasma ignition criteria, physics guidelines on plasma shape and profiles, divertor sweep scenario, and volt-seconds for an adequate burn time.

These steps are followed for several possible poloidal field configurations and scenarios, such as limiter, divertor, single null, and double null. The free-boundary equilibrium package VEQ serves as an important design tool for locating PF coils and defining current trajectories and control systems for TSC.

This report describes the equilibrium package VEQ and its role in the TSC analysis of the CIT PF system. Section 2 gives some details of the equilibrium and coil current calculations. Sections 3 through 6 give an overview of the GEM-288

CIT PF system, together with some applications of VEQ in the CIT PF analysis process. Sections 7 and 8 describe a set of reference equilibria for modeling a complete discharge in CIT and introduce the concept of a plasma shape control matrix.

2. THE EQUILIBRIUM PACKAGE VEQ

In VEQ, a free-boundary equilibrium code is called from the numerical software packages HYBRD1⁴ and VMCON⁵ to compute values of free parameters (e.g., PF coil currents) in the equilibrium problem in order to constrain certain plasma shape and profile parameters (e.g., plasma radius, X-point position, or divertor strike points). Some of these shape constraints are shown in Fig. 1. When the number

Fig. 1. VEQ equilibrium shape constraints include major radius R_0 , minor radius a , X-point coordinates (R_X, Z_X) , and divertor strike points (R_O, Z_O) and (R_I, Z_I) .

of free parameters is equal to the number of constraints, the package HYBRD1 is used to solve the nonlinear equations problem NLEQ:

$$\mathbf{F}(\mathbf{x}) = 0 \quad ,$$

where $\mathbf{F}$ and $\mathbf{x}$ are vectors of length N ($\mathbf{F}:R^N \rightarrow R^N$). Whenever the number of independent variables is greater than the number of constraints (e.g., when the problem is to find seven CIT PF coil currents to fix the plasma major radius, minor radius, X-point coordinates, and flux linkage), it is possible to minimize some objective function (e.g., the PF coil energy $W_{PF} = 1/2 \mathbf{I}^T \mathbf{M} \mathbf{I}$) while still satisfying the constraints. For this problem, the optimization package VMCON is used. VMCON is designed to solve the general nonlinear optimization problem NLOP:

$$\text{Minimize } f(\mathbf{x}) \quad ,$$

subject to

$$F_i(\mathbf{x}) = 0, \quad i = 1, \dots, k, \quad \text{and } F_i(\mathbf{x}) \geq 0, \quad i = k+1, \dots, M \quad ,$$

where $f:R^N \rightarrow R^1$ and $F_i:R^N \rightarrow R^1$ are assumed to be continuously differentiable. In addition, VMCON allows simple bounds to be placed on independent variables. Evaluating the functions $\mathbf{F}$, f , and F_i requires solving the equilibrium problem. Here we refer to the equilibrium problem as EQP and to the HYBRD1 and VMCON problems as NLEQ and NLOP, respectively.

2.1 EQUILIBRIUM CALCULATION

VEQ uses relatively standard methods to solve the free-boundary EQP for the poloidal flux function ψ in cylindrical coordinates (R, Z) on a rectangular mesh Ω . Similar methods treated in detail elsewhere (e.g., Ref. 6) are outlined here.

1. **Outer Loop.** For appropriate initial estimates, an outer loop scales the solution ψ to fix the plasma current I_p and computes boundary values ψ^b based on a knowledge of the plasma current (J_ϕ) and coil current (I_i) distributions:

$$\psi(\mathbf{x}^b) = \int_{\Omega} G(\mathbf{x}^b, \mathbf{x}) J_\phi(\mathbf{x}) d\Omega + \sum_i G(\mathbf{x}^b, \mathbf{x}_i^c) I_i \quad , \quad (1)$$

where $\mathbf{x}^b = (R^b, Z^b)$ is on the boundary of Ω , $\mathbf{x}_i^c = (R_i^c, Z_i^c)$ is the location of the i th coil center, and G relates the poloidal flux at a boundary point to a unit current at a plasma or coil filament using the Biot-Savart law.

2. **Inner Loop.** Given the boundary values, an inner loop solves the nonlinear Grad-Shafranov equation (Ref),

$$\Delta^* \psi \equiv R^2 \nabla \cdot R^{-2} \nabla \psi = -\mu R J_\phi, \quad (2)$$

by iteratively computing the limiting flux values at the magnetic axis (ψ_0) and plasma edge (ψ_L), evaluating the plasma current density based on assumed plasma pressure (P) and toroidal magnetic flux functions ($F = RB_t$), and inverting the elliptic operator Δ^* using a direct method.⁷

2.2 COIL CURRENT CALCULATION

An axisymmetric PF coil with a rectangular cross section is represented in VEQ as a set of filamentary loops. VEQ has three major options for the specification or calculation of coil currents. Subsets of coil currents may be

- Designated to have fixed current (option 01).
- Related linearly to the external poloidal flux at points on a prescribed plasma boundary contour (option 02). These currents are computed by solving a least-squares approximation problem with a regularization term. This calculation is part of the inner loop in the solution of the EQP.
- Related through a set of nonlinear equations to constraints on plasma X-point coordinates, divertor strike points, PF volt-seconds, or coil energy (option 03). With this option, the currents are treated as fixed in the solution of the EQP and as independent variables in the solution of either the NLEQ or the NLOP. Here the equilibrium subprogram is called iteratively by the appropriate software package to solve the nonlinear equation/minimization problem for the designated coil currents.

A given VEQ calculation, therefore, may have coil currents that are fixed, computed within the equilibrium loops (EQP), or determined by a software routine calling the equilibrium subprogram (HYBRD1 or VMCON).

For option O2, if $\mathbf{x}_i^p = (R_i^p, Z_i^p)$, $i = 1, \dots, N^p$ is a set of points describing a desired plasma boundary shape and including a limiter point $\mathbf{x}_L = (R_L, Z_L)$, and if $\mathbf{x}_j^c = (R_j^c, Z_j^c)$, $j = 1, \dots, N^c$ is a set of coil locations, then we solve the linear approximation problem

$$\sum_i \left[\sum_j G(\mathbf{x}_i^p, \mathbf{x}_j^c) I_j - \psi^e(\mathbf{x}_i^p) \right]^2 + \alpha \sum_j I_j^2 = \text{minimum} \quad (3)$$

for the variable coil currents I_j . The desired external flux is determined at every inner equilibrium iteration by

$$\psi^e(\mathbf{x}_i^p) = \psi_L - \psi^p(\mathbf{x}_i^p),$$

where the flux due to the plasma current distribution ψ^p is determined by interpolation in the two-dimensional mesh of known values. Thus, for this option, the coil currents I_j depend largely on coil location, plasma boundary points, and the regularization parameter α , which gives some weight to minimizing the total ampere-turns.

Option O3 may be used, for example, to constrain the separatrix flux surface of a symmetric divertor plasma boundary to pass through two points on the midplane $(R_0 - a, 0)$ and $(R_0 + a, 0)$, and to intersect prescribed inner and outer divertor strike points (R_1, Z_1) and (R_0, Z_r) , as shown in Fig. 1. In addition, we may require a given PF flux linkage Φ_0 . When this option is used, one pair of outboard vertical field coil currents is usually adjusted within the EQP using option O2 (with $\alpha = 0$) to make the separatrix flux surface pass through $(R_0 + a, 0)$. If the number of remaining free currents is equal to the number of constraints (four in this case), they are determined by HYBRD1 as roots of the equation

$$\mathbf{F}(\mathbf{x}) = 0,$$

where $\mathbf{x}^T = (I_1, I_2, I_3, I_4)$ and

$$\mathbf{F} = \begin{bmatrix} (\psi_0 - \psi_X)/\Delta_p \\ (\psi_1 - \psi_X)/\Delta_p \\ (a - a_0)/a_0 \\ (\Phi - \Phi_0)/\Phi_0 \end{bmatrix}.$$

Here a_0 is a given minor radius; ψ_I and ψ_O are the values of the poloidal flux at the inboard and outboard strike points, respectively, and $\Phi = \sum M_{i,p} I_i$ is the PF flux linkage with the plasma.

If the number of remaining free coil currents designated by O3 is greater than the number of constraints, VEQ switches to VMCON, which minimizes an objective function such as the stored energy,

$$f(\mathbf{x}) = 1/2 \mathbf{x}^T M \mathbf{x} ,$$

where $\mathbf{x}^T = (I_1, \dots, I_7)$, subject to the constraints $F(\mathbf{x}) = 0$.

2.3 PROFILE FUNCTIONS

Typical plasma pressure and toroidal magnetic flux profile functions used in HEQ in the evaluation of the plasma current distribution,

$$J_\phi = R \, dP/d\psi + F(dF/d\psi)/(\mu R) , \quad (4)$$

are given in terms of their derivatives:

$$dP/dx = P_0 [\exp(-Ax) - \exp(-A)] / [\exp(-A) - 1] , \quad (5)$$

$$dF^2/dx = 2\mu R_0^2 P_0 (1/\beta_J - 1) [\exp(-Bx) - \exp(-B)] / [\exp(-B) - 1] , \quad (6)$$

where $x = (\psi - \psi_0)/(\psi_L - \psi_0)$. The parameter P_0 is scaled in the outer loop of the equilibrium algorithm so that the total plasma current,

$$I_p = \int \int J_\phi(\mathbf{x}) \, d\Omega ,$$

is fixed at an input value. In general, β_J controls the plasma beta and determines whether the plasma is paramagnetic ($\beta_J < 1$) or diamagnetic ($\beta_J > 1$), and A and B control the width of the current profile. Plasma pressure and current density profiles are shown as a function of radius R ($Z = 0$) in Fig. 2(a) for a $\langle \beta \rangle = 3\%$ CIT divertor equilibrium with $A = B = -1.04$ and $\beta_J = 0.59$ and in Fig. 2(b) for a $\langle \beta \rangle = 5\%$ case with $A = B = -1.0$ and $\beta_J = 1.05$.

Plasma equilibrium pressure and current density profiles are characterized by the poloidal beta,

$$\beta_p = 4 \int P \, dV / (\mu R_0 I_p^2) ,$$

Fig. 2. Plasma pressure and normal current density profiles ($l_i/2 = 0.36$) for CIT divertor equilibria with (a) $\beta_p = 0.48$ and (b) $\beta_p = 0.89$.

and the plasma internal inductance,

$$l_i/2 = \int B_p^2 dV / (\mu^2 R_0 I_p^2) ,$$

respectively. A nominal value for the CIT poloidal beta at end of burn (EOB) is $\beta_p = 0.5$, and current profiles are referred to as broad for $l_i/2 = 0.3$, normal for $l_i/2 = 0.4$, and peaked for $l_i/2 = 0.5$.

3. THE CIT PF SYSTEM

The geometry considered in the following examples is based on a CIT design with major radius $R_0 = 2.138$ m, minor radius $a = 0.662$ m, field on axis $B_t = 11$ T, and plasma current $I_p = 12.3$ MA. The external PF coil system used is GEM-28 (ref. 8) (Table 1) and consists of seven coil groups, labeled PF1–PF7, that provide the equilibrium vertical field, shaping field, and inductive flux for an elongated divertor plasma. The CIT PF system includes windings internal to the toroidal field (TF) coils; these windings are reserved for dynamic control of shape variations caused by rapid changes in plasma pressure and profiles and carry much smaller currents than the external coils.

The central solenoid stack is split into three sections (PF1, PF2, and PF3) for added flexibility in providing for a field null at startup and shaping the plasma cross section through a discharge. The outer ring coils, PF6 and PF7, provide the major component of the equilibrium vertical field, and PF6 also contributes to the shaping field or higher-order derivatives of the external field. In general, all external

Table 1. Poloidal field coil set (upper half plane)

Coil	R_c (m)	Z_c (m)	R (m)	Z (m)
PF1	0.5917	0.4104	0.4064	0.8208
PF2	0.5917	1.3070	0.4064	0.7700
PF3	0.5917	2.1790	0.4064	0.7700
PF4	1.6300	2.8910	0.5629	0.4232
PF5	3.1900	2.8700	0.2913	0.2710
PF6	4.1870	2.0000	0.2753	0.3244
PF7	4.2245	0.7494	0.3543	0.4477

PF coils contribute to the equilibrium, shaping, and control of the CIT plasma, as well as to the flux change that induces the plasma current and ohmically heats the plasma.

4. LIMITER SOLUTION

In this section we demonstrate the use of VEQ to obtain a limiter solution used in the PF system analysis process. This example models the point in a discharge when the current in element PF1 of the solenoid is changing sign. In a typical limiter equilibrium problem, we are given plasma position (R_0 , a), shape parameters (κ , δ) at the plasma-vacuum boundary, current, field on axis, profile parameters (β_p , $l_i/2$), and PF volt-seconds (Φ). These requirements are listed in Table 2. In addition, we are required to find that solution for which $I_{PF1} = 0$.

Table 2. Fixed plasma parameters
for VEQ limiter solution

Major radius (m)	2.14
Minor radius (m)	0.70
Elongation	1.60
Triangularity	0.10
Plasma current (MA)	-4.96
Field on axis (T)	7.61
Internal inductance	0.40
Poloidal beta	0.10
Flux linkage (V·s)	3.00

For this problem, VEQ is executed in a mode in which the plasma boundary is defined by a limiter point at $x_L = (R_0 - a, 0)$ and eight additional points on a symmetric contour (see Fig. 3), given by

$$R = R_0 + a(\cos \theta + \delta \sin \theta) ,$$

$$Z = a\kappa(\sin \theta) ,$$

with $\kappa = 1.6$ and $\delta = 0.1$, equally spaced in the poloidal angle θ . HYBRD1 is used to solve for one coil current (I_{PF2}) and two independent profile parameters in order

Fig. 3. Plasma limiter boundary flux surface, passing through eight points on the midplane (small circles) on a given contour. Also shown are the 95% flux surface and the X-point location (indicated by the $\times$).

to prescribe volt-seconds, plasma internal inductance, and poloidal beta. The coil current distribution is computed using the following options:

1. $I_{PF1} = 0$ is treated as a fixed current in the EQP (option O1).
2. I_{PF2} is designated as an independent variable in the NLEQ (option O3).
3. I_{PF3} through I_{PF7} are determined within each EQP using option O2 with regularization parameter $\alpha = 0.05$ [Eq. (3)].

In addition, the profile parameters $A(=B)$ and β_J [Eqs. (5) and 6)] are treated as independent variables in the NLEQ. That is, the NLEQ problem is given by

$$\mathbf{F}(\mathbf{x}) = 0 ,$$

$$\mathbf{x}^T = (I_{PF2}, A, \beta_J)$$

and

$$\mathbf{F} = \begin{bmatrix} (\Phi - \Phi_0)/(\Phi_0 + 1) \\ (l_i - l_{i,0})/l_{i,0} \\ (\beta_p - \beta_{p,0})/\beta_{p,0} \end{bmatrix} .$$

With $N = 3$ for the NLEQ and the plasma boundary held relatively fixed within each EQP, convergence to a solution occurs rapidly (seven EQPs with NLEQ tolerance $\tau = 0.001$).

The solution corresponding to the data of Table 2 is given in Fig. 4. In the equilibrium plots, coil sizes are proportional to current. The variables labeled "q(edge)," "Kappa," and "Delta" are values of the safety factor, elongation, and triangularity,

Fig. 4. VEQ limiter equilibrium solution.

respectively, at the plasma edge for limiter solutions and at the 95% flux surface for divertor solutions. The variable labeled "Beta p2" is the poloidal beta as defined in Sect. 2.3.

5. DIVERTOR SOLUTION

A high-beta ($\beta = 5\%$), diverted solution provides a "highly severe," but credible, scenario as a starting point for a TSC disruption analysis.⁹ This section describes such a fixed X-point divertor solution with prescribed volt-seconds and poloidal beta. Further equilibrium constraints (Table 3) include plasma position, current, and field on axis.

Table 3. Fixed plasma parameters
for VEQ divertor solution

Major radius (m)	2.138
Minor radius (m)	0.662
R, X-point (m)	1.660
Z, X-point (m)	1.480
Plasma current (MA)	-12.300
Field on axis (T)	11.000
Poloidal beta	0.890
Flux linkage (V·s)	38.000

For this problem, VEQ is used in an X-point-limited mode, where the plasma-vacuum boundary is defined by the flux value at a magnetic separatrix. The plasma boundary is fixed at only one point $(R_0 + a, 0)$ in the EQP, and HYBRD1 is used to compute four coil currents and one profile parameter (β_J) in order to constrain X-point coordinates, minor radius, volt-seconds, and poloidal beta. That is, the NLEQ problem is given by

$$\mathbf{F}(\mathbf{x}) = 0 ,$$

$$\mathbf{x}^T = (I_{PF1}, I_{PF2}, I_{PF3}, I_{PF4}, \beta_J),$$

and

$$\mathbf{F} = \begin{bmatrix} (a - a_0)/a_0 \\ (R_X - R_{X,0})/R_{X,0} \\ (Z_X - Z_{X,0})/Z_{X,0} \\ (\Phi - \Phi_0)/\Phi_0 \\ (\beta_p - \beta_{p,0})/\beta_{p,0} \end{bmatrix} ,$$

where

- $I_{PF1}, \dots, I_{PF4}$ are independent variables in the NLEQ problem (option O3),
- $I_{PF5} = I_{PF6} = 1$ MA are fixed currents in the EQP (option O1),
- I_{PF7} is a variable in the EQP (option O2) with $\alpha = 0.0$.

The divertor solution is shown in Fig. 5. In this mode, care must be taken to ensure that the initial guess for the coil current vector produces a solution with an X-point within the computational mesh Ω . This initial guess is usually estimated from a previous solution or, in the case of a new problem, from a preliminary limiter solution.

Fig. 5. VEQ divertor equilibrium solution.

6. MINIMUM-ENERGY SOLUTIONS

A critical point in the CIT coil current trajectories is at start of flat-top (SOFT), where the 12.3-MA plasma is fully elongated and at relatively low triangularity

(starting the divertor sweep). At this point in the scenario, large currents of opposite direction in solenoid elements PF2 and PF3 produce high separating forces on the solenoid sections. VEQ is exercised in the NLOP mode to examine the dependence of the coil current and stored energy on the magnitude of I_{PF3} in a sequence of minimum-energy equilibria. Constrained parameters and values for the equilibria of this section are listed in Table 4.

Table 4. Fixed parameters for VEQ
minimum-energy solutions

Major radius (m)	2.138
Minor radius (m)	0.662
Elongation (95%)	2.000
Triangularity (95%)	0.300
Plasma current (MA)	-12.300
Field on axis (T)	11.000
Flux linkage (V·s)	36.800

As in Sect. 3, solutions are assumed to be X-point limited, and the plasma boundary is fixed at $(R_0 + a, 0)$ in the EQP by varying I_{PF7} . For the solutions in this section, however, six coil currents ($I_{PF1}, \dots, I_{PF6}$) are allowed to vary in order to minimize coil stored energy while constraining only four parameters—minor radius, elongation and triangularity at the 95% flux surface, and flux linkage. That is, the NLOP is formulated as

$$\text{Minimize } f(\mathbf{x}) = \mathbf{x}^T M \mathbf{x}, \quad \mathbf{x}^T = (I_{PF1}, \dots, I_{PF6}),$$

subject to the constraints $F_i = 0$, where

$$F_1 = (a - a_0)/a_0$$

$$F_2 = (\kappa_{95} - \kappa_{95,0})/\kappa_{95,0}$$

$$F_3 = (\delta_{95} - \delta_{95,0})/\delta_{95,0}$$

$$F_4 = (\Phi - \Phi_0)/\Phi_0$$

and M is the matrix of mutual inductances between the coils and plasma.

Table 5 gives the coil current distributions associated with a sequence of minimum-energy solutions at SOFT. For totally unbounded coil currents, as in

Table 5. Minimum-energy solutions at SOFT with current in coil PF3 unbounded, bounded by $|I_{PF3}| \leq 6$ MA, and bounded by $|I_{PF3}| \leq 2$ MA

	Coil current (MA)							Coil energy (MJ)
	PF1	PF2	PF3	PF4	PF5	PF6	PF7	
Case I, current in coil PF3 unbounded	15.62	21.87	-16.72	-7.75	3.05	3.90	2.20	1.94
Case II, current in coil PF3 bounded by $ I_{PF3} \leq 6$ MA	16.32	18.06	-6.00	-10.57	3.18	4.82	1.74	2.05
Case III, current in coil PF3 bounded by $ I_{PF3} \leq 2$ MA	16.62	16.53	-2.00	-11.46	2.89	5.51	1.46	2.17

case I of Table 5, currents in PF2 and PF3 are 21.9 MA and -16.7 MA, respectively, and magnetic stored energy is $W_{PF} = 1.94$ GJ. With PF3 bounded in magnitude by $|I_{PF3}| \leq 2$ MA, the current in PF2 is 16.5 MA and $W_{PF} = 2.17$ GJ, because of larger shaping currents in the outboard coils. Therefore, a more desirable distribution of current in the central solenoid is found with only a small increase in stored energy. The equilibria for minimum coil energy are shown in Fig. 6.

7. REFERENCE EQUILIBRIA FOR A COMPLETE DISCHARGE SIMULATION

In this section, VEQ is used to develop reference equilibria for the TSC 12.3-MA divertor fiducial discharge simulation. Preprogrammed PF coil current distributions are computed at 1-s intervals during current ramp (5.5-7.5 s) and through current flattop (7.5-12.5 s). The plasma is assumed to be diverted at 5.5 s and the 95% elongation to be held constant at 2.0 as the plasma current and toroidal field increase linearly to their maximum values of $I_p = 12.3$ MA and $B_t = 11$ T at 7.5 s. During the flattop interval, triangularity is increased in a controlled manner from 0.29 to 0.38 to produce the required divertor sweep. The outboard strike point sweeps 23-cm distance. Table 6 summarizes the values of the parameters that are held constant with VEQ for the TSC fiducial equilibrium sequence. The equilibria at SOFT and end of burn (EOB) in Fig. 7 show the endpoints of the divertor sweep. The coil current trajectories are given in Fig. 8.

Fig. 6. Minimum-energy equilibrium solutions at SOFT with current PF3 (a) unbounded, (b) bounded by $|I_{PF3}| \leq 6$ MA, and (c) bounded by $|I_{PF3}| \leq 2$ MA.

Table 6. Time points and plasma parameters held fixed for 12.3-MA divertor fiducial discharge reference equilibrium calculations

t (s)	5.5	6.5	7.5	8.5	9.5	10.5	11.5	12.5
R_0 (m)	2.138	2.138	2.138	2.138	2.138	2.138	2.138	2.138
a (m)	0.662	0.662	0.662	0.662	0.662	0.662	0.662	0.662
κ_{95}	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
δ_{95}	0.26	0.27	0.28	0.30	0.32	0.34	0.36	0.38
$l_i/2$	0.35	0.32	0.30	0.32	0.34	0.36	0.37	0.38
β_p	0.14	0.24	0.29	0.35	0.37	0.38	0.38	0.38
I_p (MA)	-9.05	-10.6	-12.3	-12.3	-12.3	-12.3	-12.3	-12.3
B_t (T)	9.4	10.2	11.0	11.0	11.0	11.0	11.0	11.0
Φ (V·s)	22.5	30.0	35.6	36.0	36.4	36.8	37.2	37.6

Fig. 7. Reference (a) SOFT and (b) EOB equilibria for a TSC discharge simulation.

Fig. 8. Reference coil current waveforms for current flattop.

8. PLASMA SHAPE CONTROL MATRICES

The TSC user often needs additional flexibility in specifying preprogrammed currents for a discharge simulation. Since the transport model in TSC evolves the plasma pressure and current density profiles self-consistently, these profiles are somewhat different from those assumed in the equilibrium calculations used to generate the coil currents, which results in differences in the plasma shape parameters. These differences are automatically corrected to some degree during a TSC calculation by feedback algorithms. It is extremely useful, however, to have the ability to make corrections between TSC runs, without recalculating the equilibria. In this section we introduce the concept of a shape control matrix A , relating correction currents in a subset of the PF coils to errors in the plasma shape,

$$d\mathbf{I} = \mathbf{A} \epsilon .$$

Shape control matrices associated with each reference equilibrium are developed by computing M additional free-boundary, unconstrained VEQ equilibria at each time point, changing the coil current distribution by $\mathbf{I}_j = \mathbf{I}_0 + d\mathbf{I}_j$ between each calculation, where $\mathbf{I}_0$ is the reference coil current distribution and the index j refers

to the equilibrium calculation. For each calculation, the error vector ϵ_j , whose elements are the changes in certain shape parameters and flux linkage between equilibrium j and the reference equilibrium, is recorded. If $d\mathbf{I}_j^T = [dI_{1j}, \dots, dI_{Nj}]$ represents the change in the current distribution associated with the j th equilibrium calculation, we set

$$\mathbf{a}_i \cdot \epsilon_j \approx dI_{ij}, \quad i = 1, \dots, N, \quad j = 1, \dots, M$$

and solve for the elements of the row vectors $\mathbf{a}_i$ of the control matrix $\mathbf{A}$. The error vector usually contains five elements, resulting in an $N \times 5$ control matrix. Typically $M > 5$, and this system is solved as a least-squares problem.

In the X-point-limited calculations for the fiducial discharge simulation, $\epsilon^T = [\Delta R_0, \Delta a, \Delta \kappa_{95}, \Delta \delta_{95}, \Delta \Phi]$, and the coil currents are changed one at a time between equilibrium problems; that is, $\mathbf{I}_j^T = (I_{10}, \dots, I_{j0} + dI_j, \dots, I_{N0})$, where I_{i0} refers to the i th reference coil current in the subset of coils chosen to be used in the control matrix. A control matrix corresponding to case III in Table 5 is given in Table 7. Each column of the control matrix corresponds to a current distribution that would change the plasma shape (or flux) parameter associated with that column while leaving the remaining parameters unchanged. To estimate the change in the current distribution of case III in Table 5 for a 0.05 change in both elongation and triangularity, for example, one would multiply the sum of columns 3 and 4 by 0.05.

Table 7. Plasma shape control matrix for the solution of case III in Table 5

Coil	R_0	a	κ	δ	Φ
PF1	17.22	-6.73	14.52	16.93	0.58
PF2	1.86	-46.65	-54.84	-43.61	0.73
PF3	-5.65	146.79	58.47	2.63	0.34
PF4	-8.29	64.47	27.23	9.85	0.22
PF5	-0.69	-11.54	-5.57	0.36	0.01
PF6	-0.22	-5.61	-2.27	-0.23	0.00
PF7	-0.29	-5.47	-1.11	-0.44	0.00

9. SUMMARY

Axisymmetric magnetohydrodynamic (MHD) equilibrium codes continue to play an important role in the design of tokamak experiments. This is particularly evident in the case of the CIT, where unique requirements in plasma shape control and divertor design have led to new developments in the computational equilibrium problem. The free-boundary equilibrium code VEQ, designed to compute the PF coil current distribution of elongated, magnetically limited tokamak plasmas with minimum stored energy and specific constraints on plasma shape, profiles, and volt-seconds, is used extensively in the CIT PF coil design process to provide preprogrammed coil currents and plasma shape control matrices for the fiducial discharge calculations and plasma disruption modeling carried out with TSC.

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