27
7/6-76
25 DUTIS

UCID- 17091

Lawrence Livermore Laboratory

TEMPERATURE-PROFILE DETERMINATION
USING FREQUENCY-DOMAIN REFLECTOMETRY

R. Jeffrey Lytle
Gregory K. Myers

April 2, 1976

MASTER

This is an informal report intended primarily for internal or limited external distribution. The opinions and conclusions stated are those of the author and may or may not be those of the laboratory.

Prepared for U.S. Energy Research & Development Administration under contract No. W-7405-Eng-48.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

CONTENTS

Abstract	1
Introduction	1
Numerical Model	3
Modeling Results	6
Conclusions	9
Acknowledgments	10
References	11
Appendix: Mathematical Equations Describing a Lossy Transmission Line	12

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

TEMPERATURE-PROFILE DETERMINATION USING FREQUENCY-DOMAIN REFLECTOMETRY

ABSTRACT

The temperature distribution along a coaxial electrical cable can be determined by means of frequency-domain reflectometry if the cable has an inner sheath of a suitable temperature-sensitive dielectric material. Such a technique would have advantages in cost and convenience over the conventional method of placing a separate thermocouple at each point where the temperature is to be measured. It would be especially useful for underground applications in assessing geothermal reservoirs and in instrumenting *in situ* coal-gasification and oil-shale-retorting experiments.

INTRODUCTION

It can be difficult to accurately measure the temperature profile in the region where oil shale is being retorted *in situ*, where coal is being gasified *in situ*, or where a geothermal reservoir is being exploited. Thermocouples can be used, but since measurements at many locations are needed (to give good spatial resolution), the number of cables and thermocouples can become prohibitive and cost-ineffective.¹ It would therefore be desirable to be able to monitor the temperature profile with a single cable, along which a detailed spatial profile could be determined and displayed almost instantaneously.

This report concerns the use of reflectometry on a coaxial cable with an inner sheath of temperature-sensitive dielectric to ascertain the temperature profile along the cable. Of the three suggested applications — geothermal assessment, coal gasification, and oil shale retorting — it was decided that the *in situ* oil-shale-retorting problem would be the most severe test of numerical simulations of the data-inversion algorithm. Characteristic temperature profiles encountered during the *in situ* oil-shale-retorting process are given in Fig. 1. It is seen that the sheath material should remain a dielectric at temperatures of up to 700°C and should have a dielectric constant that varies with temperature. For unambiguous interpretation of reflectometry data, a dielectric constant that varies monotonically with temperature for $T \leq 700^\circ\text{C}$ is preferred.

To the authors' knowledge, no such material is an off-the-shelf item. Conversations with Prof. J. D. Mackenzie of the University of California at Los Angeles indicate that, from theoretical considerations, high-silica glasses show promise of maintaining their dielectric properties over this temperature range and, in addition, may have a change in dielectric constant that is at least an order of magnitude for the temperature range of 30 to 700°C. These high-silica glasses have the extra benefit that they can

easily be drawn into fibers that will maintain their flexibility and structural integrity over this temperature range. Of course, other materials may also meet the physical and electrical requirements for the sheath.

Determining a useful algorithm for inverting reflectometry data to obtain the spatial distribution of temperature inside the temperature-sensitive cable began with the computer simulation of a transmission line with nonuniform loading (with the loading dependent on the local temperature), using a differential equation representation of the cable^{2,3} (see Appendix for the mathematical details). By using this cable model, representative reflectometry data were generated. Attention was directed to specific profiles of interest in the oil-shale-retorting process (see Fig. 1). It was thought that a knowledge of the general character of the temperature profile might greatly reduce the complexity of the data-inversion problem.

A number of algorithms were considered that might be successful in terms of inverting the cable reflectometry data:

1. Optimizing^{4,5} to determine a functional fit (e.g., a fifth-order polynomial) to the data.
2. Considering discrete loading and using a multiple-reflection formalism⁶ to fit the data.
3. Considering a multiple-discrete-cell model, applying constraints, and iterating to a best fit to the data.⁷⁻⁹
4. Using integral equation solution methods.¹⁰
5. Generalized inversion methods.¹¹

Fig. 1. The *in situ* oil-shale-retort temperature profile at various times after ignition.

The functional fit was tried first, using the sum of two Gaussian curves to fit Fig. 1. However, the optimization routine was ineffectual in that it found many local minima, but usually not the *best* functional fit. It was then pointed out to the authors that the greatest interest attached to the leading and trailing edges of the temperature profiles in Fig. 1. We surmised that fitting the leading and trailing edges separately would greatly reduce the complexity of the problem. Thus only profiles of this general character were thereafter considered. We came to the conclusion that considering a specific functional form, such as $a_1 \exp[-(x - a_2)^2/a_3]$, would be limiting the application too severely, and hence we used a multiple-discrete-cell model. This means that the transmission line was represented as composed of many segments, each of which had a constant characteristic impedance. The values of the characteristic impedance of each segment were to be the result of inverting the reflectometry data. From the characteristic impedance, the local temperature could be determined using classical transmission-line relations between the local characteristic impedance and the local dielectric constant. This procedure was successful, and sample results are given below.

The problem of using reflectometry to obtain the physical parameters in an inaccessible region is, of course, classical. Many experimental approaches have been suggested, classifiable as time-domain and frequency-domain methods. Because of the dispersion problems to be expected with pulse interrogation of the cable, we have concentrated on interrogation over a wide-bandwidth (e.g., 4:1) frequency domain. We believe this reduces the complexity of data reduction. No time-domain methods have been extensively considered in our investigations.

NUMERICAL MODEL

The cable models assume that $R = G = 0$ (the cable has no losses) and that

$$L = 2 \times 10^{-7} \ln \frac{a_0}{a_1},$$

where a_0 and a_1 are the outer and inner radii of the coaxial cable, respectively. By definition,

$$C = \frac{\epsilon_r \times 10^{-9}}{18 \ln(a_0/a_1)},$$

where ϵ_r is the local value of relative dielectric constant (which is dependent on the local temperature). We arbitrarily assumed that $\epsilon_r = (T_x/30^\circ)^{1/2}$, where T_x is the local temperature in degrees Celsius. Temperature profiles such as the one shown in Fig. 2 were used in the numerical modeling. Thus over the length of the cable $1 \leq \epsilon_r \leq 5$. This is not a dramatic change in the relative dielectric constant and is thus not a severe restriction on the desired sheath-material characteristics.

The dimensions of a_0 and a_1 were chosen as 6 and 2 mm, respectively. The load impedance Z_L at the end of the line was chosen to be 50 ohms, and the length of the line

Fig. 2. The numerically simulated "actual" temperature profile that is to be inferred by using frequency-domain reflectometry.

was arbitrarily chosen at $l = 1.4$ m. The frequencies of interrogation were over the range from 50 to 200 MHz, with 21 frequencies chosen over this range. Different cable lengths l and interrogation frequencies could yield the same results (e.g., consider using the laws of scaling). Thus, if a greater cable length l is required, then lower frequencies would (from scaling) be equivalent to the results given herein. However, the reader should keep in mind that the number of segments would remain fixed. At present, we have only considered 15 segments along a cable 1.4 m long. We suspect that the number of segments could be greatly increased and the resolution thereby increased, but this has not yet been demonstrated (or pursued).

At each of the 21 frequencies, the reflected-signal magnitude and phase were computed from a model of the actual temperature profile. This reflected signal is denoted as Q_a . In terms of the numerical simulation, this represents the data that would be measured with an actual cable in the field (if there were no noise).

An initial guess was made at the variation in dielectric properties with position. From this guess an estimate of the reflected signal (Q_e) was computed. Our objective was to make iterative guesses of the actual temperature profile, for which the reflected signal would be Q_a , such that $S = |Q_a - Q_e|$ would be minimized. This guess was computed

Fig. 3. Seven initial guesses at the temperature profile along the cable. Iteration techniques were used to improve the fit of each of the guesses to the actual temperature profile.

iteratively with an optimization routine using a modified Marquardt algorithm (available in the IMSL Library obtained from the LLL Computation Department). For each iteration a value of Q_e was computed and compared with Q_a , and the estimate of the profile was modified so that S was decreased on the next iteration. The results of this procedure are encouraging and are given below.

MODELING RESULTS

Seven initial guesses (Fig. 3) were made of the actual profile (Fig. 2). These initial guesses represent a variety of possible shapes that the actual temperature profile might exhibit. Figure 4 depicts one of the worst results. The actual temperature

Fig. 4. Results from a sample numerical simulation experiment showing the actual and the computed temperature profile after 20 iterations. The iterations started with the indicated guessed-at profile, guess 1.

profile, the initial guess (one of the seven), and the estimate of the profile after the 20th iteration (the final estimate) are shown. The actual and final profiles seem to oscillate about each other, with some significant differences. With a better initial guess (see Fig. 5), after the 20th iteration, the actual and final profiles are in quite good agreement. Using the straight average of the final estimates for the seven initial guesses, a good agreement is obtained with the actual profile (see Fig. 6). Note that the average deviation of the average of the final estimates from the actual profile is 1.59%. This is a very good fit and gives good spatial resolution.

Fig. 5. Results from a sample numerical simulation experiment show the actual and the computed temperature profile after 20 iterations. The iterations started with the indicated guessed-at profile, guess 4.

Another actual temperature profile was then considered. This was an exponentially shaped profile, as shown in Fig. 7. The same procedure as discussed above was followed, using the seven initial guesses of Fig. 3. For the first, third, sixth, and seventh initial guesses, the final profiles varied wildly and contained negative values. These results were interpreted as nonconverging and were not included in computing the average of the final estimates. The straight average of the three remaining final estimates (after 20 iterations) is compared with the actual profile in Fig. 8. Note that for this case the average deviation of the average is 0.91%.

Fig. 6. The average of the final estimates for the seven initial guesses of the temperature profile agrees very well with the actual temperature profile.

Fig. 7. A second numerically simulated "actual" temperature profile that is to be inferred by using frequency-domain reflectometry.

Fig. 8. The average of the three converging final estimates of the temperature profile agrees very well with the actual temperature profile of Fig. 7.

CONCLUSIONS

This brief study using a numerical model to simulate experimental frequency-domain reflectometry data and using a data-inversion method to infer the temperature profile looks encouraging in terms of using a single downhole cable to monitor the temperature profile. More development work needs to be done before it can be stated that such a system would be successful. Further numerical modeling is needed to include the effects of cable loss, to determine the limits of resolution obtainable, and to investigate whether this method can be used routinely in the field. A materials problem is to measure the dielectric constant and conductivity of high-silica glasses (or other materials) at these high temperatures. The fabrication of a limited number of cables (there is not a large market) also requires study. The total effort required to perform these objectives is estimated to be six man-months.

If these problems prove soluble, then a single cable could replace many thermocouples and give adequate spatial resolution of temperature in geothermal, coal-gasification, and oil-shale energy programs.

ACKNOWLEDGMENTS

Assistance on aspects of the temperature profile in an *in situ* oil-shale-retorting process was provided by Edward G. Cope and John G. Harri. Edmund K. Miller shared insights on possible data-inversion procedures. This work was performed in the LLL Electronics Engineering Research Division, under Division Leader Lynn L. Cleland.

REFERENCES

1. R. H. Cornell, A Methodical Approach to Temperature and Pressure Measurements for in Situ Energy-Recovery Processes, Lawrence Livermore Laboratory, Rept. UCID-16631 (November 14, 1974).
2. L. R. Walker and N. Wax, "Non-uniform Transmission Lines and Reflection Coefficients," J. Appl. Phys. 17, 1043-1046 (1946).
3. H. Kaufman, "Bibliography of Nonuniform Transmission Lines," IRE Trans. Antennas and Propagation AP-3, 218-220 (1955).
4. M. Mostafair and R. Mittra, "Remote Probing of Inhomogeneous Media Using Parameter Optimization Techniques," Radio Sci. 7, 1105-1111 (1972).
5. D. L. B. Jupp and K. Vozoff, "Stable Iterative Methods for the Inversion of Geophysical Data," Geophys. J. R. Astr. Soc. 42, 957-976 (1975).
6. R. J. Lytle and D. L. Lager, "Using the Natural Frequency Concept in Remote Probing of the Earth," Radio Sci. 11, 199-210 (1976).
7. L. Cagniard, "Basic Theory of the Magnetotelluric Method of Geophysical Prospecting," Geophysics 18, 605-635 (1953).
8. S. H. Ward and G. R. Jiracek, "Electromagnetic Reflection from a Plane-Layered Lunar Model," J. Geophys. Res. 73, 1355-1372 (1968).
9. Special issue on physical and computation aspects of three-dimensional image reconstruction, IEEE Transactions on Nuclear Science NS-21 (1974).
10. L. Colin, Mathematics of Profile Inversion, National Technical Information Service, N73-11585-630 (August 1972).
11. J. R. Inman, J. Ryu, and S. H. Ward, "Resistivity Inversion," Geophysics 38, 1088-1108 (1973).

APPENDIX

MATHEMATICAL EQUATIONS DESCRIBING A LOSSY TRANSMISSION LINE*

A transmission line can be described by the variation of voltage V and current I with spatial position x . The transmission line can have a nonuniform impedance and admittance variation with x , where the impedance Z_x and admittance Y_x are defined via the resistance R_x per unit length, the inductance L_x per unit length, the conductance G_x per unit length, and capacitance C_x per unit length. By definition, $Z_x = R_x + j\omega L_x$ and $Y_x = G_x + j\omega C_x$, where ω is the angular frequency of the exciting signal.

It can be shown that the voltage and current satisfy the equations

$$\frac{\partial V_x}{\partial x} = -Z_x I_x$$

and

$$\frac{\partial I_x}{\partial x} = -Y_x V_x$$

Thus the voltage and current independently satisfy

$$\frac{\partial^2 V_x}{\partial x^2} - \frac{1}{Z_x} \frac{\partial Z_x}{\partial x} \frac{\partial V_x}{\partial x} - Z_x Y_x V_x = 0$$

and

$$\frac{\partial^2 I_x}{\partial x^2} - \frac{1}{Y_x} \frac{\partial Y_x}{\partial x} \frac{\partial I_x}{\partial x} - Z_x Y_x I_x = 0.$$

Hence, V_x and I_x are governed by second-order linear differential equations. Therefore both the voltage and current have two linearly independent solutions that satisfy their governing differential equations.

We designate the two linearly independent solutions as V_x^1 and V_x^2 (for voltage) and I_x^1 and I_x^2 (for current). Since the voltage and current are composed of real and imaginary parts, we can define the spatial variations of V^{Re} , V^{Im} , I^{Re} , and I^{Im} as

$$V_x^{Re} = k_1 V_x^{1Re} + k_2 V_x^{2Re},$$

* A list of symbols is given on page 16.

$$V_x^{Im} = k_3 V_x^{1Im} + k_4 V_x^{2Im},$$

$$I_x^{Re} = k_5 I_x^{1Re} + k_6 I_x^{2Re},$$

and

$$I_x^{Im} = k_7 I_x^{1Im} + k_8 I_x^{2Im},$$

where the constants $k_1, k_2, \dots, k_8$ need to be determined. The functions $V_x^1, V_x^2, I_x^1, I_x^2$ are only specified as linearly independent solutions of the governing differential equations. Thus we can arbitrarily define

$$V_x^1(x=0) = 1 \quad \text{and} \quad I_x^1(x=0) = 1$$

and

$$\frac{\partial V_x^2}{\partial x}(x=0) = 0 \quad \text{and} \quad \frac{\partial I_x^2}{\partial x}(x=0) = 0,$$

and this does not violate any restrictions. It does leave the functions $V_x^1, V_x^2, I_x^1, I_x^2$ presently unknown in their spatial variation except at the point $x=0$. The spatial variation of $V_x^1, V_x^2, I_x^1, I_x^2$ is determined from the governing differential equations, knowing the functional values $V_x^1(x=0), I_x^1(x=0), (\partial V_x^2/\partial x)(x=0)$, and $(\partial I_x^2/\partial x)(x=0)$, and integrating forward in x .

Boundary conditions that can be imposed are that

$$V_0 = 1 + j0 = V_0^{Re} + jV_0^{Im}$$

and

$$Z_\ell = Z_\ell^{Re} + jZ_\ell^{Im} = \frac{V_\ell}{I_\ell},$$

where the subscripts 0 and ℓ stand for $x=0$ and $x=\ell$, respectively. This assumes that the generator voltage and the load impedance (at angular frequency ω) at $x=\ell$ on the transmission line are known.

With the specification of boundary conditions at $x=0$ and $x=\ell$, the problem is a two-point boundary-value problem. This means that the numerical simulation model is constrained to fit the boundary conditions at $x=0$ and $x=\ell$. Thus, for this problem, this

is equivalent to requiring the temperature profile to fit known results at $x = 0$ and $x = l$. The shape of the temperature profile between these two points is the unknown quantity to be determined.

A simple analogy is to view the temperature profile as a string secured at both ends. The shape of the string between the two ends is the quantity to be determined.

With the boundary conditions specified at $x = 0$ and $x = l$, one can show that

$$k_1 = 1, \quad k_3 = 0, \quad k_5 = -\frac{R_0}{\Delta} k_2 - \frac{\omega L_0}{\Delta} k_4,$$

$$k_7 = \frac{\omega L_0}{\Delta} k_2 - \frac{R_0}{\Delta} k_4, \quad k_2 = \frac{\delta_1 \beta_2 - \delta_2 \beta_1}{\psi},$$

$$k_4 = \frac{\delta_2 \alpha_1 - \delta_1 \alpha_2}{\psi}, \quad k_6 = -G_0, \quad \text{and} \quad k_8 = -\omega C_0,$$

where

$$\Delta = R_0^2 + (\omega L_0)^2,$$

$$\psi = \alpha_1 \beta_2 - \alpha_2 \beta_1,$$

$$\alpha_1 = -Z_l^{\text{Re}} \frac{\omega L_0}{\Delta} I_l^{1\text{Im}} + Z_l^{\text{Im}} \frac{R_0}{\Delta} I_l^{1\text{Re}},$$

$$\alpha_2 = V_l^{2\text{Re}} + Z_l^{\text{Re}} \frac{R_0}{\Delta} I_l^{1\text{Re}} + Z_l^{\text{Im}} \frac{\omega L_0}{\Delta} I_l^{1\text{Im}},$$

$$\beta_1 = V_l^{2\text{Im}} + Z_l^{\text{Re}} \frac{R_0}{\Delta} I_l^{1\text{Im}} + Z_l^{\text{Im}} \frac{\omega L_0}{\Delta} I_l^{1\text{Re}},$$

$$\beta_2 = Z_l^{\text{Re}} \frac{\omega L_0}{\Delta} I_l^{1\text{Re}} - Z_l^{\text{Im}} \frac{R_0}{\Delta} I_l^{1\text{Im}},$$

$$\delta_1 = -Z_l^{\text{Re}} \omega C_0 I_l^{2\text{Im}} - Z_l^{\text{Im}} G_0 I_l^{2\text{Re}},$$

$$\delta_2 = -V_l^{1\text{Re}} - Z_l^{\text{Re}} G_0 I_l^{2\text{Re}} + Z_l^{\text{Im}} \omega C_0 I_l^{2\text{Im}}.$$

From these relationships, if the parameters R , L , G , C , and Z_ℓ are known, then the spatial variations of V and I can be determined. This is what was done in the numerical simulation models discussed in the main body of the report for the simplified case with $R = G = 0$.

The reflection coefficient Q at frequency ω for the position $x = 0$ was evaluated using

$$Q_0 = \frac{(V_0/I_0) - Z_0^{\text{ch}}}{(V_0/I_0) + Z_0^{\text{ch}}},$$

where $Z_x^{\text{ch}} = (Z_x/Y_x)^{1/2}$ is the characteristic impedance of the transmission line at position x .

List of Symbols

a_1, a_2, a_3	Constants
a_i	Inner radius of coaxial cable
a_o	Outer radius of coaxial cable
C	Capacitance
G	Conductance
I	Current
k_1-k_8	Constants
l	Length of cable
L	Inductance
Q	Reflection coefficient
R	Resistance
S	$ Q_a - Q_e $
T	Temperature
$T_1(x), T_2(x)$	Linearly independent solutions of second-order differential equations governing voltage and current
V	Voltage
x	Distance along cable
Y	Admittance
Z	Impedance
ϵ_r	Relative dielectric constant
ω	Angular frequency

Subscripts

a	Actual
e	Estimate
i	Inner
l	At $x = l$
o	Outer
x	At position x along cable
0	At $x = 0$

Superscripts

Im	Imaginary part of complex number
Re	Real part of complex number
1, 2	Solutions T_1 and T_2
ch	Characteristic

NOTICE

"This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research & Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately-owned rights."

Printed in the United States of America

Available from

National Technical Information Service

U.S. Department of Commerce

5285 Port Royal Road

Springfield, VA 22161

Price: Printed Copy \$; Microfiche \$2.25

<u>Page Range</u>	<u>Domestic Price</u>	<u>Page Range</u>	<u>Domestic Price</u>
001-025	\$ 3.50	326-350	10.00
026-050	4.00	351-375	10.50
051-075	4.50	376-400	10.75
076-100	5.00	401-425	11.00
101-125	5.25	426-450	11.75
126-150	5.50	451-475	12.00
151-175	6.00	476-500	12.50
176-200	7.50	501-525	12.75
201-225	7.75	526-550	13.00
226-250	8.00	551-575	13.50
251-275	9.00	576-600	13.75
276-300	9.25	601-up	*
301-325	9.75		

*Add \$2.50 for each additional 100 page increment from 601 to 1,000 pages:
add \$4.50 for each additional 100 page increment over 1,000 pages.