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(p,2p) EXPERIMENTS AT THE UNIVERSITY OF MARYLAND CYCLOTRON

BY

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(p,2p) Experiments at the University of Maryland Cyclotron^{†*}

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I plan to discuss some of the (p,2p) work which has been carried out at the Maryland Cyclotron. As the speaker I will be presenting the work of a number of other physicists at Maryland as will be indicated in the references. As the first speaker on specifically (p,2p) reactions I will first present a brief introduction to the reaction, and the types of experimental techniques utilized in (p,2p) studies. Also I will give a brief introduction to the various theoretical treatments presently available to analyze (p,2p) reaction data. Secondly, I will present experimental and theoretical studies of (p,2p) on d, ^3He , and ^4He carried out by the Maryland group. Thirdly, I will discuss (p,2p) results for ^6Li , ^7Li , and ^{12}C at 100 MeV. Fourthly, I want to specifically discuss the effects of distortion on the experimental data by presenting theoretical calculations for ^{12}C and ^{40}Ca at various bombarding energies. Finally, I will briefly summarize.

I. Introduction

Figure 1 shows the kinematics of the three-body (p,2p) reaction. Assuming that we have three bodies in the final state, we have the energy-momentum conservation shown. A measurement of the two outgoing protons (energies and angles) provides a complete determination of the three body final state, i.e., the momentum $\vec{p}_3$ of the core. I might mention that if the core is unbound so that actually we have a four-body final state, the measurement of the two protons still determines the center-of-mass momentum of the core particles and their relative energy, i.e., only the angles of the core particles are undetermined. Thus the experiment consists of putting two proton detectors at a pair of angles and measuring the energies of coincident protons.

Figure 2 shows what one might expect in a typical experiment.

In a contour plot of E_1 vs. E_2 all events for a specific final state in the core lie on a 3-body kinematic locus. Thus the various states in the final nucleus lie on separate loci as I have indicated in the figure. This is pure kinematics. It is the distribution of events along these loci which reflects the reaction mechanism and therefore the "physics." At higher excitation energies in the core we reach the region for which 4-body breakup is possible as I have schematically indicated.

For each point on the $E_1 - E_2$ plane the recoil momentum and therefore recoil energy E_3 is determined. Thus, by calculating E_3 , we can use these data to form the binding energy spectrum as shown in the figure. This presentation of the data clearly shows the relative excitation of the various final states in the core. For any particular final state the data can be projected onto one of the proton energy axes forming the triple differential cross section $\frac{d^3\sigma}{d\Omega^2 dE}$. Such a spectrum as indicated in the figure is generally referred to as an energy sharing spectrum for obvious reasons. These data may also be plotted as a function of recoil momentum instead of E_1 , since the recoil momentum is uniquely determined.

Now let me turn to the theory - at least the theory of direct proton knockout which we hope is applicable in some form or other. Figure 3 shows the first order knockout diagram, which although it is a tremendous oversimplification, we still hope that this diagram dominates the reaction mechanism. I should mention that considerations based on this diagram do provide and have provided most of the experimental ideas for testing the reaction mechanism. The lower vertex represents the virtual breakup of the target nucleus into a proton and the core in some particular state. Thus

this vertex contains the target nuclear structure information. The upper vertex represents an off-the-energy shell interaction of two protons. If this diagram were applicable, one can clearly see the great wealth of nuclear physics available in such (p,2p) studies, i.e., both nuclear structure information about target nuclei, and nucleon-nucleon interaction information.

In the plane wave impulse approximation (PWIA) or spectator model the cross section for this diagram can be written as:

$$\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_1} = (\text{PSF}) \times \left. \frac{d\sigma}{d\Omega} \right|_{\text{p-p}}^{1/2} \times |\phi(-\vec{p}_3)|^2 \quad (1)$$

where $\left. \frac{d\sigma}{d\Omega} \right|_{\text{p-p}}^{1/2} \propto |\langle \vec{p}_1 \vec{p}_2 | t | \vec{p}_0, -\vec{p}_3 \rangle|^2$

and $\phi(-\vec{p}_3) \propto \int e^{i\vec{p}_3 \cdot \vec{R}} \langle \psi_C | \psi_A \rangle d^3R$

PSF is a kinematic factor which can be calculated from known quantities. The cross section $\left. \frac{d\sigma}{d\Omega} \right|_{\text{p-p}}^{1/2}$ in this factorized form is properly a half-shell cross

section as indicated. Generally one replaces $\left. \frac{d\sigma}{d\Omega} \right|_{\text{p-p}}^{1/2}$ by some on-shell prescription, the two most popular being the initial energy prescription

$\left(\left. \frac{d\sigma}{d\Omega} \right|_{\text{p-p}} \right)$ is taken to correspond to the initial state of the two protons before the collision), and the final energy prescription $\left(\left. \frac{d\sigma}{d\Omega} \right|_{\text{p-p}} \right)$ is taken to correspond to the relative energy and angle of the two protons in the final state). However,

one can calculate the half-shell cross section for various nucleon-nucleon potentials. Redish, Stephenson, and Lerner¹⁾ have shown that these prescriptions and the half-shell cross section can differ significantly - particularly at lower energies. I will show some of these effects later when comparing to experimental data.

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The quantity $\phi(-\vec{p}_3)$ represents the Fourier transform of the overlap integral between the initial and final nuclei, and is therefore simply the single particle wave function in momentum space. The simplicity of this expression arises from the assumption of non-interaction with the core, i.e., the core acts only as a spectator. Thus one can equate the momentum of the target proton before the collision to minus the momentum of the core after the collision, a quantity which is measured experimentally. It would be nice if this diagram were adequate; however, it has been shown many times that one cannot ignore the interaction of the protons with the core, even for $d(p,2p)n$. Thus, we must extend to a more sophisticated theory. One possible extension is to a multiple scattering theory²⁾ where one explicitly includes double scattering, triple scattering, etc. This procedure has been carried out for deuterium using the Fadeev equations as I will show later. However, at this time the procedure is too difficult to carry out any very realistic calculation.

The most direct, and numerically simplest procedure, is to add distorted waves in order to take account of the interaction of the protons with the core. In the distorted wave impulse approximation (DWIA)^{3,4,5)} one retains the factorization approximation. Thus the three-body cross section expression maintains the same form as the PWIA (Eq. 1). However, ϕ is now the so-called distorted momentum distribution

$$\phi \propto \int \chi_1^{(-)*}(\vec{r}) \chi_2^{(-)*}(\vec{r}) U(\vec{r}) \chi_0^{(+)}(\vec{r}) d^3r$$

where the χ 's are distorted waves for the incoming and outgoing protons calculated using potentials which fit elastic scattering and $U(\vec{r})$ is the bound proton single particle wave function. In the limit of zero distortion (zero well depths), this

expression reduces to that of the PWIA. A number of DWIA calculations have been carried out over the years and provide rather good fits to the experimental data. These calculations clearly show the necessity of including the distortion introduced by the core.

Finally, if a suitable representation of the nucleon-nucleon interaction in coordinate space could be found, one can remove the factorization approximation, put the interaction into the calculation, and carry out the full 6-dimensional calculation, a rather time consuming computation compared to DWIA.

$$\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_1} \propto \left| \int \chi_1^{(-)*}(\mathbf{r}_1) \chi_2^{(-)*}(\mathbf{r}_2) t(\vec{\mathbf{r}}_1, \vec{\mathbf{r}}_2) U(\vec{\mathbf{r}}_2) \chi^{(+)}(\vec{\mathbf{r}}_1) d^3r_1 d^3r_2 \right|^2$$

Such calculations (DWTa) have been carried out by McCarthy⁶⁾ using a pseudo-potential in the form of a sum of Yukawa interactions. The parameters were chosen to reproduce the energy dependence of the 90° p-p scattering cross section. Unfortunately this pseudo-potential does not reproduce the angular dependence of the p-p cross section, and therefore one worries about its general applicability. Later I will present some comparisons of the DWIA and DWTa to show the accuracy of the factorization approximation - at least within the framework of this particular pseudo-potential. Let me just make two comments at this point.

- (1) In my opinion one should abandon this coordinate space pseudo-potential for (p,2p) and work in momentum space where the nucleon-nucleon t-matrix could be parameterized. Calculations such as those of Koshel⁷⁾ where the distorted waves are expanded in a plane wave set should be ideal for properly investigating the factorization approximation.
- (2) I hope the factorization approximation is adequate, so that one can continue to carry out the relatively simple DWIA calculations.

Now let me turn to experimental techniques. The great advantage of 3-body breakup reactions such as (p,2p) compared to the equivalent transfer reaction lies in their greater flexibility. For example, independent of the bombarding energy one can always choose angles to provide momentum matching, so that one can investigate the target nucleon with zero momentum. Thus it is possible to choose a high energy to minimize distortion, and still investigate target nucleon momenta from 0 MeV/c to some very large value. In addition, the flexibility of a second angle allows one to make a much more detailed investigation of the reaction mechanism.

Figure 4 shows three typical experimental studies of the (p,2p) reaction (or any 3-body reaction). These studies are based on the dominance of the first order diagram (or the DWIA equivalent). The first experiment is the energy sharing experiment. In this experiment one fixes the detectors at equal proton angles such that zero recoil momentum is kinematically allowed. Such angles I will refer to as quasi-free angles. Then one measures the $\frac{d^3\sigma}{d\Omega^2 dE}$ as a function of the energy of one proton. For this particular arrangement the two-body cross section does not vary dramatically in energy or angle over the range of energy sharing, and thus the structure of the cross section is dominated by ϕ^2 (the distorted momentum distribution). [As a specific example for 200 MeV $^{12}\text{C}(p,2p)^{11}\text{B}$ 1s knockout (BE = 34 MeV) for detected proton energies from $E_1 = 15$ MeV to the equal energy point $E_1 = 83$ MeV, one needs the p-p cross section (Final Energy Prescription) over an angular range of $76^\circ \rightarrow 90^\circ$, and a lab energy from 126 MeV $\rightarrow$ 139 MeV. This range does not lead to a large variation in $\left. \frac{d\sigma}{d\Omega} \right|_{pp}$. I present what one might typically expect for $L = 0$ and $L \neq 0$ proton knockout. Thus, in this experiment one is primarily isolating the lower vertex. One inconvenient feature of this technique is the need to know the outgoing proton optical potentials at a variety of energies.

The second experiment which I call a quasi-free angular distribution, attempts to isolate the upper vertex. In this experiment one measures the 3-body cross section for a variety of quasi-free angles. Assuming distortion effects are not too angular and energy dependent (or at least that we can calculate them) so that for a fixed value of recoil momentum p_3, ϕ^2 is approximately constant, one can plot $\frac{d^3\sigma}{d\Omega^2 dE} / \text{PSF}$ versus the 2-body p-p center-of-mass scattering angle. In the impulse approximation this quantity should be proportional to $\left. \frac{d\sigma}{d\Omega} \right|_{p-p}^{1/2}$. In the sketch I have indicated what one expects for PWIA. This experiment can provide a rather nice test of the factorization approximation, especially since the flatness in angle of the p-p scattering cross section must arise from a delicate cancellation of the p and d waves, and has been rather successfully applied to α -particle knockout.^{8,9,10} Unfortunately, to my knowledge, with the exception of d(p,2p) and d(p,pn) such a (p,2p) experiment has never been performed, and clearly should be.

Finally I show the canonical arrangement of the coplanar-symmetric angular distribution. In this experiment one measures the (p,2p) cross section for $\theta_1 = -\theta_2$ and $E_1 = E_2$ for a variety of equal angle pairs. Each angle corresponds to a different value of recoil momentum which is either parallel or anti-parallel to the incident beam direction. Again considering this study from the 1st order diagram we see that both vertices vary as a function of angle, the lower vertex because of the changing recoil momentum and the upper vertex because of the changing relative energy of the two protons. In this arrangement the 2-body p-p c.m. scattering angle is always 90° . However, the p-p relative energy can change dramatically. Again using 200 MeV $^{12}\text{C}(p,2p)$ 1s knockout we find that over an angular range from $\theta_1 = -\theta_2 = 15^\circ$ to 60° the p-p energy in the final state varies from $E_{\text{lab}} = 24$ MeV to 256 MeV. Schematically I

have indicated what one might expect to see for the knockout of an s-state proton. At the larger angles the two-body cross section is fairly constant, since $\left. \frac{d\sigma}{d\Omega} \right|_{p-p} (90^\circ)$ is rather flat at higher energies. Thus the structure is dominated by ϕ^2 . However, at the smaller angles the relative energy between the two protons gets smaller therefore dramatically increasing the 2-body cross section, as I have indicated. In this small angle region one might hope to gain information on the off-shell behavior of the p-p cross section, since the reaction can be very far off-shell. However, many other effects such as distortion are extremely important in this region. I should point out that since the outgoing proton energies are nearly constant, one only needs the optical potential at one energy. Although this is the canonical experiment, I hope to convince you later that this is not the experiment to do first.

II. A = 2, 3, 4

Now let me turn to some experimental results for $d, {}^3\text{He}$, and ${}^4\text{He}$. First we consider $d(p,2p)n$ and $d(p,pn)p$, which I treat separately since we do have multiple scattering calculations, as opposed to DWIA for the other cases. Figure 5 shows the energy sharing cross sections for $d(p,2p)n$ at the quasi-free equal angle pair for three energies. The curves are PWIA calculations using a Hulthen wave function normalized to the data by the factor indicated in the figure. Clearly the PWIA is inadequate even for $d(p,2p)n$ being too large by a factor which is energy dependent, and not providing a terribly good representation of the shape. While at Maryland John Wallace performed exact 3-body multiple scattering calculations¹¹⁾ for these data. His calculations used only a separable s-wave nucleon-nucleon interaction but the strength was modified to reproduce the on-shell nucleon-nucleon cross section. Using this interaction the full multiple scattering series was summed. Let me just show some examples. Figure 6 shows the $(p,2p)$ and (p,pn) energy sharing results at 65 MeV. Again one sees the inadequacy of single scattering--particularly for $(p,2p)$. The inclusion of the full multiple scattering series provides extremely good agreement with the data. Figure 7 shows $(p,2p)$ energy sharing results at 100 MeV¹²⁾, and again the multiple scattering calculation provides excellent agreement with experiment. Next I present (Fig. 8) results for a non-quasi-free angle pair for which we have a p-n final state interaction. In this case the multiple scattering series fluctuates wildly, but the full calculation provides at least reasonable agreement. Finally I show some quasi-free angular distribution¹²⁾ results in Fig. 9. Again one sees the tremendous improvement gained by including multiple scattering, but we also see significant discrepancies. One can at least qualitatively understand them as

arising from (a) the neglect of the Coulomb interaction and (b) the use of pure s-wave for the p-n interaction (i.e., the p-n cross section is not flat as a function of angle as is the case with p-p scattering). Further results can be found in Ref. (12). In order to do any more we await a three-body calculation which includes higher nucleon-nucleon partial waves. In fact in my opinion experiment is so far ahead of three-body theory that we can wait several more years before providing any additional data on deuterium.

Now let me turn to ^3He , $^4\text{He}(p,2p)$. Figure 10 shows the energy sharing cross sections for d, ^3He and ^4He at 100 MeV.¹³⁾ The curves are PWIA calculations (Irving-Gunn ^3He wave function, and Gaussian ^4He wave function) normalized to the data by the factors indicated. One sees the pronounced increase in width as the binding energy increase to 20 MeV for ^4He . In fact at 100 MeV the energy is not sufficiently high to provide much information on the ^4He wave function. Figure 11 shows the ratio of the experimental cross section to the PWIA calculation as a function of energy. This clearly shows the reduction in multiple scattering effects as the energy increases, and the increase in multiple scattering as the A of the target increases, although detailed comparisons are not valid due to the inadequacy of the wave functions used - particularly for ^4He .

The data points were obtained using the final energy prescription for $\left. \frac{d\sigma}{d\Omega} \right|_{p-p}$. If one uses the half-shell cross section instead, one obtains the curves represented by the dashed lines. Only for ^4He which is bound by 20 MeV does the 1/2-shell cross section make an important difference, the difference decreasing with increasing energy.

The rapid energy dependence of ^4He , plus a Phys. Rev. Letter¹⁴⁾ which reported a fit to the ^4He data with the extremely simplified attenuation model prompted me to try DWIA calculations for ^4He . Without going into many details

I performed DWIA calculations¹⁵⁾ as a function of energy using the proton single particle wave function of Lim¹⁶⁾. This wave function is of the Eckart form with parameters chosen to fit electron scattering. The wave function includes short range correlations, and thereby reproduces the minimum observed in electron scattering at high momentum transfer. In PWIA the Lim wave function yields a cross section about a factor of two smaller than the Gaussian wave function used in the previous analysis, and thereby raises the ^4He curve. The optical potentials were obtained from fits to elastic scattering at lower energies, and extended to higher energies using the impulse approximation.

Figure 12 shows the results at various energies. Here I present

$\left. \frac{d^3\sigma}{d\Omega^2 dE} \right|_{\text{exp}} / \text{PSF} \times \left. \frac{d\sigma}{d\Omega} \right|_{\text{p-p}}^{1/2}$ for the equal angle zero recoil momentum point. The solid curve represents the DWIA calculation, and the dashed curve is DWIA $\times 1/2$, or if you like, assuming a proton spectroscopic factor of 1. Unfortunately the conclusions are not obvious. Clearly there is a discrepancy between the 480 MeV and 600 MeV data, and I would like to see the experiment repeated in this energy range. If the 600 MeV data is correct, then the wave function of Lim is probably reasonable, and DWIA does not work at lower energies. If the 480 MeV data is correct, then the shape predicted by the DWIA is quite reasonable although interesting detailed discrepancies arise at the lowest energies (see the talk of E. F. Redish in this same meeting). Thus perhaps we are showing the inadequacy of the single particle wave function, thereby gaining information on the single particle wave function in ^4He . I feel that a good, detailed higher energy study of the $^4\text{He}(p,2p)t$ reaction is important, and has the possibility of providing very good information on the ^4He single particle wave function.

Let me explicitly point out the interesting energy dependence predicted by the DWIA. Assuming applicability of the DWIA we find little to be gained in terms of absorption by increasing the bombarding energy beyond 100 MeV to 150 MeV. I will return to this point when I discuss distortion effects in general.

III. A = 6, 7, 12

At Maryland we have also made detailed investigations¹⁷⁾ of the (p,2p) reaction at 100 MeV on ${}^6\text{Li}$, ${}^7\text{Li}$, and ${}^{12}\text{C}$. In Fig. 13 I present the binding energy spectra for these three nuclei. The experimental resolution is only about 1 MeV due to the use of NaI detectors. The low lying states arise principally from p-shell proton knockout, although the 5/2 - and 7/2 - states in ${}^{11}\text{C}$ are clearly excited - presumably by a 2-step process. Redish will discuss these states in his talk. At high excitation energies we see the contribution from the s-state proton knockout. This stands out very clearly for ${}^6\text{Li}$ and ${}^7\text{Li}$, but is barely visible above a rather constant background in ${}^{12}\text{C}$.

Discussing first the p-state knockout, I want to give special consideration to ${}^6\text{Li}(p,2p){}^5\text{He}$ which is very special since the transition goes to an unbound state. Thus although there is a resonance in the $p_{3/2}$ n- α channel (${}^5\text{He}$ "ground state"), there are also $S_{1/2}$ and $P_{1/2}$ components in the n- α system. Therefore the ${}^5\text{He}$ ground state region is really a mixed angular momentum "state". Now if the ${}^6\text{Li}$ ground state is not pure $(1s)^4(1p)^2$ but contains configurations such as $(1s)^4(2s)^2$ as it would assuming an $\alpha + d$ cluster structure, we can have a mixed angular momentum transition. Thus we can not have just $L = 1$ transitions, but also $L = 0$. Near $p_3 = 0$ the $L = 0$ cross section is much larger than the $L = 1$ cross section ($L = 1$ is zero in

the PWIA), and thus small components of $L = 0$ will be visible. To make a long story short, this does happen and has led to much discussion over the years. We^{18,19)} have analyzed our ${}^6\text{Li}$ data basically following the procedure of Saito, et al.²⁰⁾ but including distortion effects by means of the DWIA as well as a better n - α wave function. In particular the n - α unbound final state for each $E_{n-\alpha}$ was obtained from Woods-Saxon potentials which reproduce the n - α phase shifts, including the resonances. An $\alpha + d$ cluster model was used for the ${}^6\text{Li}$ ground state, thereby introducing the configuration mixing.

Figure 14 shows calculations of the binding energy spectrum (the experimental resolution has been folded in) for a particular pair of angles. We see that although the peak contains a predominance of $P_{3/2}$ state, there is a large contribution from the s -wave. As we go to higher n - α energies, the fit is not very good, and I believe arises from lack of the model to provide the proper proton separation energy.

Figure 15 shows basically the energy sharing spectra for a number of equal angle pairs. Here we have plotted $\frac{d^3\sigma}{d\Omega^2 dE} / \text{PSF}$ versus the recoil momentum p_3 . The curves are the DWIA calculations with a constant normalization. At the quasi-free angle we see that there is only a very weak minimum, and that this is well reproduced by the calculation which includes the s -wave contribution to fill in the minimum. I should emphasize that the lack of a minimum is not due to experimental angular and energy resolution. At forward angles we see that the slope of the data increases and this effect is included in the DWIA. At the backward angles the distribution flattens out and the calculations do not provide quite as good agreement.

Taking the minimum recoil momentum point for each angle pair we obtain the coplanar symmetric angular distribution which is shown in Fig. 16.

The curves at the top represent the various prescriptions for $\left. \frac{d\sigma}{d\Omega} \right|_{p-p}$ and we see that the differences are not too large for this case which is not far off-shell. Again I present the DWIA calculation and this time show the s- and p- knockout contributions separately. I think this figure speaks for itself. The fit is magnificent, and clearly shows the necessity for including the s-state contribution. These data and calculations are probably the best evidence for configuration mixing, or the need for an $\alpha + d$ cluster model, in the ${}^6\text{Li}$ ground state. This use of the unbound state provides a very powerful technique, but unfortunately is not very common. The measurement is clearly best when one is looking for a little s-state in the presence of an $L \neq 0$ transition to an unbound state -- not such an easy case to find. (We do find the same problem in the ${}^{12}\text{C}(p,p\alpha){}^8\text{Be}$ reaction to the $2+$ region in which s-waves can contribute for the α - α system.) It also clearly points to possible difficulties in the analysis of $(p,2p)$ reactions to unbound states, especially if only a couple of final state channels are open. Thus one may get knockout contributions which are not isotropic and may lead to erroneous conclusions from attempting to subtract a constant "background".

Figure 17 shows the ${}^7\text{Li}(p,2p){}^6\text{He}(\text{g.s.})$ energy sharing distribution.²¹⁾ In this case we see a very pronounced minimum showing that the filling in of the minimum in ${}^6\text{Li}$ arises from neither experimental resolution nor distortion effects. The two curves are DWIA calculations for different proton bound state wave functions. Again the fit is reasonably good.

Figure 18 shows the coplanar symmetric angular distribution²¹⁾. We see that the fit is reasonably good although it deteriorates some at forward angles, depending upon the prescription for $\left. \frac{d\sigma}{d\Omega} \right|_{p-p}$. The spectroscopic factor is 0.24, but this can be increased by changing the bound state parameters

with no real deterioration in the fit.

As a quick aside, in Fig. 19, I show the energy sharing distribution for the 1st excited state (2+) of ${}^6\text{He}$ which is unbound, unlike the ${}^6\text{He}$ ground state. We again see that the minimum disappears and I think that this again arises from s-state contributions, although no detailed analysis has been attempted due to the 2n final state.

Finally for p-state knockout I present the results²¹⁾ for the ${}^{12}\text{C}(p,2p){}^{11}\text{C}(\text{g.s.})$ reaction. Figure 20 shows energy sharing spectra which are reasonably well described by DWIA, and Fig. 21 shows the coplanar symmetric angular distribution and the DWIA calculation with $C^2S \approx 2.8$ in good agreement with shell model predictions. The fit beyond the minimum is extremely good, but as in the case of ${}^7\text{Li}$ it deteriorates at forward angles, although it is worse in this case.

I believe that this deterioration at forward angles arises from a breakdown in the factorization approximation at forward angles at 100 MeV. In order to examine this possibility we carried out DWIA and DWTA calculations¹⁷⁾ with McCarthy's code and pseudo-potential. The results for ${}^7\text{Li}$ and ${}^{12}\text{C}$ are presented in Fig. 22. We see that at least for this not very realistic pseudo potential, the breakdown in factorization is primarily at forward angles and is in such a direction as to improve agreement with the experimental results. Thus at larger angles the factorization is probably quite good, but one should be very careful of the forward angle data. I should also mention that the shape of the calculation at these same forward angles is most sensitive to changes in the distorting potentials. Thus the forward angle data taken in the coplanar-symmetric geometry show many sensitivities, and thoughts of extracting off-shell nucleon-nucleon information from this region probably should be considered pipe dreams.

For s-state knockout we have integrated over the broad peak at high excitation energies shown in Fig. 13. Fig. 23 shows the available data in the coplanar symmetric geometry. The data for ^{12}C was not systematically analyzed, but has a nearly constant magnitude of $\sim 25 \mu\text{b}/\text{sr}^2 \text{ MeV}$ over the angular range indicated. I have also included the cross section for $^4\text{He}(p,2p)t$ which is basically the core for these other nuclei. These data show quite a nice systematic trend and are just about what one would expect due to an increase in distortion effects as the mass of the target is increased. Unfortunately, we have not performed a systematic DWIA calculation for these data. However, the calculations which have been done very nearly reproduce this behavior.

Fig. 24 shows some energy sharing spectra for the ^6Li s-state knockout. The curves are DWIA calculations. In this case we have the same trend as we had for p-state knockout; viz., at forward angles the data is steeper. Unfortunately the DWIA calculations do not reproduce this trend. However, at larger angles the agreement is reasonably good. Fig. 25 shows the coplanar symmetric angular distribution for this state. The curves above show the 2-body cross section $\left. \frac{d\sigma}{d\Omega} \right|_{\text{p-p}}$ for the various prescriptions. Here we see that at the forward angles the differences are quite pronounced. The curve through the data is a DWIA calculation normalized to the data. The normalization factor is almost the same as that used for ^4He (within 20%). The DWIA calculation uses the 1/2-shell cross section, and our initial reaction to the quality of the fit was exultation over the necessity of using this $\left. \frac{d\sigma}{d\Omega} \right|_{\text{p-p}}^{1/2}$. However, after our later experiences with the sensitivity of the forward angles to the factorization approximation and the optical potentials we were suitably deflated. Thus this data does not prove the necessity of employing the 1/2-shell cross section. The s-state

knockout data for ${}^7\text{Li}$ is very similar and I will not bother to show the results.

IV. General Distortion Effects

In an attempt to study general distortion effects in (p,2p) reactions we have carried out a number of DWIA calculations at various energies using the code of N. S. Chant.⁵⁾ In particular we have calculated ${}^{12}\text{C}(p,2p)$ for $1\text{P}_{3/2}$ and $1\text{S}_{1/2}$ knockout, ${}^{40}\text{Ca}(p,2p)$ for $2\text{S}_{1/2}$ and $1\text{S}_{1/2}$ knockout, and ${}^{208}\text{Pb}(p,2p)$ for $3\text{S}_{1/2}$ knockout. The proton bound state Woods-Saxon well parameters were taken from Elton and Swifts' shell model analysis.²²⁾ The optical model potentials for the entrance and exit channels were painfully obtained. For ${}^{12}\text{C}$ we took the geometrical parameters from a 61 MeV p-scattering analysis of Fulmer, et al.²³⁾ The well depths as a function of energy were chosen to smoothly reproduce the volume integrals (real and imaginary) of analyses²⁴⁾ of data from 30 MeV to 1 GeV. The same procedure was followed for ${}^{40}\text{Ca}$ and ${}^{208}\text{Pb}$ using the optical model analyses of Refs. 25 and 26. Thus the potentials have a nice systematic variation with energy. Where possible, or worthwhile, calculations were carried out at 50, 100, 150, 200, 300, and 400 MeV.

Fig. 26 shows the ratio of the DWIA calculation to the PWIA calculation as a function of energy. The calculations correspond only to equal angle quasi-free angle pairs at zero recoil momentum. We see the same trend as I showed in the case of ${}^4\text{He}$; viz., there is a rapid increase at lower energies, and then the ratio flattens. Thus above a certain energy, the reduction in cross section due to distortion effects remains constant. The details of course depend on the binding energy and the A of the nucleus. Thus for ${}^{12}\text{C}$ 1-S knockout (BE $\approx$ 34 MeV) the reduction is smallest. Whereas, the 1S knockout for ${}^{40}\text{Ca}$ (EB $\approx$ 60 MeV) is most strongly reduced and one must go to 300 or 400 MeV to reach the flat portion of the curve.

Based on this display one would argue that from a spectroscopy standpoint a bombarding energy of the order of 150 to 200 MeV is adequate for most (p,2p) studies and there is no need to go to a higher energy where the experiments are generally more difficult. From a reaction mechanism standpoint I think it would be interesting to study the (p,2p) reaction from 50 to 200 MeV, to see if the DWIA reproduces this behavior.

In an attempt to understand this behavior and to understand the region of sensitivity of the (p,2p) reaction, we have calculated the (p,2p) cross section contribution as a function of cutoff radius. The code prints out this information for each calculation, so no additional calculations are required. In Fig. 27 we have plotted the quantity $\Delta\sigma = \sigma(R_{\text{cutoff}} = r + 0.5) - \sigma(R_{\text{cutoff}} = r)$ as a function of radius. This plot should be at least an indication of the sensitivity of the reaction to various radial regions. The calculations correspond to zero recoil momentum for the equal angle quasi-free angle pair for the $2S_{1/2}$ and $1S_{1/2}$ states of ^{40}Ca . The top distribution corresponds to PWIA with no distortion effects. Let me note the following. First there is a large change in scale between the lower and higher energies. Secondly, at lower energies in addition to the reduced contribution, there are clearly pronounced phasing effects. This presumably arises from the importance of the real potential at these lower energies - the real potential is comparable to the energies of the particles. Thirdly, at the higher energies the $\Delta\sigma_{\text{DWIA}}$ begins to rather closely resemble the PWIA calculation. At these energies the real well is small compared to the proton energy, and the distortion effects are dominated by absorption. Thus we just have a gradual, but important, reduction as we penetrate into the nucleus. Finally, no matter what the energy the nuclear interior plays a significant role in some fashion.

The second study we made was to carry out detailed calculations at 100 MeV and 200 MeV for both the coplanar symmetric angular distribution (CSAD) and the energy sharing/distribution^(ES) at the equal angle quasi-free angle pair. The calculations were carried out for an angular range (CSAD) of roughly 15° to 60° , and an energy range (ES) from about 20 MeV to the equal energy point. In order to facilitate the comparisons I have plotted the calculated distorted momentum distributions ϕ^2 versus recoil momentum p_3 . For the energy sharing distribution at equal angles the distribution must be symmetric about $p_3 = 0$ due to the identity of the two protons. For the CSAD I have plotted the forward angles to the left of $p_3 = 0$ and the large angles to the right of $p_3 = 0$.

Fig. 28 shows the calculated ϕ^2 for $P_{3/2}$ knockout from ^{12}C . The dashed curve is the PWIA calculation (i.e., the square of the Fourier transform of the wave function) normalized by the factor indicated. At 100 MeV we see large differences between DWIA and PWIA, and we see that the minimum in the CSAD is shifted a great deal. In neither case can one say that the ϕ_{DW}^2 reflects the true momentum distribution. However, at 200 MeV the ES calculation is strikingly similar to the PWIA ϕ^2 . The CSAD is not nearly as good being too low at forward angles and too high at large angles. Fig. 29 shows the same results for $1S_{1/2}$ knockout from ^{12}C . My comments are the same as before. The 100 MeV calculation is significantly different than PWIA, and at 200 MeV the ES distribution most clearly reflects the true momentum distribution.

Fig. 30 shows the $2S_{1/2}$ knockout from ^{40}Ca at 100 MeV. Here the ES distribution is most reasonable, but shows no evidence of the nodes in the wave function. Fig. 31 shows the same state at 200 MeV. In this case we see the very lovely feature of nodes (or at least minima) in the ES distribution. In fact the ES distribution reflects most of the important features of the distribution. The CSAD on the other hand would lead to extremely misleading results,

if interpreted in a plane wave formalism.

The final figure (Fig. 32) shows the $3S_{1/2}$ knockout from ^{208}Pb at 200 MeV. Again the results are similar to those I presented previously.

I think that the conclusions from this array of figures is obvious. In terms of spectroscopy one wants to perform the ES experiment, not the CSAD, around 200 MeV. This is nice since the ES experiment is generally easier, since the data can be taken in one shot. Doing this experiment first, one can obtain information on the wave function. Then, one can carry out the CSAD in order to study the reaction mechanism, distortion effects, factorization, and/or off-shell effects. I might also add that I expect the ES experiment to be less sensitive to a factorization breakdown, since one is really using the two-body cross section over a relatively small range of energy and angle, and at a large enough angle where the breakdown does not appear to be too great. It is unfortunate that almost all experiments have concentrated on the CSAD[†], when in fact the simpler ES experiment should have been carried out first.

V. Conclusions

Very briefly I hope that I have shown a number of the interesting aspects of the (p,2p) reaction. Over the past few years I believe that too little attention has been given to these reactions. A careful, systematic investigation of the (p,2p) reaction which can now be performed with high precision accelerators

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[†] Prof. Maris pointed out at this point that the original reason for choosing the CSAD was to remove polarization effects, since the p-p scattering angle is always 90°. This is an important consideration. Recent polarized (p,2p) studies at TRIUMF²⁷⁾ should indicate the importance of polarization effects in the ES distribution.

in the 100 to 400 MeV range should provide quantitative nuclear structure information previously unattainable.

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1. Kinematics of the (p,2p) reaction.

$$\vec{P}_0 = \vec{P}_1 + \vec{P}_2 + \vec{P}_3$$

$$E_0 + Q = E_1 + E_2 + E_3 + E_{ex}$$

2. Expected experimental energy sharing results for S-state proton knockout.

3. First order knockout diagram.

Types of Experiments based on $\left\{ \frac{d^2\sigma}{dn^2 dE} = \text{PSF} \cdot \frac{d\sigma}{dn} \Big|_{p-p} \cdot |\psi(-\vec{p}_3)|^2 \right\}$

1) Energy Sharing

- a) $\theta_1 = \theta_2$ fixed
- b) $\vec{p}_3 = 0$ kinematically allowed

2) Quasi-free Angular Distribution

- a) θ_1 and θ_2 varied but always chosen to allow $\vec{p}_3 = 0$
- b) $\frac{d^2\sigma}{dn^2 dE} (p_3=0) / \text{PSF}$ extracted from data

3) Coplanar Symmetric Angular Distribution

- a) $\theta_1 = \theta_2 = \theta$ varied
- b) $|\vec{p}_1| = |\vec{p}_2|$
- c) Extract $\frac{d^2\sigma}{dn^2 dE}$ for $|\vec{p}_1| = |\vec{p}_2|$

4. Typical experimental studies of the (p,2p) reaction based on the first order diagram.

5. Energy sharing spectra at three bombarding energies for the $^2\text{H}(p,2p)n$ reaction¹²⁾ at equal symmetric quasi-free scattering angle pairs. The curves correspond to PWIA calculations normalized to the peak by the factors indicated in the figure.

6. Energy sharing spectra for the $^2\text{H}(p, 2p)n$ and $^2\text{H}(p, pn)p$ reactions¹²⁾ at 65 MeV. The curves are multiple scattering calculations^{11,12} [dashed line: single scattering; dash-dot line: truncated after double scattering; dotted line: truncated after triple scattering; solid curve: full multiple scattering calculation].

7. Energy sharing spectrum for the ${}^2\text{H}(p,2p)n$ reaction¹²⁾ at 100 MeV. The curves are multiple scattering calculations.^{11,12)}

8. Energy sharing spectrum for the ${}^2\text{H}(p,2p)n$ reaction¹²⁾ at 100 MeV with angle pairs chosen to emphasize the n-p final state interaction. The curves are multiple scattering calculations.

9. Quasi-free scattering angular distributions (see Fig. 4) for the $^2\text{H}(p,2p)n$ and $^2\text{H}(p,pn)p$ reactions.

10.

Energy sharing spectra for the $(p,2p)$ reactions¹³⁾ on ^2H , ^3He , and ^4He . The curves are PWIA calculations normalized to the data by the factors indicated in the figure.

11. Ratio of the experimental (p,2p) cross sections to the PWIA calculations as a function of bombarding energy.¹³⁾ The solid curve is a smooth curve drawn through the data. The dashed curve corrects for off-shell effects.

12. Ratio $\frac{d^3\sigma}{d\Omega^2 dE} / \text{PSF} \times \frac{d\sigma}{d\Omega} \Big|_{\text{P-P}}^{1/2}$ as a function of bombarding energy for the $^4\text{He}(p,2p)$ reaction.¹⁵⁾ The full curve is a DWIA calculation; the dashed curve is the DWIA reduced by a factor of two.¹⁵⁾

13. Binding energy spectra for the ${}^6\text{Li}$, ${}^7\text{Li}$, and ${}^{12}\text{C}(p, 2p)$ reactions at 100 MeV.¹⁷⁾

14. Comparison of the experimental and calculated binding energy spectrum¹⁷⁻¹⁹⁾ for ${}^6\text{Li}(p, 2p)$ near the ${}^5\text{He}$ $3/2^-$ resonance ($S_{1/2}$, $P_{1/2}$, and $P_{3/2}$ refer to the possible $n+\alpha$ states in the region).

15. Experimental momentum distributions $\frac{d^3\sigma}{d\Omega^2 dE} / \text{PSF}$ for the ${}^6\text{Li}(p,2p){}^5\text{He}$ ("G.S.") reaction^{17,18} at 100 MeV for various equal angle pairs. The curves are DWIA calculations.

16. Coplanar symmetric angular distribution for the ${}^6\text{Li}(p,2p){}^5\text{He}$ ("G.S.") reaction^{17,18} at 100 MeV. The curves at the bottom of the figure show the DWIA (and PWIA) calculations for the two possible angular momentum states. The curves labeled $\frac{d\sigma}{d\Omega} (p,p)$ show the two-body cross section for various prescriptions.

17. Energy sharing spectra for the ${}^7\text{Li}(p,2p){}^6\text{He}(\text{G.S.})$ reaction²¹⁾ at 100 MeV. The curves are DWIA calculations.

18. Coplanar symmetric angular distribution for ${}^7\text{Li}(p,2p){}^6\text{He}(\text{G.S.})$ reaction²¹⁾ at 100 MeV. The curves represent DWIA calculations, and the two-body cross section for various prescriptions.

19. Energy sharing spectra for the ${}^7\text{Li}(p, 2p){}^6\text{He}$ (1st excited state) reaction²¹⁾ at 100 MeV. The curves are DWIA calculations for the ground state.

20. Energy sharing spectra for the ${}^{12}\text{C}(p, 2p){}^{11}\text{B}(\text{G.S.})$ reaction²¹⁾ at 100 MeV. The curves are DWIA calculations.

21. Coplanar symmetric angular distribution for the $^{12}\text{C}(p,2p)^{11}\text{B}(\text{G.S.})$ reaction²¹⁾ at 100 MeV. The curve is a DWIA calculation.

22. DWIA and DWTA calculations¹⁷⁾ for the $^7\text{Li}(p,2p)^6\text{He}(\text{G.S.})$ and $^{12}\text{C}(p,2p)^{11}\text{B}(\text{G.S.})$ reactions at 100 MeV.

23. Available coplanar symmetric angular distribution data^{13,17,18,21)} for S-state proton knockout from ^4He , ^6Li , ^7Li , and ^{12}C at 100 MeV.

24. Energy sharing spectra for S-state proton knockout from ^6Li at 100 MeV. The curves are DWIA calculations.

25. Coplanar symmetric angular distribution for S-state proton knockout from ${}^6\text{Li}$ at 100 MeV. The curves represent DWIA calculations, and the two body cross section for various prescriptions.

26. The ratio $\sigma_{\text{DWIA}} / \sigma_{\text{PWIA}}$ as a function of bombarding energy for S-state knockout from three nuclei.

27. Plot of $\Delta\sigma(=\sigma(R_0+\Delta)-\sigma(R_0))$ versus radius for 1-S and 2-S knockout from ^{40}Ca . The results are presented for the $p_3 = 0$ MeV/c point for various bombarding energies. Also shown are the single particle wave function and the PWIA results.

28. Calculated momentum distributions (PW and DW) for $P_{3/2}$ proton knockout from ^{12}C at 100 and 200 MeV bombarding energy. Distorted wave results are presented for both the energy sharing (ES) and coplanar symmetric angular distribution (CSAD) experimental arrangements.

29. Calculated momentum distributions for $1S_{1/2}$ proton knockout from ^{12}C at 100 and 200 MeV (see Fig. 28).

30. Calculated momentum distributions for $2S_{1/2}$ proton knockout from ^{40}Ca at 100 MeV (see Fig. 28).

31. Calculated momentum distributions for $2S_{1/2}$ proton knockout from ^{40}Ca at 200 MeV (see Fig. 28).

32. Calculated momentum distributions for $3S_{1/2}$ proton knockout from ^{208}Pb at 200 MeV (see Fig. 28).