CONF-770434-3

THE ANALYSIS OF MAGNETOHYDRODYNAMIC MODES IN TOKAMAKS BY X-RAY TECHNIQUES*

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Abstract

A brief review of recent studies of fluctuations in x-ray emission from tokamak plasmas of controlled thermonuclear fusion interest is given. The origin of the x-rays, the nature of the oscillations, and measurement and analysis techniques are discussed, with emphasis on the work performed on the ST and PLT tokamaks. Areas for future research, particularly in the region of reconstruction, are stressed.

Introduction

As controlled thermonuclear fusion experiments have begun producing plasmas with temperatures in excess of a few kilo-electron volts⁽¹⁾, the emission of soft x-rays has become an increasingly important process from the diagnostic viewpoint. Due to the strong dependence of the emissivity of the plasma on local values of electron temperature and the densities of the various ion species (since the processes involved include bremsstrahlung, recombination and line radiation⁽²⁾), the intensity of x-ray emission serves as a sensitive probe of conditions inside the discharge.

Unfortunately, instead of being a representation of a truly local quantity, the radiation seen by a detector consists of the emission integrated over a chord of the plasma defined by the collimation of the detector system. Fortunately, in contrast to some other methods, the data can be inverted to yield quantitative information on the relevant local conditions. The operations involved in our inversion of the line-integrated data are the topics of this report.

Figure 1 is a schematic diagram of a tokamak. The device consists of a toroidal vacuum vessel surrounded by sets of coils which produce basically toroidal and vertical magnetic fields. The plasma current flows in the toroidal (ϕ) direction, driven by an electric field induced by a time-changing toroidal current in the windings; thus, the plasma serves as a secondary of a transformer.

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Fig. 1. Diagram of a conventional tokamak, nomenclature, and coordinate system.

* This work was supported by U. S. Energy Research and Development Administration Contract E(11-1)-3073.

The major radial ($\hat{R}$) force balance is achieved by the interaction of the plasma current with a small vertical magnetic field, which reinforces the plasma current's own so-called "poloidal" magnetic field on the outside of the plasma. In general, macroscopic stability is maintained by the use of a strong toroidal magnetic field. However, some oscillations persist despite the restraining influence of the toroidal field due either to some exceptional aspect of the normal mode structure or to non-ideal effects which allow separation of the fluid from the magnetic field.

The machine has periodicity constraints in both the toroidal and poloidal directions and a symmetry condition in the toroidal direction. Hence, in the toroidal direction, the Fourier components ($\exp(-i n t)$) of the perturbations can be expanded in harmonics of the toroidal angle ($\exp(i n \theta)$). Since the poloidal coordinate is not truly ignorable, the normal modes are superpositions of poloidal harmonics ($\exp(i m \phi)$); however, due to the small ratio of the minor radius (r) of the plasma to the major circumference of the machine ($2\pi R$), the poloidal harmonics are somewhat decoupled and the total perturbation can be treated as predominantly a fixed- m mode with a finite number of harmonics of nearby m 's.

Analyses of the fluctuation data from tokamaks have been based on this separation into poloidal harmonics. A schematic is shown in Figure 2. Generally, waves appear to propagate in the toroidal and poloidal directions either due to a Doppler shift of a structure fixed in a moving fluid⁽³⁾ or due to a real frequency in the frame of the fluid⁽⁴⁾; i.e.

$$\omega_{m,n} = \omega_{m,n}^F + m\dot{\theta}^F + n\dot{\phi}^F, \quad (1)$$

where $\omega_{m,n}^F$ is the frequency in the frame of the fluid, m and n are poloidal and toroidal mode numbers, and $\dot{\theta}^F$ and $\dot{\phi}^F$ are the rotation rates of the fluid. In the case of a rigid poloidal rotation, for instance, $\omega_{m,n}^F = 0$ (or, more generally, proportional to m), $\dot{\phi}^F = 0$, and $\omega_{m,n} = m\dot{\theta}^F$; in this case the plasma appears to simply rotate before the detectors and the poloidal components can be determined by a temporal decomposition. If the frequencies are not proportional to the mode number, the poloidal analysis is more complex; several systems are possible; e.g., (1) detectors viewing several poloidal positions allow a poloidal decomposition (the resolution in m -space being limited by aliasing due to the finite number of poloidal positions being sampled), or (2) detectors viewing several toroidal positions can give poloidal information if the helicity of the perturbation is known (which limits the analysis to cases of fixed helicity). These techniques will be discussed in a later section on coupled modes.

Fig. 2. An illustration of a mode with one toroidal and one poloidal wavelength (i.e., $n = m = 1$).

Measurements of x-ray fluctuations in tokamaks have been actively pursued since roughly 1972⁽⁵⁾ when rapid symmetric ($m = n = 0$) expansions of the plasma column, accompanied by $m = n = 1$ oscillations, were observed using a pin-hole camera arrangement composed of a lead collimator with surface barrier detectors, shown in Figure 3a. The two detectors were movable so that they could view all radial positions separated by either roughly 0 or 130 degrees; other detectors viewed different poloidal and toroidal positions; by comparing the phases of the signals, the mode numbers of the dominant components were determined.

Similar measurements have been made on other machines (6, 7, 8)

Fig. 3a. Diagram of the 2-detector system used on the ST tokamak. To make a scan, one detector was moved while the other served as a monitor.

Fig. 3b. X-ray signals from outside ($r = 3.9\text{cm}$) and inside ($r = 0$) the $q = 1$ surface in a "sawtooth" discharge.

During these so-called "sawtooth discharges", the profile of the electron temperature undergoes a cyclic process. The ohmic-heating, being maximum at the radius with the lowest resistivity (i.e., highest temperature), heats the center of the discharge preferentially; the radiative cooling at the edges reinforces the tendency for the temperature profile to become more peaked. As a result, the temperature at the center slowly rises, reducing the resistivity relative to that at the edges, and the current redistributes itself to a more peaked profile. At some point, MHD or microstability processes go unstable and the peak collapses (9, 10, 11), the energy being rapidly transported outwards. This causes a sudden drop in the x-ray emission at small radii (where the temperature is reduced) and a rise in the emission at large radii (where the heat is deposited), producing the "sawtooth", or internal disruption, shown in Figure 3b (5, 12).

Figure 4a (5, 13) shows the radial profiles of the equilibrium electron temperature and density. Assuming classical resistivity and a radially uniform toroidal electric field, the plasma current density can be calculated; from this, a quantity

$$q(r) = \frac{B_T r}{B_p(r) R} = \frac{B_T c}{2 I(r) R} \quad (2)$$

the ratio of toroidal and poloidal rotation rates of the field line, can be derived (B_T is the toroidal field, B_p the poloidal field, r the minor radius, c the speed of light, R the major radius and $I(r)$ the toroidal plasma current inside radius r , with all quantities in electrostatic units). This quantity q plays a crucial role in MHD stability criteria, the general tendency being the excitation of modes with $m + nq = 0$ (i.e., large parallel wavelength). A graph of this function $q(r)$ is shown in Figure 4b. Figure 4c shows the relative amplitude of the observed sawtooth and Figure 4d the relative amplitude of the $m = 1$ sinusoidal precursor of the drop in intensity. Given the symmetry, such line-integral data can be inverted to give information on the behavior of the local emissivity. Figure 5 is a diagram of the temporal evolution of an $m = 1$ perturbation as predicted by computer codes and theory (9); the figures show contours almost equivalent to those of constant temperature. Early in the process a banana-shaped island structure appears, due to an $m = 1$ perturbation; a flow pattern carries the island nearer the center until the original fluid at the center has been displaced by the island. By this means, the lower emissivity fluid convects to the center, effecting a sudden drop in intensity. The sinusoidal precursors of the sawtooth may be due to a rigid rotation of the island structure before a detector. Using this approximation, the various m -components can be separated by a simple temporal Fourier decomposition. Each component can then be inverted to accomplish the local measurement. The techniques will be discussed later.

Fig. 4. Radial profiles of: (a) electron temperature and density from laser data, (b) the safety factor $q(r)$ from laser data, (c) relative amplitude of the internal disruption (sawtooth), and (d) relative amplitude of the $m=1$ precursor oscillations.

Fig. 5. Magnetic flux surfaces during the phase of decreasing internal emissivity in a "sawtooth".

In different regimes of discharge parameters, the internal disruption activity is not observed. Instead, predominantly $m = 2$ oscillations are seen⁽¹³⁾. At some intermediate radius, at which the parallel wavelength is infinite, the mode structure exhibits a node accompanied by a 180 degree phase shift. Due to radial integration, another node is generally seen in the profile of the line-integrated data. Figure 6 shows recent data for an $m = 2$ oscillation from PLT, similar to that found on the ST tokamak⁽¹³⁾. By appropriate inversion, to be discussed, the local behavior was deduced. An anomaly was observed at radii near that of the outer node: i.e., the otherwise sinusoidal traces were truncated. By including the related higher- m harmonics, a flat radial region was found to exist; presumably, this corresponded to regions of the discharge over which there was exceptionally rapid transport--possibly due to ergodic magnetic structures.

Fig. 6. Fast time evolution of x-ray intensities integrated along chords through radii 0, 4, 10, 12, 14, 16, and 20 centimeters. The time scale is in milliseconds. Prior to a disruption, oscillations with $m=2$, $n=1$ symmetry are observed.

Fig. 7. Photograph of one of the surface barrier detector arrays used on PLT at Princeton.

More recently, investigations of irreproducible non-steady-state behavior have commenced (13, 14, 15, 16, 17, 18). Such studies require arrays of detectors which simultaneously view the plasma at many radial, toroidal and poloidal positions. Figure 7 is a photograph of one of the multi-detector arrays installed on PLT (18). Looking through a thin (0.5 mil) beryllium window, the detectors* view different regions of the plasma. The amplified signal from each surface barrier detector is sampled by an A/D converter, which writes into a buffer in the transient analyzer module. After the shot, the data, along with information on gain settings and sampling times, are written through a PDP 11/34 into the PDP-10, onto disk, and then accessed by analysis programs in the PDP-10. Within approximately 30 seconds, the analyzed data are displayed on CRT scopes in the control room, ready for interpretation by the physicists preparing PLT for the next pulse.

The analysis which we perform on PLT data is described in the next sections.

General Mode Structure Analysis

The local emissivity of the plasma consists of both equilibrium (steady state) and perturbed (oscillating) components. The equilibrium profile is determined by the balancing of forces due to plasma pressure and currents, evolving on a time scale dictated by transport of particles and energy, corresponding to several tens of milliseconds in PLT, except in disruptive cases. The perturbed components oscillate at fundamental frequencies of order several kilohertz. This wide separation of time scales permits a simple decomposition into equilibrium-related and wave-related effects, except in the cases of disruptions. Hence, in the analysis of the modes we sample the signals over several wave periods, and consider the temporal average to correspond to the equilibrium signal and the remaining behavior to the perturbations.

In general, the oscillating emissivity can be expressed as a sum over a complete set of basis functions:

$$\varepsilon(r, \theta, \phi, t) = \sum_{m, n, \omega} \varepsilon_{m, n, \omega}(r) \cos(\omega t - m\theta - n\phi - \vartheta_{m, n, \omega}), \quad (3)$$

where ω can be assumed to be positive. If we consider a detector viewing the plasma at tangent radius r and tangent angle ϑ_c , the total signal $I(r, \vartheta_c, \phi, t)$, which is related to power, is an integral of the emissivity along the line of sight. Using the fact that the components proportional to $\sin m(\theta - \vartheta_c)$ vanish in the integration due to symmetry, the intensity at each frequency can be expressed as

$$I(r, \vartheta_c, \phi, t) = \sum_{m, n} \left[C_{m, n}(r) \cos(\omega t - m\vartheta_c - n\phi) + S_{m, n}(r) \sin(\omega t - m\vartheta_c - n\phi) \right] \quad (4)$$

where

$$\begin{pmatrix} C_{m, n}(r) \\ S_{m, n}(r) \end{pmatrix} = 2 \int_r^{a} \frac{dr_0 r_0 \cos[m(\theta - \vartheta_c)] \varepsilon_{m, n}(r_0)}{\sqrt{r_0^2 - r^2}} \begin{pmatrix} \cos \vartheta_{m, n}(r_0) \\ \sin \vartheta_{m, n}(r_0) \end{pmatrix}. \quad (5)$$

An equivalent expression is

$$I(r, \vartheta_c, \phi, t) = \sum_{m, n} I_{m, n}(r) \cos(\omega t - m\vartheta_c - n\phi - \varphi_{m, n}(r)) \quad (6)$$

where

$$I_{m, n}(r) \begin{pmatrix} \cos \varphi_{m, n}(r) \\ \sin \varphi_{m, n}(r) \end{pmatrix} = \begin{pmatrix} C_{m, n}(r) \\ S_{m, n}(r) \end{pmatrix}. \quad (7)$$

The problem then reduces to finding the radial profiles of the amplitude $I_{m, n}(r)$ and phase $\varphi_{m, n}(r)$ for each mode and then inverting the integrals for each (m, n) .

* ORTEC Surface Barrier Type Soft X-ray Detectors.

We consider these two problems separately in the following sections. The first concerns the case of a single component (m,n) with radial profiles of $I_{m,n}(r)$ and $f_{m,n}(r)$ already known. The second deals with the determination of these components in general situations.

Single Modes

If only a single pair (m,n) exists in the plasma or if the components have already been separated, one has available a set of discrete signals I_j from the detectors:

$$I_j(\omega t) = I(r_j, \theta_{cj}, \phi_j, t) = A_j \cos(\omega t - \alpha_j), \quad (8)$$

where A_j and α_j are the amplitude and phase of the signal j . Relating these to the predicted intensities and translating in time $\omega t \rightarrow (\omega t + m\theta_c + n\phi)$ to remove phase shifts due to poloidal and toroidal displacements, we find from equation (6)

$$A_j \cos(\omega t + m\theta_{cj} + n\phi_j - \alpha_j) = I(r_j) \cos[\omega t - f(r_j)], \quad (9)$$

where the subscripts (m,n) have been suppressed and from which we can extract the relations:

$$A_j \begin{bmatrix} \cos(\alpha_j - m\theta_{cj} - n\phi_j) \\ \sin(\alpha_j - m\theta_{cj} - n\phi_j) \end{bmatrix} = \begin{bmatrix} C(r_j) \\ S(r_j) \end{bmatrix} \quad (10)$$

Considering $C(r)$ and $S(r)$ as continuous functions over a continuous domain, we need to find operators which enable us to extract information on the emissivity $\epsilon(r)$ and the phase $f(r)$ from this pair of radial quantities, using equation (5). For this purpose, we must invert integrals of the form

$$\psi(r) = 2 \int_r^a \frac{dr_0 r_0}{\sqrt{r_0^2 - r^2}} \cos m(\theta - \theta_c) \chi(r_0) \quad (11)$$

where

$$\cos m(\theta - \theta_c) = T_m(r/r_0), \quad (12)$$

$T_m(x)$ being the Chebyshev polynomials⁽¹⁹⁾. For $m = 0$ the standard Abel-inversion techniques are appropriate^(20,21). For $m > 0$ the methods have been generalized.

Coupled Modes

The extension of the theory to cases of many modes involves the violation of the previous assumptions of (a) cylindrical symmetry, or (b) linearity. To first approximation, in the limit of vanishing aspect ratio (r/R) and small amplitude, independent modes with components (ω, m, n) are found. (a) Toroidal effects introduce a term proportional to $\cos \theta$, which couples (ω, m, n) to $(\omega, m \pm 1, n)$; the frequency and toroidal mode number are conserved due to the ignorable coordinates (t, ϕ) in the frame of reference at rest with respect to the asymmetry, even if the fluid is rotating. (b) Nonlinearities generate harmonics of (ω, m, n) at $(2\omega, 2m, 2n)$ and $(0, 0, 0)$; the conservation of the ratio m/n implies a structure with helical symmetry [i.e., a function of $(\phi + m\theta/n)$].

Experimentally, periodic oscillations with double frequency components are routinely observed. The interpretation as an effect of nonlinearity in the mode structure is not unique, however: the interaction of a linear displacement with a non-uniform gradient can also generate harmonics. The analysis of the nonlinearities of the mode requires an independent measurement of the m - or n - spectrum. With sufficient detectors observing different poloidal and toroidal positions, a spatial decomposition can be accomplished. If the helicity is conserved [i.e., if all modes have a common ratio m/n], then nonlinearities in a cylindrical model may explain the observations. If the mode number n is constant over a spectrum of m 's at the same frequency, toroidal coupling is a possible mechanism.

Conclusions

Experimental research on the structure of modes in tokamaks by x-ray techniques has evolved to the point where the application of reconstruction methods is appropriate. With the advent of multi-detector arrays, data from both steady-state and rapidly-varying phenomena can be acquired; analysis techniques are under development for the interpretation of the fluctuations and comparisons with theory.

Linear MHD theory seems adequate to explain the occurrence of long wavelength fluctuations in the plasma, the theoretical thresholds for instability being in rough agreement with experimental observations. The saturation of the modes during steady-state conditions and the rapid transport of the plasma in disruptive cases are still unexplained or untested; with sufficient analysis, the data on x-ray fluctuations may afford some insight into these processes and permit the development of techniques for operating tokamaks in regimes suitable for the achievement of controlled thermonuclear fusion energy.

Acknowledgments

The author would like to thank Dr. S. von Goeler for his close collaboration in this work, W. Stodiek for his overall support, W. Mycock and J. Boychuk for technical assistance, G. Bates, G. Hovey, and R. Neindorff for data acquisition software, and P. Greenberg for the careful preparation of the manuscript.

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