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DENSITY FLUCTUATIONS IN THE
ADIABATIC TOROIDAL COMPRESSOR

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MASTER

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Density Fluctuations in the Adiabatic
Toroidal Compressor

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ABSTRACT

The frequency spectrum of density fluctuations has been measured in the ATC Tokamak by means of microwave scattering in an overdense plasma. A wide spectrum of oscillations with frequencies up to 300 kHz has been detected.

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Tokamaks are plagued by a severe anomalous transport of the electron energy. At present we have not a satisfactory explanation of the observed losses apart from the general assertion that they are caused by some kind of instability.

This paper describes an attempt at measuring the frequency spectrum of density fluctuations in the uncompressed discharge of the ATC Tokamak. For this purpose we used microwave scattering in an overdense plasma. The main reason for this choice was the limited accessibility to the discharge required by this method.

A microwave beam was launched into the plasma and reflected back from the region where the plasma frequency (ω_p) was equal to the microwave frequency (ω). At the same time electron density fluctuations produced the scattering of electromagnetic waves.

The microwave beam had a frequency which was adjustable from 27 to 38 GHz, corresponding to critical densities between 0.9 and $1.8 \times 10^{13} \text{ cm}^{-3}$. The wave, which had its electric field parallel to the toroidal magnetic field of the discharge, was launched by a horn (gain = 28 db) located on the equatorial plane of the toroidal configuration at 30 cm from the center of the plasma cross section. The scattered radiation was collected with another horn (gain = 22 db) which was located on the plane containing the direction of the incident wave and the axis of symmetry of the toroidal discharge; it was aimed at

an angle of 18° with respect to the direction of the incident wave.

A homodyne detection system was used in which the received wave was mixed with a larger reference signal directly from the microwave oscillator. The signal from a crystal detector was sampled (2000 times with a maximum sampling frequency of 5 MHz), stored in an electronic waveform recorder and analyzed by a spectrum analyzer. The spectral resolution was primarily limited by the length of the sampling window (typically 0.4 - 1 msec).

Since the propagation of waves in a strongly fluctuating medium presents formidable mathematical difficulties, we are compelled to make some simplifying assumption for interpreting the experimental results. The first assumption is to consider the plasma as a semi-infinite medium in which one coordinate can be ignored. The effect of the poloidal magnetic field is neglected and the dielectric constant is taken to be:

$$\epsilon(x, z, t) = 1 - (\omega_p/\omega)^2 = \epsilon_0(z) + \epsilon_1(x, z, t).$$

The z-axis is normal to the plasma-vacuum surface and perpendicular to the x-axis.

The frequencies of plasma fluctuations are considered to be much smaller than the wave frequency $\left(\left| \partial \epsilon / \partial t \right| \ll \left| \epsilon \omega \right| \right)$ and therefore the time variable in the dielectric constant can be treated as a constant parameter of the wave equation.

Finally, and more importantly, we shall assume that two kinds of fluctuations are present in the plasma. The first

kind is that caused by large scale MHD-instabilities (kinks, tearing mode, etc.), the second kind is that induced by micro-instabilities. The first kind of fluctuations can be quite catastrophic and therefore we cannot calculate their effect on the wave propagation by using a perturbation solution of the wave equation. Owing to their large scale, we shall assume that we may use instead the geometrical optics approximation. On the contrary for the fluctuations induced by microinstabilities we shall assume that we can limit ourselves to the weak scattering approximation.

For simplicity we shall neglect the x-dependence of MHD-fluctuations. Geometrical optics is not valid in the region where $\epsilon \rightarrow 0$ but the phase of the reflected wave can be very accurately obtained from that approximation together with the condition of reflection at the point $z=z_0$, where $\epsilon(z_0) = \sin^2 \theta_0$, θ_0 being the incident angle¹. The phase shift between the reflected and the incident wave is, apart from a term which does not depend on z or t ,

$$\phi(t) = 2 \frac{\omega}{c} \int_0^{z_0} \left(\epsilon(z,t) - \sin^2 \theta_0 \right)^{1/2} dz \quad (1)$$

If ϵ is a linear junction of z we get $\phi = 4\omega z_0 \cos \theta_0 / 3c$.

The condition for Eq. (1) to be valid is that ϵ is approximately a linear function of z around the reflection point. For the parabolic density distributions of the ATC discharge, this condition implies that the reflection point must be a few centimeters away from the center of the plasma column.

Assuming for ϕ the Fourier expansion

$$\phi(t) = \sum_{\ell=1}^N \phi_{\ell} \sin(\omega_{\ell} t + \theta_{\ell})$$

we get that the reflected wave is given by

$$\left[E \right]_{z=0} = E_0 e^{i(\omega t - kx)} \prod_{\ell=1}^N \sum_{n_{\ell}=-\infty}^{+\infty} J_{n_{\ell}}(\phi_{\ell}) e^{in_{\ell}(\omega_{\ell} t + \theta_{\ell})} \quad (2)$$

where $k = (\omega/c) \sin \theta_0$ and $J_{n_{\ell}}(x)$ is the Bessel function of the first kind of order n_{ℓ} . If $|\phi_{\ell}| \ll 1$ we can approximate Eq. (2) with

$$\left[E \right]_{z=0} \approx E_0 e^{i(\omega t - kx)} \left[1 + i \sum_{\ell=1}^N \phi_{\ell} \sin(\omega_{\ell} t + \theta_{\ell}) \right] \quad (3)$$

For a square-law detector the output signal will be proportional to

$$S(t) = \left\langle \left[R_e (E + E^{\text{ref}}) \right]^2 \right\rangle \quad (4)$$

where $E^{\text{ref}} = U e^{i(\omega t + \delta)}$ is the reference wave ($|U| \gg |E_0|$) and $\langle \rangle$ means the time average over an interval τ such that $|\omega|^{-1} \ll \tau \ll |\omega_{\ell}|^{-1}$. Substituting Eq. (2) into Eq. (4) we get

$$S(t) \approx \frac{1}{2} U^2 + 2UE_0 \prod_{\ell=1}^N \left\{ \cos \delta \left[\frac{1}{2} J_0(\phi_{\ell}) + J_2(\phi_{\ell}) \cos(2\omega_{\ell} t + 2\theta_{\ell}) + J_4(\phi_{\ell}) \cos(4\omega_{\ell} t + 4\theta_{\ell}) + \dots \right] + \right. \\ \left. + \sin \delta \left[J_1(\phi_{\ell}) \sin(\omega_{\ell} t + \theta_{\ell}) + J_3(\phi_{\ell}) \sin(3\omega_{\ell} t + 3\theta_{\ell}) + \dots \right] \right\} \quad (5)$$

where terms of the second order in E_0 have been neglected.

Eq. (5) shows how it is possible to determine the quantities Φ_l from the relative amplitude of even or odd harmonics of $S(t)$. In practice this is possible if only few modes of oscillation with linearly independent frequencies are present in the plasma.

Fig. 1 shows the spectra of the signal S at three different instants of a discharge in which a "strong" MHD-oscillation was present (probably a tearing mode with azimuthal number $m = 2$). Here "strong" means that a small increase ($\approx 10\%$) of the plasma current would have triggered the disruptive instability^{3,4}. The amplitude of the signal induced in a pickup coil was constant in time. The discharge parameters were: $R=88$ cm, $a=17$ cm, $B_T=17$ kG, $I_p=70$ kA.

From these spectra we get the values of 3.3, 2.7 and 2.4 radians respectively for the amplitude of the phase shift. Since the plasma density was increasing in time, the reflection point moved outwards during the pulse. Assuming a parabolic density profile we get that the position of the reflection point was at 8.5, 10.5 and 11.5 cm from the center of the plasma column respectively. Assuming that the ratio of the fluctuating part of the plasma density ($\tilde{n}$) to the average density ($\bar{n}$) was z -independent, we get that $\tilde{n}/\bar{n} \approx 4 \times 10^{-2}$ in all the three cases. This value must be considered a lower limit; if the mode was spatially localized the ratio $\tilde{n}/\bar{n}$ could have been considerably higher.

The approximation of geometrical optics cannot be used for studying the scattering produced by small scale fluctuations;

the wave equation must be solved. This has been done by Pitteway with the approximation that the intensity of the scattered wave is much smaller than that of the incident wave.⁵ His results may be summarized as follow: let $\tilde{E}_0 = \tilde{E}_0(z,t)\exp(-ik_0x)$ and $\tilde{E}_1 = \tilde{E}_1(z,t)\exp(-ik_1x)$ be two solutions of the unperturbed ($\epsilon_1 \equiv 0$) wave equation. Both $\tilde{E}_0$ and $\tilde{E}_1$ can be separated into an incident and a reflected wave. The incident component of $\tilde{E}_0$ represents the test wave of the experiment. The incident angles of $\tilde{E}_0$ and $\tilde{E}_1$ are $\theta_0 = \sin^{-1}(k_0c/\omega)$ and $\theta_1 = \sin^{-1}(k_1c/\omega)$ respectively. Pitteway's result is that the Fourier component $\tilde{e}_1 = \tilde{e}_1(z,t)\exp(ik_1x)$ of the scattered wave is given by

$$\left[\tilde{e}_1 \cdot \tilde{E}_{1i} \right]_{z=0} = i \frac{\omega}{2c \cos\theta_1} \int_0^\infty f(z,t) \tilde{E}_0(z,t) \tilde{E}_1(z,t) dz, \quad (6)$$

where $f(z,t)\exp(ik_sx)$ is the Fourier component of $\epsilon_1(x,z,t)$ which varies in the x-direction like $\exp(ik_sx)$ with $k_s = k_0 + k_1$. $\tilde{E}_{1i}$ is the incident component of $\tilde{E}_1$. If $\epsilon_0(z)$ is a linear function of z , $\tilde{E}_0$ and $\tilde{E}_1$ are given in terms of the Airy functions.¹ If we replace $\tilde{E}_0$ and $\tilde{E}_1$ of Eq. (6) with the solutions of geometrical optics for the case $k_s=0$ we recover Eq. (3).

From the geometry used in the experiment we get $k_s = \bar{k} = (\omega/c) \sin 18^\circ$ which corresponds to a wavelength $\lambda \approx 2.6$ cm. However, owing to the finite divergency of the radiation pattern of the launching and the receiving antenna and the fact that the plasma column had a cylindrical geometry, the system was capable of detecting the radiation scattered by plasma fluctuations in a large interval of k_s around the value $k_s = \bar{k}$ (from $k_s = 0$

to few times $\bar{k}$), that is all plasma waves with wavelength larger than ~ 1 cm were capable of scattering the microwave beam into the receiving antenna.

The frequency spectrum of the received microwave signal is shown in Fig. 2 for several different positions of the reflection point. The plasma parameters were: $R=88$ cm, $a=17$ cm, $B_T=16$ kG, $I_p=70$ kA, $\bar{n}_e=1.5 \times 10^{13}$ cm $^{-3}$, $T_e^{\max}=1200$ eV, $\bar{T}_e=450$ eV, $\bar{T}_i=200$ eV. The safety factor was 3.7 at the limiter position and 0.8 in the center of the discharge. The last value was calculated assuming a current distribution proportional to $T_e^{3/2}$. No sign of MHD activity was detected with pickup coils. The average ratio of the bounce frequency of trapped electrons to the effective collision frequency was $\approx 1/3$.

For the outer positions of the reflection point (cases i - l) the spectrum peaked in the region $50 \div 100$ kHz. As the reflection point moved deeper into the discharge (cases d $\div$ h) the amplitude of fluctuations sharply increased in their spectrum broadened. When the reflection point moved even closer to the center (cases a $\div$ c) the amplitude of fluctuations decreased again and an oscillation with a frequency of 15-20 kHz began to appear. At the same time the signal showed the sawtooth modulation which is a characteristic of the x-ray emission from the center of Tokamak discharges.^{6,7} The observed oscillation is the one that the authors of Ref. 6 identified as the $m=1$ internal kink mode.

To estimate the effect of these high frequency fluctuations on the heat conduction, we make the usual assumption that their contribution to the heat transport coefficient is of the same

order of magnitude as the ratio of their growth rate to the square of their wave number in the direction perpendicular to the magnetic field ($\chi \approx \gamma / k_{\perp}^2$). If we assume that $\gamma \approx \omega$ and take an average frequency of 100 kHz and an average wavelength of 2 cm, we get an energy confinement time close to that (≈ 3 msec) deduced from Thomson scattering measurements.

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Fig. 1. Frequency spectra of the received microwave signal (left column) at different instants of a discharge in which a MHD oscillation was present. The signals induced on a pickup coil are shown on the right column; $t_a = 13$ msec, $t_b = 20$ msec, $t_c = 23$ msec.

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Fig. 2. Frequency spectra of scattered waves for different penetrations of the microwave beam. Assuming a parabolic density profile the microwave frequency was varied in such a manner that the position of the point where $\omega = \omega_p$ moved, with respect to the center of the plasma column, from 7.8 cm (case a) to 13.2 cm (case i) with steps of 0.6 cm. The scales on the ordinates are in arbitrary units.