POWER MATCHING FOR PELLET FUSION

by

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operated under contract W-31-109-Eng-38 for the

U. S. ENERGY RESEARCH AND DEVELOPMENT ADMINISTRATION

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POWER MATCHING FOR PELLET FUSION*

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February 12, 1976

ABSTRACT

The number of beams required for optimum power transfer from a given power source to the surface of a pellet is derived. The result is valid for linear optical systems, hence, for pellet fusion by laser or high energy ion beams. The optimum number of beams turns out to be inconceivably large for any practical system. Practical pellet fusion by lasers or high energy heavy ion beams must thus compromise physical principles in favor of reduced cost and optical complexity.

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*Work performed under the auspices of the United States Energy Research and Development Administration.

The compression of small pellets of a deuterium-tritium mixture and subsequent ignition of a thermonuclear reaction require very high power levels incident on a small pellet. The possibility of achieving such power levels with laser, electron beams, and heavy ion beams has been discussed. In each case, considerable development is needed to generate the required power levels. It is the purpose of this note to develop the relationship between the source brightness and the optimum power match between the source and the pellet.

We will define brightness by

$$B = \frac{d^4 I}{d^2 A d^2 \Omega}$$

where the intensity I is the energy carried in the beam per unit time and $d^2 A$ and $d^2 \Omega$ are the four (two each) transverse dimensions of phase space. This definition ignores the other longitudinal dimension (ΔE or $\Delta P_{\parallel}$) of six dimensional phase space. For the present purpose, the omission is of little consequence since compression and heating of a pellet are relatively insensitive to energy spread or small variations of wavelength.

Any linear optics system conserves the source brightness or the product $dA d\Omega$ while varying the individual beam sizes and angles. Examples of such linear optics systems are the lenses of laser systems or the quadrupoles of ion beam transport systems. The pinch effect of relativistic electron beams is not in this category, however, so the present analysis does not apply to electron beam pellet fusion.

The source brightness of any beam in any practical case will be limited, whether for fundamental or practical reasons. Such limitations might be space charge defocussing forces for ion beams, or electric field limitations in a given media for lasers, for instance. For a given source brightness and pellet size, a simple relationship determines the maximum number of source beams which can be focussed onto the pellet. This number, however impractical in reality, represents the optimum power matching of a source to a pellet. The statement is equivalent to saying that the maximum power that can be focussed onto the

surface of a given pellet requires N_b source beams, where N is determined by the relationship we will derive.

In ion beam transport, it is convenient to talk about the emittance of a beam. Emittance is defined for each of the transverse dimensions, x and y , such that $\pi\epsilon$ is the area in the phase space of that dimension (such as Δx and $\Delta\theta_x$) occupied by the beam. For most sources, $\epsilon_x = \epsilon_y = \epsilon$ and we make that assumption here. For a single beam focussed upon the pellet with a waist in the beam envelope at the pellet, we have

so that

$$\epsilon = \theta r / 4 = \theta r / 2 \quad (1)$$

Equation (1) determines the radius to which a beam of given emittance can be focussed, where θ relates inversely to the f number of the lens system required. The solid angle contained in such a beam is

$$\Delta\Omega = \frac{\pi}{4} \theta^2 \quad (2)$$

For multiple beams incident on the pellet from different directions, with the restriction that no element of phase space of the individual beams be allowed to overlap, the effective solid angle is the area of a square rather than a circle and the effective solid angle utilized is simply

$$\Delta\Omega_e = \epsilon^2 \quad (3)$$

The number of individual beams which can be focussed onto a pellet is given by combining Eqs. (3) and (1) with the total solid angle available, 4π sr, surrounding the pellet

$$N_b = \frac{4\pi}{\Delta\Omega_e} = \frac{\pi r^2}{\epsilon^2} \quad (4)$$

where if r is in cm, ϵ is given in units of rad-cm.

As previously pointed out,¹ the power of ion beams of a given energy which can be contained in a storage ring is limited by space charge defocussing to be directly proportional to the emittance ϵ . Since, however, the number of beams which can be used is inversely proportional to ϵ^2 by Eq. (4), the total ion beam power which can be focussed on a pellet of given size is proportional to $1/\epsilon$ and optimum power matching, therefore, requires a minimum ϵ .

The emittance of an ion beam may well depend on the type of source, but acceleration of such a beam results in a reduction of the emittance inversely with the increasing momentum. For any ion species, the emittance of nonrelativistic beams is expected to be proportional to $1/\sqrt{E}^{1/2}$ where E is the kinetic energy of the ions.

In our proposal for pellet fusion with high energy heavy ion beams,² we have normalized the ion beam emittance to that of 50 MeV protons derived from a duoplasmatron, with which we have considerable experience. For ion beams of interest² for pellet fusion, say 8 GeV iodine to 86 GeV uranium, the natural beam emittance ranges from 2.0×10^{-4} to 5.6×10^{-5} rad-cm, respectively. For a pellet with $r = 0.05$ cm, the optimum power transfer to the pellet would occur with $N_b \sim 2 \times 10^5$ for 8 GeV iodine and 2.5×10^6 for 86 GeV uranium!

Such numbers stagger the imagination and appear quite unpractical from both a cost and complexity point of view, even though they appear technically

feasible. In our proposal² for the utilization of heavy ions for pellet fusion, we have severely compromised this optimum matching by considering 100 ion beams incident on the pellet from different directions, with the emittance of each beam increased from its natural emittance in the process of acceleration and storage in order to increase its energy or power content. This compromise still appears to provide adequate power for scientific breakeven of small DT pellets.

The diffraction limited divergence of a laser beam (in the Fraunhofer region) is given by

$$\theta = \frac{1.22 \lambda}{d} \quad (5)$$

where λ is the wavelength of the laser and d the diameter of a cylindrical beam. The emittance of such a beam is then

$$\pi \epsilon = \frac{\pi}{4} \theta d \quad (6)$$

$$\text{or} \quad \epsilon = 0.3 \lambda \quad (7)$$

For a laser of 1μ wavelength, $\epsilon = 3 \times 10^{-5}$ rad cm and N_b from Eq. (4), again for $r = 0.05$ cm, is 8.7×10^6 .

Again, implementation of this result is inconceivable in any practical case for cost and complexity reasons. We point out, however, that if a given diffraction limited laser beam can contain only a given maximum power for practical reasons, the power that can be transferred to the surface of a pellet is directly proportional to the number, N , of such laser beams up to the maximum number N_b determined by Eq. (4). Thus, many beams are even more advantageous for lasers than for high energy heavy ions where the power increases as $\sqrt{N}$.

The result of power matching for pellet fusion by laser beams suggests that it might be much more straightforward, and in the long run less expensive, to utilize 10^4 laser amplified beams of 100 J each rather than 10 laser amplifiers

of 10^5 J each. We believe that the abhorrence of the concept of so many beams arranged symmetrically around a spherical enclosure is nothing more than a natural mental block by human beings to visualization of complex, three dimensional objects and that this difficult problem can be solved.

References

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