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PRELIMINARY ANALYSIS AND FEASIBILITY STUDY ON HIGH SPEED TURBULENCE GENERATOR

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October 15, 1976

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Prepared for U.S. Energy Research & Development Administration under contract No. W-7405-Eng-48.

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PRELIMINARY ANALYSIS AND FEASIBILITY STUDY ON
HIGH SPEED TURBULENCE GENERATOR

ABSTRACT

This interim report is submitted for review and critique prior to preparation of a subsequent report that will complete the summary of pre-experimental turbulent design analysis. The reported work is in support of the D-Division "Rainout" program. Experimental facility design, diagnostics development, and concept evolution are under the guidance of General Chemistry. Preliminary analysis, results, predictions, and suggestions here are fluid dynamic considerations on the design of the turbulent attachment rate experiments. Timely publication of these current results is essential in order to meet experimental design schedules. The final LLL report will consist of these results and analyses, additions requested by the program reviewers, and an additional study emphasizing detailed turbulence structure and energy spectral considerations.

INTRODUCTION

The following calculations and figures address preliminary design considerations on two high-speed (1 to 20 m/s) turbulent-flow generators. One, the Comte-Bellot, is a modified wind tunnel using multiple turbulence screens and a downstream area contraction. This design provides a high quality and useful test length of almost homogeneous, quasi-isotropic turbulent flow. It is based on the well known turbulence experiments of G. Comte-Bellot and S. Corrsin.^{1,2} This is sketched in Fig. 1. The second, the Browand, is a jet plate interrupted-flow tube with a downstream area contraction as suggested by Professor F. Browand, of the University of Southern California.³ This concept is sketched in Fig. 2.

Part I of this report is a presentation of predicted pressure and total flow-volume requirements for the Comte-Bellot and Browand configurations. Calculations include 10-cm, 30-cm, and 1-m lateral test dimensions. Blowdown steady flow is considered for one-, five-, and ten-minute durations.

Relative variations in the drag of the turbulence grid and jet plate are incorporated. These calculations include the estimated wall boundary layer growth, frictional and heat transfer losses as well as the somewhat larger drag losses of the turbulence grid screens and jet plate. In addition we present estimated turbulence intensity, characteristic scales and decay of the turbulence with axial distance.

Part II of this report contains predictions of the lateral spreading rate and trajectory of the water droplets introduced into the turbulent core of the aerosol stream. These calculations of lateral droplet dispersal toward the walls necessarily incorporate estimated levels of the turbulent accelerations and longitudinal turbulent intensity decay. Droplet injection speeds of 50 and 10 m/s are considered, together with estimates on the length scale of the lateral fluctuations and diffusional properties of the predicted turbulent fields as they might affect the droplet trajectory. These calculations result in an estimate of the length of useful test section over which the walls are essentially free of coating by water droplets and aerosol particulates. This is a vital consideration in the application of a newly developed, real-time, hypergolic attachment diagnostic technique.

Part III of this preliminary analysis consists of detailed calculation on the flow uniformity, sidewall boundary-layer effects, turbulence energy spectra, and spectral dissipation characteristics for each of the two configurations. Two-dimensional, axially symmetric calculations of the turbulent core and wall boundary-layer regions as well as unsteady-flow analysis will be subsequently reported. The primary tool for later analytical efforts will be the TEULER two-dimensional unsteady-turbulent Navier-Stokes Code. The numerical procedure is outlined in Appendix A.

This third section contains data which is the basis for selecting, positioning, and calibrating the experimental turbulence hot-wire anemometry diagnostics and for preparation of the statistical data-reduction techniques. However, these details do not impact directly on the start of engineering design and plan-preparation of the test configurations. Hence, to avoid unwarranted delay in the engineering design, the results of calculations on the first two parts are presented at this time with only partial results of the third section. Results have also been communicated orally to General Chemistry and to Mechanical Engineering, experimental design principals. The detailed flow calculations and resulting analysis in Part III will be completed

in a few weeks, in time for consideration on the hot-wire turbulence anemometry diagnostics. Shake-down runs in the planned facility are scheduled for December, 1976.

PART I. RESERVOIR PRESSURE AND GAS VOLUME REQUIREMENTS

Calculations have been completed giving estimated driving pressures, flow rates, gas volumes, critical dimensions, and other data needed to select one of the two configurations examined and to initiate hardware and experimental design. The calculations include area change, turbulence generator drag, screen drag, and side-wall turbulent boundary-layer shear (heating) and displacement effects. Details of the calculations, approximations, and boundary conditions are placed at the end of this section. All calculations were based on air at standard temperature and pressure.

The Comte-Bellot configuration is essentially a rectangular cross-section steady-flow wind tunnel shown in Fig. 1.^{1,2} A square-rod lattice turbulence grid is placed as shown in the largest diameter cross-section region of the tunnel. Downstream of the grid, at a distance of about 18 grid mesh spaces is a gradual, modest area contraction (1.27:1). In effect, an axial strain is applied to the turbulence stress field by the corresponding change in mean velocity of the flow at the time it crosses the contraction. An almost isotropic turbulent field is generated with a measured deviation of about 5% in the longitudinal to lateral turbulent mean decay profiles with distance.^{1,2} For laminar flow tests the grid would be removed and the mean flow-velocity dropped (by pressure regulator in the feed line) from the nominal 10 m/s-flow described here to about 1 m/s. A fixed ratio of grid open space (M_G) between bars as compared to bar diameter (D_G) is maintained. This ratio is 5:1. For ratios less than 5:1 the generation of a cross-tunnel homogeneous turbulent field may not be possible.⁴ Both Comte-Bellot and Browand configurations begin, in our conceptual design, with a shallow-angle diffuser (about 20° in half-angle) followed by a constant area flow alignment and settling chamber. This is followed, in turn, by a gradual but severe area contraction of about 10:1 over which the flow is uniformly accelerated to the nominal speed of 10 m/s.

The Browand configuration, shown in Fig. 2, differs from the Comte-Bellot configuration, primarily, in the turbulence generator.³ In place of the turbulence grid, a jet plate is positioned. It is a solid plate in which 600 or

Fig. 1. Turbulent aerosol-droplet attachment test section, based on Comte-Bellot configuration, showing pressure-regulated supply line, eight stainless steel fine-mesh flow-alignment screens with mesh density 40/cm and wire diameter approximately 50 μm (four in half-angle diffuser section, four in settling chamber), square-rod turbulence grid, and cross-section area details.

Fig. 2. Turbulent aerosol-droplet attachment test section, based on Browand configuration, showing pressure-regulated supply line, eight fine-mesh flow-alignment screens, and cross-section area details, including detail of 600-round-hole jet plate.

more small-diameter holes have been drilled in a regular rectangular pattern. The spacing from hole rim to hole rim is maintained at three diameters. The plate exerts a larger drag loss on the flow stream than does the grid. This is manifest in the higher system pressure required to drive the stream at the steady nominal flow velocity desired. Figures 1 and 2 give the dimensions and geometric characteristics of the two configurations. They also show the positioning of eight fine-mesh wire screens used to enhance flow alignment and to promote the settling of any irregularities. The stainless steel screen characteristics suggested by Browand³ are a screen mesh density of 100 per in. (40 per cm) and a wire diameter of 0.002 in. ($\sim 50 \mu\text{m}$). The screens remain in place for the laminar flow experiments as well as the turbulent ones.

Generally, the Browand configuration promises to give more energetic turbulent flows and smaller scales with a correspondingly higher dissipation rate than the Comte-Bellot configuration. However, higher driving pressures are required for the Browand configuration. The Browand configuration may develop significantly more anisotropy ($\sim 20\%$) than the Comte-Bellot configuration ($\sim 5\%$).

The accompanying table (Table 1) is divided into two parts. The upper part contains values unique to the Comte-Bellot configuration in addition to other values common to both the Comte-Bellot and the Browand configurations. Values unique to the Browand configuration are presented in the lower part.

Lengths are all in cm units. D is the nominal cross-sectional measure in the larger area section containing the turbulence generator. The 1.27:1 area contraction downstream of the generator reduces this dimension by 1.13 in the downstream section. M , d , and L are the open grid spacing (between bars) in the turbulence grid, the diameter of the bars, and the overall length of the conceptual test section measured with origin at the turbulence generator, respectively. ΔP_0 is the overpressure requirement calculated for the configuration based on a steady state blowdown flow with test conditions at 1 atm (standard air). The flow rate $\dot{m}$ is given in g/s while the volume rate is given in liter/s. The core-flow Reynolds number based on the nominal diameter and the mean flow speed of 10 m/s is listed next. This flow speed must be reduced by about one order of magnitude to conduct the laminar flow experiments. Conceptually, the feed line pressure head is controllable by a pressure-regulator valve so that the desired steady flow speed is readily

Table 1. Summary of Comte-Bellot and Browand turbulent flow generator designs.

Comte-Bellot configuration			
	Diam, cm		
	100	30	10
Open-grid spacing			
Turbulence grid (M), cm	2.45	0.762	0.254
Bar diameter (d), cm	0.51	0.15	0.051
Overall length (L), cm	1000	300	100
Overpressure requirement (ΔP_0), atm	0.02	0.02	0.02
Flow rate ($\dot{m}$), g/s	1.63×10^4	1.46×10^3	1.63×10^2
Volume (V), liter/s	371	33.4	3.71
Core-flow Reynolds number [(Re)D]	8.7×10^5	2.6×10^5	8.7×10^4
Required as storage volume, liters			
60-s blowdown	2.23×10^4	2.00×10^3	2.23×10^2
300-s blowdown	1.11×10^5	1.00×10^4	1.11×10^3
600-s blowdown	2.23×10^5	2.00×10^4	2.23×10^3
Residence time, s	0.8	0.24	0.08
Browand configuration ^a			
	Diam, cm		
	100	30	10
No. of jet plate holes	625	625	625
Hole diam, cm	1.0	0.3	0.1
Hole spacing, cm	3.0	0.9	0.3
Mean emergent jet velocity, U_{jet} m/s	204	204	204
Jet diameter Reynolds number, (Re) _{jet}	1.4×10^5	4.2×10^4	1.4×10^4
Overpressure requirement, (ΔP_0), atm	0.6	0.6	0.6

^a Other dimensions and quantities, flow rates, etc. are identical to those estimated for the Comte-Bellot configuration above. See Figs. 1 and 2 for additional information on area contraction, cross-section dimensions, notational identification, position and placement of grid, jet plate, mesh screens, etc.

maintainable. The total storage volume of gas required for 1-, 5-, and 10-min blowdown periods, respectively, is listed next. It was assumed that gas under storage (tanks) was pressurized at the more-or-less standard 143° atm (2100 psi) for these total-volume gas estimates. That is, the figures given are for the stored volumes at these usual storage pressures. Residence time is defined as the nominal time a fluid particle remains in the test configuration during passage through the contraction subsequent to processing in the turbulence generator.

Viscous losses and wall boundary-layer displacement effects were estimated, based on flat-plate turbulent boundary-layer calculations for "open" tunnels (in the absence of a turbulence generator). Since both configurations are geometrically identical as "open" tunnels by assumption, the same calculated wall turbulent and viscous losses apply to both. In effect, the major contribution to the fluid dynamic losses (turbulence-generator drag effects) were superimposed on the open tunnel results to estimate the driving overpressure requirements.

In addition to the data common to both configurations, the Browand portion lists information on the total number of holes drilled in the jet plate and spacing, rim to rim between holes. The holes are arranged in a square array as indicated in Fig. 2. Additional information includes the mean, emergent, jet velocity (at the plate) and the emerging Reynolds number based on the jet diameter.*

Method

The Table I estimates were based on iterative numerical integration of the one-dimensional steady-flow equations with area change together with turbulent wall boundary-layer calculations. The turbulent boundary-layer calculations used in this part of the analysis are relatively primitive in comparison to the detailed turbulent field, shear layer, and wall calculations initiated in Part 3 and reserved for presentation in a later report. They are, however, appropriate for the present requirements: (1) comparison of the two candidate

* At the time this report is in preparation, additional calculations are being completed for the Browand configuration at nominal values D of 15 and 20 cm. In addition, the hole diameter and spacing for these two later cases is maintained constant: hole diam = 1 mm, spacing = 3 mm. The number of holes increases. This slightly alters the jet plate drag characteristics and, more significantly, alters the turbulence structure and turbulence decay with distance downstream of the jet plate.

configurations and (2) estimates of turbulent wall, boundary-layer frictional losses and displacement effects.

The iterative procedure is initiated with a calculation of the inviscid one-dimensional core flow with area change and estimated wall-heat loss due to boundary-layer frictional effects. The core pressure and velocity as a function of axial distance generated by this solution is in turn used to generate a fully-developed, steady, turbulent wall boundary-layer solution. The boundary-layer results give a second estimate on the wall-friction and heating loss as well as an estimate of the displacement effects of the wall boundary layers. These displacement effects force the core flow streamlines away from the wall itself so that a slight reduction results in the area open to the core flow. This "hydraulic" area as opposed to the original, open, geometric area is used in place of the original, unrestricted, geometric area. In the present calculation, these boundary-layer effects are small, but not negligible, with respect to the drag losses of the turbulence generators and the geometric contraction consequences on the core flow. A second set of one-dimensional core-flow calculations based on the revised areas and friction factors completes the sequence. The resulting second approximation to the pressure and velocity distribution with axial distance (x) are the basis of the estimated driving overpressures and other pre-design data given in Table I.

The analysis presented in Part III and that which will be presented in a later report differs. The turbulent field for the Part III analysis must be treated generally. Subsequent discussion will illustrate how it is coupled in space and time with the inviscid (inertial) field. Two-dimensional (axially-symmetric) solutions are sought and obtained to provide radial (y) distributions of the flow variables and to predict sensitive regions of irregular (inhomogeneous) flow. Based on these later results for the two-dimensional inertial (mean flow) field a more extensive prediction of the resulting turbulence field from the walls to the core results. Subsequently, a more sophisticated model will be applied to predict turbulence energetics, spectra, and structural effects for the diagnostics design and model development. It is emphasized here that these calculations, albeit sophisticated, are just predictions. The experiments to follow will be the sole basis for establishing the detailed turbulence structure.

Here we consider an iteratively corrected model of essentially inviscid core flow subject to separately predicted turbulence-energy losses and

displacement effects. For example, in one-dimensional steady flow, the equations can be cast in terms of the Mach No. $M(x)$, as given by Shapiro⁵:

$$\frac{dM^2}{M^2} = - \frac{2(1 + \frac{\gamma-1}{2} M^2)}{1-M^2} \frac{dA}{A} + \frac{(1 + \gamma M^2)(1 + \frac{\gamma-1}{2} M^2)}{1-M^2} \frac{dT_\tau}{T_\tau} + \frac{\gamma M^2 (1 + \frac{\gamma-1}{2} M^2)}{1-M^2} 4cf \frac{dx}{D^*} + \frac{2dF}{\gamma P_\infty A M^2} \quad (1)$$

In Eq. (1), A represents the cross-sectional area of the tunnel at any station x , with or without the displacement-thickness correction depending on the level of the iterative approximation. Equation (1) also implies steady, polytropic, equilibrium, nonadiabatic flow of an ideal gas with constant polytropic exponent γ . The metric coordinates are designated x in the axial direction and y in the radial direction within the tube, with the origin $y = 0$ at the wall. The subscript ∞ applies to the inviscid core flow (the "outer" boundary or "outer" infinity in the boundary layer approximation). The wall friction coefficient cf is computed as a function of x from the boundary-layer solutions. D^* is the hydraulic diameter: the geometric diameter at any station x less the displacement effects

$$D^* \triangleq D - 2 [\delta_d(x)] ,$$

where $D = D(x)$ is the geometric diameter and $\delta_d(x)$ is the calculated turbulent boundary-layer displacement thickness. F is the drag force for the turbulence generator. It is estimated from Goldstein⁴ as well as Comte-Bellot^{1,2} for the turbulence-grid generator. It is estimated from Browand³ for the jet-plate turbulence generator. The temperature T_τ is the "total" temperature and thereby characterizes the entropy state of the gas and corresponding local effects of heat loss.

Equation (1) is integrated numerically subject to the stagnation reservoir (subscript "o") conditions, which in steady state are

$$\left\{ \begin{array}{l} U_\infty(x=0) = M(x=0) = M_o = 0, \\ T = T_\tau(x=0) = T_o \triangleq T + U_\infty^2(x)/c_p, \\ P_o/\rho_o = P_t/\rho_t(x=0) = \frac{P}{\rho} + \frac{\gamma-1}{\gamma} \frac{U_\infty^2}{2}. \end{array} \right. \quad (2)$$

In the turbulent, wall boundary-layer region, a choice of the integral approximation for fully-developed turbulent boundary layers appears appropriate and consistent with the other assumptions. In accordance with the thin boundary-layer assumption applied to the turbulent boundary layer as discussed, for example, in Hinze⁶

$$\delta_d/D \ll 1,$$

hence

$$\frac{\partial p}{\partial y} = 0, \text{ for } \delta_d \geq y \geq 0, \rho = \text{constant} = \rho_\infty(x), \quad (3)$$

where

$$\delta_d \triangleq \text{displacement thickness} = \int_0^\infty \left(1 - \frac{\langle u \rangle}{U_\infty}\right) dy,$$

with $\langle \rangle$ denoting a random field-time average for a fluctuational component such as u' and

$$\frac{1}{2\delta t} \int_{t-\delta t}^{t+\delta t} u' dt = 0 \text{ and } \langle u \rangle \equiv \frac{1}{2\delta t} \int_{t-\delta t}^{t+\delta t} u dt. \quad (4)$$

The foregoing corresponds to separation of the velocity components

$$\vec{U}_\infty \triangleq u \vec{i} + v \vec{j}$$

into mean and randomly fluctuating values denoted by primes

$$u = \langle u \rangle + u', \quad v = \langle v \rangle + v'. \quad (5)$$

Use of the foregoing boundary-layer assumptions and order-of-magnitude analysis in the systematic elimination of terms of order ϵ with respect to unity results in a simplified equation of motion

$$\langle u \rangle \frac{\partial \langle u \rangle}{\partial x} + \langle v \rangle \frac{\partial \langle u \rangle}{\partial y} = - \frac{1}{\rho} \frac{\partial P_m}{\partial x} - \frac{\partial}{\partial y} \overline{R}_T + v \frac{\partial^2 \langle u \rangle}{\partial y^2}, \quad (6)$$

and one of energy

$$\overline{R}_T \frac{\partial \langle u \rangle}{\partial y} + \frac{\partial}{\partial y} \langle v' \left(\frac{p}{\rho} + q^2/2 \right) \rangle = v \frac{\partial^2}{\partial y^2} \left(\frac{\langle q^2 \rangle}{2} \right) - \epsilon', \quad (7)$$

where ν is the kinematic viscosity and q^2 is twice the specific turbulence kinetic energy

$$q^2 = \langle u'^2 + v'^2 \rangle \quad (8)$$

while

ϵ' is the specific turbulence dissipation rate

$$\underline{\underline{\Delta}} \nu \langle \frac{\partial v'}{\partial x} \frac{\partial u'}{\partial y} \rangle \quad (9)$$

The integral technique essentially treats a moment solution of (6) and (7) as opposed to the detailed structure of the differential equations. We presume a polynomial expansion represents the velocity profiles and solve the corresponding ordinary differential equations after integration over the boundary layer from $y=0$ to $y=\delta_d$ subject to a closure approximation on the Reynolds stress, $\overline{R_T}$ and the dissipation ϵ' . For the present analysis* we concern ourselves with perhaps the simplest closure approximation consistent with the simplified but usefully accurate estimate for the design comparison and analysis. The Reynolds stress is related to the mean flow through an eddy viscosity $\tilde{\epsilon}$

$$\overline{R_T} \underline{\underline{\Delta}} \langle u'v' \rangle = \tilde{\epsilon} \frac{\partial \langle u \rangle}{\partial y}, \quad (10)$$

and the dissipation rate is related to a turbulence heat transfer

$$\tilde{\epsilon} C_p (P_{RT})^{-1} \frac{\partial^2 T}{\partial y^2} = \epsilon'. \quad (11)$$

The turbulent Prandtl number in our preliminary analysis is set equal to unity, $P_{RT} = 1$, and the Eqs. (6) and (7) are formally integrated over the boundary layer with integral-profile approximations for the velocity function $\langle u(y) \rangle$. We use an iterative "shooting" method for the resulting 2-point boundary value problem subject to the boundary conditions

* The axisymmetric, turbulent-flow analysis directed to Part 3 uses a more general two-equation closure model consistent with the more detailed, two-dimensional, calculational method applied.

$$\begin{aligned}
\text{At the wall (y = 0)} \quad & \left\{ \begin{array}{l} T = \text{constant} = 288 \text{ K} \\ \langle u \rangle = \langle v \rangle = 0 \end{array} \right. \\
\text{At the outer infinity} \quad & \left\{ \begin{array}{l} T = T_\infty(x), \text{ matching results of Eq. (1)} \\ \langle u \rangle = U_\infty(x) \text{ matching results of Eq. (1)} \end{array} \right.
\end{aligned} \tag{12}$$

In the foregoing solutions, one obtains the velocity distribution $\langle u(y) \rangle$ in the boundary layer to within an integral (in-the-large) approximation. Once obtained, the boundary-layer effects are readily defined by integral relations.

The integral-displacement thickness is given by

$$\delta_d = \int_0^\infty \left(1 - \frac{\langle u \rangle}{U_\infty}\right) dy. \tag{13}$$

This is shown in Fig. 3 as a fraction of the total geometric diameter D for the three sizes computed: $D = 10, 30, 100$ cm.

In addition, the boundary-layer wall friction is given by

$$2C_f \triangleq \frac{\tau_w}{\rho U_\infty^2} = \frac{d}{dx} \int_0^\infty \left(\frac{\langle u \rangle}{U_\infty}\right) \left(1 - \frac{\langle u \rangle}{U_\infty}\right) dy, \tag{14}$$

and the heat transfer from the integral energy balance over the turbulent boundary layer

$$\frac{d}{dx} \left[\frac{\delta_e}{2} - \frac{1}{2} \int_0^\infty \frac{\langle q^2 \rangle \langle u \rangle}{U_\infty^3} dy \right] = \frac{1}{Re_\Delta} + \int_0^\infty \frac{\epsilon'}{U_\infty^3} dy, \tag{15}$$

where the Reynolds number Re_Δ , based on the dissipation thickness, tends to be a large value for our high-speed flow and is stated as

$$\Delta \triangleq \left[\int_0^\infty \left(\frac{\partial}{\partial y} \frac{\langle u \rangle}{U_\infty} \right)^2 dy \right]^{-1}. \tag{16}$$

We have introduced the turbulent boundary-layer thermal thickness δ_e in Eq. (15). It is defined by

$$\delta_e \triangleq \int_0^\infty \left(\frac{\langle u \rangle}{U_\infty}\right) \left(1 - \frac{\langle u \rangle^2}{U_\infty^2}\right) dy. \tag{17}$$

The foregoing integral approach is replaced by a more exacting numerical procedure and we relieve some of the assumptions, particularly Eqs. (10) and (11) as well as the iterative coupling to the inertial flow, Eq. (1), in the more detailed flow-field structure calculations initiated in Part III and to be presented in final form in a later report.

Fig. 3. Relative displacement effects of turbulent-wall boundary layer in high-Reynolds number flow. Comte-Bellot and Browand configurations.

PART II. DROPLET-AEROSOL DISPERSAL ENVELOPE - WALL-WETTING LIMITS

The next problem addressed was that of defining the water droplet trajectory in the turbulent stream for various injection positions, angles, and injection speeds. The water droplets are entrained by aerodynamic drag in the mean-core aerosol flow. Turbulent radial accelerations act to drive the droplets toward the wall, coating the wall of the tunnel after some length of travel depending on their injection speed, inclination, and the location of the injector. The droplets serve as the oxydizer in the hypergolic, reactive, real-time detection system so that wall coating is to be avoided if possible as the resulting wall reactions would mask the attachment events being measured in the flow. The trajectory calculations in this section provide results for estimating the radial or lateral droplet dispersal to the walls during passage down the tunnel. Hence they provide a means for deciding on placement and injection angle of the droplet injectors.

The most intense turbulent field, that generated in the Browand (Fig. 2) configuration, was used as a basis for the estimated turbulent accelerations and intensity decay. This provided the severest condition in terms of droplet lateral dispersion in comparing the two configurations. The calculated trajectories are all based on this intense turbulent field and are thereby conservative estimates. Part III of this report describes the initial turbulent field estimates.*

Generally the results indicate that wall injection is not a favorable option at any angle or at any speed. The shallow-angle results favor the axial component motion hence speed the droplet through the test section. However, the shallow angles also mean that the droplet is in closer proximity to the wall surface at which it was injected so that the lateral turbulent motions act to move it to the wall more readily. The larger incidence angles promote trajectories that carry the droplet to the opposite wall from that in which it was introduced before the droplet can leave the chamber. This appears to be the case at either injection speed examined: 10 m/s or 50 m/s. Axial injection in alignment with the axis in the flow core appears to be the

* In a subsequent report, currently in preparation, probabilistic diffusional estimates of lateral droplet dispersal are presented and compared to those given here. The rigid droplet dispersal results presented here are more conservative than the diffusional results presented in the later report. Therefore, they are more appropriate for design.

practical answer based on these estimates. Conceivably, the injector could be sting-support mounted on the grid or jet plate itself with the capillary tube extended so that the droplets emerge in the fully developed turbulent region rather than the violently sheared and distorted mixing region immediately behind the grid or plate.

Figure 4 is a plot of a 40- μ m spherical, constant-mass, non-deforming droplet injected on-axis in the fully-developed turbulent field about 20 jet-separation lengths behind the jet plate. Considered are injection velocities of 10 m/s and 50 m/s. These are labeled on the curves. Radial (lateral) position is the abscissa while axial (longitudinal) position is the ordinate measured from the point of injection. The times corresponding to the droplet positions are indicated in seconds by small, labeled circular marks on the trajectory path curves. At either speed, the droplet passes through the test section without colliding with the walls.

The wall injection is illustrated in Fig. 5 for two injection angles (inclination of the injector with respect to the side wall surface) of 20° and 45°. Droplets are predicted to collide with at least the near-wall surface for all four trajectories calculated (two injection angles and two injection speeds labeled on the figure). Again, as in Fig. 4, the trajectory times are explicitly noted on the position curves in seconds adjacent to circular marks on the curves. Radial position is the abscissa while axial position is the ordinate.

Method

The droplet trajectories are computed numerically over incremental time steps Δt . For a droplet path s in x,y space, the instantaneous droplet velocity in the turbulent field is given by

$$\vec{S}(x,y,t) = \vec{S}(x,y,t-\Delta t) + \int_{t-\Delta t}^t \frac{\vec{F}(x,y,t)}{M} dt'. \quad (18)$$

Equation (18) is integrated again for the instantaneous position vector $\vec{S}(x,y,t)$.

The instantaneous force on a droplet of mass M in a turbulent field in the absence of droplet spin, shape change, or mass change is a combination of both drag and normal force imparted by the relative velocity of the droplet with respect to the fluctuating gas velocity. The gas flow is composed of both mean gas flow $\bar{U}_\infty$ and local turbulent velocity fluctuations $U'(x,y)$. It is given by

Fig. 4. 40- μm water droplet trajectories and axial injection. Numbers on curves indicate time in seconds. Browand configuration.

Fig. 5. 40- μ m water droplet trajectories and wall injection at indicated inclination angles. Numbers on curves indicate time in seconds. Browand configuration.

$$2\vec{F}(\vec{x}, y, t) = C_D(\vec{x}, y) \rho_\infty \left[\vec{S}(\vec{x}, y, t) - (\vec{U}_\infty + \vec{U}'(\vec{x}, y, t)) \right]^2. \quad (19)$$

The drag coefficient is computed as a function of local droplet Reynolds number, using experimental spherical drag data including the wake drag (non-Stokesian).⁴ In the absence of deformation, condensation, or evaporation

$$C_D(\vec{x}, y) = f \left[U_{REL}(\vec{x}, y, t), (\text{droplet diameter}), \nu^{-1} \right],$$

where $\vec{U}_{REL}(\vec{x}, y, t)$ is the instantaneous relative droplet velocity given within the square brackets in Eq. (19) and ν is the kinematic viscosity.

The turbulent velocity fluctuations $\vec{U}'(\vec{x}, y, t)$ are estimated from the turbulent intensity decay calculations given in Part III of this report.

PART III. DETAILED TWO-DIMENSIONAL TURBULENT FLOW QUANTITIES

In this section, results are presented of calculations and experimental data extracted from the open literature on affine incompressible turbulent flow situations. The focus is on estimating the turbulent field properties in a sequence of steps. The first, presented here, is a simple estimate based on similarity properties of the flow and dimensional analysis arguments. Subsequently, we will refine these estimates through use of two-dimensional, unsteady flow-field solutions and concomitant turbulence fine-scale digital computer simulations.

The TEULER code described in Appendix A will be the primary tool in these later calculations. In future reports the focus will be on analysis of the experimentally measured turbulence field, the mixing, the energetics, and the attachment process of debris particles to droplets. In conjunction with the experimental analysis, the attachment process in spectral scale dependent turbulent field will be simulated numerically so that the relationships between the laboratory experiments and the physical cloud situation can be systematically defined, tested, and, where possible, verified by meteorological measurements.

The results presented show estimated attenuation of the turbulence with axial distance from a "virtual origin" located behind or downstream of the turbulence generator (grid or jet plate). The region at, and immediately behind, a turbulence generator is characterized by highly disorganized transient mixing, recirculation, and developing transition from initial instability to turbulence. Inhomogeneity and persistent transient changes in the flow

structure in this region make detailed measurements difficult. Characterization of the developing random field properties in this region is even more difficult.

Fortunately this near-generator, developing near-wake situation is a product of the laboratory experiment, not of the physical turbulent cloud field represented. The axial extent of the developing turbulence region and near-wake region can be estimated for the grid screen (18 mesh spacings) and with less certainty for the interactive jet mixing (perhaps 20 times the separation distance between jets).

At this distance the turbulence field is organized and contains measurable mean structural characteristics that may be approximately homogeneous in any cross-tunnel plane measured from the center line core to the edge of the wall boundary layer. At the wall boundary layer, the turbulent core flow mixes intermittently with the distinctive boundary-layer turbulent field. The characterization of the general turbulent field and the transfer of energy, mass, and momentum between the core and this wall boundary-layer region is one of many issues not well understood in theoretical modeling of aerodynamic turbulence. No attempt to address this problem is made here.

For comparing estimated qualities of the turbulent fields generated by the candidate configurations, Figs. 1 and 2, this virtual origin is the demarcation point between the developing turbulence region and the fully-developed turbulence region behind the generator. The fully-developed region is conceptually similar. Length and time scales can be removed or replaced by invariant ratios to fixed axial position x^* or equivalently, time x^*/U_∞ , where the position is measured from the virtual origin x_0 . That is, in the predictions on decaying turbulence behind the generator, the independent variable is

$$x^* = x - x_0.$$

The Taylor analogy⁷ of correspondence between time and space evolution of random turbulent structure is implicit in the decay approximations as well as in the experimental, turbulent (Eulerian), statistical field measurements, to follow. That is, over a sufficiently narrow interval in a randomly varying field we replace temporal derivatives with spatial ones

$$\frac{\partial}{\partial t} () \simeq - \vec{U}_\infty \frac{\partial}{\partial x} (). \quad (20)$$

Incrementally, $(x - x_0)/U_\infty$ corresponds to $t - t_0$, and any spatial inhomogeneity has its temporal analogy:

$$1/2 \langle u'_i u'_i(t_1) \rangle \Rightarrow 1/2 \langle u'_i u'_i(x_1/\vec{U}_\infty) \rangle.$$

A turbulence property of particular interest in the present turbulence-generation configuration comparison is the relative turbulence intensity. It is a relative amplitude measure of the i th component of rms fluctuation to the mean steady flow

$$\left(\frac{u'_i}{\vec{U}_\infty} \right) \triangleq \sqrt{\langle u'_i u'_i \rangle} / \vec{U}_\infty. \quad (21)$$

The mean core flow, $\vec{U}_\infty$, in both Comte-Bellot and Browand configurations is conceptually accelerated to 12.7 m/s by passage through the modest area contraction downstream of the turbulence generator. The contraction, ideally, is positioned at the virtual origin of the similar, self-preserving inertial range turbulent field. The turbulent intensity defined by Eq. (21) decays with axial distance from the virtual origin ($x_0 = 0$.) as shown in Fig. 6. These results are used in the trajectory calculations [see Eq. (19), Part II above] for the spatial dependence of the rms velocity fluctuations.

$$\vec{U}'(x, y, t) = u'_i \vec{i} + u'_j \vec{j}.$$

Figure 6 shows the decay of intensity with distance for three nominal diameters: 10, 30, and 100 cm. Both the Browand (solid line) and Comte-Bellot (dashed line) generator configurations are represented on this figure.

Three turbulent length scales and their decay are represented in the last two figures. Figure 7 shows the decay of the two microscales (Kolmogorov and Taylor) and the macro- or integral scale with axial distance from the virtual origin for the Comte-Bellot configuration. Results for the three nominal diameters are shown. Figure 8 illustrates the corresponding scale decay with axial distance for the Browand configuration.

The two microscales and their decay with distance provide an estimate of the turbulent fine-scale structure, which is potentially present in the test volumes of the two configurations. Turbulence scales of this order may be construed to enhance directly the transport processes, (mass, momentum, and energy) in the neighborhood of similarly sized droplets. In effect, the micro-scale size and population is a qualitative indication of the probability of

Fig. 6. Decay of turbulent intensity with distance from virtual origin. Diameters (D) in cm of two conceptual test tube cross sections appear on the figure.

Fig. 7. Estimated growth of turbulent scales with distance from the origin. Diameters (D) in cm of three conceptual test tube cross sections appear on the figure. Comte-Bellot configuration.

Fig. 8. Estimated growth of turbulent scales with distance from the origin. Diameters (D) in cm of three conceptual test tube cross sections appear in the figure. Browand configuration.

associated levels of diffusive effects in the attachment process. This is only a portion of the probable turbulence dynamics in attachment. Other mechanisms associated with larger scale accelerations, for example, collisional and grazing impact between aerosol and droplet, may be of equal or greater significance in the attachment process. To indicate the estimated range of dependence of other scales of turbulence, the integral scale is also plotted as a function of axial distance for both configurations in Figs. 7 and 8. The integral scale can be interpreted as the largest anticipated range measure for organized or connected random turbulent accelerations associated with the flow locally. It is the distance over which the statistical energy distribution of all smaller scales are interconnected. An aerosol particle captured within this range may experience any portion of the higher frequency fluctuations contained within it down to the microscale limit. The energy cascades from largest organized scales to smallest so that at least in the inertial sub-range the energy distribution with scale is of vital importance in estimating the effects of turbulence on attachment.

The Comte-Bellot values of relative intensity and scale are taken from detailed measurements of the flow conditions described here.^{1,2} The Browand results are only estimates based on dimensional and similarity arguments for decaying, homogeneous, isotropic turbulence^{3,6,8} generated in the spreading wake of a turbulent jet flow. Additional guidance is derived from published information or related experiments.^{9,10}

In general, the Comte-Bellot turbulent field is of lower intensity than that estimated for the Browand configuration. However, it is a well-studied experiment. In addition, the experimental configuration produced a useful, very gradual decay of intensity with distance and was almost perfectly isotropic. These features must be heavily weighed in deciding between the two generators. The Browand configuration can probably be modified for a closer approach to isotropy by use of the modest area contraction. The calculations are based on this modification for the Browand configuration.

Method

The approximations for predicting the decay of turbulence with distance from its source (generator) in the laboratory focus on calculating the dissipation rate of the turbulence. For two-dimensional steady flow the volumetric dissipation rate is, in general, expressed as a product of the mean strain rates which must be measured.^{4,6,8}

$$\dot{E}_v = \mu [2 \langle (\frac{\partial u}{\partial x})^2 \rangle + 2 \langle (\frac{\partial v}{\partial y})^2 \rangle + \langle (\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y})^2 \rangle]. \quad (22)$$

The calculations of the mean strain rates for the specific configuration selected require solutions of the full two-dimensional flow including viscous and fine-scale turbulence modeling. The numerical method is described in the appendix. The detailed calculations will be completed in the next several weeks in order to estimate the spatial and spectral turbulence energy fields in the experiments which are planned to start in December. The level of precision and accuracy obtained in the pre-experimental calculations is of importance in design and selection of the turbulent field diagnostics and analog-to-digital data analysis. At present the discussion is limited to the measured data and approximations used to provide the comparisons of the two turbulence configurations reported here.

First, the volumetric dissipation rate expression, Eq. (22), simplifies considerably under assumptions of isotropy^{4,6}

$$\langle (\frac{\partial u}{\partial x})^2 \rangle = \langle (\frac{\partial v}{\partial y})^2 \rangle \text{ and } \langle \frac{\partial u}{\partial x} \cdot \frac{\partial v}{\partial x} \rangle = 0,$$

as well as incompressibility

$$\langle (\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y})^2 \rangle = 0, \quad \nabla^2 P = 0.$$

In combination with the axial and radial incompressible turbulence (viscosity neglected) momentum equations this leads to the well-known isotropic dissipation expression^{4,6}

$$\dot{E}_v(x,y) = 7.5 \mu \langle (\frac{\partial u}{\partial y})^2 \rangle \quad (23)$$

The (Taylor) microscale limit to the fine-scale structure of a turbulent field is defined here in terms of the two-point statistical correlation.

If measured in one direction (y) this is expressed

$$R_y = \frac{\langle u'(y) \cdot u'(y + \Delta y) \rangle}{\sqrt{u'(y)^2} \sqrt{u'(y + \Delta y)^2}}, \quad 0 \leq R_y \leq 1. \quad (24)$$

Or, in terms of Taylor's experimentally useful equivalence assumption [see Eq. (20)]

$$R_y = \frac{\langle u'(y,t) \cdot u'(y,t + \Delta t) \rangle}{\sqrt{u'(y,t)^2} \cdot \sqrt{u'(y,t + \Delta t)^2}}$$

Taylor showed that if in the immediate neighborhood of a point, the rms value of the velocity component fluctuation can be considered invariant with y (an assumption verified by experiment), a series expansion in terms of the measured mean strain rate gives the correlation coefficient

$$R_y = 1 - \frac{1}{2!} \frac{y^2}{\bar{u}} \langle \left(\frac{\partial u}{\partial y} \right)^2 \rangle + \dots, \quad (25)$$

where

$$\bar{u}^{-2} \triangleq \sqrt{\langle u'^2 \rangle} \cdot \sqrt{\langle u'^2 \rangle}$$

The curvature of a plot of measured R_y versus y near the origin ($y = 0$) gives the mean strain rate. In the limit as $y \rightarrow 0$

$$\langle \left(\frac{\partial u}{\partial y} \right)^2 \rangle = 2 \bar{u}^{-2} \lim_{y \rightarrow 0} \left(\frac{1 - R_y}{y^2} \right) \quad (26)$$

This is a parabola in y which introduces the Taylor microscale in terms of the measured correlation distance limit

$$\frac{1}{\lambda_T^2} = \lim_{y \rightarrow 0} \left(\frac{1 - R_y}{y^2} \right) \quad (27)$$

The parabolic curve of R_y vs y near the origin gives a measurement of λ_T . This microscale measurement is the displacement in y between the origin ($R_y = 1.$) and the intercept ($R_y = 0.$) and is the smallest measurable turbulent correlation structure. The Taylor microscale distribution is identified for each of the configurations in Figs. 7 and 8.

Kolmogorov^{6,8} introduced the fundamental concept of an inertial sub-range of fine-scale turbulence structure which exhibits a dynamic equilibrium between production, dissipation, and diffusion. This concept leads to the

important consequences of similitude in length and time scales as well as isotropy within this sub-range. The inertial sub-range features are wholly determined by the dissipation rate variation with turbulence fluctuational wavelength. The dissipation rate variation with wavelength is functionally independent of the flow situation so that in the Kolmogorov concept this inertial sub-range is essentially independent of the large scale principal energy bearing structure that drives it. A limit to the Kolmogorov spectrum exists where the inertial forces imparted by turbulent accelerations are equilibrated by kinetic viscous forces. The Reynolds number that characterizes this limit is, by definition, unity

$$\text{Re}_\lambda \triangleq \frac{u_\lambda \lambda_k}{\nu} = 1. \quad (28)$$

The scale size λ_k associated with this fluctuational limit is the Kolmogorov microscale limit. Its distribution is shown in Figs. 7 and 8.

In the Kolmogorov sub-range, once the initial dissipation, Eq. (23), is measured or predicted by calculation of the mean strain rate, its spectral dependence is also immediately determined since it theoretically varies as $\kappa^{-5/3}$, $\kappa \triangleq \lambda^{-1}$. In this range, all lengths and velocities are characterized by universal scales independent of the source of the turbulence

$$\ell^* = \left(\frac{\nu^3}{\dot{E}} \right)^{1/4}, \quad u^* = (\nu \dot{E})^{1/4}. \quad (29)$$

A characteristic Reynolds number of the turbulence may be defined as

$$\text{Re}_{\ell^*} = \left(\frac{u^* \ell^*}{\nu} \right), \quad (30)$$

from which the local microscale can be estimated. Through use of dimensional analysis arguments, the Reynolds-number dependence of the microscale is

$$\frac{\lambda_k}{\ell^*} = (\text{Re}_{\ell^*})^{-3/4}.$$

In the absence of measured values the Taylor microscale can be similarly estimated as a function of the characteristic Reynolds number

$$\frac{\lambda_T}{\ell^*} = (Re_{\ell^*})^{-3/4}.$$

In the foregoing expressions, Eqs. (29) and (30), the turbulent fine-scale range is seen to be essentially characterized by the specific dissipation rate

$$\dot{E} \triangleq \dot{E}_v / \rho_\infty.$$

It remains to show how this is approximated with distance in the estimates shown in Figs. 6, 7, and 8. First, a definition is given of the largest contiguous scale of turbulence, the integral scale appearing in the figures. In terms of the measured or estimated correlation distribution in one dimension, this integral length scale is

$$L \triangleq \int_0^\infty R_y dy. \quad (31)$$

In the absence of measured mean strain rates, the turbulence energy-dissipation rate can be estimated in terms of the mean square velocity. Combining Eqs. (26) and (27), we obtain

$$\dot{E}_v = 15 \mu \bar{u}^2 / \lambda_T^2. \quad (32)$$

The distribution of λ_T can then be characterized by the distribution of the characteristic Reynolds number of turbulence once the local distribution of velocity u^* and length scale ℓ^* are known. In the present estimates similarity solutions were used to approximate these distributions.

It is useful to find a relationship for the decay of the turbulence intensity and scales in the wake of the fixed turbulence grid. In turn, this can be checked against the experimental measurements,^{1,2} to lend support to a similar development for the jet flow wake. The mean rate of loss of turbulence kinetic energy behind a grid is given by

$$- 1/2 \rho \bar{U}_\infty \frac{d}{dx} (\bar{u}^2 + \bar{v}^2 + \bar{w}^2)$$

or in isotropic turbulence

$$- 3/2 \rho \bar{U}_\infty \frac{d(\bar{u})^2}{dx}, \quad \bar{u} \triangleq \sqrt{\langle u'^2 \rangle}. \quad (33)$$

Experimentally^{4,8} it has been determined that for grid-generated turbulence where the grid spacing M is at least five times the grid bar diameter, the wake becomes almost uniform and isotropic, transverse to the primary flow direction at a distance of about 20 grid spaces behind the grid. The microscale distribution may be estimated as a fixed function of the local turbulence intensity $\bar{u}$ in accordance with

$$\frac{\lambda_T}{M} = C_1 \sqrt{\frac{\nu}{M\bar{u}}} \quad (34)$$

This is used to eliminate λ_T in the isotropic dissipation-rate expression given by Eq. (32). This dissipation-rate expression is then set equal to the kinetic energy loss, Eq. (33). The resulting expression integrates directly to give an estimate of the decay of relative intensity with distance x

$$\frac{\bar{U}_\infty}{\bar{u}} = \frac{5x}{C_1^2 M} + \text{constant}. \quad (35)$$

The constant C_1 is determined by experiment to vary within narrow limits: order of 1 for the Comte-Bellot^{1,2} (1966) experiments up to about 2.0 for experiments completed about 30 years ago.^{4,8} The simple isotropic wake-decay relationships (34) and (35) are in satisfactory agreement with the measured values of Comte-Bellot which are correlated by

$$\frac{\bar{U}_\infty}{\bar{u}} = 4.6 \left(\frac{x}{m} - 3.5 \right)^{1.12}$$

and

$$\frac{\bar{U}_\infty}{\bar{v}} = 4.5 \left(\frac{x}{M} - 3.5 \right)^{1.12}. \quad (36)$$

The other constant is determined by the measured intensity at the virtual origin of fully-developed turbulence, about 18 mesh spaces behind the grid in the Comte-Bellot experiments.

The integral scale, Eq. (31), is the driving-energy scale containing, proportionately, the largest share of turbulence energy. Its size is of the order of the characteristic dimension of the source (grid spacing or jet diameter). It should be constant, and should persist, in accordance with the simple isotropic theory of turbulence. Integration of the measured two-point correlations, however, reveals a modest growth with distance. This is

expressed by the weak (less than 1/2 power) spatial dependence of the measurements which follow the relationship^{1,2}

$$\frac{L}{M} = 0.048 \left(\frac{x}{M} - 3.5 \right)^{0.4}. \quad (37)$$

Conceptually, this growth may reflect that an even larger scale structure (of the order of the tunnel diameter) is weakly coupled with the grid-born turbulence. The weakness of the coupling is characterized by the very slow growth rate with distance and may imply the lack of energy content in these largest (tunnel diameter) wavelengths.

Analytical similarity solutions of the isolated, round, turbulent jet issuing into quiescent surroundings were obtained originally by Tollmein in the late 1920's. These solutions were later modified in the 1930's for the slight effects of transverse pressure gradient and a somewhat better approximation for the entrainment of the outer flow through use of spatially varying eddy exchange coefficients in the mixing layer.^{4,8}

Information of importance on two of the scale distributions affecting the turbulent jet decay is obtained from these analytical solutions, somewhat more comprehensive numerical solutions by the writer and others, and experiments conducted with advanced diagnostics in recent years.^{9,10}

First, we consider the isolated, round, turbulent jet spreading with axial (x) position. Second, we note the behavior of the characteristic maximum velocity (centerline velocity U_E) of the turbulent jet with axial position. All characterizations are associated with the similar, self-preserving, turbulent jet growth, hence are developed in the region $x_0 \leq x^* \leq x_c$. The origin or virtual origin x_0 of the similar turbulence features is the position at which fully-developed turbulence of maximum relative intensity conceptually commences. The developing region $x \leq x_0$ has been discussed before. The region $x \geq x_c$ has been examined theoretically and experimentally with sufficient diligence to indicate that it is no longer similar, possesses dominant features not associated with the source or near wake, and inherently resembles the decay of the turbulence at large distances behind the standard turbulence grid previously discussed, Eq. (35).

Considering the simple situation of the jet boundary growth undergoing turbulent fluctuations, one may write^{4,8}

$$\frac{d(b)}{dt} \approx v', \quad (38)$$

but mixing length hypotheses, indicate the correspondence

$$u', v' \approx \ell_M \frac{\partial \bar{u}}{\partial y}, \quad (39)$$

where, in the absence of unsteady (mean flow) effects,

$$\frac{\partial \bar{u}}{\partial y} \approx \text{constant} * \frac{U_M}{b} \ell_M, \text{ where } b \triangleq 1/2 \text{ width of jet core,}$$

and for steady flow, on integration, one obtains the simple result

$$b = C_1 * x + C_2, \quad (40)$$

where the constants C_1 and C_2 are given by the jet width and length at the virtual origin. The width is assumed to fill the hole-to-hole space.

Second, the calculations and Browand's estimates indicate that the maximum (centerline) jet velocity issuing into a quiescent surrounding falls off with increase in axial displacement from the virtual origin, x_0

$$\frac{U_E}{U_{jet}} = \frac{\text{Constant} * D_j / x}{1 - \frac{\text{Constant}}{10} \left(\frac{U_\infty}{U_j} \right)}, \text{ where } D_j \triangleq \text{issuing jet diameter.} \quad (41)$$

Browand³ estimates the value of this constant at 6.4, while in the writer's calculations this constant appeared to vary from about 5.5 to 7.0 and is very dependent on the turbulence-exchange model. For the present estimates Browand's factor is used for the region upstream of jet interaction.

With the spreading rate and velocity characterized, length ℓ^* and velocity u^* profiles with distance are determined. It follows that the characteristic turbulence Reynolds number, Eq. (30), and the dissipation rate, Eq. (29), can be estimated with distance from the virtual origin. However, this is valid to about 30 jet diameters. Past this point, the similar behavior of the jet breaks down. In this far wake-decay region, the grid-decay estimates developed for the Comte-Bellot configuration are applied to preliminary analyses presented here, Eqs. (34) and (35). Further modifications to the simple approximations include estimated relative intensities and anisotropy based on the experiments of Betchov¹⁰ together with the detailed wake-decay information from the Uberoi experiments.⁹

Additional modification was made to these estimates in applying the modest growth of the integral scale implied by Eq. (37) for the jet-plate test configuration, assumed to be of identical overall test section geometry as that of the Comte-Bellot. The results are summarized in Figs. 6, 7, and 8.

Energy-spectrum calculations and flow-nonuniformity predictions will be reported together with the results of the detailed two-dimensional flow calculations planned for the next phase.

PRELIMINARY CONCLUSIONS

1. The calculations presented are inherently one-dimensional and make considerable use of similitude and dimensional analysis arguments. The turbulence energetics and turbulent spectral predictions which will follow require much more information, particularly: two-dimensionality; unsteady, large-scale, turbulence properties; nonlinear diffusive effects; and competition of at least two distinct time scales, namely finite difference marching time step with model equations and transients induced by large-scale motions or turbulent fluctuations about the mean velocity.
2. The current estimates on peak intensity and volumetric perturbations of significant amplitude favor the Browand configuration for maximum detectable effects in aerosol-droplet attachment.
3. The Comte-Bellot experiment is well-documented, has been widely studied, and is perhaps better understood. It possesses modest intensity in comparison to the Browand jet plate, but decays more slowly and is almost ideally isotropic (about 5% deviation).
4. The system design analysis and preliminary turbulence numbers are offered to stimulate initial design and fabrication and to further discussion and revision of those estimates. The calculations serve as a basis and a primary motive for the more detailed two dimensional, unsteady calculations which will be reported subsequently.

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APPENDIX A. DESCRIPTION OF THE TEULER FINITE-DIFFERENCE CODE METHOD

General

The TEULER code incorporates the general nonsteady conservation equations for both laminar (molecular) and turbulent momentum and energy dissipation. TEULER (Turbulent Euler) is formulated in an explicit Eulerian finite-difference scheme, using a second-order time-splitting procedure developed by MacCormack (1969).¹

The TEULER code treats the unsteady conservation of mass, momentum, and energy in two-space dimensions (either axial or plane symmetry). The difference method is being extended to three-space dimensions and time. In general, TEULER is nonsteady and compressible. Steady state flow situations are obtained as a late-time consequence of an initial-value nonsteady flow process. This permits use of the hyperbolic forward-marching technique as an iteration procedure. This is particularly useful for handling mixed subsonic and supersonic flows and some classes of nonlinear boundary value problems.

Outline of Formulation

The mass, momentum, and total or internal energy-conservation relations are expanded in rectilinear cartesian computational coordinates* for random, turbulent fluctuational flow fields. Introducing the mean time-averaged density ($\bar{\rho}$), velocity ($\bar{u}_j$), pressure ($\bar{P}$), and internal energy ($\bar{I}$), the governing equations are written

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial}{\partial x_j} (\bar{\rho} \bar{u}_j) = 0, \quad (1)$$

$$\frac{\partial (\bar{\rho} \bar{u}_i)}{\partial t} + \frac{\partial}{\partial x_j} \left[(\bar{\rho} \bar{u}_j \bar{u}_i) + \bar{P} \delta_{ij} - \tau_{ij} + \langle \bar{\rho} u'_j u'_i \rangle \right] = 0 \quad (2)$$

* Transformation from computational to "natural" or "physical" coordinates may be specified to take place automatically at each integration cycle. This is described in a subsequent section. The benefits of mesh transformation: Preservation of accuracy, boundary condition and geometry integrity, and minimization of nodal requirements are becoming a standard feature in TEULER simulations.²

$$\frac{\partial}{\partial t} \left[\bar{\rho} \left(\bar{I} + \frac{\bar{u}_i \bar{u}_i}{2} \right) \right] + \frac{\partial}{\partial x_j} \left[\left(\bar{I} + \frac{\bar{u}_i \bar{u}_i}{2} \right) \bar{\rho} u_j - \bar{u}_i \tau_{ij} + \langle \bar{\rho} u'_i u'_j \rangle \bar{u}_i \right. \\ \left. - \frac{\mu}{PR_L} \left(\frac{\partial \bar{I}}{\partial x_j} \right) + \bar{P} u_j + \langle \bar{\rho} u'_j \bar{I} \rangle \right] = 0, \quad (3)$$

together with the equation of state

$$\bar{P} = \bar{P}(\bar{\rho}, \bar{I}). \quad (4)$$

We introduce from Saffman and Wilcox³ the mean by mass-density averaging

$$\bar{u}_i = \tilde{u}_i - u'_i = \langle \rho' \tilde{u}_i \rangle / \bar{\rho}, \quad \bar{I} = \tilde{I} - I' = \langle \rho' \tilde{I} \rangle / \bar{\rho}, \quad (5)$$

$$\bar{\rho} = \frac{1}{2\Delta t} \int_{t-\Delta t}^{t+\Delta t} \rho' dt.$$

The quantities designated by a tilde overhead are instantaneous values of velocity and internal energy while u'_i and I' are the differences between the mass-averaged and instantaneous values (fluctuations) as implied by Eq. (5). The term Δt is an incremental time step at the n th finite-difference explicit-time epoch

$$t = \sum_i n_i \Delta t_i. \quad (6)$$

The mean laminar viscous stress tensor is also carried, in general, for Navier-Stokes simulations at low Reynolds number and to test asymptotic limits (high wave number and at non-slip boundaries). For this we introduce as local transport variables the molecular coefficients of bulk λ and shear viscosity μ

$$\tau_{ij} = \lambda \delta_{ij} \frac{\partial \bar{u}_i}{\partial x_j} + \mu \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right), \quad (7)$$

where δ_{ij} and PR_L are the Kronecker delta tensor and mean laminar Prandtl number (the ratio of momentum diffusivity to thermal diffusivity), respectively.

The sub-grid scale^{*} Reynolds stress tensor σ_{ij} is related, as in the Saffman-Wilcox model, to an eddy viscosity ϵ and a specific turbulent energy e .

* Grid scale or computable unsteady fluctuations of the order of microsecond time intervals are coupled using an imposed explicit intermittent fluctuational model as for example in Buckingham (1972).⁴

$$e \equiv \langle \rho' u_i' u_i' \rangle / 2\bar{\rho}, \quad (8)$$

$$\sigma_{ij} \equiv - \langle \bar{\rho} u_i' u_j' \rangle = \bar{\rho} \epsilon \left[\frac{\partial \bar{u}_i}{\partial x_j} \frac{\partial \bar{u}_j}{\partial x_i} + \lambda \frac{\partial \bar{u}_k}{\partial x_k} \delta_{ij} \right] - 2/3 \bar{\rho} e \delta_{ij},$$

while the turbulent flux of internal energy is given in terms of the turbulent Prandtl number PR_T by

$$- \langle \bar{\rho} u_j' \bar{I}' \rangle = (\bar{\rho} \epsilon / PR_T) \frac{\partial \bar{I}}{\partial x_j}. \quad (9)$$

Closure requires two additional non-linear diffusion equations. These relate the localized eddy viscosity to two modeled properties of the turbulence at each finite difference grid node. These closure properties are the specific turbulent energy e and the pseudovorticity corresponding to the squared fluctuational enstrophy

$$\Omega \equiv \frac{1}{2} \left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i} \right)^2. \quad (10)$$

The two model diffusion equations are inhomogeneous, so that care must be taken in treating the source terms. These describe energetic growth of the turbulence strength through interaction with the mean rate of strain and the diffusion of the turbulence through its effect on mean velocity flux divergence

$$\frac{\partial}{\partial t} (\bar{\rho} e) + \frac{\partial}{\partial x_j} \left[(\bar{\rho} u_j e) - (\bar{\mu} + \sigma^* \bar{\rho} \epsilon) \frac{\partial e}{\partial x_j} \right] \quad (11)$$

$$= \left[\alpha^* \sqrt{\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right)^2} + 2 \left(\frac{\partial \bar{u}_x}{\partial x_k} \right)^2 \beta^* \bar{\rho} \Omega - \xi \frac{\partial \bar{u}_k}{\partial x_k} \right] \bar{\rho} e,$$

$$\frac{\partial}{\partial t} (\bar{\rho} \Omega^2) + \frac{\partial}{\partial x_j} \left[(\bar{\rho} u_j \Omega^2) - (\bar{\mu} + \sigma^* \bar{\rho} \epsilon) \frac{\partial \Omega^2}{\partial x_j} \right] \quad (12)$$

$$= \left[\alpha \sqrt{\left(\frac{\partial \bar{u}_i}{\partial x_j} \frac{\partial \bar{u}_j}{\partial x_i} \right)} - \beta \bar{\rho} \Omega \right] \bar{\rho} \Omega^2.$$

Additional theoretical-numerical effort is currently underway which is directed specifically at an automatic computational scheme to dynamically select an implicit or explicit procedure to treat the source terms during solutions.⁵

The coefficients $\sigma, \sigma^*, \alpha, \alpha^*, \beta, \beta^*$ are currently constant values as suggested by the Saffman-Wilcox (1974)² studies. The turbulent Prandtl number PR_T , and turbulent energy diffusion coefficient, ξ , are specified initially in correlation with the known state of the gas.

The structure of the turbulent boundary layer model is compatible with the integral properties for the near wall inner boundary layer and shear layer dividing streamline region through the retention of the mean molecular Navier-Stokes terms associated with the μ, λ , and PR_L coefficients retained.

Coordinate Transformation

In current Navier-Stokes simulations, the utility of automatic coordinate transformation has been amply demonstrated.² As an example, consider the viscous flow over an airfoil. Introduce a compressible stretching

$$\tilde{x}_2 \equiv \frac{1}{\delta} \int_0^{x_2'} \rho/\rho_0 \, dx_2. \quad (13)$$

Also introduce streamwise condensation of nodes about a critical point (mean 1/3 chord, etc.)

$$d\xi = [f(x)/f(\bar{x})] \, dx$$

$$f(x) = \frac{1}{1 + \left(\frac{x-x_c}{c_1} \right)^J}, \text{ where } J = 0, 2, 4, \dots \quad (14)$$

The mean value $\bar{f}(x)$ of the function is obtained by quadrature

$$f(\bar{x}) = \frac{1}{\pi} \int_{-x/3}^{2x/3} f(x) \, dx'.$$

Minimum Y normal to plate is obtained as a weighted length with respect to the steady-state boundary-layer attachment distance, namely

$$(\Delta Y)_{\min} = \frac{c_2 \int_0^{\infty} \frac{\rho(Y)}{\rho^{\infty}} dY'}{(R_e)^{1/5}}, \quad (15)$$

where $0 < c_2 < 1$ is the fraction of local steady-state boundary-layer displacement thickness. The normal transformation η is then given by

$$d\eta = [g(Y)/g(\bar{Y})] dy,$$

where

$$g(Y) = \frac{1}{1 + \left(\frac{Y}{c_3}\right)^J}, \quad (16)$$

and

$$g(\bar{Y}) = \frac{1}{\bar{Y}} \int_0^{\bar{Y}} g(Y) dY'.$$

In addition, transform the difference equations corresponding to the derivative quantities by successive use of the chain rule

$$\frac{\partial}{\partial Y} () = > \frac{\partial}{\partial \eta} () \frac{\partial \eta}{\partial y} + \frac{\partial}{\partial \xi} () \frac{\partial \xi}{\partial y}. \quad (17)$$

. . . etc.

Outline of Finite-Difference Procedure

The finite difference technique is an adaptation of the second order explicit MacCormack method.² The flow Eqs. (1), (2), and (3) are represented in the basic scheme by the vector differential equation

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x_1} + \frac{\partial G}{\partial x_2} = 0. \quad (18)$$

The transport Eqs. (11) and (12) carry, in addition, diffusive flux and source terms requiring an extended form of the vector differential Eq. (19), written

$$\frac{\partial U^*}{\partial t} + \frac{\partial F^*}{\partial x_1} + \frac{\partial G^*}{\partial x_2} = H. \quad (19)$$

The vector components are

$$U \equiv \begin{pmatrix} \bar{\rho} \\ \bar{\rho}\bar{u}_1 \\ \bar{\rho}\bar{u}_2 \\ \bar{E} \end{pmatrix}, \quad F \equiv \begin{pmatrix} \bar{\rho}\bar{u}_1 \\ \bar{\rho}\bar{u}_1^2 + \bar{\rho}\bar{u}_1\bar{u}_2 + \bar{P} - \tau_{12} + \langle \bar{\rho} u'_1 u'_2 \rangle \\ \bar{\rho}\bar{u}_1\bar{E} - \bar{u}_1\tau_{12} + \langle \bar{\rho} u'_1 u'_2 \rangle \bar{u}_1 - \frac{\mu}{PR_L} \frac{\partial \bar{I}}{\partial x_1} + P\bar{u}_1 + \langle \bar{\rho} u'_1 \bar{I}' \rangle \end{pmatrix}$$

$$G \equiv \begin{pmatrix} \bar{\rho}\bar{u}_2 \\ \bar{\rho}\bar{u}_2^2 + \bar{\rho}\bar{u}_2\bar{u}_1 + \bar{P} - \tau_{21} + \langle \bar{\rho} u'_2 u'_1 \rangle \\ \bar{\rho}\bar{u}_2\bar{E} - \bar{u}_2\tau_{21} + \langle \bar{\rho} u'_2 u'_1 \rangle \bar{u}_2 - \frac{\mu}{PR_L} \left(\frac{\partial \bar{I}}{\partial x_2} \right) + P\bar{u}_2 + \langle \bar{\rho} u'_2 \bar{I}' \rangle \end{pmatrix}, \quad (20)$$

$$E \equiv \left(\bar{\rho} \left(I + \frac{\bar{u}_1 \bar{u}_1}{2} \right) \right)$$

$$U^* \equiv \begin{pmatrix} \bar{\rho}e \\ \bar{\rho}\Omega^2 \end{pmatrix}, \quad F^* \equiv \begin{pmatrix} \bar{\rho}\bar{u}_1 e - (\mu + \sigma^* \bar{\rho} \epsilon) \frac{\partial e}{\partial x_1} \\ \bar{\rho}\bar{u}_1 \Omega^2 - (\mu + \sigma^* \bar{\rho} \epsilon) \frac{\partial \Omega^2}{\partial x_1} \end{pmatrix}, \quad (21)$$

$$G^* \equiv \begin{pmatrix} \bar{\rho}\bar{u}_2 e - (\mu + \sigma^* \bar{\rho} \epsilon) \frac{\partial e}{\partial x_2} \\ \bar{\rho}\bar{u}_2 \Omega^2 - (\mu + \sigma^* \bar{\rho} \epsilon) \frac{\partial \Omega^2}{\partial x_2} \end{pmatrix},$$

$$H \equiv \begin{pmatrix} \left[\alpha^* \sqrt{\left(\frac{\partial \bar{u}_1}{\partial x_2} + \frac{\partial \bar{u}_2}{\partial x_1} \right)^2} + 2 \left(\frac{\partial \bar{u}_1}{\partial x_1} + \frac{\partial \bar{u}_2}{\partial x_2} \right)^2 - \beta^* \bar{\rho} \Omega - \xi \left(\frac{\partial \bar{u}_1}{\partial x_1} + \frac{\partial \bar{u}_2}{\partial x_2} \right) \right] \bar{\rho} e \\ \left[\alpha \sqrt{\left(\frac{\partial \bar{u}_1}{\partial x_2} \right)^2 + \left(\frac{\partial \bar{u}_2}{\partial x_2} \right)^2 + \left(\frac{\partial \bar{u}_1}{\partial u_2} + \frac{\partial \bar{u}_2}{\partial x_1} \right)^2} - \beta \bar{\rho} \Omega \right] \bar{\rho} \Omega^2 \end{pmatrix}$$

For illustration, consider the nth time interval

$$\Delta t(n) \equiv \Delta t^{n-1} = t(n) - \sum_{j=z}^{n-1} \Delta t(j), \quad (22)$$

over which the transport Eq. (19) is integrated by time-splitting with a predictor. Actually, in the current TEULER formulation, the spatial sweeps are split into separate operations. Updating from one cycle to the next is a four-step sequence: 2 fractional times and 2 spatial sweeps. We start with a predictor step

$$\begin{aligned} \hat{U}_{k,L}^{*n} = & U_{k,L}^{*n-1} - \frac{\Delta t^{n-1}}{\Delta x_1} \left(F_{k+1,L}^{*n-1} - F_{k,L}^{*n-1} \right) - \frac{\Delta t^{n-1}}{\Delta x_2} \left(G_{k,L+1}^{*n-1} - G_{k,L}^{*n-1} \right) \\ & + \Delta t^{n-1} \left(H_{k+1/2,L+1/2}^{n-1} \right), \end{aligned} \quad (23)$$

then follow with a corrector step

$$\begin{aligned} U_{k,L}^{*n} = & 0.5 \left[\left(\hat{U}_{k,L}^{*n} + U_{k,L}^{*n-1} \right) - \frac{\Delta t^{n-1}}{\Delta k_1} \left(\hat{F}_{k,L}^{*n} - \hat{F}_{k-1,L}^{*n} \right) - \frac{\Delta t^{n-1}}{\Delta x_2} \right. \\ & \left. \left(\hat{G}_{k,L}^{*n} - \hat{G}_{k,L-1}^{*n} \right) + \Delta t^n \left(\hat{H}_{k-1/2,L-1/2}^{*n} \right) \right] \end{aligned} \quad (24)$$

The courant and Fourier diffusion-stable integration time step is computed at each cycle

$$\Delta t^{n-1} = f \left\{ \text{minimum} \left| (\Delta x_1, \Delta x_2) \right|, \hat{u}_1, \hat{u}_2, \hat{P}, \hat{\rho}^{n-1} \right\} \quad (25)$$

The inhomogeneous diffusive flux and source terms are evaluated as slowly varying averaged contributions to the transport

$$\begin{aligned} H_{k+1/2,L+1/2}^{n-1} & \equiv 0.25^* \left(H_{k+1,L}^{n-1} + H_{k,L+1}^{n-1} + 2H_{k,L}^{n-1} \right), \\ \hat{H}_{k-1/2,L-1/2}^{*n} & \equiv 0.25^* \left(\hat{H}_{k-1,L}^{*n} + \hat{H}_{k,L-1}^{*n} + 2\hat{H}_{k,L}^{*n} \right). \end{aligned} \quad (26)$$

Here,

$$H_{k,L}^n \equiv g \left(\frac{\partial u_i^n}{\partial x_j}, \frac{\partial u_j^n}{\partial x_i}, \Omega^n, e^n \right). \quad (27)$$

The cross derivatives are replaced by substitution of a second-order Taylor series expansions about the localized computational node.

Stability and convergence characteristics, examples of solutions, accuracy comparisons with theory and other specific modifications to TEULER for particular applications are being included in a separate LLL technical report currently in preparation.

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Available from
National Technical Information Service
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