Magnetohydrodynamic Equilibria and Local Stability of Axisymmetric Tokamak Plasmas

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THERMONUCLEAR DIVISION
MAGNETOHYDRODYNAMIC
EQUILIBRIA AND LOCAL STABILITY OF
AXISYMMETRIC TOKAMAK PLASMAS

by

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MAGNETOHYDRODYNAMIC EQUILIBRIA AND LOCAL STABILITY
OF AXISYMMETRIC TOKAMAK PLASMAS*

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ABSTRACT

Axisymmetric Magnetohydrodynamic equilibria are evaluated in terms of the Mercier Stability Criterion. The parameters of interest include poloidal beta (β_p), current and pressure profile widths, D-shaped and doublet plasmas with elongation (σ) and triangularity (δ), and the aspect ratio (A). For marginal local stability, the critical values of β , plasma current, and the safety factor q with fixed toroidal field at the geometric center of the plasma are obtained. It is shown that for a wide range of profiles in a D-shaped plasma with $A = 3$, the highest critical β occurs at $\beta_p = 2.4$, $\sigma = 1.65$, and $\delta = 0.5$. If the toroidal field at the coil surface is fixed, the highest critical pressure occurs near $A \approx 3-4$, given reasonable distance between the coils and the plasma edge. Calculations for a Doublet II-A plasma with $\sigma = 3$ show that with similar pressure profile the highest critical β occurs at $\beta_p = 1$ and is 84% of the highest critical β for the D-shaped plasmas. Critical values of ohmic heating power density are also found to be comparable for the two plasma shapes. A D-shaped plasma with the above parameters is suggested for use in future high- β tokamak devices.

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1. INTRODUCTION

Recent neutral-beam injection heating and gas-puffing experiments have produced substantial plasma beta (β) increases in ohmically heated tokamaks.¹ Poloidal beta ($\beta_p \propto \bar{p}/I_p^2$, I_p being the plasma current) approaching the aspect ratio is anticipated in future neutral-beam tokamaks and tokamak reactors.² It is also generally accepted that, to improve β , a tokamak device should have a noncircular cross section. It is therefore of interest to investigate the effects of such high β_p values and of plasma shaping on MHD equilibrium and stability.

A systematic analysis of axisymmetric equilibria with noncircular cross sections has been carried out by Mazzucato.³ By obtaining a class of exact solutions for low β_p equilibria, he has calculated the limiting current density and safety factor (q) at the magnetic axis for local stability as functions of β and plasma elongation.

Here we present numerically obtained equilibria for a wide range of equilibrium parameters. These include β_p , plasma pressure and current profiles, elongation (σ) and triangularity (δ) for D-shaped and doublet plasmas, and the aspect ratio (A). For a fixed vacuum toroidal field at the geometric center of the plasma, the plasma current (I_p) is increased until the Mercier Stability Criterion^{4,5} is violated somewhere in the plasma. This gives the critical values of β ($\propto \bar{p}/B_0^2$, B_0 being the vacuum toroidal field at the geometric center), I_p (or the volume-averaged ohmic heating power density), and safety factor values q_0 and q_a at the magnetic axis and the plasma edge. A systematic study of the dependence of these is presented.

The numerical equilibrium solutions are described in Section 2. These equilibria are diagnosed in terms of their stability against localized ideal MHD modes in Section 3. Applications of the results to future tokamaks are discussed in Section 4.

2. MHD EQUILIBRIA

The equations for MHD equilibria in two-dimensional axisymmetric geometry can be reduced to an elliptic nonlinear differential equation for the poloidal flux $\psi(R, Z)$,⁶⁻⁹

$$R^2 \nabla \cdot (R^{-2} \nabla \psi) = -R^2 p' - FF', \quad (1)$$

in the cylindrical coordinates (R, ϕ, Z) . Here the plasma pressure p and the toroidal field function $F \equiv RB_\phi$ are treated as given functions of ψ . The primes denote derivations with respect to ψ . The magnetic field B and current density J can then be written as

$$B = -\frac{F}{R} \hat{\phi} - \frac{1}{R} \nabla \psi \times \hat{\phi},$$

and

$$J = -J_T \hat{\phi} - \frac{1}{R} \nabla F \times \hat{\phi}, \quad J_T = Rp' + \frac{FF'}{R}, \quad (2)$$

where $\hat{\phi}$ is the unit vector in the ϕ direction.

PROFILE FUNCTION

The forms of $p(\psi)$ and $F(\psi)$ can be arbitrarily chosen; they implicitly determine the resulting profiles and β_p values of the equilibrium. Various analytical^{3,10} and numerical¹¹ solutions of Eq. (1) have been obtained when p' and FF' are constant or linear in ψ . However, this generally results in low- β_p equilibria with toroidal current profiles that are relatively flat and have relatively large current at the plasma edge. By using more elaborate forms of p and F in a numerical approach,⁹ one is able to prescribe the current and pressure profiles and the β_p value in addition to the arbitrary plasma cross section. Thus, a nonlinear function $p(\psi)$ is introduced,

$$\begin{aligned}
p &= \beta_J p_0' \left\{ (\psi - \psi_b) - \frac{\alpha}{2} \left[(\psi - \psi_m)^2 - (\psi_b - \psi_m)^2 \right] \right. \\
&\quad \left. + \frac{\gamma}{L+1} \left[(\psi - \psi_m)^{L+1} - (\psi_b - \psi_m)^{L+1} \right] \right\}, \quad L \geq 4 \quad (3) \\
\gamma &= - \left[1 + \alpha(\psi_m - \psi_b) \right] / (\psi_b - \psi_m)^L.
\end{aligned}$$

Here, α can be adjusted to narrow or broaden the pressure profile, and γ is introduced to make $p' = 0$ at the plasma boundary where $\psi = \psi_b$. The value of p_0' can be adjusted to obtain convenient values of $\psi = \psi_m$ at the magnetic axis; β_J is put in to vary β_p of the equilibrium.

Integrating Eq. (1) along a flux surface before integrating in ψ , one obtains

$$I_p(\psi) = - \frac{1}{2\pi} \int_{\psi_m}^{\psi} d\psi V' (p' + FF' \langle R^{-2} \rangle), \quad (4)$$

$$V' = 2\pi \oint d\ell / B_p, \quad \langle X \rangle = \frac{2\pi}{V'} \oint X d\ell / B_p.$$

Here B_p is the poloidal magnetic field and $I_p(\psi)$ is the toroidal current inside the flux surface at ψ . The value of β_p can be increased by increasing β_J if the toroidal plasma current is fixed. It is seen that if the integrand in Eq. (4) can be independent of β_J , $I_p(\psi)$ remains fixed.

It is difficult to do this exactly, because $\langle R^{-2} \rangle$ can be calculated only after $\psi(R, Z)$ is obtained. However, we can make I_p nearly independent of β_J by making the integrand roughly equal to p'/β_J .

This gives the following form of FF' :

$$FF' = (\beta_J^{-1} - 1) p' / \langle R^{-2} \rangle \approx (\beta_J^{-1} - 1) R_0^2 p', \quad (5)$$

where $R = R_0$ at the plasma geometric axis. With Eq. (5), it is found that the equilibrium β_p values always fall within 20% of β_J for values of β_J up to the aspect ratio A . Note that Eq. (5) also sets $J_T = 0$ at the plasma edge.

With these functions $p(\psi)$ and $FF'(\psi)$, Eq. (1) is solved⁹ by the method of successive over relaxation (SOR)¹² correction of an initially estimated solution ψ . Since Eq. (1) is nonlinear, there is no straightforward formula relating p_0' and α in Eq. (3) to the $p(R, Z)$ and $J_T(R, Z)$ profiles. However, the numerical method permits adjustment of p_0' and α during iteration to produce solutions with prescribed profile widths. This enables us to compare equilibria with the same p or J_T profile width.

Figure 1 illustrates the dependence of the J_T profiles on β_p for fixed pressure profile half-width (w_p , normalized to the plasma minor width). In addition to the well-known appearances of the reversed current⁹ at large β_p values, it is seen that the current profile becomes progressively more peaked with increasing w_p or β_p . Thus, in order to preserve "reasonable" current profiles while increasing β_p , w_p has to decrease. While appropriate transport calculations must eventually be included to determine how w_p depends quantitatively on β_p , this decrease of w_p is plausible. We find that smoothness of the J_T profile is well preserved by using fixed values for $(\psi_b - \psi_m)$ and α as β_p is increased (thus prescribing a particular way that w_p should decrease with increasing β_p ; see Fig. 5b).

It is of interest to point out some implications of the results shown in Fig. 1, where the J_T peak starts to shift substantially away from the pressure peak as β_p increases beyond 1. If we assume that the ohmic heating loop voltage, V_L , is constant over the plasma cross section, the ohmic heating power density will be proportional to J_T/R . The flux surface average and the volume average of this power density are then expressed by

$$H(\psi) = - \frac{V_L}{2\pi} (p' + FF' \langle R^{-2} \rangle)$$

and

$$\bar{H} = \frac{\int_{\psi_m}^{\psi_b} H(\psi) V' d\psi}{\int_{\psi_m}^{\psi_b} V' d\psi} \quad (6)$$

Calculations show that $H(\psi)$ generally decreases as ψ increases from ψ_m , except for $w_p \gtrsim .65$ and $\beta_p > 1.5$ A. This means that the shifted J_T profile still promotes a normal pressure profile peaked at the magnetic axis. However, J_T has to be maintained by some auxiliary means, such as neutral-beam driven current.

NONCIRCULAR CROSS SECTION

The numerical approach using SOR can accept various contour functions as the plasma boundary. "Ghost" points immediately outside the boundary are used to ensure that $\psi = \psi_b$ along the boundary. We use 30 x 60 points and 16-digit accuracy and require results with maximum relative change in ψ below 10^{-6} . This accuracy enables us to obtain smooth numerical derivatives of the surface functions used in evaluating the Mercier criterion (see Section 3).

D-shaped equilibria of various elongation (σ) and triangularity ($\delta = x_p/a$, see Fig. 2a) and doublet II-A equilibria of various σ have been obtained. Typical ψ -plots of these equilibria with $A \approx 3$ are shown in Fig. 2. The stability of these equilibria toward ideal local modes is examined in Section 3.

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3. STABILITY TOWARD LOCAL MODES

The Mercier Stability Criterion^{4,5} is a convenient diagnostic tool for evaluating the equilibria obtained in Section 2. Satisfying this necessary criterion everywhere in the plasma then defines a critical plasma β and toroidal current beyond which the plasma is assuredly unstable according to the ideal MHD theory.

In studying the effects of these equilibrium parameters on the plasma local stability, it is important to choose reasonable profiles and to change only one factor at a time.

The Mercier criterion can be symbolized as

$$M = M_s + M_w + M_p \geq 0, \quad (7)$$

the three terms representing contributions from the shear, the magnetic well, and the finite plasma pressure, respectively. These terms can be rewritten in flux surface quantities as²⁵

$$\begin{aligned} M_s &= \frac{1}{4} S^2, \quad S = (FV' \langle R^{-2} \rangle)' / V'^3, \\ M_w &= -p' FSQ_1 / V'^3 + p' V'' (F^2 Q_2 + V' \langle R^{-2} \rangle) / V'^6, \\ M_p &= -p'^2 [F^2 (Q_2 Q_3 - Q_1^2) + V' \langle R^{-2} \rangle Q_3] / V'^6, \end{aligned} \quad (8)$$

where

$$Q_1 = V' \langle R^{-2} B_p^{-2} \rangle, \quad Q_2 = V' \langle R^{-4} B_p^{-2} \rangle, \quad Q_3 = V' \langle B_p^{-2} \rangle.$$

These forms allow accurate evaluations of Eq. (7) for any given solution $\psi(R, Z)$, because only S and V'' must be obtained by numerical differentiation of flux surface averages. Calculations with different grid sizes, doubling the $\Delta\psi$ size in evaluating Eq. (8) and fitting $\psi(R, Z)$ by polynomials near the magnetic axis to calculate Eq. (7)

analytically, have indicated that numerical errors in the value of Eq. (7) fall within 4%.

Typical behavior of the M terms in D-shaped and doublet plasmas at the marginal stability point are shown in Figs. 3 and 4. It is seen that the plasma is usually most unstable near the magnetic axis, where M_s is relatively small and where M_w and M_p nearly cancel although they are larger than M_s by roughly two orders of magnitude. The relative importance of M_s drops as β_p increases.

It has been shown¹³ that Eq. (7) becomes a sufficient condition for stability against local modes if the current is nearly flat and if S (shear) is small, e.g., near the magnetic axis. Thus, for these equilibria which are least stable at the magnetic axis, the critical β and current values are likely to indicate sufficient stability against local modes.

EFFECTS OF β_p

The dependence of the critical values of β , $\bar{H}$, and q_a on β_p in D-shaped plasmas is shown in Fig. 5. It is seen that the enhancement of the critical β by high β_p is maximal at β_p values of 2-2.4. As β_p increases, w_p has been allowed to decrease by fixing $(\psi_b - \psi_m)$ and α to preserve qualitatively the current profile as flat, rounded, or peaked. If we fix w_p instead of fixing $(\psi_b - \psi_m)$ and α , these critical β values increase monotonically with β_p . In both cases, the values of critical q_a increase strongly with β_p , typically doubling when β_p increases from 0.5 to 2.5 and tripling when β_p reaches 5 (Fig. 5b). Here the values of q_0 have always been between .97 and 1.02.

This disagrees with the theoretical results of Shafranov and Yurchenko,¹⁴ which showed a strong increase of q_0 and β_p increases for circular flux surfaces. Since the total magnetic flux is not conserved when β_p increases, the present conclusions do not apply to the flux-conserving tokamak (FCT) plasmas.^{15,16}

Similar results for doublet plasmas with rounded current profiles are shown in Fig. 6. The w_p values used in comparing the doublet and D-shaped plasmas are within 2% of each other. We find that a doublet plasma has a slightly higher critical β for $\beta_p \leq 1$. But the maximum critical β in a doublet plasma occurs near $\beta_p = 1$ and is roughly 84% of the maximum in a D-shaped plasma that occurs at $\beta_p = 2.4$. Also, the critical q_0 is found to be 1.1. In both shapes, the H values are comparable and decreasing monotonically with β_p . This indicates the need of auxiliary heating in ohmically heated tokamaks to reach high β .

EFFECTS OF ELONGATION (σ)

The effects of σ on the stability of D-shaped plasmas with $\beta_p \approx 2.4$ and various values of w_p is shown in Fig. 7. It is seen in Fig. 7a that the critical β peaks near a value $\sigma \approx 1.65$ for rounded ($w_p = .515$) and peaked ($w_p = .430$) profiles. For the flat profile ($w_p = .600$), β rises monotonically with σ (Fig. 7a), but saturates at a value 12.5%. At $\sigma = 1.65$, one has already reached 94% of the saturation level. The enhancement factor of the critical β by an elongation of $\sigma = 1.65$ is found to be 1.26, 1.53, and 1.61 for peaked, rounded, and flat profiles, respectively. This reflects the fact that

effectiveness of elongation for improving β is strongly limited by the steepness of equilibrium profiles. It is found that $\bar{H}$ increases with σ only when σ is small ($\sigma < 1.3$ for $w_p = 0.515$), and then decreases with σ when σ is large. This is opposite to the scaling of $\bar{H} \propto (1 + \sigma^2)/2$, obtained if one assumes uniform J_T and loop voltage.

The correlation between the critical q_0 and the flux surface elongation near the magnetic axis (σ_0) as σ increases is shown in Fig. 7b. It is seen that for $w_p = .515$, q_0 drops to a minimum below 0.9 as σ_0 increases beyond 1 near $\sigma = 1.3$. For $\sigma > 1.3$, q_0 increases to 1.5 as σ_0 and σ increase to 2.24 and 3, respectively. Comparison with the results derived by Solovév, Shafranov, and Yurchenko,¹⁷ assuming negligible triangularity in the innermost flux surface, shows that their results indicate a higher critical q_0 and are more pessimistic than the numerical results (see Fig. 7b).

Similar results for a doublet plasma with $w_p = 0.515$ and $\beta_p = 2.0$ are shown in Fig. 8. It is seen that the critical β stops increasing when $\sigma > 2.6$, where $\bar{H}$ also peaks. This suggests that a doublet shape with an elongation of $\sigma < 3$ may lead to improved ohmic heating without sacrificing the critical β .

EFFECTS OF TRIANGULARITY (δ)

The effects of δ on stability in D-shaped plasmas are shown in Fig. 9. It is found that over a wide range in w_p , the critical β is enhanced by increasing δ and maximizes near $\delta = .5$. By increasing δ from 0 to .5, the critical β can be increased by a factor of 1.19, 1.35, or 1.40 for $w_p = .430$, .515, or .600, respectively. Comparison

with theoretical results in Ref. 17 shows that the theoretical results are more pessimistic than our numerical calculations for $w_p = 0.515$ and for a δ_0 (triangularity of the innermost flux surface) that increases from $-.07$ to 0.0 as δ increases from $-.2$ to $.6$.

EFFECTS OF ASPECT RATIO A

Figure 10 shows that for fixed w_p , the critical β and $\bar{H}$ are a strongly decreasing function of A. Therefore, for fixed B_0 at the plasma geometric center, the torus should be as tight as possible. However, if the maximum field B_M at the toroidal field coil surface is fixed, it becomes more appropriate to consider the critical values with respect to B_M . We have

$$\beta' = \beta B_0^2 / B_M^2, \quad \bar{H}' = \bar{H} B_0 / B_M \quad (9)$$

and

$$B_M = A B_0 / (A - 1 - \frac{\Delta}{a}),$$

where Δ is the distance between the toroidal field coil and the plasma edge. The results for plausible choices $\Delta = 0.5a$ and $1.0a$ are also plotted in Fig. 10, showing that $A = 3 - 4$ gives the highest critical pressure. Thus, taking into account the maximum field at the coils we arrive at conclusions different from those obtained in the JET design.¹⁸

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4. SUMMARY AND DISCUSSION

Axisymmetric MHD equilibria have been assessed by evaluating the Mercier criterion to study the effects of equilibrium parameters on stability toward ideal local modes. We have considered equilibria with different profiles in order to cover many cases that are likely to be relevant to present and future tokamaks.

Our results may be summarized as follows:

1. It has been shown that if the current profile is qualitatively unchanged as β_p increases, the highest critical β will be reached when $\beta_p \approx 0.8$ A for D-shaped plasmas and when $\beta_p \approx 0.33$ A for a doublet plasma. In D-shaped plasmas with $A = 3$, the highest critical β for local stability occurs at $\sigma = 1.65$, $\delta = 0.5$, and $\beta_p \approx 2.4$ (see Fig. 2a). This result is true over a wide range of pressure profile half-width w_p . The peak critical β is 12% at $w_p = .600$, 5.6% at $w_p = .515$, and 2.5% at $w_p = .430$. For a doublet plasma with $A = 3$ and $\sigma = 3$ (see Fig. 2b), the critical β is 4.3% at $\beta_p = 2.4$ and $w_p = 0.515$, and the peak critical β , 4.7%, occurs at $\beta_p = 1.0$ and $w_p = .545$. This result qualitatively agrees with those of Mazzurato,³ Solovév,¹³ Freidberg and Grossmann,¹⁹ Herrnegger and Maschke,²⁰ Artsimovich and Shafranov,²¹ and Dobrott and Helton,²² but disagrees with those of Coppi, Dagazian, and Gajewski²³ and Laval, Pellat, and Soule.²⁴ It is worth noting that the nonlocal instability calculations by Freidberg and Grossmann¹⁹ using a constant pressure model have shown that for $A = 3$ the optimum condition occurs with the critical $\beta = 8-11\%$ at $\sigma = 1.9-2.2$ and $\delta = 0$.

2. The critical value of the average ohmic heating power $\bar{H}$ decreases monotonically with β_p in D-shaped and doublet plasmas. This indicates that the stability requirement places limits on the plasma β in ohmically heated tokamaks. The critical $\bar{H}$ for a doublet is found to be comparable to that of the D-shaped plasma shown in Fig. 2a. The necessity of using auxiliary heating in both cases is well known.

3. It has been shown that by changing the shape of a circular plasma with $\beta_p = 2.4$ to the D-shape shown in Fig. 2a, the critical β at $\beta_p \approx 2.4$ can be improved by a factor of 1.50, 2.07, or 2.25 for $w_p = .430$, .515, or .600, respectively. We have found that further increases in elongation σ and triangularity δ with fixed β_p will decrease the critical β and $\bar{H}$. It is also found that in a doublet plasma, reducing σ from >3 to 2.6 will not decrease the critical β , although the critical $\bar{H}$ will be somewhat improved.

4. The critical β and $\bar{H}$ are found to decrease strongly as A increases. However, if the maximum toroidal field at the coil surface is fixed and if there are reasonable distances between the plasma edge and the toroidal field coils, these critical values with respect to the maximum field peak around $A = 3-4$.

Our results suggest the use of the D-shaped plasma (as shown in Fig. 2a) with $\sigma = 1.65$, $\delta = 0.5$, and $A = 3-4$ for high β and β_p tokamaks. Questions concerning the poloidal field design required to maintain the free-boundary D-shaped plasma at various values of σ and β_p must be resolved to make the D-shaped plasma an attractive candidate for future tokamak reactors.

We have begun numerical investigations of higher β equilibria which may result in a very hot tokamak where high classical plasma conductivity would "freeze" magnetic flux into the plasma. This suggests study of a flux conserving sequence of equilibria with a fixed $q(\psi)$ profile. Early results indicate that the critical β values become an increasing function of β_p . An approximate indication of this trend can be seen in Fig. 5 by connecting points of constant q_a through peaked, rounded and flat profiles. It is clear that if β_p is increased in an FCT^{15,16} fashion from 0.2-2.7, the critical β increases from 1% to nearly 12% with local stability satisfied. Detailed discussion of FCT equilibria will be presented in a subsequent paper.

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REFERENCES

1. ORMAK Group, private communications.
2. L. A. Berry et al., ORMAK Upgrade Proposal, ORNL/TM-5130 (November, 1975); M. Roberts and E. S. Bettis, Oak Ridge Tokamak Experimental Power Reactor Reference Design, ORNL/TM-5042 (November, 1975).
3. E. Mazzucato, *Phys. Fluids* 18, 536 (1975) and references therein.
4. C. Mercier, Proc. Conf. on Plasma Physics and Controlled Nuclear Fusion, Vol. II, 801 (1962).
5. J. M. Green and J. L. Johnson, *Phys. Fluids* 5, 510 (1962); *Plasma Phys.* 10, 729 (1968).
6. M. D. Kruskal and R. M. Kulsrud, *Phys. Fluids* 1, 265 (1958).
7. H. Grad and H. Rubin, Proc. 2nd International Conference on the Peaceful Uses of Atomic Energy, Vol. 31, 190 (1958).
8. V. D. Shafranov, *Zh. Eksp. Teor. Fiz.* 37, 1088 (1959) [*Sov. Phys. - JETP* 37, 775 (1960)].
9. J. D. Callen and R. A. Dory, *Phys. Fluids* 15, 1523 (1972) and references therein.
10. F. Hennecker, Proc. 5th European Conference on Controlled Fusion and Plasma Physics, Vol. I, p. 26 (1972).
11. M. S. Chu et al., *Phys. Fluids* 17, 1183 (1974).
12. G. E. Forsythe and W. R. Wasow, Finite-Difference Methods for Partial Differential Equations, p. 242, Wiley, New York 1960.
13. L. S. Solov'ev, *Zh. Eksp. Teor. Fiz.* 53, 2063 (1967) [*Sov. Phys. - JETP* 26, 1167 (1968)]; *Atomnaya Energiya* 30, 14 (1971).
14. V. D. Shafranov and E. L. Yurchenko, Proc. 4th Conf. on Plasma Physics and Controlled Nuclear Fusion Research, Vol. II, 519 (1971).

15. J. F. Clarke, Bull. Am. Phys. Soc. 20, 1228 (1975).
16. J. F. Clarke et al., "Flux Conserving High Beta Toroidal Equilibrium," presented at the Annual Meeting of Theoretical Aspects of Controlled Thermonuclear Research, Madison (1976).
17. L. S. Solov'ev, V. D. Shafranov and E. L. Yurchenko, Proc. 4th International Conf. on Plasma Physics and Controlled Nuclear Fusion Research, Vol. II, 553 (1970).
18. J. A. Wesson, Proc. 7th European Conf. on Controlled Fusion and Plasma Physics, Vol. 2, 102 (1975).
19. J. P. Freidberg and W. Grossmann, Phys. Fluids 18, 1494 (1975).
20. F. Herrnegger and E. K. Maschke, Nucl. Fusion 14, 119 (1974).
21. L. A. Artsimovich and V. D. Shafranov, Zh. Eksp. Teor. Fiz. Pis'ma Red. 15, 72 (1972) [JETP Lett. 15, 51 (1972)].
22. D. Dobrott and F. J. Helton, Tokamak Equilibrium with Arbitrary Cross Section, GA-A13641 (August 1975).
23. B. Coppi, R. Dagazian, and R. Gajewski, Phys. Fluids 15, 2404 (1972).
24. G. Laval, R. Pellat, and J. S. Soule, Phys. Fluids 17, 835 (1974).
25. L. S. Solov'ev, Zh. Eksp. Teor. Fiz. 53, 626 (1967) [Sov. Phys.-JETP 26, 400 (1968)].

FIGURE CAPTIONS

- Fig. 1 Typical correlations among pressure and current profiles at $Z = 0$ and various β_p values in D-shaped equilibria where $A = 3$. These profiles are characterized as flat, rounded, and peaked.
- Fig. 2 MHD equilibria with $A = 3$ and various cross sections, β_p and w_p . The parameters of a D-shaped cross section are defined in 2a.
- Fig. 3 The behavior of M , M_s , M_w , M_p and H (in arbitrary units) as functions of ψ at marginal stability in D-shape plasmas with $A = 3$, $\sigma = 1.65$, $\delta = 0.5$, $w_p = 0.515$ for $\beta_p = 0.53$ (3a) and $\beta_p = 2.41$ (3b).
- Fig. 4 The behavior of M , M_s , M_w , M_p and H in arbitrary units as a function of ψ at marginal stability in doublet plasmas with $A = 3$, $\sigma = 3$, $w_p = 0.515$, for $\beta_p = 0.60$ (4a) and $\beta_p = 2.26$ (4b). ψ_s is the separatrix ψ .
- Fig. 5 Critical β , $\bar{H}$ (in arbitrary units), q_a , q_0 and the prescribed w_p as functions of β_p for D-shaped plasmas with $\sigma = 1.65$, $\delta = 0.5$ (Fig. 2a), and the profiles shown in Fig. 1.
- Fig. 6 Critical β , $\bar{H}$ (in arbitrary units), $\bar{q}_a$, q_0 and the prescribed w_p as functions of β_p for doublet plasma (Fig. 2h) with the rounded profile shown in Fig. 1.

- Fig. 7 Critical β , $\bar{H}$ (in arbitrary units), and q_0 as functions of elongation (σ) for D-shaped plasmas with $A = 3$, $\beta_p = 2.4$, $\delta = 0.5$, and various values of w_p . The ellipticity of the innermost flux surface (σ_0) and the critical q_0 for $w_p = .515$ according to the theory by Solovév, Shafranov and Yurchenko¹⁷ are also plotted in 7b.
- Fig. 8 Critical β , $\bar{H}$ (in arbitrary units), q_0 and σ_0 for doublet plasmas with $A = 3$ and $\beta_p = .0515$.
- Fig. 9 Critical β , $\bar{H}$ (in arbitrary units), q_0 and σ_0 as functions of δ in D-shaped plasmas with $A = 3$, $\sigma = 1.65$, $\beta_p \approx 2.4$ and various values of w_p . The results of Ref. 17 are also plotted in 9b.
- Fig. 10 Critical β and $\bar{H}$ for the D-shaped plasma of Fig. 2a with $\beta_p = 2.4$ and $w_p = 0.515$ as functions of aspect ratio A for fixed B_0 at the geometric center of the plasma. The corresponding critical values with respect to fixed maximum B_M at the coil surface are denoted B' and $\bar{H}'$.

Fig. 1

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(a) $\sigma = 1.65$, $\delta = 0.5$, $W_p = .515$, $\beta_p = 2.44$

Fig. 2a

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(b) $\sigma = 1.65$, $\delta = 0$, $W_P = .430$, $\beta_P = .42$

Fig. 2b

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(c) $\sigma = 1.65$, $\delta = -.2$, $W_p = .600$, $\beta_p = 0.82$

Fig. 2c

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(d) $\sigma = 3$, $\delta = .5$, $W_p = .515$, $\beta_p = 1.35$

Fig. 2d

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(e) $\sigma = 2$, $\delta = .5$, $W_P = .430$, $\beta_P = 1.92$

Fig. 2e

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(f) $\sigma = 1$, $\delta = .5$, $W_p = .600$, $\beta_p = 2.47$

Fig. 2f

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 $(\gamma) \sigma = 2, W_p = .515, \beta_p = 2.95$

Fig. 2g

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(h) $\sigma=2$, $W_p=.515$, $\beta_p=3.50$

Fig. 2h

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(i) $\sigma=4$, $W_p=.515$, $\beta_p=2.77$

Fig. 2i

Fig. 3

Fig. 4

Fig. 5

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Fig. 6

Fig. 7

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Fig. 8

Fig. 9

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Fig. 10

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