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DE#0467-7

UCID- 21394

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April 18, 1988

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Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under Contract W-7405-Eng-48.

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I. INTRODUCTION

There has been recent interest in the formulation of a nonperturbative theory of quantum electrodynamics (QED). In this report we critically discuss a recent theory of nonperturbative QED. We point out a difficulty with this theory and outline a theory which is free of this difficulty. Finally, computational schemes for implementing the theory are briefly discussed.

Normally the semiclassical theory of Lamb and coworkers,¹ in which an atom and radiation field are described by quantum mechanics and by classical electrodynamics, respectively, has been appropriate to interpret many experiments in quantum optics. New experiments^{2,3} on a single absorbing and/or emitting ion, however, probe the quantum-mechanical or discrete nature of the radiation field. If the field strength is assumed to be weak or the atom and field are assumed to be out of resonance, then standard quantum electrodynamic perturbation theory provides an appropriate description of the interaction of an atom with a quantized radiation field. Perturbation theory is invalid, however, if either of these assumptions is not met. Furthermore, one or both of these assumptions is usually always violated in quantum optics. Thus a strong need exists to develop a nonperturbative theory to describe the interaction of the two quantum mechanical systems, the atom and the field.

A fully quantum mechanical theory describing the interaction of a two-level atom and a single-mode radiation field has been available for some time⁴; however, no satisfactory theory exists to describe the atom's interaction with a multi-mode, quantized radiation theory. Such a theory is needed to interpret recent experiments on single-atom spontaneous emission³

and, on a many-atom level, to understand the growth of coherent radiation, i.e., either amplified spontaneous emission or superfluorescence,⁵ from quantum noise, i.e., the spatially-temporally random light pulses of many spontaneously emitting atoms.

II. THEORY

Understanding the physical mechanism responsible for the spontaneous decay of an atomic excited state or for the shift in the positrons of the energy levels (Lamb shift) and other radiative corrections is still a stimulating problem of high current interest, although it goes back to the earliest days of quantum mechanics. The conventional explanation of the spontaneous decay of an excited state is that the electron interacts with the fluctuating vacuum field. On the other hand, an alternative explanation⁶ is that spontaneous emission is caused by the electron's interaction with its own field, the so-called radiation-reaction field. In this approach it is assumed that the electron itself is the source of radiation and of radiative corrections to atomic structure. This implies that the correct equation of motion is not the Schroedinger or Dirac equation for a bare electron but an equation with an additional nonlinear self-energy term, in analogy to the Lorentz equation for a classical, radiating electron with a Larmor term included.

The basis of the radiation-reaction explanation⁶ is found in the Heisenberg equations of motion for the electronic and field variables and on the use of an approximate, but nonperturbative solution to these equations. Since the radiation-reaction field is generated by the electron's motion, in analogy to a radiating dipole of classical electrodynamics, the theory contains a more

general electrodynamic description of the atom than standard QED perturbation theory, which makes corrections to the zeroth-order motion of a bare electron. However, the Heisenberg equations are not suited to explicit numerical solutions. Thus the unusual situation exists that this prototype problem appears not to have been studied in any great numerical detail. That is, the dynamical evolution of a bound electron under the influence of its own self field has never been explicitly described in a detailed numerical study of the appropriate equations. Of course, certain quantities as the Einstein A coefficient are calculated; however the A coefficient gives the lifetime an atomic state has on average. Recent experiments³ show that a single atom repeatedly prepared in an excited state will decay in a series of random "quantum jumps," and the average of the series of random times the state has existed before its sudden decay or jump is its "natural lifetime." Thus the A coefficient really applies to an ensemble of atoms, and the dynamical description of single-atom decay is still unavailable.

Recently Barut and coworkers⁷ have proposed a nonperturbative, nonsecond-quantized version of QED which is based on the idea of including the self-fields from the beginning in analogy with classical electrodynamics where the field is calculated directly from the current associated with the particle trajectory. This approach is substantially different from the perturbation method of quantum electrodynamics in which the self-energy is discarded in lowest approximation and afterwards added back in photon by photon. According to Barut and co-workers, the electron's self-field can be calculated from the Maxwell equation,

$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \vec{A} = - \frac{4\pi}{c} e \vec{j} \quad , \quad (1)$$

where $\vec{j}$ is the electron's current,

$$\vec{j} = \frac{\hbar}{2mi} (\psi^* \vec{\nabla} \psi - \psi \vec{\nabla} \psi^*) - \frac{e}{mc} \psi^* \psi \vec{A} \quad , \quad (2)$$

defined from the equation of continuity,

$$\frac{\partial}{\partial t} \psi^* \psi + \vec{\nabla} \cdot \vec{j} = 0 \quad (3)$$

for the Schroedinger wave-function density $\psi^*\psi$. However, if the atom is prepared in a pure stationary state at $t = 0$, then the first two terms on the right hand side of Eq. (2) cancel, giving a vanishing source. Hence $\vec{A} = 0$ for all times. This flaw in Barut's work is implied in a comment⁸ on his apparently incorrect, or incomplete, definition of the wave-function ψ .

We use the more general definition of the field,

$$\vec{A} = \vec{A}_0 + \vec{A}_{RR} \quad , \quad (4)$$

where $\vec{A}_{RR}$ is the radiation-reaction field defined by Eqs. (1) and (2) with $\vec{A}$ replaced by $\vec{A}_{RR}$. $\vec{A}_0$ is the usual quantized free field and is the homogeneous solution of Eq. (1). Analysis shows that the wave function is written

$$\psi = e^{\frac{ie}{\hbar c} \vec{A}_0 \cdot \vec{r}} \varphi \quad , \quad (5)$$

where φ obeys the Schroedinger equation containing the full interaction,

$$V_I = \frac{ie\hbar}{2mc} (\vec{A} \cdot \vec{\nabla} + \vec{\nabla} \cdot \vec{A}) + \frac{e^2}{2mc^2} A^2 \quad , \quad (6)$$

and φ obeys the Schroedinger equation,

$$\begin{aligned} i\hbar \frac{\partial}{\partial t} \varphi = & \left[-\frac{\hbar^2}{2m} \nabla^2 + V - e \vec{E}_0 \cdot \vec{r} + \frac{ie\hbar}{2mc} (\vec{A}_{RR} \cdot \vec{\nabla} + \vec{\nabla} \cdot \vec{A}_{RR}) \right. \\ & \left. + \frac{e^2}{2mc^2} (\vec{A}_0 \cdot \vec{A}_{RR} + \vec{A}_{RR} \cdot \vec{A}_0) + \frac{e^2}{2mc^2} A_{RR}^2 \right] \varphi \quad , \end{aligned} \quad (7)$$

where V is the Coulomb potential. Equation (7) is still exact; the transformation defined as Eq. (5) is necessary to remove the dependence of the current on $\vec{A}_0$. In the Coulomb gauge ($\vec{\nabla} \cdot \vec{A}_0 = 0$), $\vec{E}_0$ is calculated from,

$$\vec{E}_0 = -\frac{1}{c} \frac{\partial}{\partial t} \vec{A}_0 \quad (8)$$

Conventional theory follows by ignoring all interaction terms in Eq. (7) except $-e\vec{E}_0 \cdot \vec{r}$. If we average Eq. (7) with respect to a quantum state of the free field $\vec{A}_0$ and ignore the second line of Eq. (7) as small, then the interaction depends only on $\vec{A}_{RR}$, since on average $\vec{E}_0 = 0$, in apparent agreement with Heisenberg theory.⁶ At $t = 0$ only $\vec{A}_0$ (or $\vec{E}_0$) exists; therefore both $\vec{A}_0$ and $\vec{A}_{RR}$ contribute. From Eqs. (1) and (2), with $\vec{A}$ replaced by $\vec{A}_{RR}$, elimination of $\vec{A}_{RR}$ in Eq. (7) shows that the Schrodinger equation is nonlinear. Thus we have a problem in nonlinear dynamics which to our knowledge has not been carefully studied. It is furthermore of interest to study the influence of this nonlinear term on the possible development of quantum stochasticity.⁹ A sensitivity to initial conditions (characteristic of chaotic systems) of how the atom is prepared in an initial excited state could possibly explain why an excited atom lives a nonreproducible length of time before it jumps to another state, since an experiment would not precisely replicate the same set of initial conditions for the prepared state. Finally, we believe that the present approach is general enough to address a wider scope of problems in nonperturbative field theory involving the fundamental restructuring of the vacuum state in the presence of very strong fields (see e.g., Ref. 10).

III. Numerical Methods

In conventional theory, Eq. (7) (with $\vec{A}_{RR} = 0$) is solved by expanding

the wave-function,

$$\varphi = a_e(t) |e\rangle |0\rangle + \sum_{\mathbf{k}\sigma} a_{g\mathbf{k}\sigma}(t) |g\rangle |1_{\mathbf{k}\sigma}\rangle, \quad (9)$$

for ground (g) and excited (e) atomic eigenstates and free radiation-field states containing 0 and 1 photons. The sum is over $\mathbf{k}$ frequency modes and σ polarizations of the free field. In order to calculate the Lamb shift, however, Eq. (9) must be augmented by including all other states of the atom. Elimination of $\vec{A}$ then means that the Schrodinger equations for the probability amplitudes $a_e(t)$ and $a_{g\mathbf{k}\sigma}(t)$ are nonlinear in these dependent variables. It would be interesting to study these coupled, nonlinear ordinary differential equations both analytically and numerically in order to find the effect of $\vec{A}_{RR}$ on the emission process.

Eq. (9) is too restrictive, since obviously the Lamb shift depends on the complete set of atomic states. Rather we could solve Eq. (7) and Eqs. (1) and (2) (with $\vec{A}$ replaced by $\vec{A}_{RR}$ as a pair of coupled, nonlinear partial differential equations. PDE methods are being vigorously investigated at the Laboratory in optical and molecular physics¹¹ and in laser and collision physics.¹² Thus computer algorithms are already available which could possibly be adapted to the present problem. Note that the theory is nonperturbative, such that it may offer a powerful and reliable method for calculating the Lamb shift, especially for high-Z ions.

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