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AND THE PROOF OF UNITARITY

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SOFT-GROUP-MANIFOLD STRUCTURE OF SUPERGRAVITY, AND THE PROOF OF UNITARITY

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We review the Soft Group Manifold (SGM) method of gauging a non-internal group. Applications to three different versions of Extended Supergravity are presented. We then summarize the geometric identification of the ghost fields and BRS equations, another aspect of the Group Manifold method. We apply the results to Supergravity and provide a completion of the proof of Unitarity of Stermann, Townsend and Van Nieuwenhuizen.

INTRODUCTION

To celebrate Einstein's Centennial and the role of Marcel Grossmann in the introduction of Differential Geometry in Physics, a review of the more recent work in Supergravity is certainly appropriate. This theory has led to important progress in Quantum Gravity and in another part of Einstein's spiritual inheritance, the search for further unification. It has also advanced an aspect of the application of Differential Geometry to Physics: the clarification of the Geometrical structure of gauging a non-internal local symmetry group. Both Gravity and Supergravity exploit the geometry of Soft Group Manifolds, i.e. Manifolds whose tangents are rigid Lie Group Manifolds (in this case of the Poincaré and the "Super"-Poincaré groups).

The development of Supergravity drew on three interests,

- (a) Exploring local supersymmetry, after the interest in global supersymmetry (the "Poincaré Supergroup") in 1971-74.
- (b) Advancing Quantum Gravity, after the first results of 't Hooft and Veltman in 1974, and the negative conclusions regarding the effects of the presence of matter.
- (c) A renaissance of Einstein's Unification program, based upon the allowed combination of Poincaré or Conformal supersymmetry with internal symmetries $O(N)$ or $U(N)$ respectively).

All three motivations have already been vindicated by at least partial success:

- (a) by a complete solution of the riddle posed by spin $J = \frac{3}{2}$ fields: Velo-Zwanziger faster-than-light propagation, Johnson-Sudarshan indefiniteness etc.. and a new facet of the uses of $J = 2$.
- (b) by proof of 1-loop renormalizability in the presence of matter, provided that the matter fields form a supersymmetry multiplet with the graviton ($J = \frac{3}{2}$ for $N = 1$ supersymmetry, $J = \frac{3}{2}$ and $J = 1$ for $N = 2$ etc, checked in detail up to $N = 3$ but probably true up to $N = 8$).
 - by similar proof of 2-loop renormalizability - up to anomalies.
 - by proof of the positive-definiteness of the gravitational hamiltonian (otherwise mostly an unproved conjecture, including some recent geometrical arguments).
- (c) by the construction of $O(N)$ supergravity,

- either with ungauged internal symmetry and no further problems
- or with gauged internal symmetry and a resulting huge cosmological constant, which may either be taken care of by Wheeler "foam" or vanish when spontaneous breakdown is invoked.

Both results go much further than Einstein could ever get with the methods he was using.

O(8) supergravity is represented here by Dr. Julia's paper. It should come closest to the phenomenological requirements.

PROGRAM

In this paper, we would like to touch upon several aspects in which the geometrical understanding plays an important role.

- 1) We shall review the main features of the Soft Group Manifold, where the gauging of the Poincaré and Super-Poincaré groups is taking place.
- 2) We discuss some of the problematics of Extended (O(N)) Supergravity.
- 3) We discuss the geometrical identification of the renormalization ghost fields, and provide a completion of the proof of Unitarity for Quantum Supergravity.

THE SOFT (DALI) GROUP MANIFOLD

This geometrical arena of Gravity and Supergravity has been investigated and systematized in work with T. Regge¹⁾ and J. Thierry-Mieg²⁾. To gauge an n-dimensional group containing space-time translations (or their pre-contraction precursors, such as the rotations into a fifth dimension, when using $\bar{O}(3,2)$ or $\bar{O}(4,1) = Sp(4,R)$ prior to a Wigner-Inönü contraction), we first consider the rigid Group Manifold, the Lie group itself as our vacuum. In the Poincaré case, we thus have 6 Lorentz dimensions, in addition to 4-dimensional space time. In the "super" case, we add 4 extra (Grassmann) spinor dimensions. We now consider as a vielbein over G the Left-invariant: Cartan forms ω^A , satisfying the Maurer-Cartan equation,

$$R = d\omega + \frac{1}{2} [\omega, \omega] = 0 \quad (1.1a)$$

The ω are dual to the Left-invariant vector-fields (or covariant derivatives) L_A

$$[L_A, L_B] = f_{AB}^C L_C \quad (1.2)$$

where

$$[\omega, \omega]^C = -f_{AB}^C \omega^B \wedge \omega^A \quad (1.1b)$$

We now define the Soft Group Manifold (SGM)G. This bears the same relationship to G as a curved space-time does to Minkowski's except that the existence of a metric is now replaced by assumptions about the vielbein. The term "soft" is inspired³⁾ by Salvador Dalí's "Soft Self Portrait". Over G, we now define an arbitrary vielbein ρ^A , replacing ω^A . However, no assumption is made regarding the two-form of Curvature,

$$R^C = d\rho^C - \frac{1}{2} f_{AB}^C \rho^B \wedge \rho^A \quad (1.3a)$$

Writing the generalized curvatures as

$$R^C = \frac{1}{2} R_{AB}^C \rho^B \wedge \rho^A \quad (1.4)$$

we also have

$$d\rho^C - \frac{1}{2} (f^C_{AB} + R^C_{AB}) \rho^B \wedge \rho^A = 0 \quad (1.3b)$$

and the vector-field basis $\tilde{L}_A$ orthogonal to the ρ^A obeys field-dependent commutation relations,

$$[\tilde{L}_A, \tilde{L}_B] = (f^C_{AB} + R^C_{AB}) \tilde{L}_C \quad (1.4)$$

The most general non-internal gauge transformation in a SGM corresponds¹⁾ to a local action of the $\tilde{L}_A$ (this is a Lie-derivative in Differential Geometry). It corresponds to an Anholonomic General Coordinate Transformation (AGCT) $Z^A \rightarrow Z^A + E^A$ where Z is the G n -dimensional variable. Although the custom in General Relativity, for instance, is to deal with x^μ , with μ the (holonomic) space-time variable, only $x^a = x^\mu e^a_\mu$ is an invariant concept, the tetrad frame e^a supplying a local base for the Poincaré group. Under infinitesimal AGCT we have,

$$(\delta\rho)^A = (D\varepsilon)^A + \tilde{\varepsilon} \lrcorner R^A, \quad \tilde{\varepsilon} = \varepsilon^A \tilde{L}_A \quad (1.5)$$

$$\delta R = D(\tilde{\varepsilon} \lrcorner R) - [\varepsilon, R] \quad (1.6)$$

and the Bianchi identity

$$DR = 0 \quad (1.7)$$

where

$$D\phi = d\phi + [\rho, \phi] \quad (1.8a)$$

$$DD\phi = [R, \phi] \quad (1.8b)$$

Note that internal gauge transformations, as defined for a Yang-Mills field, would read

$$(\delta\rho)^A = (D\varepsilon)^A$$

Such variations occur in (1.5) only if $R^A = 0$, or if we restrict the variation to a subset of the ε , such that $\tilde{\varepsilon} \lrcorner R = 0$. In a Principal Bundle (representing the geometry of a Yang Mills gauge), we do not represent horizontal variations, i.e. translations of the Bose manifold. The ε are thus only vertical, and indeed $\tilde{\varepsilon} \lrcorner R = 0$.

The non-internal gauge Lagrangian is not invariant under local G . For Gravity and Supergravity, its local invariance is reduced to the Lorentz subgroup F . In general, if the algebra g of G is Weakly-reducible

$$g = f \oplus h, \quad \begin{aligned} [f, f] &\subset f \\ [f, h] &\subset h \end{aligned} \quad (1.9a)$$

and "symmetric" (WRS)

$$[h, h] \subset f \quad (1.9b)$$

and the Lagrangian is only f -invariant locally, spontaneous fibration occurs²⁾, i.e. ($\underline{\circ}$ denotes mass-shell equations)

$$R^h \underline{\circ} 0 \quad (1.10a)$$

$$R^f_{ff} \underline{\circ} 0, \quad R^f_{fh} \underline{\circ} 0 \quad (1.10b)$$

All forms now factorize¹⁾ the f variables and the ρ^A and R^A reduce on shell to forms on the h variables only, i.e. on the Manifold $H = G/F$. This H is Min-

kowski $M^{3.1}$ in Gravity, Superspace

$M^{3.1/4}$

in Supergravity, $S^{3.1}$ for $Sp(4R)$ etc... Supergravity also offers the possibility of taking $F_{L/R} = SL(2,C) \times \theta^2_{L/R}$, where θ^2 represents a complex 2-dimensional fermionic 2-spinor representation. The θ_L and θ_R are the chiral components of the supersymmetric variable. Working in simple pre-contraction $OSp(4/1)$ or $OSp(4/N)$ makes this a WRSS, and parity-breaking Extended Supergravity exists²⁾ for any N . For $N=1$, it was shown¹⁾ that (1.9), (1.10) when inserted in (1.5) indeed reproduce the local supersymmetry variations, when ϵ reduces to the Majorana ϵ^α . This occurs because $N=1$ is the only case where one can use first $F_{L/R}$ in $OSp(4/1)$, contract each case separately, add up both Lagrangians and recover a parity-conserving geometric theory. We have shown that for $N > 1$, the resulting field equations do not have as a solution the original rigid group manifold or its summed contractions. This implies that the vacuum solution is no more in the tangent.

Working with the SGM provides a direct insight into the system. For example, the constraint equations (2.5), (2.8) and (2.14) of ref.⁴⁾ are a direct consequence of our equation of motion.

An independent use of this formalism to Extended Supergravity has been presented in this meeting by S. MacDowell⁵⁾. We understand his construction as follows. Starting from the group manifold $OSp(N/4)$, factorization of the subgroup $O(N) \times$ Lorentz is assumed, leaving an extended superspace as physical space. Those curvatures involving superspace directions are now constrained to be of low spin only, and may then be parametrized by the low-spin fields of the Extended Supersymmetry multiplet of the graviton. We may now read the second term of (1.5) as the contribution of the auxiliary fields to the supersymmetry transformation of the gauge fields and (1.6) as describing their own transformations. The resulting geometry is that of a bundle with flat $O(N)$ fibers and softly (low spin) curved Grassmann fibers. The auxiliary fields of various authors⁶⁾ are also components of R in (1.5), including the off mass shell contributions needed for quantization⁷⁾.

UNITARITY OF QUANTUM SUPERGRAVITY

Another advantage of working in the full dimensionality of the S.G.M. is the geometric identification⁸⁾ of the renormalization ghost fields⁹⁾. These were shown to consist locally of the "residual pieces" C^A of the connections ρ^A , beyond the potentials ϕ^A_μ . For an internal gauge, represented by a Principal Bundle P with base manifold M , picking a section Σ (i.e. making a gauge choice) locally diffeomorphic to M , we can choose a base (x, α) where x^μ is lifted from M (by π^{-1}) and α^i from G (Π is the P projection, α a group parameter or coordinate, G the structure group):

$$\Sigma : \alpha^i(x) = 0, \quad i = 1 \dots n \quad (\text{the group dimensionality}) \quad (2.1)$$

$$\frac{\partial}{\partial x^\mu} \lrcorner \rho^A = \phi^A_\mu, \quad \frac{\partial}{\partial \alpha^i} \lrcorner \rho^A = \chi^A_i \quad (2.2)$$

$$\rho^A = \phi^A_\mu dx^\mu + \ell C^A, \quad C^A = \chi^A_i d\alpha^i \quad (2.3)$$

ℓ is a constant length. This yields the BRS equation¹⁰⁾

$$s C^A = - \frac{1}{2} [C, C]^A \quad (2.4)$$

$$s \phi_\mu^A = D_\mu C^A \quad (2.5)$$

where $s, d^{(\Sigma)}$ are given by

$$\begin{aligned} df &= \lambda s f + d^{(\Sigma)} f; \quad \lambda s f = d\alpha^i \frac{\partial}{\partial \alpha^i} f; \quad d^{(\Sigma)} f = dx^\mu \frac{\partial}{\partial x^\mu} f \\ d^{(\Sigma)^2} &= s d^{(\Sigma)} + d^{(\Sigma)} s = s^2 = 0 \end{aligned} \quad (2.6)$$

The covariant quantization path-integral can be given a geometrical form. The Feynman diagrams involve non-integrated exterior forms (the ghosts) together with anticommuting Lagrange multipliers (the antighosts). The minus sign required by ghost loops, which has led to their traditional interpretation as spin zero fermions, is then indeed just the sign due to self-anticommutation of one-forms (since $C^A = \chi_1^A d\alpha^1$).

With a SGM instead of a Principal Bundle, the geometric identification of the ghosts still stands, even though the BRS equations may differ⁷⁾. In the simplest case of spontaneous fibration (1.10), everything is exactly the same - on mass shell. Off mass shell, the (1.5) gauge transformations will give rise to additional variations for the (m,n) horizontal directions (in Supergravity, super-space GP/SL(2C) according to equation (1.5).

$$\delta \rho^A = D\epsilon^A + \rho^m \epsilon^n R_{nm}^A \quad (2.7)$$

and the BRS transformations will accordingly involve such additional terms⁷⁾ which vanish under the equations of motion (here we also take care of dimensions)

$$s e_\mu^a = \kappa^2 D_\mu C^a + \kappa e_\mu^b C^\alpha R_{ab}^\alpha \quad (2.8)$$

This "non-closure of the algebra of local supersymmetry" as it is usually described, has been recognized in the component formalism to lead to quartic ghost terms in the effective quantum Lagrangian, and modified BRS transformations. The construction is also further hybridized by the fact that the space-time horizontal ("parallel-Transport") AGCT transformation of (1.5) is treated holonomically by most authors (notwithstanding its resulting lack of an operational definition), whereas the spinor part ("local supersymmetry") is treated anholonomically...

In this note we shall only discuss one result of the geometric approach: fixing signs of loops (and commutation or anticommutation relations) for the Supergravity ghosts, in the proof of Unitarity. To enable direct adaptation to existing calculations¹¹⁾, we use the fields h_μ^a , from the expansion of the vierbein potential,

$$e_\mu^a = \delta_\mu^a + \frac{1}{2} \kappa h_\mu^a + \dots \quad (2.9)$$

and ψ_μ^α (we write the spinor index for formal reasons), and their ghosts C^a (spin one fermion) and C^α (spin half boson);

$$\begin{aligned} \rho^a &= e_\mu^a dx^\mu + \kappa^2 C^a \\ \rho^\alpha &= \kappa \psi_\mu^\alpha dx^\mu + \kappa^2 C \end{aligned} \quad (2.10)$$

There are two different mechanisms for anticommutation. One $Z(2)$ corresponds to the anholonomic indices (a, α) in $M^{3,1/4}$, i.e. the grading in Supersymmetry. This ensures opposite commutation properties to e_μ^a versus ψ_μ^α and to C^a versus C^α . The second $Z(2)$ is that of the Grassmann algebra of forms, ensuring opposite commutation properties for e_μ^a (a 0-form) versus C^a (a 1-form) or for ψ_μ^α (a 0-form) versus C^α (a 1-form). In Feynman diagrams, this corresponds to a sign reversal when a gauge field is replaced by its ghost. We thus have a $Z(2) \oplus Z(2)$ gradation,

$$\eta^{pu} \wedge \xi^{qv} = (-1)^{pq+UV} \xi^{qv} \wedge \eta^{pu} \quad (2.11)$$

where η^{pu} and ξ^{qv} are respectively a p-form and a q-form and UV are the respective gradings of the components u and v of the superalgebra. This fixes the sign when commuting

$$\begin{aligned} [h_\mu^a, h_\nu^b] &= [h_\mu^a, C^b] = [h_\mu^a, C^\alpha] = [h_\mu^a, \psi_\nu^\alpha] = \\ &= [\psi_\mu^\alpha, C^a] = [C^a, C^\beta] = 0 \end{aligned} \quad (2.12)$$

$$\{\psi_\mu^\alpha, \psi_\nu^\beta\} = \{\psi_\mu^\alpha, C^\beta\} = \{C^a, C^b\} = \{C^a, C^\alpha\} = 0$$

Some of these signs come as a surprise to the non-geometrist: the commutation of the two fermions $[\psi_\mu^\alpha, C^a]$ and the anticommutation of the fermion-boson pairs $\{\psi_\mu^\alpha, C^\beta\}$ and $\{C^a, C^\alpha\}$. The conventional formalism does not fix the signs, and a proof of the Unitarity of renormalized Supergravity¹¹⁾ did not specify the sign to be attached to closed ghost loops (see line (e) of fig. 10 of ref.¹¹⁾).

Sterman, Townsend and van Nieuwenhuizen prove Unitarity, i.e. that only 2 components of the graviton h_μ^a and of the gravitino ψ_μ^α are physical. They work with an effective Lagrangian,

$$\begin{aligned} L_{\text{eff}} &= L_{\text{mv}} + L_{\text{fix}} + L_{\text{gh}} - \frac{5}{8} \kappa^2 (B_Y^{\nu\hat{B}}) (\bar{C} \gamma_\nu C) + O(\kappa^3) \\ L_{\text{mv}} &= (2\kappa^2)^{-1} e^{\rho} - \frac{1}{2} \epsilon^{\lambda\rho\mu\nu} \bar{\psi}_\lambda \gamma_5 \gamma_\rho D_\mu \psi_\nu \\ L_{\text{fix}} &= -\frac{1}{4} (\partial_\mu \tilde{h}^{\mu\nu})^2 - \frac{a}{\kappa^2} (h_{a\mu} - h_{\mu a})^2 + \frac{1}{4} \bar{\psi}_\mu \gamma^\mu \gamma^\rho \partial_\rho (\gamma^\nu \psi_\nu) \\ L_{\text{gh}} &= \bar{C}_A (M)_B^A C^B \end{aligned} \quad (2.13)$$

They use second order formalism and thus also eliminate C^{ab} and C^{*ab} , the ghost and antighost for ω_{μ}^{ab} . $h_{\mu\nu}$ is symmetric. The linear parts of their BRS transformations follow (2.4-2.5).

$$\begin{aligned} s h_{\mu\nu} &= \kappa (D_\nu C_\mu + D_\mu C_\nu) \\ s \psi_\mu^\alpha &= 2\kappa (D_\mu C)^\alpha \\ s C^\nu &= 0 \\ s C^\alpha &= 0 \end{aligned} \quad (2.14)$$

and for the antighosts, they involve the gauge-fixing terms

$$\begin{aligned} s \bar{C}^{\nu} &= \kappa F^\nu, & F^\nu &= -\frac{1}{2} (\partial_\mu \tilde{h}^{\mu\nu}) \\ s \bar{C}^\alpha &= \kappa F^\alpha, & F^\alpha &= \frac{1}{2} (\bar{\psi}_\mu \gamma^\mu (\gamma^\rho \partial_\rho))^\alpha \end{aligned} \quad (2.15)$$

The commutation relations (2.12) are consistent with the on-shell Ward identities for cut Green's functions, expressing current conservation (fig. 5 in ref.¹¹⁾).

In our fig. 1, we insert our relative signs (from (2.12)) of the one loop contributions to the self-energies of h_μ^a (wavy line) , ψ_μ^α (curly line) , C^a (dashes ----) and C^α (full line). In our fig. 2 we give the relative signs of the

two-loop vacuum bubbles. The graphs in both figures are ordered so as to display explicitly the compensation mechanism yielding unitarity.

The proof of Unitarity of Sterman et al is given first for a two particle cut ($N = 2$), then extended by induction from N to $N + 1$ (their fig. 10). In our fig. 3, we provide the relative sign of the ghost contributions to this $N + 1$ th order unitarity relation, thus completing the proof in ref.¹¹).

CONCLUSION

Using the basic tools of differential geometry, the exterior forms, we have described gravitation and its extensions as theories over a new kind of space, the Soft Group Manifold. This space is uniquely united to Einstein's ideas, in the sense that all of its geometry is controlled by the dynamics, including the fibration of the Lorentz subgroup. Its construction is not just a matter of geometric simplicity and elegance (as in the A.G.C.T. identification of the variation of ρ^{ab} as derived in ref.¹). It provides an understanding of the auxiliary fields of supergravity and implies modified BRS relations. As an explicit application of the impact of Differential Geometry in Physics, we have provided for the first time the sign rules for higher-order perturbative Supergravity, and have thus completed the proof of Unitarity of Supergravity.

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(a)

(b)

(c)

(d)

Figure 1.
Relative signs of the one loop contribution to the self energy.

Figure 2.
Relative signs of the two loops vacuum bubbles.

Figure 3-

Relative sign of the ghost contribution to the $N + 1$ order unitarity relation.

Note Added

In a recent study ¹²⁾ Van Nieuwenhuizen has shown that the BRS invariance of supergravity can also be displayed using the conventional $Z(2)$ gradation (i.e. considering the supersymmetry ghost as an authentic boson) rather than the $Z(2) \times Z(2)$ gradation advocated here, or even a $Z(2) \times Z(2) \times Z(2)$ gradation, where the third grading refers to the BRS parameter (a trivial consideration, since the BRS parameter is useless as soon as the geometric $Z(2) \times Z(2)$ gradation is used for the gauge and ghost fields).

The Lagrangian and the Feynman rules are exactly the same in all 3 cases. However, as the mixed ghost interaction is of the form $(\bar{C}^a \psi C^\mu + \bar{C}^\mu \bar{\psi} C^a)$ (eq. 1.7 of ref. ¹¹⁾ and eq. 2.1 of ref. ¹³⁾, the sign rules derived by Wick analysis, even in the case of the $Z(2)$ gradation, are those depicted in our figure. Thus we recognize that unitarity does not depend on the explicit use of the $Z(2) \times Z(2)$ gradation. Our misunderstanding of the results of ref. ¹¹⁾ arose from 3 facts: a) In figure 10.e of ref. ¹¹⁾ no signs were provided, b) In formula 4.10-4.11 of ref. ¹⁴⁾ our $Z(2) \times Z(2)$ gradation is explicitly used whereas $Z(2)$ gradation is used in formula 3.7 of the same reference, c) For any vacuum diagrams, the correct sign rule always follows the mnemonic of the $Z(2) \times Z(2)$ gradation, i.e. that one separate minus sign must be associated to each closed ghost or Fermi loop. The sign associated with open diagrams is then found by cutting the necessary propagators and changing the sign for any cut self-anticommuting field (ψ^a or C^μ). In this sense, $Z(2) \times Z(2)$ is the most natural gradation.

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