Runaway Modeling in ORMAK and TNS

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FUSION ENERGY DIVISION

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ABSTRACT

A zero-dimensional tokamak start-up code is used to investigate the generation of runaway electrons in the Oak Ridge Tokamak (ORMAK) and The Next Step (TNS) and the effect of the runaways on the plasma breakdown, current, and energy balance. The runaways, which are considered collisionless, are treated self-consistently in the circuit equation by including a runaway current component (determined from a runaway rate equation). During the earliest stages of a discharge, the total current in the plasma is so low that closed flux surfaces do not exist. Runaways produced are lost almost instantly because they follow magnetic field lines that intersect the limiter. Once the current in the tokamak builds up sufficiently, closed flux surfaces start forming which improves runaway containment. The time τ when this occurs is uncertain. Results on ORMAK and TNS are displayed for different values of τ (before which any runaway created is assumed to be lost instantaneously). The assumption of flux surface closure after some arbitrary time τ is then justified by modeling an ORMAK discharge which includes the effects on runaway confinement of error fields generated by currents flowing in the leads to the toroidal field coils. Also shown are ORMAK simulations in different density regimes and TNS results for different loop voltage pulses.

1. INTRODUCTION

Most studies of the initiation of a discharge in a tokamak¹⁻³ have either evaluated the current carried by the runaway component of the electron distribution function in a manner that is decoupled from the dynamic evolution of the plasma or neglected it altogether. A self-consistent analysis of the start-up problem requires that the runaway current be included in the circuit equation. Its inclusion can then dynamically modify the way the plasma current evolves, which in turn influences the electron temperature. This influence can affect the plasma breakdown and the energy balance and, consequently, the conditions for further runaway generation.

An attempt is made here to investigate the start-up phase of a tokamak operation with emphasis on the effects runaways have on the plasma current, formation, and energy balance. For this purpose, a very simple set of zero-dimensional, time-dependent, coupled rate equations for atomic hydrogen are solved numerically in conjunction with a circuit equation which contains a runaway term. Molecular physics processes are ignored and coronal equilibrium is assumed in the treatment of all impurities.

These equations are described in more detail in Sects. 2-4 and applied to model discharges in the Oak Ridge Tokamak (ORMAK) and The Next Step (TNS) in Sects. 5 and 6. ORMAK and TNS discharges are simulated for a set of different initial times early in each discharge (prior to which all magnetic field lines are considered to spiral out and strike the limiter, so any runaway created is assumed to be lost instantaneously). In addition, ORMAK simulations are carried out for different filling pressures, and therefore different final electron densities, and TNS discharges are modeled for different loop voltage pulses.

2. BASIC EQUATIONS

The study in zero dimensions of the plasma evolution in a tokamak requires solving a set of coupled rate and circuit equations. The coupled rate equations examined here include simple versions of the ion, neutral atom, electron energy, and ion energy balance equations. The electron density is obtained from the condition of plasma quasi-neutrality.

The ion balance equation consists of an ionization rate term ($n_e n_o \langle \sigma v \rangle$) and a confinement loss rate term (n_i / τ_p)

$$\frac{dn_i}{dt} = n_e n_o \langle \sigma v \rangle - n_i / \tau_p, \quad (1)$$

where n_i , n_e , and n_o are the proton, electron, and neutral atom densities, respectively, and τ_p is the particle confinement time. The neutral atom balance equation is composed of a loss rate due to ionization, a gas puff rate g_p , and a recycling rate off the toroidal chamber walls. It is assumed here that all ions lost to the walls are recycled back as cold neutral atoms;

$$\frac{dn_o}{dt} = -n_e n_o \langle \sigma v \rangle + g_p + n_i / \tau_p. \quad (2)$$

The electron energy balance equation considered here can be written as

$$\begin{aligned} \frac{3}{2} \frac{d}{dt} (n_e T_e) = & \eta J^2 - n_e n_o \langle \sigma v \rangle U_o - \frac{3}{2} n_e T_e / \tau_e \\ & + 1.2 \times 10^{-18} n_i n_o (T_i - T_e) / T_e^{3/2} \\ & - 3 \times 10^{-21} Z_{\text{eff}} n_e^2 T_e^{1/2} - P_z - P_i, \end{aligned} \quad (3)$$

which includes ohmic heating power (ηJ^2), ionization power ($n_e n_o \langle \sigma v \rangle U_o$, $U_o \sim 20$ eV includes molecular dissociation, radiation, and ionization²), heat conduction to the walls ($n_e T_e / \tau_e$, τ_e = energy confinement time),

ion electron energy transfer rate [$1.2 \times 10^{-18} n_i n_e (T_i - T_e) / T_e^{3/2}$ in kiloelectron volts per cubic meter per second, where T_i and T_e are the ion and electron temperatures], bremsstrahlung radiation power loss ($3 \times 10^{-21} Z_{\text{eff}} n_e^2 T_e^{1/2}$ in kiloelectron volts per cubic meter per second), impurity line radiation power (P_z), and impurity ionization power (P_i).

The ion energy balance equation consists of the ion electron energy transfer rate and heat conduction to the walls,

$$\frac{3}{2} \frac{d}{dt} (n_i T_i) = -1.8 \times 10^{-18} n_i n_e (T_i - T_e) / T_e^{3/2} - \frac{3}{2} n_i T_i / \tau_e \quad (4)$$

The circuit equation is modified to include the current component carried by runaways,

$$L \frac{dI_T}{dt} = V - I_P R_p \quad (5)$$

where $I_T = I_p + I_R$. The runaways are considered to be collisionless and hence perfectly conducting, so there is no resistivity associated with their motion. The resistance is $R_p = (2R_0/a^2\epsilon)\eta$, where R_0 = major radius, a = minor radius, ϵ = ellipticity of a cross-sectional cut of the tokamak, and η = resistivity. The resistivity η is given by

$$\eta = A\eta_{sp} + \eta_n \quad (6)$$

where η_{sp} is the Spitzer resistivity for a $Z_{\text{eff}} = 1$ plasma [$\eta_{sp} = 1.65 \times 10^{-9} / T_e^{3/2}$ (Ωm , keV)], and A is the anomaly factor,^{4,5} introduced by the presence of impurities, which enhances the resistivity. For the sake of simplicity, one species of impurity (oxygen) is considered in modeling the plasma behavior in this analysis;

$$A = 1 + n_z/n_e \{z^2 - z - z \ln z / \ln \Lambda + [(z_t^2 - z^2)/4 \ln \Lambda] \ln [1 + 1.5g^4 (T_e/Ry)^2]\} \quad (7)$$

where z is the average charge of the impurity ions, n_z is the impurity density, z_t is the total number of electrons per neutral impurity atom

(8 for oxygen), $g = 1.5 (z_t - z)^{3/2}/z_t$, $\ln \Lambda$ is the Coulomb logarithm and Ry is the Rydberg constant ($Ry \equiv 13.6$ eV).

η_n is an effective resistivity introduced by electron neutral collisions which are dominant when the plasma is weakly ionized. In gas discharge experiments, the electron drift velocity is approximated by

$$v_D = 1.24 \times 10^{24} E/n_0 \text{ (m/sec, V/m, m}^{-3}\text{)} . \quad (8)$$

The current density is $j = en_e v_D = E/\eta_n$; therefore, the neutral resistivity is

$$\eta_n = 5.04 \times 10^{-6} n_0/n_e . \quad (9)$$

3. IMPURITY TREATMENT

As mentioned, oxygen is used to model all impurity behavior in the plasma. In addition, it is assumed that coronal equilibrium conditions exist and results obtained from the average ion model⁶ are used to determine the average charge state z of the oxygen. The average ion model replaces the fractional population of the different ionic states of an impurity with a fictitious average impurity ion.

The presence of a large impurity concentration when a plasma is still cold can extinguish a discharge by line radiation. For modeling tokamak discharges, a desorption model is used which assumes that all oxygen impurities are attached to the chamber walls at first and are desorbed linearly as a function of temperature once the plasma becomes sufficiently hot (>50 eV). When T_e exceeds 500 eV, it is assumed that all impurities have been cleared off the walls. The impurity density can then be written as

$$n_z = \begin{cases} 0 & T_e < 0.05 \text{ keV} , \\ n_{zt} [(T_e - 0.05)/0.45] & 0.05 \text{ keV} < T_e < 0.5 \text{ keV} , \\ n_{zt} & T_e > 0.5 \text{ keV} . \end{cases} \quad (10)$$

n_{zt} is the total impurity density chosen to be desorbed into the plasma. With expressions for z and n_z obtained, the resistive anomaly factor A can be determined.

Coronal equilibrium calculations for oxygen⁷ give the line radiation power as

$$P_z = 10^{-13} n_e n_z L_z \text{ (W/m}^3\text{)} \quad , \quad (11)$$

where L_z is expressed as a function of the electron temperature.

To obtain an expression for the ionization power of the oxygen impurities, some further simplifications are made. When $j < z < j + 1$,

$$\begin{aligned} z &= \sum_{k=1}^8 k(n_z^k/n_z) \approx j(n_z^j/n_z) + (j+1)(n_z^{j+1}/n_z) \\ &\approx j[(n_z - n_z^{j+1})/n_z] + (j+1)(n_z^{j+1}/n_z) \\ &= j + (n_z^{j+1}/n_z) \quad , \end{aligned} \quad (12)$$

where (n_z^k/n_z) is the fraction of the oxygen with charge state k . Similarly, an average ionization potential i can be written as

$$\begin{aligned} i &= \sum_{k=1}^8 i_k(n_z^k/n_z) \approx i_j(n_z^j/n_z) + i_{j+1}(n_z^{j+1}/n_z) \\ &\approx i_j + (i_{j+1} - i_j)(n_z^{j+1}/n_z) \quad , \end{aligned} \quad (13)$$

where i_k is the ionization potential of the k^{th} oxygen state.

The oxygen ionization power is approximated by

$$P_i \approx I_p \frac{dn_z}{dt} + n_z \frac{di}{dt} \quad . \quad (14)$$

The first term represents the power required to get the oxygen stripped off the chamber walls to the average charge state z . The second term is the impurity density times the rate of change of the average ionization potential. For an average charge z ,

$$I_p = \sum_{k=1}^j i_k + i_{j+1}(n_z^{j+1}/n_z) \quad (15)$$

From Eqs. (12), (13), and (15), one can obtain expressions for i and I_p as functions of z so that Eq. (14) for $j < z < j + 1$ is

$$P_i = \left[\sum_{k=1}^j i_k + i_{j+1}(z-j) \right] \frac{dn_z}{dt} + n_z (i_{j+1} - i_j) \frac{dz}{dt} \quad (16)$$

where z and dz/dt are obtained from results of coronal equilibrium calculations.⁶

The rough approximations made to obtain an expression for the oxygen ionization power may not have a very great impact because the line radiation power P_z is a more dominant term in the electron energy balance equation. During the steady-state portion of a discharge the assumption of coronal equilibrium is relatively good, but unfortunately this approximation is not too reliable during the plasma breakdown phase.

4. RUNAWAY PHYSICS

The equation of motion of an electron in an electric field is given by

$$\frac{dp}{dt} = eE - m\nu v \quad (17)$$

where acceleration of the electron by the field E is balanced by the drag caused by Coulomb collisions. The electron-electron collision frequency is

$$\nu_{ee}(v) = (4\pi e^4 n_e / m^2 v^3) \ln \Lambda \quad (18)$$

where e is the electron charge, n_e the electron density, m the electron mass, $\ln \Lambda$ the Coulomb logarithm, and v the electron velocity. The collision frequency decreases with increasing electron velocity. This implies that above a certain critical velocity given by

$$v_c^2 = (4\pi e^3 n_e / mE) \ln \Lambda \quad , \quad (19)$$

the electron is continuously accelerated to even higher energies as eE becomes larger than mv in Eq. (17). A runaway electron is that which has a velocity greater than v_c .

The electric field required for the critical velocity to be equal to the thermal velocity is defined as the Dreicer field and is given by

$$E_D = (4\pi e^3 n_e / mv_{th}^2) \ln \Lambda \quad . \quad (20)$$

In this analysis it is considered that all electrons, once they run away, are essentially collisionless. Only if they strike the limiter of the toroidal chamber are they assumed to be lost from the runaway distribution. In addition, the effects of microturbulence are neglected.

The orbit shift model⁸ is used to determine the confinement of runaways in a tokamak. It is based on the principle of conservation of angular momentum of the particles in the device, i.e.,

$$P_\phi = \gamma m_0 v_\phi R - \frac{e}{c} R A_\phi = pR - \frac{e}{c} \psi \quad ,$$

where

$$R = R_0 + r \cos \Theta \quad . \quad (21)$$

For an approximately type-B current profile $\{j(r) \approx j_0[1 - (r/a)^3]\}$, the flux function ψ is in Gaussian units for $r < a$,

$$\psi(r) = \int_0^r B_\theta R dr \approx \frac{5}{6} (\pi R_0 I_T / \epsilon E c) (r/a)^2 [1 - (4/25)(r/a)^3] \quad , \quad (22)$$

where

R_0 = major radius

a = minor radius

ϵ = ellipticity of a cross-sectional cut of the tokamak

E = complete elliptical integral of the first kind

I_T = total current

c = speed of light.

Neglecting $(4/25)(r/a)^3$ and writing $r^2 = (R - R_0)^2 + y^2$, the canonical angular momentum is

$$P_\phi \approx -\frac{5}{6} (\pi R_0 I_T e / \epsilon E a^2 c^2) \left[\left(R - R_0 - \frac{3}{5} \frac{\epsilon E a^2 c^2}{\pi R_0 I_T e} p \right)^2 + y^2 - \left(\frac{3}{5} \frac{\epsilon E a^2 c^2}{\pi R_0 I_T e} \right)^2 \right]. \quad (23)$$

Because P_ϕ is a constant over the orbit of a particle, this equation represents a circle shifted outward in major radius.

The magnitude of the orbit shift of a runaway is proportional to its momentum and inversely proportional to the total current,

$$\Delta = \frac{3}{5} (\epsilon E a^2 c^2 / \pi R_0 e) (p / I_T) \quad (24)$$

A runaway born on a certain orbit gains energy from the electric field and shifts outward until it strikes the limiter.

In this zero-dimensional analysis, it is assumed that on the average, all runaways born in a tokamak are confined if their orbit shifts are less than half the minor radius, and that they strike the limiter once, $\Delta = a/2$. One can then obtain from Eq. (24) an expression for the maximum momentum a runaway can have and still remain contained,

$$p_{\max} = \frac{5}{6} (\pi R_0 c / \epsilon E c^2 a) I_T \quad (25)$$

Because electrons are essentially collisionless once they run away, the equation of motion of an electron that becomes a runaway at time t_1 can be integrated to

$$p = p_c(t_1) + \int_{t_1}^t eE \, dt' \quad , \quad (26)$$

where $p_c(t_1)$ is the critical momentum ($p_c = \gamma m_0 v_c$). All runaways born at t_1 have the same momentum p at any time $t \geq t_1$. This is more clearly illustrated in Fig. 1. All runaways created at time t_1 are accelerated by the electric field until $t = t_2$ when their momenta $p = p_{\max}$. At this time their orbits have shifted by half the minor radius and therefore are considered to be striking the limiter. At any given time, the runaway population rate of change is

$$\frac{dN}{dt} = [S_R(t) - L(p_{\max})] V_T, \quad (27)$$

where

$$S_R(t) = c_1 n_e v_{ee}(v_{th}) \left(\frac{E}{E_D}\right)^{-\frac{3}{16}} (Z_{eff} + 1) \exp \left\{ -\frac{E_D}{4E} - \left[(1 + Z_{eff}) \frac{E_D}{E} \right]^{1/2} \right\} \times \exp \left\{ -\frac{T_e}{m_0 c^2} \left[\frac{1}{8} \left(\frac{E_D}{E}\right)^2 + \frac{2}{3} \left(\frac{E_D}{E}\right)^{3/2} (1 + Z_{eff})^{1/2} \right] \right\} \quad (28)$$

is the runaway creation rate,⁹

the constant $c_1 = 0.35$ (Ref. 10),

$$v_{th} = \sqrt{T_e/m_0},$$

$$Z_{eff} \equiv (n_i + z^2 n_z)/n_e,$$

V_T = toroidal chamber volume, and

$L(p_{\max})$ = loss rate of runaways whose momenta $p = p_{\max}$ at time t .

As illustrated in Fig. 1, these runaways were all created at some earlier time t_0 , so in essence

$$L(p_{\max}) \approx S_R(t_0). \quad (29)$$

If the runaway velocity is taken to be approximately c , the runaway current is $I_R = e(\pi a^2 \epsilon) n_R c$, so the rate of change of runaway current is

$$\frac{dI_R}{dt} = e c \pi a^2 \epsilon [S_R(t) - S_R(t_0)]. \quad (30)$$

The determination of t_0 is done by numerically solving Eq. (26) for $p = p_{\max}$ at each time step. This requires keeping track of $p_c(t)$ and

$\int_0^t j_e E dt$ at each step j . One must also keep track at each time step of all the variables that contribute to the runaway creation rate S_R . If

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Fig. 1. When the momentum p of a runaway exceeds p_{MAX} , it strikes the limiter.

t_0 falls between t_k and t_{k+1} , then the value of each variable at t_0 is determined by a linear interpolation of its values at t_k and t_{k+1} .

The circuit equation is solved (in conjunction with the coupled rate equations) using Eq. (30) for the rate of change of runaway current. The runaway creation rate used is only approximate, as Eq. (28) is obtained from a quasi-steady-state solution of the Fokker-Planck equation. A more accurate treatment of this problem would require solving the very complex, fully time-dependent kinetic equation for the runaway production rates.

5. NUMERICAL RESULTS FOR NORMAL ORMAK DISCHARGES

An Ordinary Differential Equation (ODE) solver¹¹ is used to solve the coupled rate and circuit equations. A typical loop voltage for a normal ORMAK discharge is shown in Fig. 2 and consists of a capacitor-driven component during the first 15 msec and a battery-driven component of ~ 2.4 V from then on. ORMAK parameters are $R_0 = 0.8$ m, $a = 0.23$ m, $B_T = 2.5$ T, and $\ell_i = 1.5$, where ℓ_i is the internal self-inductance. The initial conditions chosen are $T_i = 0.05$ eV, $T_e = 2$ eV, and $n_e = 10^{16}$ m⁻³. The energy confinement time¹² is $\tau_e = 10^{-21} n_e \sqrt{q/Z_{eff}}/3$ (sec, m⁻³) and the particle confinement time is chosen to be $\tau_p = 3\tau_e$. To minimize numerical difficulties, τ_e is not allowed to be less than about 2 msec.

The equations are first solved for a filling pressure of 1.35×10^{-4} torr ($\sim 9 \times 10^{18}$ neutral atoms/m³). A smaller filling pressure is required to obtain a certain final electron density, as it is inherent in a zero-dimensional analysis that the plasma does not screen out the neutral atoms. It is assumed that runaway electrons get scraped off at the limiter only if their orbits shift by at least half the minor radius. No other loss mechanism is taken into consideration. The evolution of densities and currents is shown in Fig. 3. The plasma does not break down at all and virtually all the current is carried by the runaways. The input power is almost totally deposited in the runaway component of the electron distribution function. It appears that because the runaways are so well confined at early times, the discharge becomes runaway-dominated at the expense of the plasma current.

TYPICAL APPLIED
ORMAK
LOOP VOLTAGE

Fig. 2. Typical applied ORMAK loop voltage.

Fig. 3. Filling pressure $p = 1.35 \times 10^{-4}$ torr.

To obtain simulations that are in better agreement with experimentally observed normal discharges (which break down in the first few milliseconds and carry only about 1 kA of runaway current), additional loss mechanisms for runaway electrons must be considered. One to be discussed in further detail later is that the magnetic field structure can exhibit inhomogeneities during the very early stages of a discharge, which allows runaways to be lost instantaneously by following field lines that strike the limiter. Another possible explanation that could account for a poorer runaway confinement at early times is the existence of a skin effect, which would imply that most runaways created early are located near the plasma edge. Then the condition that runaways are lost only if the shifts of their orbits are greater than half the minor radius becomes too restrictive.

Figure 4 shows the effect on plasma and runaway currents when all runaways created before a time τ are considered to be lost instantaneously. For $\tau = 20 \mu\text{sec}$, the plasma and runaway currents are comparable in magnitude after 10 msec. The $\tau = 200 \mu\text{sec}$ simulation appears to give relatively good agreement with experiments with $I_p \sim 200 \text{ kA}$ and $I_R \sim 2 \text{ kA}$. A good confinement of runaways during the early stages of a discharge results in a high runaway current as well as a delayed rise of the plasma current and consequently a delayed breakdown.

The evolutions of runaway currents for different filling pressures, and hence different steady-state electron densities, are shown in Fig. 5 for the case when all runaways created in the first 200 μsec are considered to be lost instantaneously. At high density all runaways are created prior to breakdown, while at low density most runaways are produced after the plasma has ionized. The runaway creation rate S_R is proportional to $\exp [-1/4(E_D/E) - \sqrt{(1 + Z_{\text{eff}})/E_D/E}]$. When the plasma is weakly ionized, $E/E_D \propto (I_p T_e n_0)/n_e^2$, so for comparable I_p , T_e , and n_e , a higher filling pressure implies a higher E/E_D and hence a greater production rate. But once the plasma ionizes, E/E_D scales as $(Z_{\text{eff}} I_p)/(T_e^{1/2} n_e)$. The dominant difference in E/E_D between high density and low density simulations comes from n_e . Therefore, the lower the final electron density, the larger the creation rate. All this accounts for the fact that the runaway current in the low density simulation tends to be smaller prior to breakdown but

Fig. 4. Filling pressure $p = 1.35 \times 10^{-4}$ torr.
The impurity fraction is 0.1675.

RUNAWAY CURRENTS

Fig. 5. Dotted curve: $n_0 = 6.0 \times 10^{18}$ neutral atoms/m³; dashed curve: $n_0 = 9.0 \times 10^{18}$ neutral atoms/m³; solid curve: $n_0 = 1.8 \times 10^{19}$ neutral atoms/m³. All runaways born prior to $\tau = 200$ μ sec are assumed lost. The impurity fraction is 0.1675.

increases afterwards, while runaways in the high density simulations are born only when the gas is weakly ionized.

To illustrate the effect a typical magnetic field inhomogeneity has on runaway containment in a tokamak, a modified flux function developed by J. A. Rome¹³ for ORMAK that includes a contribution from the currents flowing in the leads to the toroidal field coils is examined:

$$\psi(x,y) = \left[\frac{\mu_0 N_L I_L b_L}{4\pi^2 R_0} \ln \left[\frac{(x - r_L - a_L)^2 + y^2}{(x - r_L + a_L)^2 + y^2} \right] - \frac{\mu_0 I_T}{4\pi a^2} (x^2 + y^2) \right] = \psi_1 - \psi_0, \quad (31)$$

where

I_L = current in leads to the toroidal field coils

N_L = number of toroidal field coils = 56

b_L = width of error coil = 0.015 m

a_L = length of error coil = 0.09 m

r_L = distance between toroidal chamber and error coil loop center = 0.455 m

ψ_1 = flux due to error coils

ψ_0 = flux due to the total current in the plasma.

I_T is the total current in the plasma, R_0 the major radius, and a the minor radius. This flux function is applicable at the early stages of an ORMAK discharge when the current density can be assumed to have a somewhat flat profile.

When $\psi_1 \ll \psi_0$, the flux surfaces form concentric tori in the chamber. However, early in a discharge ψ_1 can be of the same order as ψ_0 and all (or at least a significant portion) of the magnetic field lines can spiral out and strike the limiter. Consequently, most runaways born at this time are lost almost instantly. As the plasma current builds up, closed flux surfaces start forming and runaways begin to be confined. When the toroidal field and current are parallel, the largest flux surface contained strikes the outer limiter at the midplane of the torus¹³ (at the inner limiter when the field and current are antiparallel). Using

Eq. (31), one can evaluate the flux at the outer limiter (at $x = a$, $y = 0$) and then evaluate the other position at the midplane of the torus which gives the same ψ . That determines the size of the largest contained flux surface. With the assumption that this flux surface has roughly the same shape as the tokamak cross section, all runaways created within its volume are considered to be contained, while those born outside are assumed to strike the limiter immediately after their birth without contributing to the runaway current.

Since the runaways are contained in only part of the toroidal chamber volume, the expression for I_R and hence dI_R/dt must be slightly modified:

$$I_R = \frac{ec\pi a^2 \epsilon}{V_T} N = \frac{ec}{2\pi R_0} N \quad , \quad (32)$$

where N is the total number of runaways in the torus.

Therefore,

$$\frac{dI_R}{dt} = \frac{ec}{2\pi R_0} \frac{dN}{dt} \quad (33)$$

and

$$\begin{aligned} \frac{dN}{dt} &= \text{creation rate} - \text{loss rate} \\ &= \frac{d(n_R V)}{dt} - \left. \frac{dN}{dt} \right|_{\text{loss}} \end{aligned} \quad (34)$$

The rate of runaways being lost at time t is equal to their creation rate at an earlier time t_0 , i.e.,

$$\left. \frac{dN}{dt} \right|_{\text{loss}} = \left. \frac{dN_0}{dt_0} \right|_{\text{creation}} = \frac{d}{dt_0} \left[n_R(t_0) V(t_0) \right] \quad (35)$$

Equation (34) then becomes

$$\begin{aligned}
\frac{dN}{dt} &= \frac{d}{dt} (n_R V) - \frac{d}{dt_0} [n_R(t_0) V(t_0)] \\
&= VS_R + n_R \frac{dV}{dt} - V_0 S_{R_0} - n_{R_0} \frac{dV_0}{dt_0} \\
&\approx VS_R - V_0 S_{R_0} ,
\end{aligned} \tag{36}$$

where the volumes V_T , V , and V_0 are illustrated in Fig. 6 and $S_{R_0} = S_R(t_0)$. The term $n_R \frac{dV}{dt} \left(n_{R_0} \frac{dV_0}{dt_0} \right)$ corresponds to the rate of volume increase of the largest contained flux surface times the density of runaways in the incremental volume at time t (t_0). This density is very small, as all runaways created (in the incremental volume) prior to containment are assumed to be instantaneously lost. So $n_R \frac{dV}{dt} \left(n_{R_0} \frac{dV_0}{dt_0} \right)$ is neglected compared to $VS_R - V_0 S_{R_0}$. Then

$$\frac{dI_R}{dt} = \frac{ec}{2\pi R_0} \left(VS_R - V_0 S_{R_0} \right) . \tag{37}$$

The coupled rate equations and the circuit equation are then solved using Eq. (37) for the rate of change of runaway current. A current $I_L = 9$ kA flowing in the toroidal field coils gives $B_T \approx 2$ T on axis. The initial conditions and parameters are otherwise the same as those chosen in the results of Fig. 3. The evolution of densities and currents is shown in Fig. 7. The plasma breaks down within a millisecond, and at steady state $I_p > 200$ kA and $I_R \sim 1$ kA. This simulation is in good agreement with experiments and with the $\tau = 200$ μ sec simulation of Fig. 4. During the early stages of the discharge the field lines hit the limiter so there is no runaway buildup. This allows enough of a plasma current and breakdown to develop before the runaways created start to be contained. In this case the effect of runaways is not as crucial as if they were well confined from the very beginning of the discharge.

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Fig. 6. V_T = toroidal chamber volume; V = volume of largest contained flux surfaces at time t ; V_0 = volume of largest contained flux surface at time $t_0 < t$.

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Fig. 7. $I_L = 9$ kA; $p = 1.35 \times 10^{-4}$ torr.

6. TNS DISCHARGES

The parameters for the TNS tokamak are $R_0 = 5$ m, $a = 1.25$ m, $\epsilon = 1.6$, $B_T = 4.3$ T, and $\ell_i = 1.21$. The initial conditions chosen are $T_i = 0.5$ eV, $T_e = 5$ eV, $n_e = 10^{16}$ m $^{-3}$, and $n_0 = 2 \times 10^{19}$ m $^{-3}$. The energy confinement time is 1 sec and the particle confinement time is 3 sec.

The voltage pulse is chosen to rise at a rate of 100 v/msec until it reaches 500 V, where it remains until the plasma is formed (i.e., when sufficient ionization has occurred that the ion density exceeds the neutral atom density by a factor of 100). After that, the voltage is decreased to 50 V at the rate of 100 V/msec. Discharges are simulated for a set of different initial times from 0 to 1 msec, before which all runaways created are considered to be lost instantaneously. As shown in Fig. 8, the earlier confinement of runaways results in a larger runaway current and a delayed plasma formation.

The set of discharges simulated in Fig. 9 assume that all runaways produced prior to 0.5 msec are lost instantly. The runaway current at breakdown and the time of plasma formation are shown for different voltage pulses. The rates of voltage increase and decrease are kept the same as in the previous figure, but the maximum voltage is allowed to vary from 100 to 1000 V. Lower voltages produce a smaller runaway current but have greater difficulty in ionizing the plasma.

In the simulations in Figs. 8 and 9, once the plasma breaks down, the electron density is sufficiently high that no further runaways are produced.

7. CONCLUSIONS

A more appropriate modeling of the behavior of impurities in the plasma would require solving the nonequilibrium oxygen state coupled rate equations rather than assuming coronal equilibrium. In large tokamaks in particular, the skin effect may be important, so a one-dimensional analysis may give more realistic results for the runaway generation and its effect on the plasma formation.

RUNAWAY CURRENT and BREAKDOWN TIME

Fig. 8. All runaways born prior to τ are assumed instantly lost.
 $V_{\text{MAX}} = 500$ V. Solid curve: I_R ; dotted curve: t_B .

RUNAWAY CURRENT and BREAKDOWN TIME

Fig. 9. All runaways born prior to $\tau = 0.5$ msec are assumed instantly lost. Solid curve: I_R ; dotted curve: t_B .

In general, a good confinement of runaways during the early stages of a discharge results in a high runaway current and a delayed plasma breakdown. In high density regimes, runaways are generated prior to bulk ionization while in low density regimes most runaways are formed after the plasma breakdown. This agrees qualitatively with ORMAK experimental evidence.¹⁴ Finally, higher applied loop voltages for TNS achieve a faster plasma formation but are accompanied by larger runaway currents.

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REFERENCES

1. F. B. Marcus and S. E. Attenberger, *Bull. Am. Phys. Soc.* 21, 1125 (1976).
2. R. J. Hawryluk and J. A. Schmidt, *Effects of Low Z Impurities during the Startup Phase of a Large Tokamak*, MATT-1201, Princeton Plasma Physics Laboratory, Princeton, New Jersey (1976).
3. R. Papoular, *The Genesis of Toroidal Discharges*, EUR-CEA-FC-769, CEA Centre d'Etudes Nucleaires de Fontenay-aux-Roses, France (1975).
4. V. D. Kirillov, B. A. Trubnikov, and S. A. Trushin, *Fiz. Plazmy* (Sov. J. Plasma Phys.) 1, 117 (1975).
5. D. L. Shaeffer, *Impurity Electron Core Effect in ORMAK*, ORNL/TM-5745, Oak Ridge National Laboratory, Oak Ridge, Tennessee (1977).
6. D. E. Post, (Princeton Plasma Physics Laboratory), private communication.
7. R. V. Jensen, D. E. Post et al., *Calculations of Impurity Radiation and Its Effects on Tokamak Experiments*, PPPL-1334, Princeton Plasma Physics Laboratory, Princeton, New Jersey (1977).
8. D. A. Spong, *Equilibrium, Confinement, and Stability of Runaway Electrons in Tokamaks*, ORNL/TM-5147, Oak Ridge National Laboratory, Oak Ridge, Tennessee (March 1976).

9. J. W. Connor and R. J. Hastie, Nucl. Fusion 15, 415 (1975).
10. R. M. Kulsrud, Y. C. Sun et al., Phys. Rev. Lett. 31, 690 (1973).
11. L. F. Shampine and M. K. Gordon, *Computer Solution of Ordinary Differential Equations: The Initial Value Problem*, W. H. Freeman & Co., San Francisco, California, 1975.
12. L. A. Berry et al., *Plasma Physics and Controlled Nuclear Fusion Research 1976*, Vol. 1, p. 49 (1977).
13. J. A. Rome, *Effects of Magnetic Field Errors in ORMAK*, ORNL/TM-3880, Oak Ridge National Laboratory, Oak Ridge, Tennessee (June 1972).
14. H. Knoepfel and S. J. Zweben, Phys. Rev. Lett. 35, 1340 (1975).

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