TOPICS IN LEPTON-HADRON INTERACTIONS

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Contents

	<u>Page</u>
1. Introduction	i
2. Scaling in Electron and Muon Inelastic Scattering on Nucleons	9
2.1 Introduction to Electron-Nucleon Inelastic Scattering	9
2.2 Experimental results on Scaling in Electron-Nucleon Inelastic Scattering	15
2.3 Muon-Nucleon Inelastic Scattering	17
3. Scaling in Neutrino Interactions at Medium Energies ($E < 30$ GeV).	22
3.1 Neutrino Cross-Sections in the Quark-Parton Model	25
3.3 Sumrules for Neutrino Scattering	26
3.4 Conclusions	35
4. Theoretical Predictions for Breakdown of Scaling	36
4.1 Sources of Scaling Breakdown	36
4.2 Renormalizability and Divergences in QED	37
4.3 Gauge Theories of Strong Interactions	39
4.4 Application to Nucleon Structure Factors	41
4.5 Comparison With Experiments	45
4.6 Summary	47
5. New Quantum Numbers and New Couplings in High Energy Neutrino Reactions	49

	<u>Page</u>
5.1 Dimuon Events in Counter Experiments	50
5.2 Dilepton Events in Bubble Chamber Experiments	52
5.3 Discussion of Dilepton Events in Terms of Charm	57
5.4 Anomalies in γ Distributions at High Energy	61
5.5 The Total Cross-Section Ratio	65
5.6 Interpretation of Anomalies in Terms of New Quarks and Couplings	67

1. INTRODUCTION

These lectures are aimed at a discussion of lepton-hadron interactions at high energy-momentum transfers. The subjects discussed include the inelastic scattering of leptons (e, μ, ν) by nucleons and the process $e^+e^- \rightarrow$ hadrons. The selection of material is arbitrary, non-comprehensive and dictated by what I felt were the most interesting developments in the field.

The general background to the subject of lepton-hadron interactions has been covered in a number of excellent reviews. From the viewpoint of the material presented here, I should mention the reviews by Friedman and Kendall (Ann. Rev. Nucl. Science 22, 203 1972) on inelastic electron-nucleon scattering; by Llewellyn-Smith (Physics Reports 3C, 261, 1972) and by Sehgal (ANL-HEP-PR 75-45) on neutrino-nucleon scattering; and by Perl and Feldman (Physics Reports 19C, 233, 1975) on e^+e^- annihilation to hadrons.

The basic motivation for studying lepton-hadron interactions is well known; the structure of hadrons - about which we know relatively little - is probed using beams of leptons, the properties of which are, relatively, well understood. The form of the interaction is particularly simple:- because the leptons have small (electromagnetic or weak) couplings to other particles, and because lepton number and baryon number are conserved, the interaction is described as the coupling of two currents (lepton currents and quark currents) by single vector particle exchange. The vector (axial-vector) nature of the exchanged particle is a consequence of the spatial transformation properties

(observed experimentally) of the weak and electromagnetic interactions. The relevant diagrams for lepton-nucleon scattering and e^+e^- annihilation to hadrons are shown below, where the symbol "u" stands for a hadronic constituent (quark). In (a), (b) and (c) the momentum

(a) EM CURRENT ($\Delta Q=0$)

(b) WEAK CURRENT ($\Delta Q=1$)

(c) WEAK CURRENT ($\Delta Q=0$)

(d) e^+e^- ANNIHILATION

transfer is spacelike ($q^2 > 0$) and in (d) it is timelike ($q^2 < 0$).

The differential cross-sections for the above processes depend on the product J_μ (lepton) J_μ (hadron). Leptons are pointlike and J_μ (lepton) has the simple form $\bar{\ell} \gamma_\mu \ell$ for electromagnetic interactions and $\bar{\ell} \gamma_\mu (1 + \gamma_5) \ell$ for weak interactions. The hadron current in general may have a very complicated form. However, the assumption of single vector particle ($J = 1$) exchange has the happy result that J_μ (hadron), summed over possible polarization states, can be described in terms of not

more than $3(= 2J + 1)$ independent, unknown structure functions $W_i(q^2, \nu)$, where $i = 1, 2, 3$. W_i are functions of the Lorentz scalars $q^2 = q \cdot q$ and $\nu = q \cdot P$ where P is the four momentum of the target nucleon in (a), (b), (c) and of a produced hadron in (d). Experimentally, this means one has to determine only a small number (2 or 3) arbitrary functions for a given target. The structure functions $W_i(q^2, \nu)$ are in principle different for each type of lepton/antilepton and for proton and neutron targets, but there are relations between them based on general hypotheses such as charge symmetry, CVC, chiral symmetry and hermiticity (for neutral weak currents).

The important conclusion from electron, muon and neutrino scattering experiments on nucleons, as well as from the e^+e^- annihilation data, of the past years is that the structure of the hadronic currents, as measured by the different types of probe at high energy-momentum transfers, can be described, at least approximately, in terms of a very simple and powerful model; the hadrons are composed of quasi-free, pointlike, spin $1/2$ quark constituents. This "naive" quark-parton model describes a tremendous range of experimental data: inclusive (Bjorken) scaling, $[W(q^2, \nu) = W(q^2/2M\nu) = W(x),]$ resulting in linearly rising neutrino cross-sections, the constancy of R [$R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$] at high energy; ratios of magnetic to electric coupling in electron-nucleon scattering, and angular distributions of "jet" directions in e^+e^- annihilation, both consistent with spin $1/2$ constituents; similarity of hadron energy spectra in the different processes, as expected from the fragmentation products of the quark constituents; relations between weak and

electromagnetic cross-sections as predicted for the fractional charges assigned to quarks, and the general verification of sum rules based on quark quantum number assignments.

In these notes, I shall start by reviewing briefly the experimental evidence for the quark-parton picture, and then discuss in more detail the recent modifications and developments. Briefly, there have been three outstanding (and astounding) new classes of phenomena discovered in the past year or so. These are:

(i) Observation of "charmed" mesons and baryons - that is narrow resonances decaying weakly, predominantly to strange particles, and with substantial leptonic branching ratios. The experimental verification of the new charm quantum number is a major triumph for theoretical physics.

Recall that, in 1964, Bjorken and Glashow introduced charm on symmetry arguments, namely that to match the 2 lepton doublets one should introduce

<u>Leptons</u>		<u>Quarks</u>	
<u>Q/e</u>		<u>Q/e</u>	
0	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$	+ 2/3	$\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}$
-1		- 1/3	
	$\Sigma Q/e = -2$		$3 \Sigma Q/e = +2$

a fourth quark to provide two quark doublets of fractional charge. This model possessed also the nice feature that "triangle anomalies" arising, for example, in the decay $\pi^0 \rightarrow 2 \gamma$, might disappear. The Adler-Bell-Jackiw^(1, 2) anomaly arises because PCAC plus gauge invariance together predict that the amplitude

for $\pi^0 \rightarrow 2\gamma$ should vanish for zero pion mass, contrary to the result from a perturbation calculation using fermion-antifermion intermediate states. The

condition for the vanishing of the amplitude is that the total charge of the fundamental fermions should vanish; $\sum Q(\text{leptons}) + \sum Q(\text{quarks}) = 0$. The introduction of the fourth quark, together with three colour codes, accomplishes this.

However, the most compelling argument for a fourth quark, and one which specified much more closely its properties, was that proposed by Glashow, Iliopoulos and Maiani (GIM) in 1970⁽³⁾. They were concerned with the weak couplings of quarks, and the experimental absence of $\Delta S = 1$ neutral weak currents, which were allowed in the Gell-Mann/Zweig/Cabibbo model. The weak quark current doublets they introduced included

$$\begin{pmatrix} u \\ d_c \end{pmatrix}_L + \begin{pmatrix} c \\ s_c \end{pmatrix}_L$$

$$d_c = d \cos \theta + s \sin \theta \quad s_c = s \cos \theta - d \sin \theta$$

that is, the old Cabibbo-rotated combination d_c and a new combination s_c

(rotated through $\theta + \pi/2$ instead of 0). These two currents together suppress the purely weak $\Delta S = 1$ neutral current amplitude for $K_L^0 \rightarrow \mu^+ \mu^-$: it vanishes to all orders for a quark mass $M_c = M_{u,d,s}$ and gives a sufficient suppression factor for $M_c^2 \ll M_W^2$. A more stringent limit $M_c < 5 \text{ GeV}$ is obtained from the smallness of the $(K_L - K_S)$ mass difference. ⁽⁴⁾

Thus, two predictions of the GIM model are:

- (a) Charmed hadrons should exist, with a new quantum number, $C = \pm 1$, conserved in strong interactions.
- (b) The lowest-lying charmed hadrons will decay weakly ($\Delta C = 1$) into strange or non-strange hadrons, with rates in the ratio $\cos^2 \theta / \sin^2 \theta$, and with appreciable ($> 10\%$) leptonic decay modes.

The recent observations of new narrow meson states (D, D^*) in e^+e^- collisions at SPEAR, ⁽⁵⁾ decaying to kaons ($D \rightarrow K\pi, K3\pi$); the evidence for a new baryon decaying to $\bar{\Lambda}3\pi$ ⁽⁶⁾ in photoproduction experiments at FNAL; the observation of "dilepton + V" events in neutrino experiments in bubble chambers at CERN and FNAL ^(7,8) - all seem to be evidence for new hadrons with exactly the properties expected of charmed particles.

(ii) Violations of Bjorken Scaling

The exact scaling of the "naive quark-parton model" was expected to hold in the limit ($q^2, \nu \rightarrow \infty$), and one of the embarrassments or surprises of the early (1968-70) experiments on inelastic electron - and neutrino-nucleon scattering was the observation that scaling held approximately, and precociously, down to values of $q^2 \sim 1 \text{ GeV}^2$ or less. In a field theory of

interacting, and bound, quark constituents, exact scaling cannot hold, and the question then is; how is the Bjorken limit approached? The asymptotically - free field theories, which are infra-red unstable (connected with quark confinement?) predict that scaling will be exact in the extreme ultra-violet limit, the deviations at finite q^2 going as $\sim (\ln q^2 / \mu^2)^{-1}$, where μ is an unknown scale parameter.

Recent analyses of electron and muon inelastic scattering in SLAC and FNAL experiments appear to indicate variations of $F(q^2, x)$ with q^2 at fixed x which are indeed logarithmic, and of a magnitude that suggests μ is small ($\mu^2 \ll 1 \text{ GeV}^2$); the scale of the logarithmic deviations is accessible with present accelerators. This is tremendously encouraging; it means that, over the next few years, precise experiments with muon and neutrino beams will provide enough data to serve as the input for what one hopes may be a real field theory of hadrons.

(iii) New Quantum Numbers and Couplings

Another significant development has been the observation, in a high energy counter neutrino experiment by the HPWF group at Fermilab, of very large deviations from scaling and charge-symmetry; these are manifested by anomalously flat y ($= \nu/E$) distributions in antineutrino scattering, and by a sharp rise in the ratio R of total cross-sections for antineutrino and neutrino scattering above 30 GeV energy. The results come from a single experiment and badly need to be confirmed before they can be accepted as of noninstrumental origin.

Although opinions are divided at the present time, the majority view is that these phenomena cannot be accounted for simply in terms of excitation of the new charm quantum number, or of the logarithmic deviations from scaling mentioned above, if one sticks to the conventional (V-A) form of weak coupling of leptons and quarks. The antineutrino cross-sections can only be enhanced to the degree required by introducing right-handed currents, perhaps carried by new types of quark. ⁽⁵⁰⁾

Recall that the V-A theory of weak (charge-changing) currents works over a huge range of energy, from $\sim$ keV in nuclear β -decay to $\sim$ 30 GeV in high energy neutrino scattering-a range of 10^7 in (CMS energy)²-and the enormity of a breakdown is apparent. The existence of further quarks is perhaps more comprehensible; there is evidence for new heavy leptons by Perl⁽⁵⁵⁾ at SLAC, and thus the previous symmetry arguments would insist on it. Thus a possible new scheme for the weak couplings might be! -

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu' \\ l \end{pmatrix} \dots$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} u \\ b \end{pmatrix}_R, \begin{pmatrix} t \\ d \end{pmatrix}_R \dots$$

OLD PHYSICS

NEW PHYSICS ?

2. SCALING IN ELECTRON AND MUON INELASTIC SCATTERING ON NUCLEONS

2.1 INTRODUCTION TO ELECTRON-NUCLEON INELASTIC SCATTERING

The spin-averaged inelastic scattering cross-section for an electron (muon) on a nucleon is written as

$$\frac{d^2\sigma}{dq^2 d\nu} = \frac{4\pi\alpha^2}{q^4} \frac{E'}{EM} \left[W_2(q^2, \nu) \cos^2 \theta/2 + 2W_1(q^2, \nu) \sin^2 \theta/2 \right] \quad (1)$$

where $q^2 = 2EE' (1 - \cos\theta)$

is defined as a positive quantity and the mass W of the final hadronic state is given by

$$W^2 = M^2 + 2M\nu - q^2 \quad (2)$$

and $\nu = P \cdot q/M$ is the energy transfer measured in the nucleon rest-frame.

The functions W_2 and W_1 are the analogues of the electric and magnetic

form-factors, respectively, in elastic scattering. It is common to express the cross-section (1) in terms of the absorption cross-sections of the mediating photons, σ_L (longitudinal) and σ_T (transverse). In these terms

$$\frac{d^2\sigma}{dq^2 d\nu} = \Gamma [\sigma_T(q^2, \nu) + \epsilon \sigma_L(q^2, \nu)] \quad (3)$$

where Γ is the equivalent flux of transverse virtual photons and ϵ is the polarization parameter, given by

$$\Gamma = (\alpha/2\pi) (K/q^2 E^2) (1/(1-\epsilon))$$

$$\epsilon = [1 + 2(1 + \nu^2/q^2) \tan^2 \theta/2]^{-1}$$

The quantity

$$K = (W^2 - M^2)/2M$$

is the equivalent (real) photon energy required to excite the final state

W by photoabsorption. σ_T and σ_L are related to W_1 and W_2 by

$$\begin{aligned} W_1(q^2, \nu) &= \frac{KM}{4\pi^2 a} \sigma_T(q^2, \nu) \\ W_2(q^2, \nu) &= \frac{KM}{4\pi^2 a} \frac{q^2}{(q^2 + \nu^2)} [\sigma_T(q^2, \nu) + \sigma_L(q^2, \nu)] \end{aligned} \quad (4)$$

Since σ_T , σ_L must be positive definite it follows that

$$(1 + \frac{\nu^2}{q^2}) W_2 \geq W_1 \quad (5)$$

As $q^2 \rightarrow 0$, $\sigma_L \rightarrow 0$ and $\sigma_T \rightarrow \sigma_Y(\nu)$, the real photon cross-section at energy ν .

The relations (4) are often described in terms of the cross-section ratio.

$$R = \sigma_L / \sigma_T = \frac{W_2}{W_1} (1 + \nu^2 / q^2) - 1 \quad (6)$$

For elastic scattering, $W = M$ and $q^2 = 2M\nu$ in (2), and the variable ν is redundant. This dependence on q^2 alone will hold for any exclusive channel where W is well-defined. Experimentally; it is found that at low energy/momentum transfers ($q^2 < 1 \text{ GeV}^2$, $\nu < 2 \text{ GeV}$), the cross-section is dominated by baryon resonance production. In addition to elastic scattering, excitation of at least the first four pion-nucleon resonances has been observed, and these exclusive channels have in common that they are described by form-factors falling off very rapidly with q^2 , and parameterized by the dipole formula

$$F(q^2) \approx (1 + q^2/M_V^2)^{-2}$$

$$M_V \sim 0.84 \text{ GeV} \quad (7)$$

This behaviour of the exclusive cross-sections has a simple interpretation in the constituent (quark-parton) model of the nucleon. An exclusive (resonance) channel corresponds to a modest excitation of the nucleon to a state with well-defined quantum numbers (spin, isospin, parity, mass) and thus involves the coherent scattering of the quarks, with severe restrictions on their space/spin wave function. The form-factors are the (squares of) the Fourier transforms of these wavefunctions, and thus measure the probability that the recoiling baryon will "hold together" under the impact, q . For elastic scattering, for example, the probability of coherent recoil is only $\sim 10^{-6}$ at $q^2 = 25 \text{ GeV}^2$.

Scale invariance and partons

We shall be mostly concerned with inclusive processes where no restrictions are placed on the recoiling hadronic state i. e. q^2 and ν are independent variables. This continuum region of high q^2 , ν is usually associated with the asymptotic region of Bjorken scaling⁽⁹⁾, $q^2 \rightarrow \infty, \nu \rightarrow \infty$, $q^2/2M\nu = x$ finite. Bjorken speculated that, with the definitions

$$\frac{\nu}{M} W_2(q^2, \nu) = F_2(q^2, \nu)$$

$$W_1(q^2, \nu) = F_1(q^2, \nu)$$

either

$$\text{Lt } F_i(q^2, \nu) \rightarrow F_i(x) \text{ finite} \\ q^2 \rightarrow \infty$$

or

$$F_i(q^2, \nu) \rightarrow 0$$

In elastic scattering, for example ($x = 1$) $F_i \rightarrow 0$ at large q^2 , while in deep inelastic scattering it is found experimentally that, at least for $q^2 \sim 1 \rightarrow 20 \text{ GeV}^2$, $F_i(x)$ is finite. This scale invariant behaviour, $F_i(q^2, \nu) \rightarrow F_i(x)$, was first demonstrated in experiments by the SLAC/MIT group in 1968.⁽¹⁰⁾ Recent, more precise electron data, as well as the results from high energy muon scattering, show that Bjorken scaling in the accessible region up to $q^2 \approx 20 \text{ GeV}^2$ is only approximate. It is however, a sufficiently good description of the data as to warrant a simple physical interpretation in terms of incoherent, elastic scattering of high energy leptons by the pointlike quasi-free parton constituents introduced by Feynman.⁽¹¹⁾

Interpretation of F_2 and F_1 in Parton Picture

(i) The parton model views the collision in a reference frame where the nucleon target has a very high momentum $P \rightarrow \infty$, along the z-axis. In this frame, the parton will carry a longitudinal 3-momentum xP . If $xP \gg P_T$, the transverse momentum of a parton, we can treat the parton as quasi-free. Since the energy/momentum components of the proton are $p = (P, 0, 0, P)$, those of the photon must be $q = (0, 0, 0, -2Px)$ so that $q \cdot q/2p \cdot q = x$ as required. Thus, absorption of the photon by the parton simply reverses its 3-momentum:-

COLLISION IN PARTON BREIT FRAME

The parton model has, strictly, no meaning in the proton rest-frame, since x measures a fractional 3-momentum. However, if the "effective mass" of the parton is m , then it will indeed carry (m/M) of the proton momentum in the high-momentum frame, and in the proton rest-frame elastic kinematics give

$$q^2 = 2m\nu$$

or

$$x = q^2 / 2M\nu = m/M$$

The quantity m here refers to the equivalent mass of a (hypothetical) free parton.

(ii) Re-writing (1) in terms of

$$x = q^2 / 2M\nu$$

$$y = \nu / E$$

we obtain

$$\frac{d^2\sigma}{dx dy} = \frac{4\pi\alpha^2}{q^4} \cdot 2ME \left[F_2(x) \left[1 - y - \frac{Mxy}{2E} \right] + 2xF_1(x) \frac{y^2}{2} \right] \quad (9)$$

or

$$\frac{d^2\sigma}{dq^2 dx} = \frac{4\pi\alpha^2}{q^4} \left[\frac{F_2(x)}{x} \left[1 - y - \frac{Mxy}{2E} \right] + 2F_1(x) \frac{y^2}{2} \right]$$

In the limit $y \rightarrow 0$ ($\theta \rightarrow 0$), the second term vanishes and

$$\boxed{\frac{d\sigma}{dq^2} = \frac{4\pi\alpha^2}{q^4} \int \frac{F_2(x)dx}{x}}$$

which is simply the Rutherford pointlike scattering formula, so that we

interpret $\int \frac{F_2(x) dx}{x} = \sum_{\text{partons}} Q_i^2$ = sum of squares of parton charges

(iii) Comparing (1) with the Dirac cross-section for elastic scattering of an electron by a spin 1/2 pointlike constituent of mass m and $g = 2$,

$$\left(\frac{d\sigma}{dq^2} \right)_{\text{Dirac}} = \frac{4\pi\alpha^2}{q^4} \frac{E'}{E} \left[\cos^2 \frac{\theta}{2} + \frac{q^2}{2m^2} \sin^2 \frac{\theta}{2} \right] \quad (10)$$

we obtain

$$\frac{2W_1}{W_2} = \frac{q^2}{2m^2}$$

or

$$\boxed{2xF_1(x) = F_2(x)} \quad (12)$$

Thus, $2xF_1/F_2$ measures the ratio of magnetic to electric scattering from partons. The ratio is unity for a Dirac parton (spin $1/2$, $g = 2$) and zero for spin 0 partons. The relation (12) is the Callan-Gross relation originally derived from current algebra. ⁽¹²⁾

2.2 EXPERIMENTAL RESULTS ON SCALING IN ELECTRON-NUCLEON INELASTIC SCATTERING.

(i) $R = \sigma_L / \sigma_T$

In this section we discuss the evidence ⁽¹³⁾ for or against scaling in the SLAC/MIT data of Riordan et al. First we note from (6) and (12) that, in the naive parton model with spin $1/2$ partons, we expect $(R = \sigma_L / \sigma_T)$: -

$$\frac{1 + R}{1 + q^2/\nu^2} = \frac{F_2}{2xF_1} \quad (13)$$

so that, if asymptotic scaling holds, $R = q^2/\nu^2$. This prediction is in clear contradiction with data in the region $q^2 = 1 - 16 \text{ GeV}^2$. This is made most transparent by plotting $2xF_1/F_2$ as a function of x for various q^2 values (see Fig. 1). This diagram shows data for proton targets; deuterium data indicates very similar behaviour. For $x > 0.3$, the data is consistent with the Callan-Gross relation, but there are significant departures, of order 20%, at small x (and predominantly at $q^2 < 4$).

These results are generally discussed in terms of behaviour of

ELECTRON PROTON SCATTERING (RIORDAN *et al* SLAC-PUB

$$\frac{2xF_1}{F_2}$$

1634 (1975))

Fig. 1 Data on $2xF_1(x)/F_2(x)$ for e-p scattering, from Riordan *et al*. They indicate substantial deviations from the (asymptotic) Callan-Gross relation $2xF_1/F_2 = 1$ for free, spin 1/2, pointlike (Dirac) partons. The deviations are a function of both x and q^2 .

$R(x, q^2)$: (a) First, the average value of R over all data points is the same for protons and neutrons: $\langle R_p \rangle = 0.136 \pm .017$ and $\langle R_d \rangle = 0.137 \pm .013$. A constant value of R in fact yields acceptable fits to the data. (b) Vector dominance models ($R \propto q^2$) give unacceptable fits. (c) In conventional field theories, or those with asymptotic freedom, we expect $R \rightarrow 0$ as $1/q^2$ or $1/\log q^2$. A fit of the type

$$R = f(x)/q^2 \quad (14)$$

with $f(x) = a + b/x^2$ yields an excellent fit to the data. Fig. 2 shows the data on R_p, R_d as a function of q^2 for various x values.

In summary, the electron data on R appear to support scaling, at least as an asymptotic concept, and are consistent with the expectations from spin $1/2$ constituents in the Bjorken limit. Even at moderate q^2 , the Callan-Gross relation is good to within 10-20%.

(ii) Scaling in F_1 and F_2

Early SLAC data ⁽¹⁰⁾ supported the Bjorken scaling hypothesis - $F_2(\nu, q^2) = F_1(x)$ - for the range $q^2 > 1 \text{ GeV}^2$, $W^2 > 4 \text{ GeV}^2$, within the errors, which were considerable ($\sim 10\%$). For some time thereafter, any possible deviations from scaling were taken care of by changing the scaling variable, for example from $x = q^2/2M\nu$ to the Bloom-Gilman variable $x' = q^2/(2M\nu + M^2)$. The scaling variable has, unambiguously, the value x only in the asymptotic region; at finite q^2 , any variable tending asymptotically to x is (in principle) good, and some of the variables proposed have had the merit of bridging the gap between the deep-inelastic region and photoproduction ($q^2 = 0$). Mercifully,

Fig. 2 SLAC data (Riordan et al) on $R = \sigma_L / \sigma_T$ for proton and deuteron targets. The lines represent fits of the general form $R = f(x)g(q^2)$, with $f(x) =$ constant at fixed x and $g(q^2) = q^2 / (q^2 + d^2)^2$ or $g(q^2) = (\ln q^2 / \beta^2)^{-1}$.

further discussion of scaling variables has been aborted with the discovery that structure functions do NOT exactly scale at finite q^2 in any of the variables. Therefore, I propose to stick simply to the Bjorken variable x (or occasionally $\omega = 1/x$).

Figure 3 shows SLAC data⁽¹³⁾ on F_1 and F_2 at a few values of x , as a function of q^2 , for both hydrogen and deuterium. There is a marked decrease of F_1, F_2 with increasing q^2 for $x > 0.4$, and some evidence for an increase in F_2 with q^2 for $x \leq 0.1$. These deviations become clearer when considered in conjunction with the higher energy data on muon scattering, which we discuss next.

2.3 MUON-NUCLEON INELASTIC SCATTERING

There have been two experiments testing scaling in muon-nucleon scattering at FNAL. These are the Cornell-Michigan State-UCB-USCD experiment, using an iron target⁽¹⁴⁾ and an iron solenoid spectrometer, and the Chicago-Harvard-Illinois-Oxford experiment using H_2/D_2 targets and a large air-gap magnet.⁽¹⁵⁾

The principal of the first experiment was to compare the ratio of the (presumed) scale-invariant cross-sections at two muon energies. From (9)

$$E \frac{d^2\sigma}{dx dy} = \frac{4\pi q^2}{2Mx y^2} \left[F_2(x)(1-y) + 2xF_1(x) \cdot \frac{y^2}{2} \right] \quad (15)$$

is invariant if Bjorken scaling holds. The experiment was run at one incident muon energy $E_0 (= 56 \text{ GeV})$ and then at a higher energy $\lambda E_0 (= 150 \text{ GeV})$ i. e. scaled up by $\lambda = 8/3$, with the following changes in the experimental

Fig. 3 SLAC data on the structure functions $2xF_1$ and F_2 for protons and deuterons, as a function of q^2 for a few values of x . In all cases, the data is consistent with a simple linear dependence of $\ln 2xF_1$ or $\ln F_2$ on $\ln q^2$.

disposition: -

- (i) Counting rate $\propto \frac{1}{E_0}$; scale target thickness by λ from 233 to 622 gm/cm².
- (ii) Scale target-spectrometer distance by $\lambda^{\frac{1}{2}}$; the scattering angle θ at fixed radius scales as $\lambda^{-\frac{1}{2}}$.
- (iii) Scale magnetic field integral $B\ell$ by factor $\lambda^{\frac{1}{2}}$, by including more toroids. Since the magnetic deflection $\propto B\ell / E$, the trajectories of the scattered muons and counting rates remain the same at the higher energy, for fixed x and y , with, of course, $v \rightarrow \lambda v$ and $q^2 \rightarrow \lambda q^2$. Since the multiple scattering deflection $(\theta \text{ Coulomb})/(\theta \text{ magnetic}) \propto \frac{1}{\sqrt{t}} \propto \lambda^{-1/4}$, this was compensated by including extra toroids with the magnetic field switched off (see Fig. 4).

Thus, the experimental acceptance, count rates and resolution should be identical for the two settings, and the ratio

$$r(x, q^2) = \frac{(Ed^2\sigma/dx dy) \text{ at } E = 150}{(Ed^2\sigma/dx dy) \text{ at } E = 56} \quad (16)$$

should be unity if scaling holds, for all q^2, x . The data (Fig. 5) show r plotted as a function of $\omega = 1/x$, and also separated into three regions of $v = q^2/2ME$. The results appear to show clear deviations from scale invariance, but can be fitted with a simple power-law dependence of r on x (or ω): -

$$\begin{aligned} r &= (\omega/\omega_0)^n \\ n &= 0.096 \pm .028 \\ \omega_0 &= 6.1^{+8.9}_{-3.6} \end{aligned} \quad (17)$$

Fig. 4 Apparatus of Watanabe et al., (FNAL E26) for investigating inelastic muon scattering in Fe targets, using a solid iron spectrometer. The scales in the upper and lower halves of the diagram are in the ratio of $\sqrt{\lambda} = \sqrt{8/3}$, where $\lambda = 150/56$ is the ratio of beam energies employed in the two configurations.

Fig. 5

The ratio r of cross-sections observed at 150 and 56 GeV measured in the apparatus of Fig. 4. If Bjorken scaling holds, r should be unity at all q^2 , x . The x or ω dependence implies a q^2 dependence at fixed x .

A second test of scaling is to compare the muon scattering data at high energy with the SLAC electron data at low energy (assuming $e-\mu$ universality). From SLAC experiments on heavy targets, it was already known that the scaling functions per nucleon were very similar on deuterium and on more complex nuclei. Because of the inherently poorer resolution of the muon experiment (the multiple Coulomb scattering in the iron magnet limits $(\Delta p/p)$ to $\pm 14\%$) and there are also acceptance limitations, the connection between the two experiments rests on a Monte-Carlo calculation of the event rates to be expected in the muon experiment, using the SLAC data as input. For this purpose, the authors took values for $F_2(\omega')$ evaluated at $q^2 = 3 \text{ GeV}^2$ in deuterium. For $\omega' > 10$ there is no data from SLAC and they assumed $F_2(\omega' > 10) = 0.30 = \text{constant}$.

Fig. 6 shows the ratio, s , of (observed/expected) event rates as a function of ω and q^2 . For each ω value, the quantity $\ln s$ was fitted with a linear dependence on $\ln q^2$. Furthermore, the slopes of these lines change systematically from positive values at large ω to negative values at small ω . These results can be summarized by the empirical relations

$$F_2(\omega', q^2) = \left[F_2(\omega') \right]_{\text{SLAC}} \cdot \left(\frac{\omega}{\omega_0} \right)^a \ln(q^2/q_0^2) \quad (18)$$

with

$$q_0^2 = 3 \text{ GeV}^2$$

$$a = 0.099 \pm .018$$

$$\omega_0 = \left\{ \begin{array}{l} 6.1 + 3.9 \\ - 2.4 \end{array} \right\}$$

Fig. 6 Comparison of E26 data (μ -Fe scattering) with SLAC data (e - D). The SLAC data at $q^2 = 3$ is wed in a Monte Carlo calculation which simulates the acceptance, resolution, etc. of the E26 experiment.

Fig. 7 $F_2(q^2)$ for $\omega \sim 10$ in E26 and SLAC experiments. The curve indicates the trend expected in VDM models.

This treatment assumed ω' as the relevant variable to describe the SLAC data (it minimizes scaling violations in the low ω region). Other scaling variables were tried; scaling deviations can be reduced in one part of the ω range but they are correspondingly increased in other parts of it.

Fig. 7 shows a more recent analysis⁽¹⁶⁾ of the muon scattering data, in which $F_2(x, q^2)$ is plotted as a function of q^2 at fixed $\omega > 9$, and includes the SLAC data at lower q^2 (and to which it is normalized). A steady increase in F_2 with q^2 appears to indicate a smooth approach to scaling in the asymptotic region. At low q^2 we note from (4) that $F_2(q^2, \nu) \rightarrow 0$ as $q^2 \rightarrow 0$. This feature is also clear in the electron data of Stein et al⁽¹⁷⁾ on hydrogen at $q^2 < 1 \text{ GeV}^2$ - see Fig. 8.

The muon scattering data of Anderson et al in deuterium⁽¹⁵⁾ are of poorer statistical weight than that of the results using iron targets, but nevertheless appear to show similar non-scaling effects (Fig. 9). Perhaps the most impressive thing about this high energy data is, not so much the scale-breaking but rather the approximate constancy of F_2 with q^2 , for $q^2 > 2$ and for ω as large as 100. There is indeed little evidence in this data for an increase of $F_2(x, q^2)$ with q^2 at small, fixed x . In the large $x(x > 0.2, \omega < 5)$ region, there is again a very clear indication for a decrease in $F_2(x, q^2)$ as q^2 increases. Anderson et al parameterize their results in the form

$$F_2(\omega, q^2) = F_2(\omega, q_0^2) \left[1 + a \ln \left(\frac{q^2}{q_0^2} \right) \ln \left(\frac{\omega}{\omega_0} \right) \right] \quad (19)$$

Fig. 8 Low q^2 data on $F_2(q^2)$ in SLAC e-p data.

Fig. 9 $F_2(q^2, \omega)$ in eD and μ D scattering. Open circles, SLAC eD data; solid circles, E98 data.

Fig. 10 Curves representing best-fits to ep and μp data, of the form $A(1-x)^3 + B(1-x)^4 + C(1-x)^5$, where A , B and C are fitted in each band of q^2 . From report by T. Wuirk, Tbilisi Conf. (Aug. 1976).

which, for small deviations, is equivalent to that used in (18) by Watanabe et al. For the choice $\omega_0 = 6, q_0^2 = 3 \text{ GeV}^2$, they find

$$a = 0.072 \pm .038 \quad (20)$$

consistent with the value $0.099 \pm .018$ from the previous experiment. This group also gives the value of $F_2(\omega)$ integrated over all q^2 (Fig. 10). For $q^2 > 2$, there is some evidence for a decrease in $F_2(\omega)$ for $\omega > 10$.

The origin of the breakdown of scaling in electron and muon scattering is not clear at the present time. Possible origins are: -

- (i) Bjorken scaling is expected to be exact, if at all, in the high q^2 limit.

The deviations are a measure of the approach to Bjorken scaling. There are theoretical predictions regarding this from theories of asymptotic freedom etc. This is discussed in Section 4.

- (ii) The deviations are associated with the onset of new couplings at increasing q^2 and ν , due to excitation of new hadronic quantum numbers, for example charm. There is really no clear evidence to support this proposition.

Fig. 11 shows the result of a search for "steps" in the cross-sections at particular threshold values of W in the experiment of Anderson et al. (15)

At each value of ω , the value of $F_2(\omega, W^2)$ was found as a function of W , relative to the value averaged over the whole range of W . Then, the data were combined to obtain the overall ratio as a function of W^2 . No steps in W^2 are observed, and any such effects are below the 20% level.

SEARCH FOR A STEP OR PEAK IN VW_2 AT
ANY FINAL HADRONIC STATE MASS

Fig. 11 Search for thresholds in hadronic mass W , in E98 experiment.

3. SCALING IN NEUTRINO INTERACTIONS AT MEDIUM ENERGIES ($E < \text{GeV}$)

The Bjorken scaling cross-section for neutrino and antineutrino nucleon scattering, analogous to (1) and (15) for electromagnetic scattering, is

$$\frac{d^2 \sigma^{\nu, \bar{\nu}}}{dx dy} = \frac{G^2 4E}{\pi} \left[\left(1 - y - \frac{Mxy}{2E}\right) F_2^{\nu, \bar{\nu}}(x) + \frac{y^2}{2} \left[2xF_1^{\nu, \bar{\nu}}(x) \right] \right. \\ \left. \pm y \left(1 - \frac{y}{2}\right) \left[xF_3^{\nu, \bar{\nu}}(x) \right] \right] \quad (21)$$

where F_1 , F_2 and F_3 are arbitrary structure functions. Again the positivity of the scalar, left-handed and right handed virtual W-absorption cross-sections $\sigma_S, \sigma_L, \sigma_R$ places the following constraints on the F_i : -

$$F_2(x) \geq \frac{2x F_1(x)}{(1 + q^2/\nu^2)} \geq \frac{(xF_3(x))}{(1 + q^2/\nu^2)^{1/2}} \geq 0 \quad (22)$$

While F_2^{ν} and F_1^{ν} are the weak analogues of the electric and magnetic structure functions (see (12)), F_3 is a parity-violating interference term with opposite magnitude for neutrinos and antineutrinos. In general, the values of F_i^{ν} and $F_i^{\bar{\nu}}$ will be different, and of course each will depend on the nature of the target. Thus, altogether, for neutrinos, antineutrinos, neutron and proton targets, 12 independent functions are involved.

For $\Delta S = 0$ processes only (accounting for $\cos^2 \theta_c \sim 0.95$ of σ (total)), charge symmetry gives

$$F_i^{\nu n} = F_i^{\bar{\nu} p} \quad F_i^{\nu p} = F_i^{\bar{\nu} n} \\ F_i^{\nu N} = \frac{1}{2} (F_i^{\nu p} + F_i^{\nu n}) = \frac{1}{2} (F_i^{\bar{\nu} n} + F_i^{\bar{\nu} p}) = F_i^{\bar{\nu} N}$$

so that on an isoscalar target

$$\left(\frac{d^2 \sigma^{\nu, \bar{\nu}N}}{dx dy} \right)_{\Delta S=0} = \frac{G^2 \cos^2 \theta_c ME}{\pi} \left[(1-y) F_2^{\nu N} + \frac{y^2}{2} (2xF_1^{\nu N}) \right. \\ \left. \pm y(1-\frac{y}{2}) x F_3^{\nu N} \right] \quad (23)$$

and only 3 functions are involved.

Integration over x and y gives for the total cross-sections, in the approximation $\theta_c \sim 0$, $\sigma(\Delta S = 1) \ll \sigma(\Delta S = 0)$: -

$$\sigma^{\nu, \bar{\nu}N} = \frac{G^2 ME}{\pi} \int F_2^{\nu N} dx \left[\frac{1}{2} + \frac{A}{6} \pm \frac{B}{3} \right] \quad (24)$$

$$R = \sigma^{\bar{\nu}} / \sigma^{\nu} = \frac{3 + A - 2B}{3 + A + 2B} \quad (25)$$

where

$$A = \int 2xF_1 dx / \int F_2 dx \\ B = \int xF_3 dx / \int F_2 dx \quad (26)$$

Early data⁽¹⁸⁾ from the CERN Gargamelle heavy liquid chamber (freon CF₃ Br filling, neutron-proton ratio 1.19) and the CERN PS wideband neutrino beam, related to neutrino energies in the range 1-10 GeV, and gave

$$\sigma^{\nu}(E) = 0.74 \cdot 10^{-38} E \text{ cm}^2 \text{ GeV}^{-1} \text{ nucleon}^{-1} \\ R = \sigma^{\bar{\nu}} / \sigma^{\nu} = 0.38 \pm 0.02 \\ \langle q^2 \rangle^{\nu} = 2ME \langle x \rangle \langle y \rangle = 0.20 E \quad (27)$$

These results are shown in Figs. 12 and 13, and are consistent with, although not proving, scaling. Note that the average value of q^2 at the average

NEUTRINO AND ANTINEUTRINO TOTAL CROSS-SECTIONS

EVENTS $E > 2 \text{ GeV}$
 $\left. \begin{array}{l} 1700 \bar{\nu} \\ 2490 \nu \end{array} \right\}$

Fig. 12 Total ν , $\bar{\nu}$ cross-sections from CERN-Gargamelle experiments.

Fig. 13 Ratio $R = \sigma_{\bar{\nu}}/\sigma_{\nu}$ from CERN-Gargamelle experiments.

energy of the data, $E \sim 4 \text{ GeV}$, is $\langle q^2 \rangle \sim 0.8 \text{ GeV}^2$, so that most of the events are below the scaling cuts used at SLAC ($q^2 > 1$, $W^2 > 4 \text{ GeV}^2$). If, nevertheless, we interpret this data in terms of scaling, we obtain limits on A and B :-

$$1 \geq A \geq \frac{3(1-R)}{(3R+1)} = 0.87 \pm 0.05$$

$$0.90 \pm .04 = \frac{2(1-R)}{(1+R)} \geq B \geq \frac{3(1-R)}{(3R+1)} = 0.87 \pm .05 \quad (28)$$

These results suggest $A \sim B \sim 1$, supporting the Callan-Gross relation $2xF_1 = F_2$ expected for spin $\frac{1}{2}$ constituents and verified approximately in electron scattering. The result $B \approx +1$ implies maximum V/A interference. In the V-A model, the sign of the interference term corresponds to scattering off particle (rather than antiparticle) constituents. This becomes clear if we re-write (23) with $2xF_1 = F_2$:-

$$\frac{d^2\sigma}{dx dy} = \frac{G^2 ME}{\pi} \left[F_2 (1 - y + y^2/2) + xF_3 (y - y^2/2) \right]$$

$$\left. \frac{d\sigma}{dy} \right|_x = \frac{G^2 ME}{\pi} dx \left[\frac{(F_2 + xF_3)}{2} + \frac{(F_2 - xF_3)}{2} (1-y)^2 \right] \quad (29)$$

and compare this with the formula for $\nu + e^-$ and $\bar{\nu} + e^+$ elastic scattering (for $E > m$, the electron mass): -

$$\frac{d\sigma^{\nu e}}{dy} = \frac{2G^2 mE}{\pi} \quad \frac{d\sigma^{\bar{\nu} e}}{dy} = \frac{2G^2 mE}{\pi} (1-y)^2 \quad (30)$$

If we denote $q(x)dx$ and $\bar{q}(x)dx$ as the parton and antiparton densities in the nucleon, and write $m = Mx$ for the parton mass, then (29) and (30) give

$$\frac{F_2 + xF_3}{2} dx = 2xq(x)dx$$

$$\frac{F_2 - xF_3}{2} dx = 2x\bar{q}(x)dx$$

and thus

$$\begin{aligned} F_2(x) &= 2x [q(x) + \bar{q}(x)] \\ xF_3(x) &= 2x [q(x) - \bar{q}(x)] \end{aligned} \quad (31)$$

so that $B \approx +1$, $\int xF_3 dx \approx \int F_2 dx$ implies $\bar{q}(x) \approx 0$, that is, the parton density is large in comparison with that of antipartons. Quantitatively, the ratio of the momentum fraction carried by quarks and antiquarks is:

$$\frac{\bar{Q}}{Q} = \frac{\int x\bar{q} dx}{\int xq dx} = \frac{1 - B}{2} = \frac{3R - 1}{2(1 + R)} \sim 0.05 \quad (32)$$

3.1 NEUTRINO CROSS-SECTIONS IN THE QUARK-PARTON MODEL

Tentatively identifying the partons with the quarks u, d, s, c of the GIM model, we can with the help of (31), write down the functions F_2 and xF_3 in terms of the quark densities, as shown in the accompanying table. In this table, $u, \bar{u}$ etc. stand for $u(x), \bar{u}(x)$ and the correct Cabibbo factors are included. The quark densities refer, by convention, to those in the proton. Note that, even assuming $2xF_1 = F_2$, there are still 8 independent structure functions to determine; $F_2^{\nu n}, F_2^{\nu p}, F_2^{\bar{\nu} n}, F_2^{\bar{\nu} p}$ and the corresponding ones for xF_3 .

3.2 EVALUATION OF THE STRUCTURE FUNCTIONS F_2 AND xF_3

The actual evaluation of F_2 and xF_3 in the scaling region has been made

in the CERN Gargamelle experiments, ⁽¹⁹⁾ with a freon target (CF_3Br , $n/p = 1.19$).

As indicated before, in the approximation that charmed quarks can be neglected at low energy, and that $\sin^2 \theta_c \sim 0$, we have $F_2^{\nu N}(n) = F_2^{\bar{\nu} N}$, $xF_3^{\nu N} = xF_3^{\bar{\nu} N}$ in an isoscalar target and hence from (23)

$$\begin{aligned} F_2^{\nu N}(x) &= \frac{\pi}{2G^2ME} \frac{1}{(1-y+y^2/2)} \left[\frac{d^2\sigma^{\nu N} + d^2\sigma^{\bar{\nu} N}}{dx dy} \right] \\ xF_3^{\nu N}(x) &= \frac{\pi}{2G^2ME} \frac{1}{(y-y^2/2)} \left[\frac{d^2\sigma^{\nu N} - d^2\sigma^{\bar{\nu} N}}{dx dy} \right] \end{aligned} \quad (33)$$

The selection of events was made for $q^2 > 1$, $W^2 > 4 \text{ GeV}^2$, as in the SLAC data, the results being given in Fig. 14. The curve in this plot of $F_2^{\nu N}$ is taken from SLAC data on F_2^{eN} , multiplied by a factor 18/5 as discussed below. The curve in the xF_3 plot was computed from an empirical fit⁽²⁰⁾ of quark and antiquark density distributions to electron scattering data.

3.3 SUMRULES FOR NEUTRINO SCATTERING.

Neutrino sumrules were originally derived from the commutation relations of current algebra (based on a particular hadron symmetry; SU3, SU4, ...). Using the expressions given in the table, we can equally derive them from the quark model, using the known constraints on quark densities. For example, for the proton, we have

$$\begin{aligned} \int (u - \bar{u}) dx &= 2 \\ \int (d - \bar{d}) dx &= 1 \\ \int (s - \bar{s}) dx &= 0 \\ \int (c - \bar{c}) dx &= 0 \end{aligned} \quad (34)$$

STRUCTURE FUNCTIONS FOR EVENTS IN THE SCALING REGION

$$q^2 > 1 \text{ GeV}^2$$

$$W^2 > 4 \text{ GeV}^2$$

Fig. 14 Separated values of $F_2(x)$, $xF_3(x)$ tracted from CERN-Gargamelle data of $q^2 > 1$, $W^2 > 4 \text{ GeV}^2$.

Table I.

Values of F_2 and xF_3 In Quark-Parton Model

		F_2	xF_3
$\Delta S = 0$	$\nu p \rightarrow \mu^- + \dots$	$2x(d + \bar{u}) \cos^2 \theta$	$2x(d - \bar{u}) \cos^2 \theta$
$\Delta C = 0$	$\bar{\nu} n \rightarrow \mu^+ + \dots$		
	$\nu n \rightarrow \mu^- + \dots$	$2x(u + \bar{d}) \cos^2 \theta$	$2x(u - \bar{d}) \cos^2 \theta$
	$\bar{\nu} p \rightarrow \mu^+ + \dots$		
$\Delta C = 0$	$\nu p \rightarrow \mu^- + \dots$	$2x(s + \bar{u}) \sin^2 \theta$	$2x(s - \bar{u}) \sin^2 \theta$
$\Delta S = 1$	$\nu n \rightarrow \mu^- + \dots$	$2x(s + \bar{d}) \sin^2 \theta$	$2x(s - \bar{d}) \sin^2 \theta$
	$\bar{\nu} p \rightarrow \mu^+ + \dots$	$2x(u + \bar{s}) \sin^2 \theta$	$2x(u - \bar{s}) \sin^2 \theta$
	$\bar{\nu} n \rightarrow \mu^+ + \dots$	$2x(\bar{s} + d) \sin^2 \theta$	$2x(d - \bar{s}) \sin^2 \theta$
$\Delta S = 0$	$\nu p \rightarrow \mu^- \dots$	$2x(d + \bar{c}) \sin^2 \theta$	$2x(d - \bar{c}) \sin^2 \theta$
$\Delta C = 1$	$\nu n \rightarrow \mu^- \dots$	$2x(u + \bar{c}) \sin^2 \theta$	$2x(u - \bar{c}) \sin^2 \theta$
	$\bar{\nu} p \rightarrow \mu^+ \dots$	$2x(c + \bar{d}) \sin^2 \theta$	$2x(c - \bar{d}) \sin^2 \theta$
	$\bar{\nu} n \rightarrow \mu^+ \dots$	$2x(c + \bar{u}) \sin^2 \theta$	$2x(c - \bar{u}) \sin^2 \theta$
$\Delta S = 1$	$\nu p \rightarrow \mu^- \dots$	$2x(s + \bar{c}) \cos^2 \theta$	$2x(s - \bar{c}) \cos^2 \theta$
$\Delta C = 1$	$\nu n \rightarrow \mu^- \dots$		
	$\bar{\nu} p \rightarrow \mu^+ \dots$	$2x(\bar{s} + c) \cos^2 \theta$	$2x(c - \bar{s}) \cos^2 \theta$
	$\bar{\nu} n \rightarrow \mu^+ \dots$		

A. Comparison of Neutrino and Electron Scattering

(i) In the approximation $\sin^2 \theta \approx 0$, the table gives us for a nucleon target in the low energy region

$$F_2^{\nu N}(x) = x(u + d + \bar{u} + \bar{d}) \quad (35)$$

where charm effects are neglected. With the same assumptions, and assuming the fractional charges $+2/3$ for the u quark and $-1/3$ for the d and s quarks, the electron-scattering structure function is given by

$$\begin{aligned} F_2^{eN}(x) &= \frac{x}{2} \left\{ \frac{4}{9} (u + \bar{u} + d + \bar{d}) + \frac{1}{9} (d + u + \bar{d} + \bar{u}) \right. \\ &\quad \left. + \frac{1}{9} (2s + 2\bar{s}) \right\} \\ &= \frac{5}{18} x \left\{ (u + d + \bar{u} + \bar{d}) + \frac{2}{5} (s + \bar{s}) \right\} \end{aligned} \quad (36)$$

Hence the quark charge assignments give

$$F_2^{\nu N}(x) \leq \frac{18}{5} F_2^{eN}(x) \quad (37)$$

where the equality holds if, as (32) suggests, the antiquark population is small over most of the range of x . Fig. 14 shows this relation to be valid within the errors. Alternatively, the relation may be checked in integral form using the total cross-section data. Experimentally

$$\int_0^1 F_2^{eN} dx = 0.15 \pm .01 \quad \left\{ \begin{array}{l} \text{SLAC 16 GeV electrons} \\ \text{FNAL 150 GeV muons} \end{array} \right.$$

$$\begin{aligned} \int F_2^{\nu N} du &= \frac{3\pi}{4G^2 M^2 E} (\sigma^{\nu} + \sigma^{\bar{\nu}}) \\ &= 0.48 \pm 0.05 \quad \text{CERN data} \end{aligned} \quad (38)$$

with a ratio 3.2 ± 0.5 , compatible with 18/5.

(ii) A comparison between electron and neutrino scattering can also be made on more general grounds, not involving the quark assumptions directly: -

If we assume first-class weak currents only, the isotriplet current hypothesis relates the ($\Delta S = 0$) part of the weak vector coupling to the isovector electromagnetic coupling: -

$$F_2^{\nu N}(\text{vector}) = 2 F_2^{eN}(\text{isovector})$$

Secondly, chiral symmetry suggests equal V and A coupling of the weak current so that

$$F_2^{\nu N} = F_2^{\nu N}(V) + F_2^{\nu N}(A) = 4 F_2^{eN}(\text{isovector})$$

Finally, experiments on meson photoproduction (ϕ, ω, ρ) give a measure of the relative isoscalar/isovector couplings. If these results apply also to virtual photons in inelastic scattering, we obtain

$$F_2^{eN}(\text{isovector}) \approx 0.9 F_2^{eN}(\text{total})$$

and hence

$$F_2^{\nu N}(x) = 3.6 F_2^{eN}(x)$$

as before. If, on the other hand, the weak coupling includes second-class currents of strength λ , we expect

$$F_2^{\nu N}(x) = 3.6 (1 + \lambda^2) F_2^{eN} \quad (39)$$

so that (38) sets a limit $\lambda^2 < 0.2$, or $\lambda < 0.5$ - not very stringent. Note, finally, that the quark model incorporates CVC and chiral symmetry auto-

matically, and predicts an isoscalar/isovector coupling ratio of 1/9.

We note from (35), (36) that $\int F_2^{\nu N}$ or $(18/5) \int F_2^{eN} dx$ measure, approximately, the total fractional momentum of the nucleon carried by the active quarks/antiquarks, and that this is of order 50%. The remainder has to be ascribed to the inactive neutral gluons responsible for quark-quark binding forces. Theoretical predictions regarding the energy/momentum equipartition between quarks and gluons are discussed in Section 4.

(B) The Adler Sumrule

The most famous current-algebra sumrule for neutrinos is that of Adler,⁽²¹⁾ and is obtained from the equal-time commutation relation of the currents taken between, say, two identical proton states when these states have infinite momentum - a proviso necessary to ensure that q be spacelike, rather than timelike. The sumrule has the form (keeping track of the Cabibbo angle for later reference)

$$\lim_{E \rightarrow \infty} \frac{\pi}{G^2} \left(\frac{d\sigma^{\nu}}{dq^2} - \frac{d\sigma^{\bar{\nu}}}{dq^2} \right) = - \left(\underset{\substack{\uparrow \\ \Delta S = 0}}{2I_3 \cos^2 \theta_c} + \left(\frac{3}{2}Y + I_3 \right) \underset{\substack{\uparrow \\ \Delta S = 1}}{\sin^2 \theta_c} \right) \quad (40)$$

assuming SU3 commutators, and applies to cross-sections on any hadron specified by the quantum numbers Y, I_3 . It is to be emphasized that the Adler sumrule is a fixed q^2 sumrule which does not depend on scaling (although it inspired it) but which does hinge on both the infinite momentum assumption as well as the validity of the commutation relations. The above relation may also be cast in terms of the scaling functions

$$\lim_{x \rightarrow 0} \int \frac{dx}{x} (F_2^{\nu} - F_2^{\bar{\nu}}) = -2 (2I_3 \cos^2 \theta_c + (\frac{3}{2} Y + I_3) \sin^2 \theta_c) \quad (41)$$

The expressions on the right-hand side of (40) and (41) depend on the hadron symmetry assumed. For example, in the SU4 charm scheme, the RHS of (40) is $-2I_3$ exactly (i. e. it is obtained by setting $\theta_c = 0$). Observations are generally made on nucleon targets, of $Y = 1$, $I_3 = 0$. Generally we may write

$$\lim_{E \rightarrow \infty} \frac{\pi}{G^2} \left(\frac{d\sigma^{\nu N}}{dq^2} - \frac{d\sigma^{\bar{\nu} N}}{dq^2} \right) = A \quad (42)$$

where A is given in the table for a few hadron models: -

Table II

Adler Sumrule for Nucleon Target

Symmetry	A
SU3 (Triplet quark)	$-\frac{3}{2} \sin^2 \theta_c = -.075$
SU4 (Quartet charm) ⁽³⁾	0
SU4 + extra (V + A) coupling ⁽²⁵⁾	$+\frac{3}{2}$

As will be discussed in a later section, there are reasons to believe that fundamentally new symmetries become important at very high neutrino energies, so that the SU3 version of the Adler sumrule could never be tested!

The practical conditions for the applicability of the Adler sumrule are unknown; we do know however that the infinite momentum frame concept of

Fig. 15 Differential antineutrino cross-section as function of $v = q^2 / 2ME$, in different energy ranges, from Gargamelle experiments. The data fit roughly on a universal curve.

scaling holds approximately in practice in the GeV energy range of neutrino interactions. A low-energy test has been made in the CERN experiments. ⁽¹⁹⁾ For this purpose, it is useful to employ the variable

$$v = q^2/2ME$$

Neutrino cross-sections are known to scale in this variable, even at low energies (see Fig. 15). In terms of v ,

$$\frac{\pi}{G^2} \frac{d\sigma}{dq^2} = \frac{1}{N} \frac{dN}{dv} \frac{a\pi}{2MG^2} = \beta \frac{1}{N} \frac{dN}{dv},$$

where N is the number of events and a is the coefficient of the energy dependence of the total cross-section. From Fig. 12 we obtain $\beta = (\pi a/2MG^2) = 0.24$ for neutrinos and 0.09 for antineutrinos. Then the Adler limit becomes

$$\lim_{E \rightarrow \infty} \frac{\pi}{G^2} \left(\frac{d\sigma^{\nu N}}{dq^2} - \frac{d\sigma^{\bar{\nu} N}}{dq^2} \right) = \lim_{\substack{v \rightarrow 0 \\ q^2 \text{ fixed}}} \left[\left(\beta \frac{1}{N} \frac{dN^{\nu}}{dv} \right) - \left(\beta \frac{1}{N} \frac{dN^{\bar{\nu}}}{dv} \right) \right] \quad (43)$$

In the scaling limit, the deviations from the Adler sumrule at finite v , i. e. the deviations from (41) when $x \neq 0$, depend on $F_3(x)$ and can be calculated from empirical fits to electroproduction. ⁽²⁰⁾ Fig. 16 shows the data from the CERN experiments, for three different energy ranges. The dashed curve indicates the expected form of convergence to the Adler point, assuming scaling. It is observed that, for $E < 10$ GeV, the data seem to indicate an intercept at $v = 0$ consistent with the SU3 (or SU4) predictions $A \sim 0$ of Table II. Also shown on this graph are some data from high energies suggesting a large value for A , corresponding to large deviations from charge symmetry. ⁽²²⁾ These HPWF data have not been published.

In summary, we cannot claim that low energy experiments have

Fig. 16 The difference of differential cross-sections for neutrinos and antineutrinos, from CERN GGM and HPWF experiments. The Adler sumrule, in the limit of large E, predicts the intercept a $\nu = 0$.

verified the SU3 version of the Adler sumrule, but they are certainly consistent with it and with the low energy hadron symmetries to which it applies.

(C) The Gross-Llewellyn-Smith Sumrule

A second asymptotic sumrule of current algebra-based on equal-time commutation relations of the space components of the V and A currents - is that of Gross and Llewellyn-Smith.⁽²³⁾ This refers to the structure function $F_3(x)$. As indicated in (31), and Table 1,

$$\frac{1}{2} \int \left(F_3^{\bar{\nu}} + F_3^{\nu} \right) dx$$

measures the difference in numbers of quarks and antiquarks in a target.

Formally, the sumrule has the form, in the scaling limit and assuming SU3 symmetry:-

$$\int_0^1 \frac{1}{2} (F_3^{\bar{\nu}} + F_3^{\nu}) dx = 2B + Y + (I_3 - \frac{3}{2} Y) \sin^2 \theta_c \quad (44)$$

where B, Y and I_3 are the baryon number, hypercharge and third component of isospin of the target. For an isoscalar (nucleon) target

$$\int_0^1 F_3^{\nu N} dx = \frac{3}{2} (1 + \cos^2 \theta_c) \approx 3 \quad (45)$$

a similar result holding in SU4. The integral comes from small x - where there is little data in the scaling region. Using bins of $\Delta x = 0.1$, the CERN data gave, for $q^2 > 1$, $W^2 > 4$ GeV:-

Table III.Gross-Llewellyn Smith Sumrule

<u>Experiment</u>	<u>F_3 Fit to Electroproduction</u>
$\left(\sum_0^1 F_3(x) \Delta x \right)_{\Delta x = 0.1} = 2.8 \pm 0.8$	2.445
$\left(\sum_{0.1}^1 F_3(x) \right)_{\Delta x = 0.1} = 1.1 \pm 0.3$	1.277
$\left(\sum_0^1 F_3(x') \Delta x' \right)_{\Delta x' = .02 \quad (E > 2 \text{ GeV})} = 3.0 \pm 0.6$	2.720

where the theoretical values are taken from an empirical fit of quark momentum distributions to electroproduction⁽²⁰⁾; --

$$xF_3(x) = 1.74x^{\frac{1}{2}}(1-x)^3(1+2.3x) + 1.11x^{\frac{1}{2}}(1-x)^{3.1} \quad (46)$$

Clearly it would be desirable to evaluate the sumrule with better statistics at small x and in smaller bins, to make the result independent of the form of $F_3(x)$. An attempt in this direction was made in the CERN Gargamelle experiments by using all events $E > 2 \text{ GeV}$ (without q^2 or W^2 cuts) and employing the Bloom-Gilman variable⁽²⁴⁾ $x' = q^2/(2M\nu + M^2)$ which is found to extend the scaling behaviour down to the low q^2 region. This result is also given in Table 3. Although data at higher energy are desirable it is fairly obvious that the experiments so far are telling us that there are indeed 3 valence quarks in the nucleon.

Again, the sumrule is a reflection of the fundamental commutators of the hadron symmetry, and Table IV gives the predictions for the same

models as in Table 2: -

Table IV.

GLS Sumrule Predictions On Nucleons

<u>Symmetry</u>	
SU3	$\frac{3}{2} (1 + \cos^2 \theta) = 2.925$
SU4	3.000
SU4 + (V + A) current	1.500

(3.4) CONCLUSIONS

The general features of inclusive neutrino reactions on nucleons at medium energies support the concept of scaling and partons, and the magnitudes of the cross-sections and structure functions are in excellent agreement with the expectations from SU3 symmetry and the triplet quark model.

The y distributions in neutrino/antineutrino interactions are of course specified once the parameter $B = \int xF_3 / \int F_2$ has been measured. The general subject of y distributions will however be deferred to a later section.

(4) THEORETICAL PREDICTIONS FOR BREAKDOWN OF SCALING

(4.1) SOURCES OF SCALING BREAKDOWN

We recall that, in the "naive parton model" of scaling, all masses (nucleons, partons) as well as the parton-parton interactions were neglected. This was justified in a reference frame where the nucleon three-momentum $\underline{P} \rightarrow \infty$, which is equivalent to the requirement $q^2, M_\nu^2 \gg M^2$.

At finite q^2 , corrections to exact Bjorken scaling can arise from three sources: -

(i) Mass Correction Terms. Since quarks are bound and hence "off mass shell", the mass corrections pose some technical difficulties and at the present time, there is no general agreement on how such effects should be treated over the whole range ($x = 0 \rightarrow 1$) of kinematic variables. The reader is referred to the papers by Georgi and Politzer⁽⁶⁶⁾, Nachtmann⁽⁶⁷⁾, Ellis et al⁽⁶³⁾, A. De Rújula et al⁽⁶⁴⁾ and Barbieri et al⁽⁶⁵⁾ - to name a few. In the last reference, it is proposed that the mass corrections to scaling can be accommodated by defining a new scaling variable, in which there would be exact scaling if quark-quark interactions were neglected: -

$$\xi = x \frac{[1 + (m_f^2 - m_i^2)/q^2 + \sqrt{[1 + m_f^2 - m_i^2)/q^2]^2 + 4m_i^2/q^2}]}{1 + \sqrt{1 + q^2/\nu^2}}$$

Here m_i, m_f are the initial and final quark masses. For $m_f = m_i \approx 0$, this reduces to

$$\xi = \frac{x}{(1 + M^2 x^2/q^2 - (M^2 x^2/q^2)^2 + \dots)}$$

This formula is expected to hold for ξ not too near 0 or 1. Since, for $q^2 > M^2$, $x > \xi > x'$ where $x' = x/(1 + M^2 x/q^2)$ is the Bloom-Gilman variable, it is clear that because we know scaling is broken in both x and x' , it cannot hold in ξ . Barbieri et al indeed conclude that mass connections can account for only a minor part of the observed scale breaking. In what follows, it seems best to ignore mass effects-especially as they are not well understood.

(ii) Heavy Quark Thresholds, when excited, as they could be in neutrino scattering, will lead to a breakdown of scaling. Thus, in the transition $m_i \rightarrow m_f$, the correct scaling variable at high q^2 will become $\xi = x[1 - (m_f^2 - m_i^2)/q^2]$, a result which follows using the parton selection $(\xi p + q)^2 = -m_f^2$. Excitation of new thresholds is dealt with in a later section of the report.

(iii) Quark-Quark Interaction

If exact scaling applies in asymptopia, the quark coupling constant $\alpha(q^2)$ must decrease with increasing q^2 . In some models discussed below, the variation in $\alpha(q^2)$ goes as $(\log q^2)^{-1}$, that is more gentle than the $1/q^2$ mass correction effects, (i). It is then expected that interaction effects should dominate the scaling deviations at moderately large q^2 (say $> 3 \text{ GeV}^2$).

(4.2) RENORMALIZABILITY AND DIVERGENCES IN QED

In QED, deviations from the Coulomb potential between massive test charges are expected on account of vacuum polarization (Gell-Mann and Low 1954, Landau 1954⁽²⁶⁾): -

$$V(r) = \frac{QQ'}{r} \left[1 + \frac{2a}{3\pi} \left(\ln \frac{\lambda}{r} - \frac{5}{6} - \ln \gamma \right) + O(a^2) + \dots \right] \quad (47)$$

where Q, Q' are the renormalized (physical) values of the charges, a is the renormalized fine structure constant, $\ln \gamma = 1.781$ and $\lambda = \hbar/mc$ is the electron Compton wavelength. When $x \gg \lambda$, the potential is less than the Coulomb value; this effect is due to screening of the test charge by virtual $e^\pm$ pairs (vacuum polarization).

At distance r , the effective charge is

$$Q = Q_0 \text{ (bare)} - \delta Q$$

with $Q \rightarrow Q_0$ as $r \rightarrow 0$. In this limit,

the potential must be $Q_0 Q_0' / r$ and therefore

$$Q_0 Q_0' = QQ' \left\{ 1 + \frac{2a}{3\pi} \left(\frac{\ln \lambda}{r} \right) + O(a^2) + \dots \right\} \quad (48)$$

so that logarithmic divergences are introduced whenever one tries to express physical quantities (Q, Q', a, m) in terms of bare quantities Q_0, e_0, m_0 etc.

These divergences cancel whenever observable quantities (e.g. (g-2), Lamb shift) are expressed in terms of the renormalized parameters.

The main point is that the effective coupling constant or charge depends on r (for a static potential) or on q^2 for a scattering process. By inverting the

above expression, the coupling constant $g(q^2)$ can be expressed in the form

$$g^2(q^2) \approx \frac{g_0^2}{1 + (g_0^2/3\pi) \ln \Lambda^2/q^2} \sim \frac{3\pi}{\ln \Lambda^2/q^2} \text{ for } \Lambda^2 \gg q^2 \quad (49)$$

Here, g_0 = coupling constant at $q^2 = \Lambda^2$, and Λ is some arbitrary cut-off parameter. Note that we cannot set $\Lambda = \infty$ or $g \rightarrow 0$ at finite q^2 .

Thus, as a field theory, QED is stable in the infra-red (IR) region but requires a cut-off (Λ) in the ultra-violet (UV). Experimental limits on Λ are

above 10 GeV. In the IR region of the real world of electrodynamics, the effects are essentially undetectable; a 10% effect for example requires a q^2 -range of $\exp(0.3\pi/\alpha) \sim 10^{40}$!

(4.3) GAUGE THEORIES OF STRONG INTERACTIONS

QED is an Abelian gauge theory, that is, the gauge transformations (corresponding to the arbitrariness of the classical potential) amount to multiplying wavefunctions by a phase factor: $\psi \rightarrow \psi e^{i\phi}$ where ϕ is arbitrary. Non-Abelian gauge theories involve more complicated unitary transformations, since they describe both charged and neutral fields. These are related by "isospin" transformations e. g. $\psi \rightarrow \psi e^{i\vec{\tau} \cdot \vec{\Lambda}}$, where $\vec{\tau}$ is an isospin vector and $\vec{\Lambda}$ is arbitrary. Since they are renormalizable, such gauge theories have been introduced to describe electromagnetic, weak and strong interactions. In the latter case, the theory is one of fermions (quarks) interacting via vector bosons (gluons). It was shown by Gross and Wilczek⁽²⁷⁾ that such theories may be "asymptotically free" in the UV region because of an antiscreening of the (strong) charge: -

$$g^2(q^2) = \frac{g_0^2}{(1 - Bg_0^2 \ln \Lambda^2/q^2)} = \frac{g_0^2}{(1 + Bg_0^2 \ln q^2/\mu^2)} \quad (50)$$

Here B = constant depending on the gauge group (SU2, SU3...), μ is an arbitrary parameter analogous to Λ in QED, $g^2(q^2)$ is a running coupling constant, having the value g_0^2 at $q^2 = \mu^2$. Because the interactions are strong,

g_0^2 or Bg_0^2 are of order unity, rather than α . The important point is that the sign in the denominator of (50) is opposite to that in (49). So, the theory is UV stable (asymptotically free) but divergent in the IR region. Another difference from QED is that, in calculating physical quantities in terms of the renormalized parameters, the divergences do not cancel. Hence, there is a fundamental problem in the low q^2 region, which may be connected with quark confinement.

Since Bg_0^2 , the slope of the $\log q^2$ dependence, is expected to be large (~ 1), rather than small (α) as in the QED case, we expect to be able to obtain measurable effects in scattering processes in modest ranges of q^2 , depending of course on the magnitude of the parameter μ^2 . It should also be emphasized that the formula (50) makes sense only in the range where $(g^2(q^2)) \ll 1$ so that higher order terms can be neglected. Finally, it is found that UV stability of the model is guaranteed only if there are not too many fermions (not more than 16 quark flavours) and no Higgs scalars. Therefore, if the theory is to work at all, it is essential that the spontaneous symmetry breaking accounting for particle masses should have a dynamical origin.

(4.4) APPLICATION TO NUCLEON STRUCTURE FACTORS

So far, we have discussed only how the intrinsic coupling constant of strong interactions will behave in the high q^2 limit, for asymptotically free theories where exact Bjorken scaling will apply as $q^2 \rightarrow \infty$.

The nucleon (or other hadron) is assumed to be composed of quarks, antiquarks and neutral vector gluons mediating the interaction between the quarks. The interactions are described by matrix elements of certain operators. These operators have the character of singlets or non-singlets under $SU(m)$, where m = number of quark flavours. There are 3 classes of operator, two singlet classes and one non-singlet (i. e. octet etc.). They determine essentially the momentum distributions of the gluons, quark-anti-quark pairs and valence quarks, respectively, and together fix the q^2 dependence of the moments of the nucleon structure functions. (27, 29) Thus the n^{th} moment of the structure function $F(x, q^2)$ has the general form

$$M(n, q^2) = \int_0^1 x^n F(x, q^2) dx = \sum_i C_i^n(q^2) \quad (53)$$

where $i = 1, 2, 3$ and, as $q^2 \rightarrow \infty$, the C_i exhibit a $\log q^2$ dependence to some power:-

$$C_i^n(q^2) = A_i^n / (\log q^2)^{a_i^n} \quad (54)$$

where the a_i^n are positive "anomalous dimension" factors of order unity, and the constants A_i^n are not determined by the theory; they have to be taken from experimental data on $F(x, q^2)$ over the whole x range, for 3 values ($i = 1, 2, 3$) of q^2 . For the first moment, $n=0$, one singlet operator contains the energy momentum tensor and has $a_1^0 = 0$ (i. e. makes a constant, q^2 independent, contribution). For large n , $a_i^n \sim \ln(n)$. The actual expressions for a_i^n are complicated but they can be fitted by approximate formulae fulfilling the

boundary conditions: -

(i) Energy-momentum conservation (first-moment):-

$$\left(\sum \vec{p} \right)_{q^2 = q_1^2} = \left(\sum \vec{p} \right)_{q^2 = q_2^2}$$

so that

$$a_i^0 \rightarrow 0 \text{ as } q^2 \rightarrow \infty$$

(ii) Positivity:-

$$a_i^n > 0 \quad (n \geq 0)$$

$$M(n) > M(n+1)$$

Qualitatively therefore, as q^2 increases, the higher moments (large n) become progressively smaller and thus the distribution $F(x, q^2)$ shrinks towards the origin, $x = 0$. This is clearly consistent with a picture whereby as q^2 increases, more and more "revealed" quarks, antiquarks and gluons share the momentum of the nucleon and

$$\lim_{q^2 \rightarrow \infty} \langle x_N \rangle \propto \int x F(n, q^2) dx \rightarrow 0$$

$$\text{where } \int F(n, q^2) dx \rightarrow \text{constant}$$

One can perhaps visualize what is happening in terms of the analogy of an electron-photon cascade in QED. As q^2 increases, the valence quarks undergo "gluon bremsstrahlung", the gluons converting to quark-antiquark

pairs. Thus, there is a logarithmic increase with q^2 in the quark-antiquark sea/gluon population at the expense of the momentum of the valence quarks, the contribution of which tends to zero.

In the asymptotic limit of extremely high q^2 , an equilibrium between the quarks, antiquarks and gluons is reached; the total momentum content of the various components is determined by the statistical weight factors. For the standard coloured quark model of $SU(m) \times SU3^1$, there are m quark flavours and 3 colour codes, and the inert gluons form a colour octet. Thus

$$r(q^2 \rightarrow \infty) = \frac{\langle \vec{p}_{\text{quarks}} \rangle}{\langle \vec{p}_{\text{total}} \rangle} = \frac{N_Q}{N_Q + 2N_G} = \frac{3m}{3m + 16} \quad (56)$$

where the factor 2 for the gluons arises through the equilibrium $G \rightleftharpoons Q + \bar{Q}$; on average, a gluon must carry twice the momentum of a quark. It is straightforward to compute the integrals of the structure functions from (56), using the usual quark charge assignments in the electron scattering case, and neglecting the Cabibbo angle in weak scattering. The results are given in Table V.

Table V.

Structure Function Integrals at Equilibrium, $q^2 \rightarrow \infty$

	$\int F_2^{\nu N} dx$	$\int F_2^{eN} dx$	$\int F_2^{(ep-en)} dx$
SU3	6/25 = 0.24	2/25 = .08	0
SU4	3/7 = 0.43	5/42 = 0.11	0
SU6	9/17 = 0.53	5/34 = 0.15	0
Expt. (low q^2)	0.48 ± .05 (CERN)	0.14 (SLAC)	0.045 (SLAC)

At finite q^2 , the actual integrals will be larger. For example

$$\int F_2^{ep, en}(q^2, x) dx = \left[\frac{\sum Q_i^2}{N_Q + 2N_G} + \frac{A^{p, n}}{(\ln q^2)^\alpha} + \frac{B}{(\ln q^2)^\beta} \right] G(q^2) \quad (57)$$

where for the non-singlet term $\alpha \sim 0.2$ and for the singlet, $\beta \sim 0.7$. The singlet (gluon-gluon) terms dropout in some combinations of structure functions e.g. xF_3 , $F_2^{ep} - F_2^{en}$. The function $G(q^2)$ is a correction factor which is of order unity in the range where the theory applies: -

$$G(q^2) = 1 + \frac{\alpha(q^2)}{\pi} + \frac{\alpha^2(q^2)}{\pi^2} + \dots \quad (58)$$

with

$$\frac{\alpha(q^2)}{\pi} = \frac{12}{25 \ln(q^2/\mu^2)} \quad (59)$$

in the standard (SU4) model. Thus, for $\mu^2 = 0.1 \text{ GeV}^2$, $\alpha/\pi \sim 0.2$ only at $q^2 = 1 \text{ GeV}^2$. On the other hand, the terms in (57) involving small powers of $\ln q^2$ make a substantial contribution, even at quite large values of q^2 .

4.5 COMPARISON WITH EXPERIMENTS

The general form of the deviations from scaling observed in inelastic electron and muon scattering exhibit the qualitative features predicted from the FGT described above—namely a slow shrinking of the $F_2(x, q^2)$ towards the origin as q^2 increases. A direct comparison with experiment generally is not easy, since the theory involves a large number of unknown parameters (μ, a_1^n). Some of the predictions are however more direct e. g.

$$R(e^+e^-) = \frac{\sigma_e(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = \sum Q_i^2 (1 + \frac{a}{\pi} + \dots)$$

$$\int F_3^N dx = 3 (1 + \frac{a}{\pi} + \dots)$$

$$\frac{\sigma_L}{\sigma_T} \xrightarrow{x \rightarrow 1} \text{const. } \frac{a}{\pi} (\frac{1}{x} - 1) \quad (60)$$

As an example, Fig. 17 shows the variation of $R(e^+e^-)$ with q^2 expected in the standard $SU_4 \times SU_3'$ model, compared with SPEAR data in the region above 5 GeV energy, where one may be in a continuum region, well above narrow charmed resonances. In this graph, allowance has been made for a heavy lepton by increasing R by one unit. If these results prove anything at all, they indicate a small value for μ (≤ 1 GeV).

Regarding the structure factors in inelastic lepton-nucleon scattering, we expect a much slower convergence with increasing q^2 because of the smallish powers of $\log q^2$ in (57). In particular, non-singlet difference terms like $\int F_2^{ep} - F_2^{en}$, $\int xF_3$ are expected to converge extremely slowly.

Fig. 17 Ratio $R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$. The curves show predictions of asymptotically free theories, in the standard 4 flavour - 3 colour model. The formula, if valid at all, can only hold when the correction term $[12/25 \ln(q^2/\mu^2)] \ll 1$.

Recent data⁽³¹⁾ from the FNAL muon experiments seems to indicate no measurable change in $\int (F_2^{\text{ep}} - F_2^{\text{en}}) dx (\approx 0.045)$ in going from the SLAC energy region ($q^2 \sim 3$) to high $q^2 (\sim 20)$.

Attempts have been made to predict the behaviour of the structure factors themselves from AFGT, by evaluating moments with approximate formulae, neglecting non-leading terms and using Mellin transform techniques. A recent paper by Johnson and Wu-ki Tung⁽³⁰⁾ is thus able to reproduce the empirical $\log q^2$, $\log x$ dependence observed in inelastic muon scattering (Figs. 7 and 9), at small x . In Fig. 18, the predicted linear dependence is fitted to the data and thus μ is determined. A very small value, $\mu \sim m_\pi$, is obtained, a result following from the smallness of the departure from scaling at modest q^2 -or perhaps to the approximations made in the calculation.

Following these encouraging results, an attempt has been to measure scaling violations in neutrino scattering, using bubble chamber data. Since a wideband spectrum is involved, it is in fact only practicable to analyze the x -dependence in bands of E , rather than q^2 . Since $q^2 = 2MxyE$, the E dependence represents the q^2 dependence averaged over the y distribution. Assuming a q^2 dependence of the form suggested from muon scattering

$$F_2^{\nu}(q^2, x) = F_2^{\nu}(q_0^2, x) \exp \left[a \ln(q^2/q_0^2) \ln(x_0/x) \right] \quad (61)$$

one obtains

$$F_2(E, x) = F_2(q_0^2, x) \left(\frac{2MEx}{q_0^2} \right)^{\mu} g(\beta) \quad (62)$$

where

Fig. 18 Plot of the logarithmic change of $F_2(\omega, q^2)$ with $\log q^2$ as a function of $\omega = 1/x$ from FNAL μ -d scattering experiments.

$$\beta = a \ln(x_0/x)$$

$$g(\beta) = (\beta + 1)^{-1} \text{ for } f(y) = \text{const.}$$

$$g(\beta) = 3 \left\{ (\beta + 1)^{-1} + (\beta + 3)^{-1} - 2(\beta + 2)^{-1} \right\} \text{ for } f(y) = (1 - y)^2 \quad (62)$$

Fig. 19 shows the variation of $F_2^{\nu}(E, x)$, relative to $F_2^{\mu}(q_0^2, x)$, assuming $q_0^2 = 3 \text{ GeV}^2$, $x_0 = 0.2$ and $a = 0.10$ (as in muon scattering). If μ is indeed small, we might expect this dependence to hold down to quite low energies—perhaps a few GeV only. Fig. 20 shows a comparison of the inelastic data from Gargamelle at low energy ($E = 1 - 3$) and high energy ($E = 3 - 11 \text{ GeV}$), for νN collisions, while Fig. 21 is from a comparison of νp data in the ANL 12' chamber ($E = 2 - 6 \text{ GeV}$) and FNAL 15' chamber ($E = 10 - 200 \text{ GeV}$). Both distributions show the fall-off of dN/dx with increasing x at high energies relative to low. The slopes of these distributions are roughly compatible with the q^2 -dependence of the muon data, indicated by the dashed lines. However, the $\ln x$ dependence in muon scattering is a result for small x . At larger x , it is noticeable that, although the deviations in the SLAC data depend accurately on $\ln q^2$, they are more nearly linear in x —see Fig. 21A. The neutrino data show their most pronounced effects in the $x > 0.3$ region, and are in excellent agreement with the stronger x -dependence of $\partial \ln F / \partial \ln q^2$ of the SLAC data.

4.6 SUMMARY

The deviations from scaling observed in electron and muon inelastic scattering have a natural explanation in the asymptotically free (AF), or the fixed point (FP) theories of quark-quark interactions. [Although we have discussed the $(\ln q^2)^{-1}$ approach to scaling of the AF theory, this is not

Fig. 19 Expected dependence of the value of $F_2(x, E)$ in neutrino (antineutrino) scattering experiments, relative to that at a reference value $q^2 = 3$. Scale breaking of the form observed in muon scattering (see Fig. 18) with $\partial \ln F_2(q^2, x) / \partial \ln x = -\alpha = 0.10$ was assumed. The average over y was made assuming $d\nu/dy = \text{const.}$, and $d\bar{\nu}/dy = (1-y)^2$.

in practice distinguishable from the $(q^2)^{-1}$ dependence in the conventional (FP) theory, since the parameters of the 2 theories can always be fixed up to give similar predictions⁽³²⁾. The most impressive thing about the data is its nearness to exact Bjorken scaling, rather than the deviations from it. Because scale-breaking effects are small, and because their theoretical description involves a host of unknown parameters, it will be a long time before experiments can be sufficiently precise and detailed to give real guidance in the search for a quantitative theory of strong interactions. Nevertheless the preliminary results are encouraging.

Finally, it should be emphasized that although, in the region of small x (< 0.2), the general form of the theoretical predictions—that $\partial^2 \ln F(n, q^2) / \partial \ln q^2 \partial \ln x$ should be a constant—are unambiguous, those at large x are not. Experimentally, the only data available at $x > 0.2$ come from the very precise SLAC (ep, en) experiments, and the much less precise neutrino experiments, both involving modest values of energy transfer, ν . Here, as discussed before, there is ambiguity about the mass corrections and the appropriate scaling variable. The SLAC data show clearly that $\partial \ln F(n, q^2) / \partial \ln q^2 = f(x)$, where $f(x) \sim 0.25 - x$, and certainly not $\ln(x/x_0)$. Thus, violations of scaling in x , for $x > 0.2$ are large, and may be difficult to reproduce with the AFFT models, using only the first-order corrections.

I wish to advance the viewpoint of an experimentalist. The worst thing that can happen is that different experimentalists will use different variables and it will be impossible to compare data. The Bjorken variable

Fig. 20 Ratio of structure factors in GGM experiments, observed in two energy ranges ($\Delta \ln E = 1.25$). The line is expected for a logarithmic dependence of $\partial \ln F_2 / \partial \ln E$ on x . The total in the two ranges are absolute and not normalized.

$x = q^2/2M_p = q^2/2P \cdot q$ is a ratio of well defined Lorentz scalars, understood and used by everyone. If arbitrary masses are introduced into the data analysis stage, one quickly gets into a mess, and the confusion of the last years, when every thesis from SLAC seemed to use a different variable is testimony enough to this. Scaling is broken, in any or all variables, and it seems to be most that the data be plotted in x and q^2 , and not in x' , ξ and what have you. If the theory is worth anything, it must be able to predict the behaviour in x and q^2 , even if the resulting curves do look ugly.

5. NEW QUANTUM NUMBERS AND NEW COUPLINGS IN HIGH ENERGY NEUTRINO REACTIONS

During the last two years, the high energy neutrino programme, particularly at Fermilab, has uncovered several new phenomena indicating radical changes in the nature of the couplings of neutrinos to hadrons. The results observed so far are complex, no single coherent picture has emerged, and probably a number of different processes are involved. I shall discuss in this section: -

- (1) Dilepton events produced at the percent level by neutrinos and anti-neutrinos, namely the dimuon events in counters at FNAL, and

Fig. 21A As in Fig. 20, for νp data at ANL (E=1-6 GeV) and FNAL (E=10-200 GeV). The total distributions at the two energies are normalized to the ratio $\alpha = 0.10$.

Fig. 21B As in Fig. 21A, but with νp data from μp at $\alpha = 0.1$ in the FNAL.

Fig. 21C $d \ln F_2 / d \ln q^2$ is a function of the true experiments of SLAC $ep+ed$ (RIORDAN et al) and νp data of Fig. 21B. Only the SLAC experiments are plotted since they demonstrate that $d \ln F_2 / d \ln q^2$ is over the range 0.1 to 0.25. The curve indicates a $\ln$ dependence of $d \ln F_2 / d \ln q^2$ has been suggested from experimental data. The curve is $0.1 \ln(X/0.17)$ and the dashed curve is $(0.25-X)$. These curves are shown for comparison with the scaling variable.

the $\mu^- e^+ V^0$ events observed in bubble chambers operating at FNAL and CERN. These phenomena may be interpretable in terms of excitation of a single new hadronic quantum number (charm).

(2) Major anomalies in the charged current cross-sections above ~ 30 GeV, particularly in the σ_+/σ_- ratio and the y distributions, indicating a substantial revision of our (V-A) model of neutrinos coupling to left-handed hadron currents, and almost certainly a quite different phenomenon from (1).

5.1 DIMUON EVENTS IN COUNTER EXPERIMENTS

Neutrino events containing a pair ($\mu^+ \mu^-$) of muons were first reported by the HPWF group⁽³³⁾ and later confirmed by the CalTech group⁽³⁴⁾. The muons are identified on the basis of their penetration through iron. The fact that they originate direct from the neutrino interaction vertex has been established experimentally by the HPWF group by comparing the rates in the same mass of two targets of different density (iron and liquid scintillator), showing that (the majority) cannot be due to π , K decay in flight.⁽³⁵⁾

The origin of the extra muon has also been fairly convincingly tied down to the hadron vertex. In the three diagrams, (a), (b) and (c) represent leptonic decay of an intermediate boson W^+ , of a hypothetical

heavy lepton L^0 and of hadron H respectively. The characteristic of process (a) is that the μ^- and W^+ tend to be produced with small relative velocity in their common centre-of-momentum frame. Hence the mean energies of W^+ and μ^- in the LS are in the ratio of their masses, with the consequence that $\langle p_{\mu^-} \rangle / \langle p_{W^+} \rangle < 1$. For the decay of a heavy lepton L^0 , the ratio of the average momenta lies between 0.5 and 2.0 assuming V and A coupling, or between $(9 - 4\sqrt{2})/7 = 0.48$ and $(9 + 4\sqrt{2})/7 = 2.10$ for any admixture of V, A, S, T, or P couplings. ⁽³⁶⁾ The only assumption made is spin $\frac{1}{2}$ for the lepton L^0 . The experimental data, Fig. 22, show that for incident neutrinos, $\langle p_- \rangle / \langle p_+ \rangle = 3.7 \pm 0.65$, or 6.1 ± 0.8 if probable antineutrino background events are removed. The converse is true for incident antineutrino beams. $W^\pm$ and L^0 as sources of dimuon events can also be eliminated on the basis of the distribution in mass, W , of the accompanying hadron shower.

The statistical weight of the HPWF data on dimuons is 56 events attributed to $\nu_\mu \rightarrow \mu^- \mu^+ + \dots$ and 16 to $\bar{\nu}_\mu \rightarrow \mu^+ \mu^- + \dots$. On the basis of these numbers, they derive ⁽³⁷⁾

$$\left(\frac{\sigma_{\mu\mu}}{\sigma_{\mu}} \right)_{\text{neutrino}} = (0.8 \pm 0.3) \cdot 10^{-2}$$

and

$$\langle E_\nu \rangle \sim 100 \text{ GeV}$$

$$\frac{(\sigma_{\mu\mu})_{\text{antineutrino}}}{(\sigma_{\mu\mu})_{\text{neutrino}}} = 0.8 \pm 0.6 \quad (63)$$

The CalTech group employ a dichromatic beam and hence can investigate the energy dependence of the dimuon rate. Their most recent data is based

FNAL DIMUON EVENTS

$\bar{\nu}$ ○ HPWF } Benvenuti *et al* 1975
 ν ● HPWF }
 ν ⊗ CALTECH Barish *et al* 1975

Fig. 22 Momenta of positive and negative muons from FNAL dimuon events.

on 21 events, where both muons go through the magnet spectrometer. The rates observed at ~ 100 GeV are similar to those in (63). The values of $(E_{\mu})_{\min}$ are 4 GeV and 3 GeV in the HPWF and Caltech experiments respectively. The HPWF group observe no events of $E_{\nu} < 30$ GeV and the Caltech group also find a strong energy dependence.

Both counter experiments find a preponderance of opposite sign over like sign, dimuons: -

$$\begin{array}{rcl} \frac{(\mu^+ \mu^+) + (\mu^- \mu^-)}{\mu^+ \mu^-} & \leq 0.1 & \text{HPWF} \\ & \leq 1/21 & \text{CALTECH} \end{array} \quad (64)$$

The inequality sign indicates that the numbers have not been corrected for background.

Both these experiments find large values, $W > 4$ GeV, for the (minimum) mass of the state recoiling from the μ^- (in neutrino running). Initially also, a result from a neutrino experiment at Serpukhov indicated⁽³⁹⁾ an upper 90% confidence limit to dimuon production at 10-20 GeV of $< 10^{-3}$ of the total cross-section, thus giving further evidence for a threshold in W mass. More recently however,⁽⁴⁰⁾ revised rates have been given, which are $\sim 5 \cdot 10^{-3}$ for $E_{\nu} > 20$ GeV. In any case, there is, as discussed below, evidence that thresholds for dilepton phenomena in bubble chambers are as low as $W \sim 2$ GeV.

5.2 DILEPTON EVENTS IN BUBBLE CHAMBER EXPERIMENTS

Dilepton events have also been observed in bubble chambers operated at CERN and FNAL. At CERN the experiments have been carried out in the

Gargamelle chamber filled with heavy freon ($\text{CF}_3 \text{ Br}$) of radiation length $X_0 = 0.11 \text{ m}$, and at FNAL in the 15' chamber filled with 20% Ne/H mixture ($X_0 \sim 2 \text{ m}$). The dilepton events to be discussed consist of $e\mu$ pairs, and the use of heavy liquid allows identification of the electron by its characteristic bremsstrahlung and "spiralization" in the magnetic field.

(A) The CERN Gargamelle Experiment⁽⁷⁾ (25 GeV PS wideband neutrino beam)

In neutrino running, a total of 34,000 neutrino interactions of $E > 1 \text{ GeV}$ have been examined for lepton pairs. The table gives a list of the candidates observed to date.⁽⁴¹⁾ In all cases, the electron energy exceeds 0.2 GeV.

Table VI

DILEPTON CANDIDATES IN CERN GARGAMELLE EXPERIMENT:

(Neutrino runs: total 34,000 events of $E > 1 \text{ GeV}$)

<u>Category</u>	<u>(No. of events ($E_e > 0.2 \text{ GeV}$))</u>	<u>Background estimate</u>
$\mu^- e^+ V^0 + \dots$	3	9.10^{-2}
$\mu^- e^+ + \text{all}$	19	5.4 ± 1.5
$\mu^- e^- + \dots$	23	26 ± 8
$e^- V^0 + \dots$	5	3 ± 1
$e^+ + \dots$	6	5.5 ± 1.5

The background in the μe candidates comes from two sources:

- (i) $\nu_\mu N \rightarrow \mu^- \pi^0 + \dots$, where there is a very asymmetric Dalitz pair or very asymmetric γ converting very close to the vertex. The asymmetry probability and close pair conversion probabilities have been determined

experimentally for several thousand γ 's from normal events. Using these numbers, and the multiplicity of γ rays of $E > 0.2$ GeV from normal events and from events with V 's, the background contribution has been computed. This is the principal (90%) source of background in $\mu^- e^+$ and $\mu^- e^+ V^0$ events.

(ii) $\nu_e N \rightarrow e^- \pi^- + \dots$, $\bar{\nu}_e N \rightarrow e^+ \pi^- + \dots$, where the π^- leaves the chamber without interaction, thus simulating a μ^- . The probability can be found from the rate for $\nu_\mu N \rightarrow \mu^- \pi^-$ (interacting), $\bar{\nu}_\mu N \rightarrow \mu^+ \pi^-$ (interacting), the pion escape probability and the ν_e/ν_μ and $\bar{\nu}_e/\nu_\mu$ flux ratios. This background source accounts for 30% of the total background in $\mu^- e^-$ events. To summarise, the $\mu^- e^-$ events can all be accounted for in terms of known background, whereas a significant excess is observed in the $\mu^- e^+$ and $\mu^- e^+ V^0$ events. The validity of the numbers used for the background calculations is supported by the observed number of $e^+ + \dots$ and $e^- V^0$ events in the sample.

Of the 3 V^0 events, one gives an unambiguous fit to the Λ^0 hypothesis and the other two are Λ^0/K^0 ambiguous. Among the 16 $\mu^- e^+$ (no V^0) events, there is one consistent with the decay $\Lambda^0 \rightarrow \pi^0 + n$ (neutron star + 2 γ 's pointing a few cms. from the main vertex) and one where a positive particle interacts, and a $K_S^0 \rightarrow \pi^+ \pi^-$ points to the secondary interaction (and hence, it is conceivably a K^+).

(B) The FNAL (Wisconsin-Hawaii-LBL-CERN) experiment ^(8, 42)

(300 GeV wideband beam, 20% Ne/H)

This experiment is not yet completely analyzed, and the data given here is as of the date of the Madison Conference (April 1976). 14 $\mu^- e^+$ candidates

were observed among ~ 5000 interactions of $E_{\text{vis}} > 5$ GeV. 11 of these have μ^- identified by the external muon identifier (EMI). Of the 14, 11 have accompanying V 's, 2 of these being Λ^0 and the rest, K_S^0 or K_L^0 . 3 events have no V . The calculated background amounts to 0.9 events (see Table below).

In the Gargamelle experiment, the efficiency for detecting electrons is 100%. In the FNAL chamber filled with 20% Ne/H however, the radiation length is $\sim 2\text{m}$ and the probability to recognize the electron by bremsstrahlung is $\sim 50\%$ for $1 \text{ GeV} < E_e < 10 \text{ GeV}$. For $E_e < 1 \text{ GeV}$, $\pi^+ \rightarrow \mu^+ \rightarrow e^+$ decay presents a problem, and for $E_e > 10 \text{ GeV}$, the track curvature is so small that bremsstrahlung by an electron may be confused with γ 's from the main vertex. The total neutrino event rate peaks at 20-30 GeV and the $\mu^- e^+$ events seem to be fairly uniformly distributed in this spectrum.

An interesting feature of the data is that the mean V^0 multiplicity > 1 . One event contains 3 V 's, and, as in the CERN data, there is

Table VII

DILEPTON CANDIDATES IN FNAL
(WISC-HAWAII-LBL-CERN) EXPERIMENT ⁽⁴²⁾

(Based on total ~ 2300 neutrino events of $E > 5 \text{ GeV}$)

<u>Category</u>	<u>No. of events ($1 < E_e < 10 \text{ GeV}$)</u>	<u>EMI hits</u>	<u>Background estimate</u>
$\mu^- e^+$ + all	14	11	0.9
$\mu^- e^+ \Lambda + \dots$	2	--	--
$\mu^- e^+ (K_S^0 \text{ or } K_L^0) + \dots$	?	--	--

evidence for $K^\pm$ production. The authors quote a mean multiplicity of strange particles, per dilepton event, of order 3, when allowance has been made for neutral decay modes, the efficiency for detecting $K^\pm$ etc. The observed rate of dilepton events, allowing for positron detection efficiency, is

$$\frac{\text{Dilepton Rate}}{\text{Total Rate}} = 0.6 \pm 0.2\% \quad (65)$$

The group also states that, ⁽⁴²⁾ if half of all the single K^0 's observed in neutrino events in the film are attributed to a new channel (for example charm meson production), then the observed "dilepton + V" rate gives a semi-leptonic branching ratio

$$\frac{\text{charm} \rightarrow \text{semileptonic} (e^+, \mu^+)}{\text{charm} \rightarrow \text{all}} \sim 30\%$$

(C) The BNL/Columbia experiment ⁽⁴³⁾

(300 GeV wideband beam; 15' FNAL chamber 64% Ne/H)

Preliminary results have been given on the basis of a sample of 10,000 neutrino events of $E > 5$ GeV. 40 $e^+ \mu^-$ events were observed, with an estimated ~ 3 event background. The positron detection efficiency was given as $\sim 70\%$. 5 "dilepton + V" events are observed; 3 $K_S^0 \rightarrow \pi^+ \pi^-$ and 2 $\Lambda \rightarrow p \pi^-$.

In Table VIII I compare the data in the different bubble chamber runs. Clearly, there are discrepancies in the proportions of dilepton events with V's in the 3 experiments

Table VIII

Dilepton Rates in 3 BC Neutrino Experiments (August 1976)

CERN (E _p = 25 GeV)		FNAL (E _p = 300 GeV)	
$\eta \sim 1$		20% Ne/H ($\eta = 0.75$)	64% Ne/H ($\eta = 0.7$)
$\mu^- e^+ / \mu^-$	$\frac{14}{4000} (> 4 \text{ GeV}) \approx 0.3\%$	0.6%	0.6%
$\mu^- e^+ + \nu$			
$\mu^- e^+$	3/14	12/15	5/37

(D) Antineutrino Experiments

Finally, limits have been set on the dilepton rate observed in antineutrino experiments in the 15' chamber (20% Ne/H) at FNAL.⁽⁴⁵⁾ An upper limit (based on 1 possible event) to the ratio $\sigma(\bar{\nu} N \rightarrow \mu^+ e^- + \dots) / \sigma(\bar{\nu} N \rightarrow \mu^+ + \text{anything}) \leq 0.8\%$ has been reported.

(5.3) DISCUSSION OF DILEPTON EVENTS IN TERMS OF CHARM

The dilepton events appear, superficially at least, to have an explanation in terms of leptonic decay of charmed hadrons. The production of charmed states in neutrino interactions can be understood in terms of the quantum numbers in the table, obtained from the formula $Q/e = I_3 + (B + S)/2 + C/3$.

Table IX

Label	<u>Quark quantum numbers</u>				
	B	I	S	C	Q/e
u	1/3	1/2	0	0	+2/3
d	1/3	1/2	0	0	-1/3
s	1/3	0	-1	0	-1/3
c	1/3	0	0	1	+2/3

The weak interactions of leptons and quarks are described in terms of the coupling of the doublets:-

Leptons:

$$\begin{bmatrix} \mu \\ \nu_\mu \end{bmatrix}, \quad \begin{bmatrix} e \\ \nu_e \end{bmatrix}$$

Quarks:

$$\begin{bmatrix} u \\ s \sin \theta + d \cos \theta \end{bmatrix}, \quad \begin{bmatrix} c \\ s \cos \theta - d \sin \theta \end{bmatrix} \quad (66)$$

The cancellation⁽³⁾ of the unwanted $\Delta S = 1$ neutral currents allowed by the triplet quark model, is achieved by replacing the Cabibbo angle θ involved in the $s \rightarrow u$ and $d \rightarrow u$ coupling, by $(\theta + \pi/2)$ for the $c \rightarrow s$ and $d \rightarrow c$ coupling. Thus, the elementary neutrino-quark scattering processes leading to charmed particle production (for $E > E_{\text{threshold}}$) are:-

Table X

	<u>Coupling</u>	<u>Relative rate</u>
$\nu + d \rightarrow c + \mu^-$	$\sin \theta$	0.05 (67)
$\nu + \underset{(+s)}{s} \rightarrow c + \underset{(+s)}{\mu^-}$	$\cos \theta$	$\leq .05$ (68)
$\bar{\nu} + \underset{(+\bar{c})}{c} \rightarrow n + \underset{(+c)}{\mu^+}$	$\sin \theta$	~ 0 (69)
$\bar{\nu} + \underset{(+\bar{c})}{c} \rightarrow s + \underset{(+c)}{\mu^+}$	$\cos \theta$	$\leq .05$ (70)

The neutrino reaction (67) is the only one which can take place off valence quarks, and is suppressed by a factor $\sin^2 \theta$. The three other reactions take place off quarks/antiquarks of the "sea", which, according to (32), is also suppressed by the factor ~ 0.05 (or less). In the weak decay of c or $\bar{c}$, into non-charmed quarks, the Cabbibbo factor enhances decay to strange particles. For example

$$\begin{aligned}
 c &\rightarrow s + \bar{d}u & (\cos^2 \theta) \\
 &\rightarrow d + \bar{d}u & (\sin^2 \theta)
 \end{aligned}
 \tag{71}$$

As a result, production of single charmed particles ($C = +1$) by neutrinos should lead to decay products of $S = -1$, and the charge and strangeness of the hadrons involved are related by the selection rule $\Delta S = -1$, $\Delta Q = +1$ (since $\nu \rightarrow \mu^-$), or $\Delta S / \Delta Q = -1$. This is quite the opposite of single particle production not involving charm, for which $\Delta S / \Delta Q = +1$. Finally, reactions with neutrinos or antineutrinos (68, 69, 70) involving sea quarks will generally

$K^0 e^+$ invariant mass spectra CERN/FNAL
events

82

Fig. 23 Mass of $K^0 e^+$ system observed in $\mu^- e^+ \nu$ events in CERN-Gargamelle experiment and in the 15' FNAL chamber with 20% Ne/H filling (LBL, Hawaii, Wisconsin, CERN). Ambiguous CERN events are classified as K^0 . The curve is the predicted mass spectrum for V, A coupling in the decay of a pseudoscalar charmed meson $D \rightarrow K^0 e^+ \nu$ of mass 2.0 GeV.

result in associated production of strange particles.

The dilepton events, on this picture, have to be interpreted as leptonic decays of charmed particles, $c \rightarrow s + \ell^+ + \nu$. A necessary consequence of the charge assignments in Table IX is the appearance of leptons of opposite sign ($\mu^- \ell^+$ for neutrinos, $\mu^+ \ell^-$ for antineutrinos). The expected leptonic decay rates depend on the assumptions made, but are generally⁽⁴⁶⁾ believed to be of order $1/R$, where $R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ above charm threshold ($R \sim 5$). Thus, the cross-section for production of charmed hadrons undergoing leptonic decay, hence dilepton events, is expected to be in the region of 1% of the total cross-section above threshold, and strange particles should be present in most of these events.

Qualitatively, these expectations are borne out by the experiments described above. However, there are many qualifications on both the theoretical and experimental sides. For example, the charmed quarks are much heavier (by ~ 1.5 GeV) than the uncharmed quarks and it is not clear how slowly (or quickly) above threshold the charm production cross-section will start to "scale", like the total cross-section. The data from all the experiments is biased - towards low ℓ^+ energies in the bubble chamber experiments and towards high ℓ^+ energies in the counter experiments. The invariant mass distribution of the $K_0 e^+$ combinations in the bubble chamber events is shown in Fig. 23, and it is clearly compatible with the V-A decay of a charmed particle of mass around 2 GeV. The low dilepton rates at CERN energies compared with the 10^{-2} rate at $E > 20$ GeV at FNAL,

are also consistent with a threshold mass in the region of $W \sim 2$ GeV.

Which of the possible reactions (67)-(70) contribute to the dilepton events is also unclear. The 15' bubble chamber events, or some of them, are consistent with associated production of strange particles via the quark/antiquark sea, and there is some evidence that the values of $x_{\max} = q^2/2M_{\nu}^2$ in both counter and bubble chamber data are, on average, somewhat smaller than in inelastic reactions as a whole, again consistent with a substantial $q\bar{q}$ contribution (Fig. 24). However, none of this evidence is conclusive at the present time.

To summarize therefore, the GIM model predicts single charmed particle production by neutrinos at a definite rate (5% of the total rate) well above all thresholds for Λ_c , D , D^* , F^* etc. Semi-leptonic decays are expected to yield dilepton events of opposite sign at around 1% of the total event rate, and they should be accompanied mostly by strange particles. This is what is observed. However, the room for maneuver is perilously small. Should the experimental rates turn out to be underestimated because of biases, or should rescaling be "slow" instead of "fast", or should the leptonic branching ratio be $< 10\%$ - any or all these things could destroy the apparent agreement between theory and experiment.

(5.4) ANOMALIES IN γ DISTRIBUTIONS AT HIGH ENERGY

Perhaps the most dramatic and exciting development in neutrino physics in the last year or so has been the observation of substantial anomalies in the total and differential cross-sections at high energy.

x Distributions in dilepton events

Fig. 24 x distributions in dilepton events.

Evidence for changes in the y -distributions comes mainly from one experiment: the HPWF experiment.^(22, 47, 48) Three different types of

anomaly in antineutrino reactions have been reported: -

- (i) A small y , small x anomaly⁽²²⁾
- (ii) A large y , small x anomaly^(47, 48)
- (iii) A large y anomaly, not restricted to small x , at $E > 30$ GeV.⁽⁴⁹⁾

Any attempt to report on these several effects is bound to be somewhat confused. The small y effect has not been published, has been mentioned in Fig. 8 and section 3.3 and will not be discussed further. To begin with, we recall from our definitions in (21) - (23) that, assuming charge independence, and neglecting the Cabibbo angle, the y distribution has the form for an isoscalar target: -

$$\frac{d^2\sigma^{\nu\bar{\nu}}}{dydx} = \frac{G^2ME}{\pi} [F_2(x)] \left[1 - y + \frac{y^2}{2} \right] \pm B(x) \left(y - \frac{y^2}{2} \right) \quad (72)$$

where

$$B(x) = xF_3(x)/F_2(x) \quad , \quad B = \int xF_3 dx / \int F_2 dx$$

Thus, the y distributions for neutrinos and antineutrinos at a given x , or integrated over x , are specified by one parameter, $B(x)$ or B . This prediction, that the y distributions for both neutrinos and antineutrinos should be specified by one parameter-which follows directly from charge symmetry and the use of an isoscalar target - seems to be well verified in the energy range $E < 30$ GeV. Fig. 25 shows the neutrino and antineutrino y distributions from the HPWF experiment; Fig. 26, those from the FNAL-

ITEP-Michigan-Serpukhov Ne/H antineutrino experiment⁽⁵⁰⁾ and the CERN-Gargamelle data in the scaling region.^(19, 51) All the data can be represented by a value of $B^\nu = B^{\bar{\nu}} \sim 0.8-0.85$, as shown in Table XI: -

Table XI

Values of B from low-energy data

Experiment	Energy Range	B^ν	$B^{\bar{\nu}}$
HPWF ⁽⁴⁹⁾	10 - 30	0.90 ± 0.60	0.95 ± 0.10
FNAL 15' chamber ⁽⁵⁰⁾	10 - 30		0.78 ± 0.07
CERN GGM ^(19, 51)	$\sim 4 - 10$	$\leftarrow 0.80 \pm 0.06 \rightarrow$	

HPWF ⁽⁴⁹⁾	> 50 GeV	$0.75^{+0.2}_{-0.1}$	$0.45^{+0.15}_{-0.10}$
FNAL 15' ⁽⁵⁰⁾	> 30 GeV	---	0.69 ± 0.10

At the higher energies (Fig. 27) however, the value of $B^{\bar{\nu}}$ found by the HPWF group is substantially different from 0.80, as judged by the quoted errors (statistical only). If this is a genuine effect it implies (a) a breakdown of charge independence ($B^{\bar{\nu}} \neq B^\nu$), (b) a breakdown of scaling ($B^{\bar{\nu}}$ depends on energy).

Even bigger variations are observed if cuts are made in x as well as in E .^(22, 50) The quantity $B(x)$ gives a measure of the antiquark content as function of x , since

$$\frac{\bar{Q}(x)}{\bar{Q}(x) + Q(x)} = \frac{1 - B(x)}{2}$$

Fig. 25 y distributions in HPWF neutrino and antineutrino data, 10 < E < GeV.

and at $x \sim 0$ we expect $Q \sim \bar{Q}$ and $B(0) \sim 0$. Equally, we expect to find few antiquarks at large x and thus, $B(x) \sim 1$ in that region. Table XII gives the fits obtained in the bubble chamber experiment. For $x < 0.1$, $E < 30$ GeV, $B^{\bar{\nu}}(x) \sim 0.5$, implying an antiquark contribution in this region

Table XII

FNAL-ITEP-Michigan-Serpukhov data ⁽⁵⁰⁾

<u>Values of $B^{\bar{\nu}}(x)$</u>		
	<u>$E = 10 - 30$</u>	<u>$E = 30 - 200$</u>
$x = 0 - 1$	0.78 ± 0.07	0.69 ± 0.10
$x > 0.4$	$0.95^{+0.05}_{-0.09}$	1.00 ± 0.04
$x = 0.2 - 0.4$	$0.75^{+0.12}_{-0.15}$	$0.84^{+0.09}_{-0.16}$
$x = 0.1 - 0.2$	$0.78^{+0.13}_{-0.18}$	$0.73^{+0.19}_{-0.28}$
$x < 0.1$	$0.46^{+0.19}_{-0.27}$	$-0.55^{+0.60}_{-1.14}$

of $\sim 25\%$, instead of 10% or so when averaged over all x . However, for $E > 30$, $B^{\bar{\nu}}(< 0.1)$ is very small or negative. This is illustrated in Fig. 28, where the bubble chamber data and the HPWF data are compared. The y distribution is rather flat, which is an unphysical situation in terms of conventional coupling, (V-A), where $B_{\min} = 0$ and the flattest possible y distribution still falls off as $(1 - y + \frac{y}{2})$ - see (72).

A large increase in the antiquark contribution at high energies is unexpected, breaks scaling and is not supported by muon scattering data at FNAL energies, which suggest an average quark-antiquark component at

Fig. 26 y distributions in FNAL antineutrino and CERN neutrino/ antineutrino bubble chamber data.

the usual 10% level. ⁽⁵²⁾

(5.5) THE TOTAL CROSS-SECTION RATIO

Until recently, the available evidence indicated that total neutrino and antineutrino cross-sections were linear with energy and in a constant ratio, from about 2 GeV up to around 100 GeV. ^(18, 53) In the light of the large deviations in the antineutrino y distributions, described above, one might expect departures from this simple picture, and the HPWF group have recently reported new results suggesting a systematic change in the ratio $R = \sigma^{\bar{\nu}} / \sigma^{\nu}$ with energy. ⁽⁵⁴⁾

Fig. 29 shows the ratio R obtained, based on the measurement of the parent π , K fluxes in a quadrupole triplet beam. Absolute cross-sections, or details of the flux measurements, have not been given. Also shown on this graph are the previous datum points published by the CalTech group. ⁽⁵³⁾ It should be emphasized that the HPWF data is of higher statistical weight than the CalTech data; of order 80 antineutrino events above 100 GeV, compared with 9.

The same authors have given values of R , using all their data (quad triplet and horn-focussed beams), by a method independent of external flux measurements. This method assumes that on isoscalar targets, neutrino and antineutrino cross-sections for excitation of low-lying resonances (so-called quasi-elastic events) tend to the same constant value at high energy. This is a consequence of the Lee-Yang theorem, which states that,

Fig. 27 y distributions in HPWF experiment, $E > 50 \text{ GeV}$.

Fig. 28

Antineutrino y distribution, for $x < 0.1$, $E > 30 \text{ GeV}$, from HPWF counter experiment and 15' Ne/H bubble chamber experiment.

Fig. 29 Ratio of antineutrino/neutrino total cross-sections, using external flux measurements.

as a result of a charge symmetry and a current-current interaction mediated by single vector particle exchange, the neutrino/antineutrino cross-sections on nucleon targets can have an E-dependence containing at most quadratic terms:

$$\frac{d^2\sigma_{\nu, \bar{\nu}}}{dq^2 dW^2} = \frac{A(q^2, W^2)}{E^2} \pm \frac{B(q^2, W^2)}{E} + C(q^2, W^2) \quad (73)$$

where W = mass of final hadron state, and A, B and C are arbitrary, finite functions. Thus, as $E \rightarrow \infty$, the total cross-sections

$$\sigma_{\nu, \bar{\nu}}(W^2) \rightarrow \int C(q^2, W^2) dq^2 = \text{constant} \quad (74)$$

since C contains form-factors for any exclusive channel, W .

Fig. 30 shows ANL/CERN data⁽⁵¹⁾ on the elastic cross-sections for $\nu + n \rightarrow \mu^- + p$, $\bar{\nu} + p \rightarrow \mu^+ + n$, as a function of energy. Note that, at 5 GeV, there is still an appreciable difference; the B term in (73) has not died out. No data on other quasi-elastic processes (such as $\nu, \bar{\nu} + N \rightarrow \mu + \Delta$) exists; so that the assumption that (74) holds well enough at $E \sim 20 - 100$ GeV to serve as a useful flux monitor, for $W < 2$ GeV say, is entirely a matter of faith and hope.

Although it is doubtless true that the elastic cross-sections saturate for $E > 15$ GeV, these are too small, and too difficult to identify, to be useful. The HPWF group have therefore been compelled to extend the range of values of W up to 2 or 3 GeV. The resulting values of R are given in Fig. 31, together with their previous result,⁽⁵⁷⁾ using both this method and Hagedorn-Ranft flux estimates. The new data agree in their general trend with the data of Fig. 29.

Elastic cross-sections as determined
at ANL and CERN

Fig. 30 Elastic ν , $\bar{\nu}$ cross-sections measured at ANL and CERN.

Fig. 31 Cross-section/ratio in HPWF experiment, using the quasi-elastic normalisation of the flux, with W ranges as shown.

(5.6) INTERPRETATION OF ANOMALIES IN TERMS OF NEW QUARKS AND COUPLINGS.

The new data on the antineutrino y distributions and the ratio $\bar{\sigma}/\sigma$ at high energy appear to lead inexorably to new quarks and new couplings, simply because other mechanisms are inadequate:-

- (i) Charm threshold effects. Apart from the fact that charm thresholds are well below 30 GeV, this mechanism is no use since it produces only small modifications ($< 5\%$) to total cross-sections and y distributions and the main effects would be in neutrino, rather than antineutrino, reactions.
- (ii) Scale-breaking. In the asymptotic limit, $q^2 \rightarrow \infty$, AFFT predicts that lepton-nucleon scattering will be dominated by the quark-antiquark sea, so that $R \rightarrow 1$ in this limit, and the y -distributions for both neutrino and antineutrino will tend to a common form $(1-y + y^2/2)$. This explanation of the anomalies can be discarded immediately, firstly since the neutrino y distribution is unaffected; secondly because the approach to asymptopia is very slow- indeed it has been estimated that R would change by less than 10% by $E = 300 \text{ GeV}$; ⁽⁵⁸⁾ and thirdly, because this mechanism would not account for the threshold mass of $W \sim 5 \text{ GeV}$ which seems to be involved with the phenomenon.

We are therefore left with other hypotheses; briefly, the E/W threshold implies new heavy hadron production, and the fact that the anomaly is associated with antineutrino-rather than neutrino interactions also implies a different weak coupling ($V + A$ instead of $V-A$). The resulting y distributions are

indicated in Fig. 32.

(A) A number of authors⁽⁵⁹⁾ have proposed vector models involving 6 quarks-2 new heavy quarks b, t in addition to u, d, s and c-arranged, as far as weak coupling is concerned, in L.H. and R.H. doublets:-

<u>Q/e</u>	L. H.	R. H.
+ 2/3	$\begin{pmatrix} u \\ d_0 \end{pmatrix}_L$	$\begin{pmatrix} u \\ b \end{pmatrix}_R$
- 1/3	$\begin{pmatrix} c \\ s_0 \end{pmatrix}_L$	$\begin{pmatrix} c \\ s \end{pmatrix}_R$
	$\begin{pmatrix} t \\ b \end{pmatrix}_L$	$\begin{pmatrix} t \\ d \end{pmatrix}_R$

Below b, t threshold, parity violation in charged weak currents occurs, but it disappears at high energy. Thus parity violation, distinguishing weak couplings from the other fundamental interactions, is simply a low energy phenomenon. Although this model can account for the anomalies, it unfortunately predicts that neutral currents will be parity-conserving; there is good evidence for V/A interference in neutrino-induced neutral current phenomena and this model has to be discarded.

(B) A model by Gursey and Sikivie⁽⁶¹⁾ introduces one new quark, b, together with an arbitrary mixing angle ϕ , and the weak currents consist of four doublets (as well as a singlet, which is not involved here):-

<u>Q/e</u>	L. H.	R. H.
+ 2/3	$\begin{pmatrix} u \\ d_0 \end{pmatrix}_L$	$\begin{pmatrix} u \\ b_\phi \end{pmatrix}_R$
- 1/3	$\begin{pmatrix} c \\ s_\theta \end{pmatrix}_L$	$\begin{pmatrix} c \\ s \end{pmatrix}_R$

Fig. 32 Form of y distributions for (top) V-A coupling with 10% antiquark component, and (bottom) an extra (V + A) coupling to u quarks only. The latter affects only the antineutrino cross-sections.

Since the u-quark, not the d-quark, has a R.H. coupling above b threshold, the anomalies are accounted for. Furthermore, the valence $\rightarrow$ charmed quark coupling can be fixed up in this model to give a suppression factor $\tan \theta_c$ instead of $\tan^2 \theta_c$ in the GIM model, thus allowing much greater freedom on the leptonic branching ratio of charmed hadrons. Above b threshold, the neutral current contains strangeness- and charm-changing pieces, but there is as yet no evidence against this.

(C) Achiman, et al⁽⁶⁰⁾ have also introduced one new quark, b, but there are only 3 current doublets:-

$$\begin{pmatrix} u \\ d_0 \end{pmatrix}_{L} \quad \begin{pmatrix} c \\ s_0 \end{pmatrix}_{L} \quad \begin{pmatrix} u \\ b \end{pmatrix}_{R}$$

L.H. R.H.

This model also accounts for the anomalies as well as all the features of the standard (GIM) model.

The predictions of the above models have been compared by Albright and Schrock⁽⁶²⁾, using $m_c = 1.5$ GeV, $m_b = 4$ GeV and $m_t = 5$ GeV. Their results are shown in Fig. 33, for the cross-section ratio.

In order to understand the anomalies (and to establish them beyond doubt), more experimental data is needed. If new quarks are being excited, it follows that new quantum numbers are involved, and these must manifest themselves in the characteristics of the decay of the new hadronic states produced. Charm, eventually, was discovered because of the association with strangeness in the weak ($\Delta C = 1$) decay of the lowest-mass charmed

Fig. 10

Fig. 11

Fig. 33. Cross-sections and cross-section ratios, calculated in a paper by Albright and Schrock in the standard GIM quark model (W-S) with (V-A) coupling, and in the vector-like (V) and Gursev-Sikivie (G-S) models incorporating (V+A) couplings and extra quarks b, t with masses ~ 5 GeV.

hadrons. For the new states, we do not know what to look for; if they exist, their signatures may be readily apparent, or they may be very apparent, or they may be very obscure. The search promises a further leave of life for the bubble chamber technique.

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