OPERATIONAL SAFETY
SYSTEM RELIABILITY

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PROGRESS REPORT
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1. Introduction.

The current research effort is roughly divided into two areas, (1) the determination of the protection system reliability and its probability distribution based on the failure distribution of the components and (2) the study of statistical characteristics of protection system sensors in order to extract the maximum information and to better anticipate off-normal conditions to promote safety and possibly avoid costly plant outages. This report summarizes the work to date in these two areas. The goal is to develop methods and models suitable for analysis of fast breeder protection systems during the operational phase, with emphasis on practical reliability results and realistic determination of their confidence levels.

2. System Reliability.

The calculation of system reliability and the uncertainty in the result of this calculation are complicated by several factors; among them are:

1. Typical reactor systems are large and contain many components.
2. Component failure data are not well established; however this situation is improving (for example, the N.P.R.D. System, in-house programs by reactor vendors, I.E.E.E. manual.)
3. Methods of determining system reliability, its uncertainty and the associated implications, are not available in forms readily usable by engineers in general.

Because of the large number of components in typical systems, various approximations have been used to determine the reliability (or conversely, the failure probability). These include the evaluation of fault trees for postulated accidents by the use of cut sets and Monte Carlo sampling.

A. Exact methods (convolution and transformation of variables)

Approximate methods do not account for all components in a system since they usually are based on certain assumptions concerning failure data or the relative likelihood of various events. The lack of precise component failure rate data is well-known and has required the use¹ of log-normal probability density functions to include the observed spread of data. This data uncertainty, of course, increases the uncertainty in the calculated or observed system reliability.

Our approach to these problem areas is to concentrate on specific systems; calculate certain subsystems exactly (this is explained in later sections), including all components; propagate the uncertainty in component reliability or failure rate by convolution or transformation of variable techniques; and to make the maximum use of available failure data, anticipating improved data in the future. After careful consideration of specific systems, it appears reasonable and realistic to lump or group associated components together into a sub-system which may be characterized by its own failure rate. This sub-system failure rate is then composed of partial failure rates corresponding to the individual components and their failure rates, if known. The advantages of this "lumping" process are threefold: it reduces the number of individual entities to be dealt with; it also works to overcome the scarcity of adequate failure data since one can now consider sub-system failures, which may be just as valid as attempting to work with estimated component failure rates; and lastly, "lumping" is consistent with the trends towards increased use of integrated circuits and integrated sub-systems.

The probability density function (pdf) for the system reliability can be obtained in principle by exact methods^{2,3,4,5} in terms of the component's

failure rate distribution parameters. From the expression for the pdf, one can calculate confidence limits, make comparisons to approximate calculations and simulations, and evaluate uncertainty in system reliability. The well-known difficulty in the application of convolution and transformation techniques is that one usually encounters intractable integrals to evaluate; this situation is rapidly complicated as systems become larger. Some of the mathematical relations have already been developed; for example, products of independent random variables for lognormal, Cauchy, Gaussian and monomial distributions have been previously considered^{6,7} and the treatment of sums of random variables is well-known.^{2,4} However, to our knowledge, the application of the exact methods to the problem of propagation of component reliability uncertainty into system reliability uncertainty has not been fully explored. We have started by calculating the pdf for the reliability of simple series and parallel combinations of components. The results for these basic sub-systems can be extended to larger systems, in some cases by making use of the types of failure rate probability density functions involved (i.e. products of log-normal pdf's are also lognormal).

The uncertainty in reliability can also be obtained approximately by various means such as the method of moments;^{3,8,9,10,11} invoking the "central limit" theorem; or calculating "best" case and "worst" case estimates by varying the component failures by a few standard deviations from their mean values in both directions. The results of the exact calculation, mentioned previously, can then be used as the benchmark against which the approximate methods are to be compared.

The possibility of using transformation of variables to propagate component uncertainties is attractive in spite of the generally pessimistic viewpoint (for application to large systems) expressed by many authors. Our approach is to build on available transformations, start with simple subsystems

and extend the approach as far as possible.

Results to Date

(a) Single Component The simplest configuration is one component with a reliability of $R = \exp(-\lambda t)$. Assuming that the failure rate, λ , is normally distributed, one finds the pdf of the component reliability is (details of the following results are in Appendix A):

$$f(R) = \frac{1}{R \sigma t \sqrt{2\pi}} \exp \left[- \frac{(-\ln R - \mu t)^2}{2\sigma^2 t^2} \right]$$

where: μ, σ = mean and standard deviation of the pdf of λ

t = time

The transform of a normally distributed variate in an exponential relation is tabulated and is a lognormal pdf.² (We have not seen this result, for $f(R)$ in the literature). As noted before, the choice of a normal pdf for λ means that, strictly speaking, σ must be less than $\mu/3$ to effectively avoid the artificial problem of negative λ 's since the normal distribution extends to minus infinity.

(b) Two Components in Series For two components in series, with failure rates λ_1 and λ_2 and other parameters similarly denoted, the pdf of the overall reliability is:

$$f(R) = \frac{\exp \left[- \frac{(\ln R + (\mu_1 + \mu_2)t)^2}{2 t^2 (\sigma_1^2 + \sigma_2^2)} \right]}{\sqrt{2\pi} R t \sqrt{\sigma_1^2 + \sigma_2^2}} \left[\frac{1}{2} + \frac{1}{\sqrt{2\pi}} \int_0^{\sqrt{2} M_R} e^{-t^2/2} dt \right]$$

where:

$$M_R = \frac{\mu_1 t \sigma_2^2 - \mu_2 t \sigma_1^2 - \sigma_1^2 \ln R}{\sigma_1 \sigma_2 \sqrt{2} t \sqrt{\sigma_1^2 + \sigma_2^2}}$$

For $\sigma_i \ll \lambda_i$, the integral in the brackets becomes 1/2 thus reducing $f(R)$ to lognormal form:

$$f(R) = \frac{\exp \left[\frac{(\ln R + (\mu_1 + \mu_2) t)^2}{2t^2(\sigma_1^2 + \sigma_2^2)} \right]}{\sqrt{2\pi} R t \sqrt{\sigma_1^2 + \sigma_2^2}}$$

(c) N Series Components This pattern extends to N components in series and thus we have $f(R)$ for the obviously useful series configuration in typical systems. If each series component reliability is lognormally distributed, as in the above, we know this distribution will also be lognormal⁶ (assuming the upper limit M_R is large). Also, under certain conditions, as N becomes large, even for component pdf's other than lognormal, the series system pdf asymptotically approaches a lognormal distribution. This "central limit" conclusion holds up provided none of the individual variances dominates the system reliability.

The pdf for 2 series components is plotted in Fig. 1 and Fig. 2 for both the exact and approximate methods. For the relatively smaller standard deviation, Fig. 1, there appears to be little difference. However as shown in Fig. 2, where $\sigma = 0.75 \lambda$ there is a greater difference, which is best expressed by the cumulative probability. This is the probability that the reliability will be equal to or less than P:

$$P_c = \int_0^P f(R) dR$$

This integrated probability, P_c , is given in Table 1 for several cases for both the exact and approximate methods. The next step, now in progress, is to make the same comparison for additional series components to see the effect in a larger system. The same calculations will then be done using another pdf for the component failure rate λ , (e.g. the Weibull function). It is worth noting that cumulative or percentage tabulations are available¹² for some distributions of the Pearson family.

(d) Two Components in Parallel Success is defined as:

$$P_s = P_1 + P_2 - P_1 P_2$$

where:

P_i = success probability of i th component

This expression for P_s is viewed as a product term (already discussed) and as the sum and difference of random variables, P_i . The convolution of sums and differences of random variables is shown, in principle at least, in many texts.^{2,5} Thus, for certain probability distributions, some of the work has been developed.

(e) An extensive literature search is being made in order to make use of previous results for the convolution and transformation of random variables. Useful results already available include a combination of normal⁷, Cauchy⁷ and lognormal⁶ pdf's. Fourier transforms of univariate density functions are given in tables by Oberhettinger¹³. Giffin⁵ describes transforms of interest, in particular, the Mellin transform, for products and quotients and gives several tables of the more common distributions (normal, exponential, gamma, rectangular, binomial and Poisson).

Work in Progress

1. We are continuing the simple series and parallel cases to a larger number of elements using results and methods already developed for the combination of independent random variates. We are comparing the pdf of system reliability and its uncertainty bounds to approximate results (see next section) for larger systems. Comparison with operational experience of light water reactors will be made to the extent possible with existing data. As noted later, we are receiving extremely helpful cooperation from several utilities in gathering data from operating plants.

TABLE I
Cumulative Probability P_c

P	Exact	Approximate
	(%)	(%)
.9980	55	49
.9990	89	84
.9995	97	95

B. Approximate Methods to Determine the Probability Distribution of the System Reliability

The simplest and most obvious estimate would be to assume that the component reliabilities are all normally distributed, thus yielding a normal pdf for the system reliability. For very large systems, one might use a central limit theorem, even for mixtures of component distribution types, to claim an approximate pdf for the system reliability.

The method of moments^{3, 15} is one which can be adapted for our present purpose. The general method has been reviewed by Evans^{8, 9, 10} and has been described in many articles^{11, 14}. Estimates of the first few moments (usually four) are obtained experimentally or otherwise and used to select a form of probability distribution describing the system reliability.

The moments method involves a Taylor series expansion about the expected value of the system reliability. An expansion to second order can be written:

$$E(R) = h(E(\lambda_1), E(\lambda_2), \dots, E(\lambda_n)) + \frac{1}{2} \sum_{i=1}^n \frac{\partial^2 h}{\partial \lambda_i^2} \text{Var}(\lambda_i)$$

where:

$E(R)$ = mean system reliability

h = system reliability evaluated at each of the expectation values of the component failure rates.

λ_i = failure rate of component i

The practical application of this method is (like the transformation method) rapidly encumbered by the number of components in large systems and the need to include higher terms when the variances of the component failure rates are not small. Possible ways to evaluate the partial derivatives, dependent on the availability of analytic expressions or data for R , include linearized Taylor series expansion (classical propagation of errors); extended

Taylor series to sixth order (non-linear propagation of errors); numerical integration (where no analytic expression between R and the λ 's exists); and Monte Carlo simulation, sampling assumed distributions for the λ 's .

In our case we first apply the simplest approximation (linearized Taylor series), the classical propagation of errors. Taking the case of two elements in series:

$$E(R) = e^{-(\lambda_1 + \lambda_2)t} = e^{-(\mu_1 + \mu_2)t} ; \quad E(R) = \text{expectation value of } R$$

and

$$\sigma_{\Delta} = \sqrt{\left(\frac{\partial R}{\partial \lambda_1} \sigma_1\right)^2 + \left(\frac{\partial R}{\partial \lambda_2} \sigma_2\right)^2}$$

where we assume, for comparison to exact results:

$$\bar{\lambda}_1 = \bar{\lambda}_2 = 1 \times 10^{-6} \text{ /hr}$$

$$\sigma_1 = \sigma_2 = 0.1\bar{\lambda} \quad (\text{Case 1})$$

$$\sigma_1 = \sigma_2 = 0.75\bar{\lambda} \quad (\text{Case 2})$$

It is recognized that $E(R)$ and σ_{Δ} , the mean and standard deviation of this normal pdf, are the only population parameters available by this approach. The approximate results are shown in Fig. 1 and Fig. 2 where the exact results are also shown for comparison. For the case where the standard deviation is equal to 0.1μ , the approximate and exact forms are quite similar and symmetrical. For a standard deviation comparable to μ , the exact function becomes asymmetrical as in Fig. 2. (noting that in Fig. 2, $\sigma = 0.75\mu$ and that using a normal pdf for the estimate of λ is not as satisfactory as for the case of $\sigma = 0.1\mu$ since λ cannot be negative).

As pointed out by Pearson¹⁶, there are difficulties in the application of this method in practice. The uncertainty in the estimates of the first four moments inevitably produces an uncertainty in the proper choice of the pdf to be used. (See Fig. 3, a sketch of Fig. 6-11, Ref. 3, where β_1 and β_2 are obtained from the estimates of the first four moments and used to select the general form of the pdf.) Since β_1 and β_2 are estimates, (via the maximum-likelihood method, for ex.), they will have inherent uncertainties which lead to a zone, rather than a precise point of intersection, on Fig. 3. This introduces the question as to whether the proper form of distribution has been determined. A zone is drawn on Fig. 3 merely for illustration showing the possible ambiguity in selection.

Questions which we are addressing are:

1. Given the uncertainty in reliability data, what is the degree of uniqueness in the pdf resulting from the moments method?
2. How does the lack of uniqueness in the pdf affect the calculation of confidence limits? (What is the confidence in the confidence limits?)
3. If one wishes to attach confidence limits to an observed reliability, how do the confidence limits via the moments method compare to those obtained from the exact distributions?

Further work in progress includes the assessment of operational experience of plant systems. The observed experience will reveal typical variations in system reliability which will be reflected in the uncertainty in the estimates of β_1 and β_2 . We are using several distributions and methods of parameter estimation¹⁷ and are presently collecting relevant plant data on protective systems.

C. Operational Data

Since the only source of nuclear plant data is from light water reactors, we will use this data for testing our methods in preparation for application to the LMFBR types.

Several visits have been made to utilities and to one of the reactor vendors to become better acquainted with operational methods in nuclear power plants, plans for reliability, and current methods for maintenance, tests, calibrations and component performance recording. The basic purpose was to see what component data are available, the form, quantity and related information such as system diagrams, reports to the N.R.C. and the Nuclear Plant Reliability Data System. Our reliability models are now at a stage where we can make use of selected component and system performance histories.

Material, Information Obtained

1. Several hundred component calibration and test records for a PWR.
2. Abnormal occurrence reports, letters, personal contacts and system histories, necessary to analyze and model the reliability of the subsystem in question have been provided by two utilities and promised by two others so far.
3. A spectrum of abnormal occurrence reports and related information from NEL-PIA covering a range of reactor plants and component failures.
4. Technical specifications, maintenance procedures and forms for maintenance, calibration and surveillance from the utilities.
5. Estimates of component failure data for protection systems contained in the PSAR's for two PWR's, the FFTF (FPC-1421)¹⁸ and the CRBRF PSAR¹⁹.

6. Reporting Procedures Manual for the N.P.R.D.S., which includes the descriptions, formats, input, output forms for this data system.
7. Requested and received a "special request" form from Southwest Research Institute for accessing selected types of information (a specific component type, system or a particular model) from the N.P.R.D.S.

For our purposes we need specific component information such as model number, environmental conditions, installation date, number of such components in service, maintenance intervals, test and calibration procedures, the other components related to it in the system and many other factors. Conversations with plant engineers and technicians are indispensable. On the basis of our experience so far, some conclusions can already be made.

1. Much of the information we need already is documented but is spread around in many places.
2. Different groups within a given organization perform varying parts of the maintenance and testing, also contributing to the diversity in location.
3. General, extensive data collections do not (and probably cannot) provide the mass of specific information required and noted above.
4. The trend to computerized output formats in some reporting procedures has caused a loss of the type of information (descriptive) needed to fully understand operational performance, both success and failure.

Work in Progress

We are presently studying this data with regard to its proper use for testing our methods to determine reliability and the uncertainty in reliability.

Questions being considered include the statistical quality of the data and the proper pdf to assign to components. We will use standard statistical techniques to make the most of the available component experience, including, as required, histories of similar components in the same plant, zero failure estimates and maximum-likelihood estimates of population parameters.

More visits to nuclear plants and engineering headquarters are presently being scheduled. These will be carefully planned (as in the past) to use our host's time, as well as ours, efficiently, since every trip represents a significant perturbation in their normally tight schedule.

We will access selected component and system information from the N.P.R.D.S. for use in system reliability modeling. We will also compare and supplement this information with that received directly from the utilities.

D. Codes

Several small computer codes have been adapted or written to do reliability calculations, sensitivity, analysis and simulation. In addition, the A.N.L. code center supplied us with their version of the PREP-KITT²⁰ code system. The nature and applications of these codes are briefly noted in Table II. Monte Carlo routines will be incorporated into some of them for making comparisons to results obtained by convolution and transformation.

TABLE II

<u>Code</u>	<u>Purpose</u>	<u>Computer</u>	<u>Additional Uses</u>
1. BOUNDS ²¹	Estimates reliability by use of cut sets; for small systems	PDP-15, CDC-6600	rapid sensitivity and testing on small systems
2. RELCOMP ²²	Deterministic calculation of single system reliability, including maintenance intervals.	IBM-360/67	sensitivity checks on small systems
3. RELIA ²³	Deterministic reliability calculation, with maint. intervals.	PDP-15	More flexible than 1,2; can handle larger systems; will probably be enlarged to include Monte Carlo routines for simulation studies.
4. PREP-KITT ²⁰	Fault tree analysis; obtained from ANL Code Center	360/67	Testing, simulation

Accomplishments to Date

The codes in Table II have been debugged and are now running on the computer indicated. The code system, PREP-KITT is also being set up to run on a CDC-6600. Codes 1-3 have been intercompared and checked by running the same problems.

Work In Progress

Since the BOUNDS code uses the cut set method, we will use it to compare to the deterministic codes, RELCOMP and RELIA, to look at sensitivities to component failure rates and changes in system logic. RELIA will be used, after adding a Monte Carlo option, for simulation to compare to the system reliability pdf obtained by transformation methods. The advantage of these small codes is economy, especially when used to do deterministic calculations. The PREP-KITT system will be used more sparingly for comparison to exact reliability calculations because of its larger setup and running time.

In order to computerize the calculation of the uncertainty in system reliability by convolution or transformation methods, we are writing a Fortran program in two parts which will later be combined into one larger program. The first part of the code will set up the probability expression for a given system in terms of the system components. The second part will take this expression as a starting point, for the combination of the probability density functions of the components, to obtain the pdf of the system reliability. The system pdf will then be used to calculate the mean, variance, confidence limits, point reliability, sub-system pdf's sensitivity analyses and comparison to plant experience. The first part of this code is based on the Boolean algebra techniques of Bennetts.²⁴ The basic idea here is to enable an engineer to input the system structure directly from system block diagrams (not necessarily fault trees) into

a computer. The single input form enables the computer to construct a Boolean expression for the system and reduce it using standard Boolean algebra to an exclusive form. This exclusive form is equivalent to a probability expression giving the point reliability. We thus have an efficient means to arrive at the probability equation which is the starting point for our real goal, the determination of the uncertainty in system reliability. This code is now in the programming stage. As noted in the other sections, work is proceeding in parallel on the methods to determine the system reliability pdf (and uncertainty) in terms of the component failure rates and their pdf's.

3. Sensor Analysis

Methods to enhance the sensitivity of sensor and signal processing component and system quality checks are under study. Two general methods are available to determine the extent of instrument degradation. First, the universal practice of checkout with a calibrated source or comparison of readings under known operating conditions in successive time periods. Second, the observation and analysis of system "noise"; this includes the reactor signature when applied to an operating plant. It is the purpose of the present study to study the latter by Monte Carlo simulation³ and statistical analysis¹⁵, and to compare these results with information derivable from the more common maintenance checks and calibrations.

(A) Accomplishments to date

A Monte Carlo simulation code has been written and debugged and various runs made with it to show the effects of random noise and random uncertainty of instrument response amplitudes. The code provides for multiple frequencies (five in the present version) simulating various reactor or instrument signature inputs, each with an adjustable amplitude broadened by random sampling from a normal uncertainty distribution. The code provides as output the power spectral density function, by a conventional procedure^{25, 26, 27}, a fast Fourier transform and autocorrelation function.

Sample runs which demonstrate the usefulness of the code are shown in Figures 4 to 13. Input frequencies in the range of 0 - 32 Hz have been used so far of the form $A_i \sin(\omega_i t + \theta_i)$, where $A_i = a_i (1 + \epsilon_i)$ with a random number and normal distribution subroutine used for each simulation to determine ϵ_i in the expression,

$$P(\epsilon_i) = \frac{1}{\sigma_i \sqrt{2\pi}} e^{-\frac{\epsilon_i^2}{2\sigma_i^2}}$$

Here the subscript i refers to a particular frequency. With various σ_i and no background noise the plots of power spectral density in Figures 6 to 9 show fluctuations caused by random solution of response uncertainties in the ranges indicated by σ_1 , σ_2 and σ_3 . For reference, Figure 4 shows the power spectral density with all $\sigma_i = 0$. The successive figures, starting with 5, are grouped to show the following effects:

- (a) Figures 5 and 6 have similar input, as indicated on the figures, but the simulations were run with different random number sequences (indicated by the different "seeds") corresponding to different random uncertainties for the amplitudes of the input frequencies (1, 5 and 25 Hz respectively). These provide a reference indicating a degree of pattern variation which one can normally expect if sensor responses are not drifting (these are examples only; for the determination of limits on these random variations one would require more simulations). The significance of the Δt values are discussed in (c).
- (b) Figures 7 - 9 show that changes in σ_i (for the input frequencies) produce changes which can probably be perceived as a sensor change if this change corresponded to the indicated increase in response uncertainty. However, these particular examples are all simulations made with the same random number sequence (indicated by the same seed) and the minimum degree of change which is detectable is being determined by work in progress involving more simulations and statistical analysis.

(c) In Figures 10 and 11 the effect of background noise only is shown, in the absence of amplitude response uncertainty (again these examples are run with the same random number sequence). This noise is simulated by

$$A_5 = a_5(1 + \epsilon_5) \quad ,$$

by letting $\theta_5 = \pi/2$ and $\omega_5 = 0$ in the general expression for input frequencies. The comparison between the two figures is for different noise amplitudes ($\sigma_5 = .033$ and $.33$ respectively) to indicate the sensitivity to this parameter. It is to be noted that a fairly large random noise amplitude in this case (relative to the input frequencies) may correspond to a relatively small background in the physical case where the amplitudes of the frequencies involved (corresponding to e.g. vibrations in the reactor) are themselves small compared to the amplitudes of the steady state reactor parameters for which the sensors are designed.

(d) Figures 12 and 13 show the combined effects of background and sensor response uncertainty. The differences between these two cases arise from the use of different random number seeds, corresponding to different experimental runs.

(e) Another significant effect (well known in principle but most readily understood in practice through simulations) is that of the time relationship between the input signals and discrete intervals used in numerical autocorrelation and Fourier transform calculations. Figures 6 and 7 show the effect of a slight change in interval width Δt (from $1/63$ to $1/64$). In our case $1/64$ corresponds to exact synchronism and leads to theoretically expected peak amplitudes in the power spectral density. With $\Delta t = 1/63$ the higher frequency amplitudes are likely to be depressed, and this is exhibited in this case (Fig. 7) by the 25 Hz peak. Note that in a physical experiment one cannot adjust the phase relationships of the

signals and, therefore, the idealized case of Figure 6 is not typical.

All of the other simulations are made with the more realistic $\Delta t = 1/63$.

(B) Work in progress and tentative conclusions.

Current efforts are concentrated on statistical aspects of the Monte Carlo results with the objective of developing a useful sensitivity analysis of instrument degradation or "partial failure", as described below. Although this amounts to a problem in pattern recognition, for our present purpose it is useful to apply standard statistical tests²⁸ for "identification" or "discrimination". In fact there is much overlap with the general approaches to pattern recognition.^{29, 30}

The sensitivity analysis amounts to the following test for sensor response: assume that there is a small signal which is consistently observable when the sensor and associated signal processing system are in good condition. As demonstrated graphically by the simulations cited above, an increase in random background noise or in (amplitude) uncertainty of instrument response will degrade the signal to noise ratio. A quantitative test to measure this ratio or the observability of the test or reference signal provides a measure of instrument performance. Work in progress is aimed at optimizing such a test and comparing the results with available plant data (see Section 2B-C) in an effort to determine whether such a test will give better indications of impending instrument failure than routine calibration and maintenance procedures.

In addition to the statistical work relating to an guided by the simulations already described, a step by step error propagation analysis is being made of the numerical transformations and approximations involved in the power spectral density calculations. Some of the pattern features are predictable by this process, which makes the analysis of random effects more tractable.

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Over the past two years, we have visited many places to discuss all aspects of reliability, failure data, and plans to initiate reliability programs or enlarge upon those already in existence. We wish to acknowledge the cheerful, prompt cooperation and patience which we have received from personnel of the following organizations (in alphabetical order): Combustion Engineering, Connecticut Yankee (Haddam Neck Plant), Consolidated Edison, E.P.R.I., Millstone I Plant, N.E.L.-P.I.A., Niagara Mohawk Power Corporation and Northeast Utilities.

PERSONNEL

In the nine month period during which this research has been carried out, the Principal Investigator, Dr. R. W. Hockenbury has devoted 45% of his time during the Academic Year and five weeks during the summer. For the remaining three month period, he will devote one week in the summer and 45% of his time during the Academic Year.

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Publications (Abstracts submitted)

1. "Reliability Models for Plant Protection Systems" - R. W. Hockenbury and M. L. Yeater. Accepted for presentation at International Meeting on Fast Reactor Safety and Related Physics, Oct. 6-8, 1976, Chicago. (Originally two papers; combined into one at request of Review Committee).
2. "Comparison of An Exact and Moments Calculation of Reliability", R. W. Hockenbury, M. L. Yeater, J. M. Hawkins and J. W. Wilkinson, submitted to A.M.S. for presentation at the Nov. 1976 meeting, Washington, D.C.*
3. "Error Propagation Analysis for a Sensor System", M. L. Yeater, R. W. Hockenbury, J. Hawkins and J. W. Wilkinson, submitted to ANS for presentation at the Nov., 1976 meeting, Washington, D.C.*

* We have requested reconsideration of these two papers since we believe that the intent of the papers was misconstrued by the reviewers in that we were stressing the application and comparison of methods to data as opposed to presenting the methods themselves.

Figure Captions

- Fig. 1 Exact and approximate reliability distributions for a
2 component series system - $\sigma = 0.1\lambda$
- Fig. 2 Exact and approximate reliability distributions for a 2
component series system - $\sigma = 0.75\lambda$
- Fig. 3 Parameters β_1 and β_2 and appropriate regions for dis-
tributions in the Pearson family.
- Fig. 4 Reference power spectral density, no background noise or response
uncertainty.
- Fig. 5 Power spectral density showing effect of random number sequence
and Δt (of Fig. 6).
- Fig. 6 Psd showing effect of random number sequence and Δt (of Fig. 5).
- Fig. 7 Psd with variation in σ_1 (of Figs. 8 and 9).
- Fig. 8 Psd with variation in σ_1 (of Figs. 7 and 9).
- Fig. 9 Psd with variation in σ_1 (of Figs. 7 and 8).
- Fig. 10 Psd showing effect of background noise and its amplitude
(of Fig. 11).
- Fig. 11 Psd showing effect of background noise and its amplitude
(of Fig. 10).
- Fig. 12 Psd including both response uncertainty and background noise,
showing effect of varying random number sequence (of Fig. 13).
- Fig. 13 Psd including both response uncertainty and background noise,
showing effect of varying random number sequence (of Fig. 12).

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

Figure 7

Figure 8

Figure 9

Figure 10

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Figure 12

Figure 13

APPENDIX A

Exact Method^{*} - Uncertainty in System Reliability Due to Uncertainty in Component Failure Rate

Single Component

Assume the "constant" failure rate (λ):

$$R = e^{-\lambda t} \quad (1)$$

and assume that the estimate of λ is normally distributed

$$f(\lambda) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left[-\frac{(\lambda - \mu)^2}{2\sigma^2} \right] \quad (2)$$

We wish to find the resulting probability density function (pdf) of R , $f(R)$.
(see comment in main report on restrictions regarding normality assumption).

Using the transformation of variables method, we solve for λ ,

$$\lambda = -\frac{1}{t} \ln R \quad (3)$$

find the Jacobian : $\frac{d\lambda}{dR} = -\frac{1}{tR}$

and its absolute value $\left| \frac{d\lambda}{dR} \right| = \frac{1}{tR}$ (4)

Thus we have:

$$f(R) = \left| \frac{d\lambda}{dR} \right| f[\lambda(R)] \quad (5)$$

Substituting:

$$f(R) = \frac{1}{tR} \frac{1}{\sigma \sqrt{2\pi}} \exp \left[-\frac{\left(-\frac{1}{t} \ln R - \mu \right)^2}{2\sigma^2} \right] \quad (6)$$

or

$$f(R) = \frac{1}{tR\sigma\sqrt{2\pi}} \exp \left[-\frac{(\ln R - \mu t)^2}{2\sigma^2 t^2} \right] \quad (7)$$

^{*} See the references by Shooman and also Hahn and Shapiro.

This is the pdf for a single component reliability at time t in terms of the mean, μ , and standard deviation, σ of the component failure rate pdf. The pdf of R , $f(R)$, is lognormal in form which was expected, of course, since λ was assumed to be normally distributed. The mean of R is found as follows:

$$\bar{R} = \int_0^1 R f(R) dR = \int_0^1 \frac{1}{\sigma t \sqrt{2\pi}} \exp \left[- \frac{(\ln R - \mu t)^2}{2\sigma^2} \right] dR \quad (8)$$

$$\text{let } u = \frac{\ln R + \mu t}{\sqrt{2\sigma}} \quad du = \frac{1}{\sqrt{2\sigma}} \frac{dR}{R}$$

Substituting and rearranging:

$$\bar{R} = \frac{\exp \left[- \left(\mu t - \frac{\sigma^2 t^2}{2} \right) \right]}{\sqrt{\pi}} \int_{-\infty}^{\frac{u}{\sqrt{2\sigma}}} \exp \left[- \left(u - \frac{\sigma t}{\sqrt{2}} \right)^2 \right] du \quad (9)$$

Substituting again: $\frac{m}{\sqrt{2}} = u - \frac{\sigma t}{\sqrt{2}}$

$$dm = \sqrt{2} du$$

$$\bar{R} = \frac{\exp \left[- \mu t + \frac{\sigma^2 t^2}{2} \right]}{\sqrt{2\pi}} \int_{-\infty}^{\frac{u}{\sigma} - \sigma t} \exp \left(- \frac{m^2}{2} \right) dm \quad (10)$$

$$\bar{R} = \exp \left(- \mu t + \frac{\sigma^2 t^2}{2} \right) \left[\frac{1}{2} + \frac{1}{\sqrt{2\pi}} \int_0^{\frac{u}{\sigma} - \sigma t} \exp \left(- \frac{m^2}{2} \right) dm \right] \quad (11)$$

Note that if the standard deviation of λ is small compared to the mean, μ ,

and $\frac{u}{\sigma} \gg \sigma t$, then $\bar{R}$ reduces to*:

$$\bar{R} = \exp \left(- \mu t + \frac{\sigma^2 t^2}{2} \right) \quad \sigma \ll \mu \quad (12)$$

This is of the same form tabulated in many places for the first moment of a lognormal distribution. In this case, since R is restricted to be between

*Also assumes $t=1000$ hours

0 and 1, it was necessary to assume $\sigma \ll \mu$ to obtain the result in Eq. (12).

For the more general case, the variate ranges from 0 to ∞ , and the first moment is obtained directly in the form of Eq. (12), the mean reliability as a function of time in terms of μ and σ .

Two Components in Series

Let components 1 and 2 be connected in series; the reliability is then

$$R = R_1 R_2 = e^{-\lambda_1 t} e^{-\lambda_2 t} \quad (13)$$

If we assume the λ 's to be normally distributed and follow the transformation of variables approach, we transform to the variable R and S from the variables R_1 and R_2 . We are only interested in R, S is a dummy variable which will go out in the integration. From the single element case, we know the pdf's of R_1 and R_2 are of the form:

$$f(R) = \frac{1}{tR \sigma \sqrt{2\pi}} \exp \left(- \frac{(-\ln R - \mu t)^2}{2\sigma^2 t^2} \right) \quad (14)$$

$$\begin{aligned} \text{Let } R &= R_1 R_2 & \text{or} & & R_1 &= S \\ S &= R_1 & & & R_2 &= \frac{R}{R_1} = \frac{R}{S} \end{aligned}$$

Forming the Jacobian for the two variable case

$$\begin{aligned} \frac{\partial R_1}{\partial R} &= 0 & \frac{\partial R_1}{\partial S} &= 1 \\ \frac{\partial R_2}{\partial R} &= \frac{1}{S} & \frac{\partial R_2}{\partial S} &= -\frac{R}{S^2} \end{aligned} \quad (15)$$

The Jacobian is then:

$$J = \begin{vmatrix} 0 & 1 \\ \frac{1}{S} & -\frac{R}{S^2} \end{vmatrix} = \left| -\frac{1}{S} \right| = \frac{1}{S} \quad (16)$$

The density function of the tranformed variables R, S is:

$$\psi(R, S) = \phi[R_1(R, S), R_2(R, S)] |J| \quad (17)$$

Substituting:

$$\psi(R, S) = \phi\left(S, \frac{R}{S}\right) \frac{1}{S} \quad (18)$$

We want the pdf of R, thus:

$$f(R) = \int_S \psi(R, S) dS = \int_S \frac{1}{S} \phi\left(S, \frac{R}{S}\right) dS \quad (19)$$

If the components are independent

$$f(R) = \int \frac{1}{S} f_{R_1}(S) f_{R_2}\left(\frac{R}{S}\right) dS \quad (20)$$

or

$$f(R) = \frac{1}{\sigma_1 \sigma_2 t^2 2\pi} \frac{1}{R} \int_0^1 \frac{1}{S} \exp\left[-\frac{(-\ln S - \mu_1 t)^2}{2t^2 \sigma_1^2}\right] \exp\left[-\frac{(-\ln \frac{R}{S} - \mu_2 t)^2}{2\sigma_2^2 t^2}\right] dS \quad (21)$$

This integration may be performed after a substitution and some manipulations to yield:

$$f(R) = \frac{\exp\left[-\frac{(\ln R + \mu_1 t + \mu_2 t)^2}{2t^2(\sigma_1^2 + \sigma_2^2)}\right]}{\sqrt{2\pi} R t \sqrt{\sigma_1^2 + \sigma_2^2}} \left[\frac{1}{2} + \frac{1}{\sqrt{2\pi}} \int_0^{\frac{\sqrt{2} m_R}{t}} e^{-\frac{t^2}{2}} dt \right] \quad (22)$$

where

$$m_R = \frac{\mu_1 t \sigma_2^2 - \mu_2 t \sigma_1^2 - \sigma_1^2 \ln R}{\sigma_1 \sigma_2 \sqrt{\sigma_1^2 + \sigma_2^2} \sqrt{2t}}$$

If we again assume σ_1 and σ_2 are much smaller than μ_1 and μ_2 ,

the integral in Eq. (22) becomes $\frac{1}{2}$, and we again have a lognormal type distribution, for the two-element series case.

$$f(R) = \frac{\exp\left[-\frac{(\ln R + \mu_1 t + \mu_2 t)^2}{2t^2(\sigma_1^2 + \sigma_2^2)}\right]}{\sqrt{2\pi} \, Rt \, \sqrt{\sigma_1^2 + \sigma_2^2}} \quad (23)$$