TRANSPORT OF PLASMA ACROSS
A BRAIDED MAGNETIC FIELD

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Transport of Plasma Across a Braided Magnetic Field

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TRANSPORT OF PLASMA ACROSS
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ABSTRACT

Transport rates are calculated for a plasma immersed in a region through which magnetic lines of force meander in a stochastic fashion and in which the magnetic surfaces are destroyed. Such a magnetic condition, termed magnetic braiding, may be brought about by asymmetric magnetic perturbations, perhaps quite weak, which typically produce overlap of two sets of magnetic islands. Plasma transport is calculated for this environment, using both a fluid and a kinetic drift model. The latter gives an appreciably higher rate, namely, a fast-particle diffusion coefficient equal to $(1/2)D_M \langle |v_{\parallel}| \rangle$, where D_M is the coefficient of spatial diffusion for the magnetic lines of force. Correction terms, due to polarization-associated $E_{\parallel}$ fields, are small unless components of the braiding field resonate with ion-acoustic or drift waves. Insertion of a Bhatnager-Gross-Krook collision term shows the diffusion rate is unaffected by weak collisions. Diffusion due to magnetic braiding is of interest for tokamaks, particularly with respect to enhanced electron heat transport, enhanced current penetration, plasma disruption, and internal sawtooth oscillations.

1. INTRODUCTION

We wish to study the rates for transport of particles and heat across a locally braided magnetic field. Magnetic braiding in a region refers to a condition where magnetic surfaces are destroyed and magnetic lines wander about in a stochastic fashion. In a tokamak, magnetic braiding would typically be due to overlap of magnetic islands [1-3], perhaps produced by asymmetric current perturbations, or the interaction of toroidal effects with a set of islands, for example around the $q = 2$ surface [3].

Under conditions of magnetic braiding, one may look for several effects [4,5]. First, fast particles such as electrons or fast injected ions would undergo rapid radial diffusion throughout the braided region. The resulting fast heat transport will lead to the flattening of the electron temperature profile, while braiding of the current channels would flatten the parallel current distribution, lending the appearance of enhanced current penetration. Special effects should accompany sudden onset of braiding: inductive effects from the quick redistribution of the parallel plasma current will lead to positive or negative spikes in the toroidal loop voltage signal, while rapid redistribution of the electrons will produce large space-charge accumulations which would be neutralized by ions displaced by polarization. The polarization process will in turn set up poloidal plasma rotation, but such rotation is quickly damped by ion magnetic pumping [6-9], with the ions remaining in their displaced position, maintaining

quasineutrality. In Section 14 it is suggested that this process of electron and ion displacement could provide the mechanism for the fast-expansion phase of the internal sawtooth oscillations [10].

In this paper, plasma transport across a braided magnetic field [11] is calculated using both fluid and kinetic drift models. The essential difference in the two models lies in the nature of the flow along $\vec{B}$ -- in the first model, it is assumed that the plasma-fluid and heat both flow in a macroscopic manner along the wandering $\vec{B}$ lines, while in the kinetic drift model it is each individual particle that threads its way, at its own speed, along $\vec{B}$. The results from the two models are similar in structure but quite different in magnitude: the fluid model provides a provocative scaling for radial heat flow with a characteristic time $t \sim n_e a^4 / T^{5/2}$, but the kinetic drift model shows much more rapid transport. The quasilinear diffusion coefficient (D_{xx}) according to the kinetic drift theory is $(1/2)D_M \langle |v_{\parallel}| \rangle$ for particles which are fast compared to the parallel phase velocity of the perturbations which produce the magnetic braiding, while for slow particles, the calculation recovers Spitzer's result [12], Eq. (33). D_M is the spatial diffusion coefficient for the wandering magnetic lines of force.

It is interesting to note that neither of these diffusion processes is a resonant-particle process in the usual sense that $\omega - k_{\parallel} v_{\parallel} = 0$ for finite ω and $k_{\parallel}$. Rather than originating in the zeros of this usual Landau denominator, braiding and Spitzer diffusion originate in the simple zeros of k and ω ,

respectively, as these two Fourier coordinates appear in the denominator of the quasilinear-theory integrand. Ordinary resonant-particle diffusion with finite ω and $k_{\parallel}$, discussed in Section 11, can occur, but as it is magnetic perturbations which are under consideration, the $E_{\parallel}$ fields are quite small. These $E_{\parallel}$ fields are seen to come from the neutralization of ion polarization current, and the resonant-particle diffusion effect is only large if the magnetic perturbation happens to resonate with ion-acoustic or drift waves.

The kinetic drift calculation of the quasilinear diffusion coefficients was repeated with the introduction of weak collisions via a Bhatnager-Gross-Krook collision term. However, to first order in the collision frequency, the results are identical to the collisionfree calculation. This outcome is not difficult to understand inasmuch as collisions in the drift model can only change $v_{\parallel}$ whereas, except for polarization drift, particles in the model stick to the moving lines of force for all values of $v_{\parallel}$. Thus, the macroscopic quantities such as density, fluid velocity, and temperature are transported with the lines of force and do not undergo diffusion due to collisions.

As a final remark of an introductory nature, it should be pointed out that cross-field transport for the fluid and nonresonant-kinetic braiding cases both depend on the magnetic field only through the coefficient for spatial magnetic diffusion, D_M , defined in Eq. (1). The processes which produce magnetic braiding can include magnetic field errors, instabilities, plasma turbulence, and so forth, but it is beyond the

scope of this work to ascribe amplitudes to the perturbations. On the other hand, from a knowledge of the spectral intensity of the magnetic perturbations it is possible, as indicated in Eq. (24), to determine D_M for the case of a zero-order magnetic field without shear.

2. FLUID MODEL FOR ELECTRON TRANSPORT

We take a simple model for the diffusion of magnetic lines of force. Let $\vec{B}$ be predominantly in the $\hat{z}$ direction, but perturbed by stochastic x -fields such that the average rate of displacement in the x -direction is zero, $\langle \Delta x \rangle / |\Delta z| = 0$, while the mean square displacement can be characterized by a diffusion coefficient, D_M , which is constant through the region of interest,

$$D_M = \frac{\langle (\Delta x)^2 \rangle}{|\Delta z|} \quad (1)$$

The x -direction in this slab model would correspond to the direction of the minor radius in a torus.

Now consider a group of lines of force which pass through a line extending in the y direction with coordinates $x = x_0$, $z = z_0$. Following these lines of force to the right of z_0 , we write down an equation of the Fokker-Planck form,

$$\frac{\partial g}{\partial \xi} = \frac{1}{2} \frac{\partial^2}{\partial \xi^2} (g D_M) \quad (2)$$

where $\xi \equiv x - x_0$, $\zeta \equiv z - z_0$, and $g(\xi, \zeta)$ is the relative density of the chosen lines of force, averaged over y .

Next, we immerse a plasma in this braided magnetic field and follow, for a brief time interval, the evolution of the electron flow along $\vec{B}$. Accordingly, in our model each line of force is loaded at $t = 0$ with current-carrying electrons of density $n(x)$ which are moving along $\vec{B}$ to the right with fluid velocity $v(x)$. At a slightly later time Δt , the electron density, averaged over y , is

$$n(x, z, \Delta t) = \iint dx_0 dz_0 n(x_0) g(x-x_0, z-z_0) \delta[z - z_0 - v(x_0) \Delta t] ,$$

integrated over z_0 ,

$$n(x, \Delta t) = \int dx_0 n(x_0) g[x-x_0, v(x_0) \Delta t] ,$$

and using Eq. (1),

$$\frac{\partial n}{\partial t} = \frac{1}{2} D_M \frac{\partial^2}{\partial x^2} (nv) .$$

For electron flow at fluid velocity v to the left, we would choose $\zeta = z_0 - z$, positive. The preceding relation can then be rewritten as a conservation equation valid for both directions of flow,

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x} \Gamma_x = 0 ,$$

$$\Gamma_x = - \frac{1}{2} D_M \frac{\partial}{\partial x} |nv| . \quad (3)$$

In terms of current flow, we would have

$$j_x = \frac{1}{2} D_M \frac{\partial}{\partial x} |j_{||}| \quad (4)$$

Turning our attention now to heat flow, we write down the familiar equations for electrons:

$$\frac{3}{2} n \frac{dT_e}{dt} - T_e \frac{dn}{dt} + \nabla \cdot \vec{Q}_e = 0 \quad (5)$$

$$\vec{Q}_e = \hat{b} (-\kappa \nabla_{||} T_e + \alpha n T_e v_{||}) \quad (6)$$

where $d/dt = \partial/\partial t + \vec{v} \cdot \nabla$, $\vec{v}$ is the fluid velocity for the electrons, κ is the coefficient for heat conductivity, and the parallel electron thermal force coefficient is $\alpha = 0.71$. We again assume that n , $\vec{v}$, and T_e are initially functions only of x , but on this round we follow the evolution of the plasma temperature as heat both diffuses along $\vec{B}$ and convects with fluid velocity $\vec{v}$ away from each point in the plasma.

Looking at Eq. (5), the convective portion of the first term is $(3/2)(nv_x \partial T_e / \partial x + nv_z \partial T_e / \partial z)$. By assumption $\partial T_e / \partial z = 0$, and Eq. (3) gives us $nv_x = -(D_M/2) \partial |nv| / \partial x$. Similarly, $-T_e \vec{v} \cdot \nabla n \rightarrow -T_e nv_x (\partial n / n \partial x)$, $\partial n / \partial t = -n \nabla \cdot (\vec{n} \vec{v} / n)$, and $\nabla \cdot \vec{Q}_e \rightarrow \partial Q_x^{(e)} / \partial x$, which are to be evaluated by the same reasoning. In this connection, the convective portion of $Q_{||}^{(e)}$ may be taken immediately from Eq. (6). To compute the diffusive contribution, we first write an average over y for the temperature at $z + \Delta z$ along lines of force which all pass

through the line x, z ,

$$\langle T_e(z+\Delta z) \rangle = \int dx_o T_e(x_o) g(x-x_o, \Delta z) .$$

With this expression to evaluate $\langle \nabla_{||} T_e \rangle = \partial \langle T(z+\Delta z) \rangle / \partial (\Delta z)$ and using Eq. (2), we find the y -averaged diffusive contribution, $Q_{||}^{(de)}$, to parallel electron heat flowing to the right,

$$Q_{||}^{(de)} \equiv -\langle \kappa \nabla_{||} T_e \rangle = -\frac{1}{2} D_M \kappa \frac{\partial^2 T}{\partial x^2} . \quad (7)$$

After summing the contributions to $\nabla \cdot \vec{Q}^{(e)}$ from diffusion to the right and left, and from convection, the electron energy conservation equation becomes

$$\begin{aligned} \frac{3}{2} n \frac{\partial T}{\partial t} - \frac{3}{4} D_M \frac{\partial |nv|}{\partial x} \frac{\partial T}{\partial x} - \frac{1}{2} D_M T \frac{\partial^2 |nv|}{\partial x^2} + \frac{1}{2} D_M T \frac{\partial |nv|}{\partial x} \frac{1}{n} \frac{\partial n}{\partial x} \\ - \frac{1}{2} D_M \frac{\partial^2}{\partial x^2} |\alpha n T v| - \frac{1}{2} D_M^2 \frac{\partial^2}{\partial x^2} \left| \kappa \frac{\partial^2 T}{\partial x^2} \right| = 0 . \end{aligned} \quad (8)$$

The reader will note that Eq. (8) has rather arbitrarily invoked the absolute-value convention on a term-by-term basis. In the case of right and left diffusion, these two processes are physically separate and one may expect them to be additive. Moreover, one notes that $\nabla \cdot \vec{Q}_{||}^{(de)} \sim \partial^2 \langle T(z+\Delta z) \rangle / \partial z^2$, and iteration of the derivative calculation leads to the same final sign for both positive and negative Δz . Again in the kinetic treatment [cf., Eqs. (27), (31)] the sum of absolute values of velocities will show up in the transport coefficients.

3. DESCRIPTION OF THE KINETIC MODEL

We turn now to a treatment of the same problem for the long mean-free-path case. In place of the two-fluid equations, we use the collisionfree kinetic drift equations for both the ions and electrons. Although the actual details of the weak-collision calculation are not presented in the paper, we will discuss, in Section 12, the results of a kinetic-drift calculation carried out in the same way, but using a Bhatnager-Gross-Krook collision term. We will see there that, to first order in the collision frequency, the quasilinear collisionfree transport coefficients for cross-field diffusion remain unaffected by weak collisions. On the other hand, the collisionfree diffusion is found to be much more rapid than the fluid model transport, just calculated in Section 2. The two formulations are compared and discussed in Section 13.

To model the braided magnetic field for the drift-kinetic calculation, we select a source current able to induce magnetic and electric fields in the plasma of the type in which we are interested, namely:

$$\vec{j}_{\text{source}} = \frac{c}{4\pi} \nabla \times [\hat{x} b_x(y, z, t)] \quad . \quad (9)$$

The plasma and the source current are again imbedded in a uniform magnetic field, $\vec{B}_0 = \hat{z}B$, and $B \gg |b_x|$. In this simple model, b_x represents a stochastic variable with an $\omega, \vec{k}$ power spectrum filtered to contain only frequencies well below the ion cyclotron frequency, and to contain only phase velocities small compared to the electron-thermal and Alfvén

velocities. It may be considered that $\vec{j}_{\text{source}}$ is driven by magnetic field errors, instabilities, plasma turbulence, etc. The specific description of the $\vec{j}_{\text{source}}$ power spectrum is beyond the scope of this work, but what can be usefully determined is the enhancement of plasma transport due to given current fluctuations, as well as a qualitative assessment of the relative contributions to transport from different regions of $\omega, \vec{k}$ space in the $\vec{j}_{\text{source}}$ power spectrum.

An elementary model is also used to describe particle motions -- simple drifts along $\vec{B}$ with the one-dimensional velocity $v \equiv v_{\parallel}$. The evolution of the ion and electron velocity distribution functions, $f(x, y, z, v, t)$, is given in both cases by the relevant terms of the kinetic drift equation, namely

$$\frac{\partial f}{\partial t} + (v\hat{b} + \vec{v}_E) \cdot \nabla f + \nabla \cdot (\vec{v}_p f) + \frac{q}{m} \hat{b} \cdot \vec{E} \frac{\partial f}{\partial v} = C(f) \quad (10)$$

Here, $\hat{b} \equiv \vec{B}/B$, $\vec{v}_E = (\vec{E} \times \vec{B})c/B^2$, and the polarization drift is $\vec{v}_p = (mc^2/qB^2)d\vec{E}_{\perp}/dt$. With the choice of a uniform $\vec{B}_0$ field plus a shear modulation, the magnetic gradient drifts and $\mu\nabla B$ accelerations are only of second-order in the $b_{\perp}$ perturbation and need not be considered. For the initial calculation, $C(f)$ will be taken zero, but the use of a Bhatnager-Gross-Krook collision term is discussed in Section 12.

4. FIRST-ORDER SOLUTION

The equilibrium solution of Eq. (10) is $f_0 = f_0[\alpha(\vec{r}), mv^2/2 + q\phi(\alpha)]$, where $\vec{E} = -\nabla\phi = -(d\phi/d\alpha)\nabla\alpha$, and $\alpha(\vec{r})$ satisfies $\vec{B}_0 \cdot \nabla\alpha = 0$. Clearly, $\alpha(\vec{r}) = \text{constant}$ represents a magnetic surface. The use of $\phi(\alpha) \neq 0$ would give rise to finite zero-order cross-field drift velocities; for simplicity, we will choose $\phi(\alpha) = 0$ in equilibrium and $f_0 = f_0(x, v)$.

Linearizing Eq. (10), the first-order quantities will be $f^{(1)}$, $\vec{E}^{(1)}$, and $\vec{B}^{(1)}$, but we will use $-\partial\vec{B}^{(1)}/c\partial t = \nabla \times \vec{E}^{(1)}$ to express $\vec{B}^{(1)}$ in terms of $\vec{E}^{(1)}$. It is consequently somewhat obscured in the final equation but interesting to note in passing that $(v\hat{b} + \vec{v}_E)^{(1)} = v(B_x^{(1)}/B)\hat{x} + \vec{E}^{(1)} \times \vec{B}_0 c/B^2 = [\vec{E}^{(1)} + (v/c)\hat{z} \times B_x^{(1)}\hat{x}] \times \vec{B}_0 c/B^2$.

Using intervals $2Y$, $2Z$, and $2T$ which are long compared to the wavelengths and time-periods of characteristic components of the $b_x(y, z, t)$ spectrum, we Fourier analyze all the perturbed quantities as in the example:

$$\tilde{b}_x(k_y, k_z, \omega) = \left(\frac{1}{2\pi}\right)^{3/2} \int_{-Y}^Y dy \int_{-Z}^Z dz \int_{-T}^T dt b_x(y, z, t) \exp(-ik_y y - ik_z z + i\omega t) \quad (11)$$

In this fashion we obtain the equation for the Fourier components of the linearized Eq. (10),

$$\begin{aligned} -i(\omega - k_z v) \tilde{f} = & -\frac{c}{B} \left[\tilde{E}_y \left(1 - \frac{k_z v}{\omega} \right) + \tilde{E}_z \frac{k_y v}{\omega} \right] \frac{\partial f_0}{\partial x} - \frac{q}{m} \tilde{E}_z \frac{\partial f_0}{\partial v} \\ & - \frac{mc^2}{qB^2} (\omega - k_z v) \left[-i \frac{\partial}{\partial x} (\tilde{E}_x f_0) + k_y \tilde{E}_y f_0 \right] \quad (12) \end{aligned}$$

The $(\omega - k_z v)$ factor in the polarization drift term stems from the total derivative, $d\vec{E}_1/dt$, in $\vec{v}_p$.

The next step of the analysis is the computation of the self-consistent plasma response to the excitation $\vec{j}_{\text{source}}$, and for this purpose we need to find the first-order plasma current $\vec{j}_{\text{plasma}}^{(1)}$ in terms of $\vec{E}^{(1)}$. We have

$$\begin{aligned}\vec{j}_{\text{plasma}}^{(1)} \cdot \hat{x} &= \frac{R_x^{(1)}}{B} j_0 + \frac{\rho c^2}{B^2} \frac{dE_x^{(1)}}{dt}, \\ \vec{j}_{\text{plasma}}^{(1)} \cdot \hat{y} &= \frac{\rho c^2}{B^2} \frac{dE_y^{(1)}}{dt}, \\ \vec{j}_{\text{plasma}}^{(1)} \cdot \hat{z} &= \int q f^{(1)} v dv, \end{aligned} \quad (13)$$

where the total derivatives in the expressions for polarization current here are $d/dt = \partial/\partial t + \langle \vec{v} \rangle \cdot \nabla$, incorporating the total plasma macroscopic fluid velocity $\langle \vec{v} \rangle$. ρ is the macroscopic mass density and j_0 is the zero-order plasma current along $\vec{B}_0 = \hat{z}B$.

The integration of $f^{(1)}$ is straightforward. By conventional analysis we find

$$\begin{aligned}\tilde{j}_z &= \frac{i}{k_z} \frac{dj_0}{dx} \frac{\tilde{B}_x}{B} - \frac{i\omega}{4\pi} (K_{zz} - 1) \tilde{E}_z \\ &\quad - \frac{c^2}{B^2} \left[\frac{\partial}{\partial x} (\rho \langle v \rangle \tilde{E}_x) + ik_y \rho \langle v \rangle \tilde{E}_y \right], \end{aligned} \quad (14)$$

$$K_{zz} = 1 + \frac{4\pi}{k_z} \sum_{i,e} \frac{q_i^2}{m} \int dv \frac{\partial f_o / \partial v}{\omega - k_z v} + \frac{4\pi k_y c}{k_z^2 B} \frac{d}{dx} \sum_{i,e} q_i \int dv \frac{f_o}{\omega - k_z v} , \quad (15)$$

where the integrals are to be carried out over the usual Landau contour.

Maxwell's induction equation provides, of course, that $\tilde{B}_x = n_y \tilde{E}_z - n_z \tilde{E}_y$, where $\vec{n} \equiv \vec{k}c/\omega$, but the explicit use of $\vec{B}_x$ in Eq. (14) reveals the simple physical interpretation of the first term on the right, namely, the contribution to $j_z^{(1)}$ that results from bending the channels for $j_{||}$. The middle term in Eq. (14) is the usual dielectric contribution to $j_z^{(1)}$, while the last term originates in the polarization terms and can obviously be dropped by choosing the frame of reference in which the macroscopic fluid velocity, $\langle v \rangle$, is zero.

Continuing the use of conventional methods of analysis, we obtain familiar forms for K_{zz} in two regimes, A and B,

$$A. \quad |\omega/k_z| \ll s_i, s_e$$

$$K_{zz} \approx 1 + \frac{4\pi n_e e^2}{k_z^2 k T_e} + \frac{4\pi n_i z^2 e^2}{k_z^2 k T_i} - \frac{4\pi^{3/2} i k_y n_e e c}{k_z^2 |k_z| B s_i} \left(\frac{1}{n} \frac{dn}{dx} - \frac{1}{s_i} \frac{ds_i}{dx} \right) ,$$

$$B. \quad s_i \ll |\omega/k_z| \ll s_e$$

$$K_{zz} \approx 1 + \frac{4\pi n_e e^2}{k_z^2 k T_e} - \frac{4\pi n_i z^2 e^2}{\omega^2} + \frac{4\pi k_y c}{\omega k_z^2 B} \frac{dn_e}{dx}.$$

In passing, we note that the zeros of K_{zz} in Case B correspond to the familiar drift and ion acoustic waves.

Proceeding now to calculate the self-consistent plasma response to the excitation $\vec{j}_{\text{source}}$, we use Maxwell's equations in a mixed form,

$$\frac{\partial B_z^{(1)}}{\partial y} - \frac{\partial B_y^{(1)}}{\partial z} = \frac{4\pi}{c} j_x^{(1)}, \quad (16)$$

$$\frac{\partial^2 B_z^{(1)}}{c \partial x \partial t} - \frac{\partial^2 E_y^{(1)}}{\partial z^2} + \frac{\partial^2 E_z^{(1)}}{\partial y \partial z} = - \frac{4\pi}{c^2} \frac{\partial}{\partial t} [j_{y,pl}^{(1)} + j_{y,s}^{(1)}],$$

$$- \frac{\partial^2 B_y^{(1)}}{c \partial x \partial t} + \frac{\partial^2 E_y^{(1)}}{\partial y \partial z} - \frac{\partial^2 E_z^{(1)}}{\partial y^2} = - \frac{4\pi}{c^2} \frac{\partial}{\partial t} [j_{z,pl}^{(1)} + j_{z,s}^{(1)}],$$

the terms on the right designating components of the current density coming from the plasma (pl) and the source (s).

Now, if there is a zero-order parallel current, $j_0(x)$, in the equilibrium, there will be a corresponding zero-order magnetic field $B_y^{(0)} = (4\pi/c) \int j_0(x) dx$ which will move as the current channels move with the $B_x^{(1)}$ perturbation. If we assume that B_y remains constant along the new magnetic surfaces, we can write the component of $B_y^{(1)}$ which is due to perturbing the j_0 current channels,

$$\frac{dB_Y^{(1)}}{dz} = \frac{\partial B_Y^{(1)}}{\partial z} + \frac{B_X^{(1)}}{B} \frac{dB_Y^{(0)}}{dx} = 0 ,$$

hence,

$$\tilde{B}_Y(x) = \frac{4\pi i}{k_z c} \frac{\tilde{B}_X}{B} j_0(x) . \quad (17)$$

Using Eqs. (13), (14), and (17), we see that the terms explicitly dependent on $j_0(x)$ in set (16) cancel against the component of $B_Y^{(1)}$ which comes from perturbing $B_Y^{(0)}$, Eq. (17). Beyond this effect, the x equation in set (16) refers to the excitation of compressional modes which we will not consider here, and for the same reason the remaining $B_Y^{(1)}$ and $B_Z^{(1)}$ terms in set (16) may be dropped. We have left just

$$(n_z^2 - K_{YY})\tilde{E}_Y - n_Y n_Z \tilde{E}_Z = -n_Z \tilde{b}_X ,$$

$$- n_Y n_Z \tilde{E}_Y + (n_Y^2 - K_{ZZ})\tilde{E}_Z = n_Y \tilde{b}_X ,$$

where $\tilde{\hbar} = \tilde{k}c/\omega$, $K_{YY} \equiv 1 + 4\pi\rho c^2/R^2$, and $\tilde{b}_X$ is given by Eqs. (9) and (11). We thus find, with $\tilde{B}_X = n_Y \tilde{E}_Z - n_Z \tilde{E}_Y$,

$$\tilde{B}_X = \frac{\tilde{b}_X}{\Delta} (n_Y^2 K_{YY} + n_Z^2 K_{ZZ}) ,$$

$$\tilde{E}_Y = \frac{b_X}{\Delta} (-n_Z K_{ZZ}) ,$$

$$\tilde{E}_Z = \frac{\tilde{b}_X}{\Delta} (n_Y K_{YY}) ,$$

$$\Delta \equiv n_Y^2 K_{YY} + n_Z^2 K_{ZZ} - K_{YY} K_{ZZ} . \quad (18)$$

Provided $n_z^2 \gg K_{yy}$, which is to say that the parallel phase velocity is small compared to the Alfvén velocity, we see that $\tilde{B}_x \approx b_x$. Moreover, it would be approximately correct in this slow-wave region to write $\vec{E} = -\partial \vec{A}/c\partial t - \nabla\phi$ with $\nabla \cdot \vec{A} = 0$, and

$$\vec{A} = \frac{icb_x}{n^2\omega} (\hat{y}n_z - \hat{z}n_y)$$

$$\tilde{\phi} = \frac{icn_y n_z b_x}{n^2\omega\Delta} (K_{yy} - K_{zz})$$

$\vec{A}$ is the vector potential that would be induced in vacuo by $\vec{j}_{\text{source}}$, and the self-consistent plasma response turns out not surprisingly to be substantially electrostatic.

5. QUASILINEAR DIFFUSION COEFFICIENTS

We proceed now to the quasilinear formulation of Eq. (10).

The quasilinear equation will be of the form

$$\frac{\partial \bar{f}}{\partial t} + v \frac{\partial \bar{f}}{\partial z} + \frac{q}{m} \bar{E}_z \frac{\partial \bar{f}}{\partial v} - C(\bar{f}) \quad (19)$$

$$= \frac{\partial}{\partial x} D_{xx} \frac{\partial \bar{f}}{\partial x} + \frac{\partial}{\partial x} D_{xv} \frac{\partial \bar{f}}{\partial v} + \frac{\partial}{\partial v} D_{vx} \frac{\partial \bar{f}}{\partial x} + \frac{\partial}{\partial v} D_{vv} \frac{\partial \bar{f}}{\partial v}$$

in which $\bar{f}(x, z, v, t)$, which has been averaged over the intervals $2Y$, $2Z$, and $2T$, replaces the original $f_0(x, v)$ and now evolves on very slow time and space scales with respect

to z and t . For application to a tokamak, $\partial \bar{f} / \partial z$ would disappear due to toroidal periodicity, while classical resistive effects plus any electron drag induced by the turbulence will both balance against $\bar{E}_z$. It has been assumed that $\bar{E}_y = 0$, since it is otherwise implied that $\partial \bar{B}_z / \partial t \neq 0$.

The quasilinear form (19) is derived by carrying the perturbation calculation of (10) to second order in b_x , averaging over $2Y$, $2Z$, $2T$, and replacing $f_0(x,v)$ by $\bar{f}$. Following this procedure gives similar structures for the four diffusion coefficients,

$$D_{jk} = \frac{1}{8YZT} \left(\frac{1}{2\pi} \right)^3 \int_{-Y}^Y dy \int_{-Z}^Z dz \int_{-T}^T dt \int_{-\infty}^{\infty} d\Omega' \int_{-\infty}^{\infty} d\Omega \frac{\tilde{L}_j(\Omega') \tilde{L}_k(\Omega)}{-i(\omega - k_z v)} \\ \cdot \exp i[(k_y + k'_y)y + (k_z + k'_z)z - (\omega + \omega')t] ,$$

where Ω is shorthand for (k_y, k_z, ω) , Ω' for (k'_y, k'_z, ω') , and the integrals over $d\Omega$ and $d\Omega'$ are actually each triple integrals over $dk_y dk_z d\omega$ and $dk'_y dk'_z d\omega'$. Also, from inspection of Eq. (12), it is clear that

$$\tilde{L}_x = - \frac{c}{B} \tilde{E}_y - \frac{v}{B} \tilde{B}_x , \\ \tilde{L}_v = - \frac{q}{m} \tilde{E}_z . \quad (20)$$

Our interest from here being almost solely in quasilinear electron transport, we have dropped the contribution to $\bar{f}(x, z, v, t)$ from the polarization terms.

The use of set (18) in the expressions for $L_j(\Omega)$ eliminates the explicit dependence on $\tilde{E}_y$, $\tilde{E}_z$, and $\tilde{B}_x$ in favor of their relation to the source term $\tilde{b}_x$. We obtain

$$\tilde{L}_x(\Omega) = P_x(x, v, \Omega) \tilde{b}_x(\Omega) ,$$

$$\tilde{L}_v(\Omega) = P_v(x, v, \Omega) \tilde{b}_x(\Omega) ,$$

$$P_x(x, v, \Omega) = - \frac{1}{B} \left[\left(v - \frac{\omega}{k_z} \right) \left(1 + \frac{K_{yy} K_{zz}}{\Delta} \right) + \frac{\omega n_y^2 K_{yy}}{k_z \Delta} \right] ,$$

$$P_v(x, v, \Omega) = - \frac{q n_y K_{yy}}{m \Delta} . \quad (21)$$

Moreover, orthogonality relations in Ω and Ω' greatly simplify the expression for D_{jk} . For Fourier transforms defined, for example, with $-Z, Z$ window functions as in Eq. (11), the relation

$$\int_{-Z}^Z dz \exp[i(k_z + k'_z)z] = 2\pi \delta(k_z + k'_z)$$

is valid for integration over k_z or k'_z from $-\infty$ to ∞ .

Thus, D_{jk} becomes simply

$$D_{jk} = \frac{1}{8YZT} \int_{-\infty}^{\infty} d\Omega \frac{P_j(-\Omega) P_k(\Omega) \tilde{h}_x(-\Omega) \tilde{h}_x(\Omega)}{-i(\omega - k_z v)} . \quad (22)$$

6. DIFFUSION OF MAGNETIC LINES

At this point in the calculation, it is interesting to examine a related but much simpler problem, namely, the stochastic diffusion of magnetic lines of force. Taking the magnetic field suggested in Eq. (9), we follow a line of force along z for some $t = t_0$ and $y = y_0$. The x displacement over an interval of $2Z = B - A$ is

$$\Delta x = \frac{1}{B} \int_A^B b_x(z) dz ,$$

and $(\Delta x)^2$ may be written

$$(\Delta x)^2 = \frac{1}{B^2} \int_A^B dz \int_{A-z}^{B-z} d\zeta b_x(z) b_x(z+\zeta) .$$

Making use of an average over positions, z , the magnetic diffusion coefficient defined in Eq. (9) can be expressed

$$D_M = \frac{\langle (\Delta x)^2 \rangle}{2Z} = \frac{1}{B^2} \int_{-Z}^Z d\zeta \langle b_x(z) b_x(z+\zeta) \rangle ,$$

where $2Z$ must, of course, be much larger than the self-correlation length for $b_x(z)$. Now, we may also note that the Fourier transform of $b_x(z)$, namely, $\tilde{b}_x(k)$, is related in a simple fashion to the transform for $b_x(z+\zeta)$,

$$\left(\frac{1}{2\pi}\right)^{1/2} \int_{-Z}^Z dz b_x(z+\zeta) \exp(-ikz) \rightarrow \tilde{b}_x(k) \exp(ik\zeta)$$

for large Z . Then:

$$\langle b_x(z) b_x(z+\zeta) \rangle = \frac{1}{2Z} \frac{1}{2\pi} \int_{-Z}^Z dz \int_{-\infty}^{\infty} dk \int_{-\infty}^{\infty} dk' \tilde{b}_x(k) \tilde{b}_x(k') \exp(ik'\zeta) \quad (23)$$

to which we may again apply the orthogonality relation, yielding in turn

$$\begin{aligned} D_M &= \frac{1}{2ZB^2} \int_{-Z}^Z d\zeta \int_{-\infty}^{\infty} dk \tilde{b}_x(-k) \tilde{b}_x(k) \exp(-ik\zeta) \\ &= \frac{\pi}{B^2 Z} |\tilde{b}_x(0)|^2 \end{aligned} \quad (24)$$

This evaluation will help us in the interpretation of the quasilinear plasma diffusion coefficients, Eq. (22).

7. GENERAL PROPERTIES OF DIFFUSION COEFFICIENTS

Returning to discuss Eq. (22) now in more detail, we first note from (11) that $\tilde{b}_x^*(\Omega) = \tilde{b}_x(-\Omega^*)$, from (15) that $K_{zz}^*(\Omega) = K_{zz}(-\Omega^*)$, from (21) that $P_j^*(\Omega) = P_j(-\Omega^*)$, and hence, that the integrand in (22) also obeys $d_{jk}^*(\Omega) = d_{jk}(-\Omega^*)$. As the integration in (22) is over the entire domain of real Ω , the contributions from Ω and $-\Omega$ may always be paired off as $[d_{jk}(\Omega) + \text{complex conjugate}]$ in the integrand. Thus, D_{jk} is always real. Moreover, it is easily shown from (22), using the

Landau contour, that D_{xx} and D_{vv} are both positive diffusion coefficients. Finally, if P_x and P_v are each real, as in Case B, then D_{jk} forms a symmetric tensor in x, v space with one positive eigenvalue and one eigenvalue equal to zero [14].

Another feature of the quasilinear diffusion coefficients in Eq. (22) is that the contributions to D_{jk} from the different parts of the $|b_x(\Omega)|^2$ power spectrum are clearly algebraically additive. As mentioned earlier, the details of this power spectrum -- which depends on the field errors, instabilities, turbulence, etc., present in the plasma -- are outside the scope of this work. What can be usefully discussed, however, is the qualitative nature of the contributions to plasma transport from different regions of Ω -space.

8. FREE-STREAMING WITHOUT SHIELDING

It will turn out that collective effects, acting through the self-consistent electric fields which accompany the magnetic perturbation $\tilde{b}_x(\Omega)$, play an influential role in the transport picture. It is instructive, however, to look first at the simple model in which the only dynamics are the free-streaming of particles along the wandering magnetic lines of force, without Coulomb interaction. The relevant kinetic drift equation would be simply $\partial f / \partial t + v \hat{b} \cdot \nabla f = C(f)$, and in first-order, invoking a simple relaxation form for $C(f)$, $(-i\omega + ik_z v + \nu) \tilde{f} = -v(\partial f_0 / \partial x) \tilde{b}_x / B$. Eqs. (21) would reduce to $P_x = -v/B$ and

$P_v = 0$, and the only diffusion coefficient would be D_{xx} in Eq. (22). Evaluating this integral in the $v \rightarrow 0^+$ limit, we write

$$\frac{1}{-i(\omega - k_z v + i\nu)} \rightarrow -\frac{i}{v} P \left[\frac{1}{k_z - (\omega/v)} \right] + \frac{\pi}{|v|} \delta \left(k_z - \frac{\omega}{v} \right) , \quad (25)$$

where P denotes the Cauchy principal value, and where the absolute value of v must be introduced to preserve the correct path -- essentially the Landau contour -- around the pole in the complex k_z plane. Then, we have

$$D_{xx} = \frac{\pi |v|}{8YZTB^2} \int_{-\infty}^{\infty} d\Omega \tilde{b}_x(-\Omega) \tilde{b}_x(\Omega) \delta \left(k_z - \frac{\omega}{v} \right) , \quad (26)$$

and for a spectrum in $\tilde{b}_x(\Omega)$ peaked at the lowest frequencies, we can invoke relation (24) to obtain the common-sense result

$$D_{xx} = \frac{\pi |v|}{2ZB^2} |\tilde{b}_x(0)|^2 = \frac{|v|}{2} D_M . \quad (27)$$

9. DOMINANT TERMS FOR DIFFUSION

The complete quasilinear diffusion coefficients, as calculated in this paper, are given upon substituting the expressions for P_x and P_v , Eq. (21), into the integrand of (22). That many of the resulting terms are small compared to the dominant terms will now be shown.

First, in the expression of P_x , we may observe that $K_{yy}K_{zz}/\Delta \sim K_{yy}/n_z^2 \ll 1$ for waves well below the Alfvén velocity. Similarly, the $\omega n_y^2 K_{yy}/k_z \Delta$ term in P_x is small compared to the $-\omega/k_z$ term by approximately the ratio

$$\frac{n_y^2 K_{yy}}{n_z^2 K_{zz}} \sim \frac{k_y^2 z k T_e}{\omega_{ci}^2 m_i} \sim k_y^2 \rho_{Li}^2 \quad (28)$$

It is assumed in both of these estimates that $\Delta \sim n_z^2 K_{zz}$. Components in the power spectrum of the source, $|\tilde{b}_x(\Omega)|^2$, which are resonant with $\Delta \approx 0$ natural modes (i.e., resonant with ion acoustic or drift waves) will produce anomalously large contributions to the diffusion coefficients from these otherwise sub-dominant terms which, from (21) and (22), are of order Δ^{-1} or Δ^{-2} . We will not consider this special case of accidental resonance, but the sub-dominant terms will be discussed in another context in Section 11.

Finally, from (19), (21), and (22), it is clear that terms of order $P_v \partial f / \partial v$ compete with other terms $\sim P_x \partial f / \partial x$. The latter will be of order $[v - (\omega/k_z)] n_e / B a$, where a is the plasma radius while, using (28), the former will be of order

$\omega k_y \rho_{Li} n_e / B$. Scaling $k_y a \sim 1$, we see that the $P_x \partial f / \partial x$ term is larger than the velocity derivative term by approximately $[(\omega_{ci} / \omega) - (k_z \rho_{Li})^{-1}]$.

Summarizing, the dominant diffusion term is due to

$$P_x \approx -\frac{1}{B} \left(v - \frac{\omega}{k_z} \right); \quad P_v \approx 0.$$

Hence,

$$D_{xx} \approx \frac{1}{8YZT} \int_{-\infty}^{\infty} d\Omega \frac{[v - (\omega/k_z)] |\tilde{b}_x(\Omega)|^2}{ik_z B^2}, \quad (29)$$

and compared to D_{xx} , the effects of D_{xv} , D_{vx} , and D_{vv} may be neglected. An obvious exception, however, is the case for collisionfree resonant diffusion, discussed immediately below and also in Section 11.

Repeating the conditions under which D_{xx} in (29) represents the dominant diffusion, we have made use of $\vec{B}_0 = \hat{z}B$, $\omega \ll \omega_{ci}$, $|k_y \rho_{Li}| \ll 1$, $|\omega/k_z| \ll v_{Alfven}$, $\Delta \neq 0$, and $\omega \neq k_z v$.

Some discussion of the form of Eq. (29) is in order. The numerator, originally $[v - (\omega/k)]^2$, comes from $|P_x|^2$, Eq. (21), and originates in the two effects which contribute to the x-velocity of particles, namely, $v\tilde{B}_x/B \approx v\tilde{b}_x/B$ and $c\tilde{E}_y/B$. In this vein, the most important collective effect by the plasma for the diffusion under consideration in this paper is to "short out" the E_z field induced by current source. From (18) and (28), the voltage over a distance of one radian across the

plasma, $V_y = E_y/k_y$, may be compared to the voltage over one radian along B , $V_z = E_z/k_z$, showing $|V_y/V_z| \approx n_z^2 k_{zz}/n_y^2 k_{yy} = (k_y \rho_{Li})^{-2}$. Thus, $c\tilde{E}_y/B \approx -(\omega/k_z)\tilde{b}_x/B$, accounting for the appearance of ω/k_z in the numerator of (21). That the total x -velocity goes to zero when $v = \omega/k_z$ is then easily understood -- $\tilde{E}_z$ is effectively shorted out, the velocity of the particle relative to the wave is zero, and the particle senses no inductive E_y -field due to a changing B_x . Thus, all the effects which contribute to v_x disappear for resonant particles. It should be remembered, however, that the perturbation under consideration here is primarily magnetic. For perturbations which are mainly electrostatic, such as drift wave instabilities, the $c\tilde{E}_y/B$ component remains uncanceled [15,16].

10. NON-RESONANT DIFFUSION

In accordance with the discussion in Section 7, we note that the integration in (29), taking contributions from Ω and $-\Omega$ in pairs, provides that only the real part of the integrand, d_{xx} , contributes to D_{xx} . Thus, there are no contributions to D_{xx} from finite values of ω and k_z . On the other hand, there are two special cases in which D_{xx} is not zero.

10.1 ELECTRON MEANDERING

We consider the case where the bulk of the power in the excitation spectrum appears at phase velocities, ω/k_z , which

are small compared to the electron thermal velocity. Then, for all except the very slowest electrons, we can write $|v| \gg |\omega/k_z|$ and evaluate the pole at $k_z \rightarrow 0$, using the relation consistent with causality for an initial value problem in space,

$$\frac{1}{k_z} = \lim_{v \rightarrow 0} \left[\frac{1}{k_z - (i|v|/\bar{v})} \right] = P\left(\frac{1}{k_z}\right) + i\pi \frac{v}{|v|} \delta(k_z) , \quad (30)$$

to find

$$D_{xx} = \frac{|v|}{2} D_M , \quad (31)$$

exactly as in the case for free streaming, Eq. (27).

10.2 ION DIFFUSION: SPITZER'S RESULT

Equation (31) makes sense in that fast-streaming particles follow along lines of force and will be diffused just as the magnetic field lines are diffused. However, ions also "stick" to lines of force, and ion cross-field diffusion appears in another limit of Eq. (29). Recalling the discussion at the end of Section 9, pointing out specifically that $c\tilde{E}_Y/B \approx -(\omega/k_z)\tilde{b}_X/B$, and that the dominant ion motion is $\tilde{v}_X = c\tilde{E}_Y/B$, we rewrite the integrand in (29) as

$$d_{xx} \approx \frac{[1 - (k_z v/\omega)] |\tilde{E}_Y(\Omega)|^2 c^2}{-i\omega B^2} .$$

Particularly with respect to the ions, we consider now that the bulk of the power in the excitation spectrum belongs to phase velocities, ω/k_z , large compared to particle thermal velocities. Then for such slow particles, Eq. (29) can be

integrated using causality in time,

$$\frac{1}{\omega} \lim_{v \rightarrow 0} \left(\frac{1}{\omega + i|v|} \right) = P\left(\frac{1}{\omega}\right) - i\pi\delta(\omega) ,$$

to find

$$D_{xx} = \frac{\pi}{2T} \left| \frac{c\tilde{E}_Y(0)}{B} \right|^2 . \quad (32)$$

The use of (23) and (24) shows the equivalence of this result to that given in 1960 by Spitzer, [12]

$$2D_{xx} = D = \frac{2c^2}{B^2} \int_0^\infty d\tau \langle E_Y(t) E_Y(t + \tau) \rangle . \quad (33)$$

11. COLLISIONFREE RESONANT DIFFUSION

Although for both electrons and ions we found diffusion coefficients from the collisionless formalism, it is clear in the two derivations that neither the electrons nor the ions were resonant with the excited waves in the usual $\omega = k_z v$ sense with ω, k_z finite. On the other hand, true resonant collisionfree diffusion does occur for finite ω and k_z , but it is clear from (29) and the discussion following this equation that the contribution from this usually dominant term then drops to zero. Going back to (21) and (22), we have left

$$d_{xx} = \left| \frac{\omega n_Y^2 K_{YY}}{k_z \Lambda} \right|^2 \left| \frac{\tilde{b}_x}{B} \right|^2 \frac{\pi}{|k_z|} \delta\left(v - \frac{\omega}{k_z}\right) , \quad (34)$$

again a positive quantity. Similar expressions can be obtained for d_{xv} , d_{vx} , and d_{vv} . However, none of the resonant diffusion processes are apt to be important unless the excitation spectrum has strong components where Δ is especially small, i.e., near ion acoustic or drift wave resonance. The major significance of Eq. (34), perhaps, is just to indicate the magnitude of the corrections to the dominant diffusion, Eqs. (31) and (32).

12. WEAK COLLISIONS

The forms of the diffusion coefficients in (27) and (32) are somewhat remarkable in their dependence on zero-wavenumber or zero-frequency components of the excitation spectrum. In some ways these results are pleasing, namely in that normal wave motions with finite ω or k do not contribute to this non-resonant transport, and also in that it is precisely $|\tilde{b}_x(0)|^2$ that determines D_M , Eq. (27). On the other hand, it would also be satisfying if, by introducing weak collisions one could, for example, push the $k_z = 0$ and $\omega = 0$ poles into the imaginary part of the plane. Such turns out not to be the case and physically, that the theory is quite impervious to weak collisions is not hard to understand. The drifting particles move along $\vec{B}$ with two x-components to their velocity: $v_{\perp}(b_x/B)$, which guides the particles up or down along the instantaneous $\vec{B}$ line, and ce_y/B , which gives the

particles the same x -velocity (for $E_z \approx 0$) as the $\vec{B}$ line. Thus the particles, in this model, really stick to the $\vec{B}$ lines no matter what their parallel velocity may be -- both ions and electrons. Moreover, except for gyro-radius or banana-type displacements which already contribute to diffusion without magnetic stochasticity, the major collisional effect is to change $v_{||}$ which, as just argued, will not "unstick" the particle from $\vec{B}$. Therefore, weak collisions should not be expected to change the cross-field transport rates.

This inference is borne out in putting weak collisions into the theory with the Bhatnager-Gross-Krook collision model. Specifically, in first order let

$$C(f) = -v \left[f^{(1)} - n^{(1)} \frac{\partial f_0}{\partial n} - v^{(1)} \frac{\partial f_0}{\partial v} - T^{(1)} \frac{\partial f_0}{\partial T} \right] .$$

Now, in the two limits $|\omega/k_z| \gg v_{th}$ and $|\omega/k_z| \ll v_{th}$, it turns out that density, parallel fluid velocity, and temperature are, to a high approximation, transported with $\vec{B}$. This result is again not hard to understand -- both the ions and electrons move with their magnetic surfaces and carry the macroscopic parameters along. Thus, the leading term in computing $n^{(1)}$ is

$$\tilde{n} = \frac{-i\vec{B} \cdot \nabla}{k_z B} \frac{dn}{dx} + \dots = -\tilde{\Delta} x \frac{dn}{dx} + \dots$$

and similarly for $\tilde{v}$ and $\tilde{T}$. Therefore, $C(f)$ takes the form

$$C(f) \rightarrow -v \left(\tilde{f} + \tilde{\Delta x} \frac{\partial f_0}{\partial x} + \dots \right) ,$$

and writing $\tilde{\Delta x} = (v\tilde{b}_x + c\tilde{E}_y)/(-i\omega + ik_z v)B$, it is clear on comparison with (10) that, except for finite E_z contributions, the two collision terms cancel out.

13. COMPARISON OF FLUID AND KINETIC TRANSPORT

In Section 2 we examined electron transport in a braided magnetic field using a fluid picture of the electrons, while Section 10 and, in particular, Eq. (31) give the coefficient for cross-field electron diffusion according to both the Vlasov and the weak-collision formulations. To make a direct comparison of the transport rates, we use (19) and (31) to form and calculate

$$\begin{aligned} n\langle v_x \rangle &= - \int dv \left(D_{xx} \frac{\partial \bar{f}}{\partial x} + D_{xv} \frac{\partial \bar{f}}{\partial v} \right) \\ &\approx - \frac{1}{2} D_M \frac{d}{dx} (n\langle |v| \rangle) , \end{aligned} \quad (35)$$

and similarly

$$\begin{aligned} Q_x &= - \int dv \left(\frac{mv^2}{2} + kT_\perp \right) \left(D_{xx} \frac{\partial \bar{f}}{\partial x} + D_{xv} \frac{\partial \bar{f}}{\partial v} \right) \\ &\approx - \frac{1}{2} D_M \frac{d}{dx} (nkT_\perp \langle |v| \rangle + \frac{nm}{2} \langle |v|^3 \rangle) , \end{aligned} \quad (36)$$

for the long mean-free-path case. For both heat and particle

flux, the long mean-free-path transport rate is much faster than the fluid transport. Actually, the forms for particle transport are quite similar -- compare Eqs. (3) and (35). However, in (3) it is the absolute value of the fluid velocity that is used and in (35) it is the average of the absolute particle velocities. Had we, in the first model, considered electrons moving to the right and to the left as independent fluids, then (3) would have read

$$\Gamma_x = - \frac{1}{2} D_M \frac{\partial}{\partial x} (|n_R v_R| + |n_L v_L|) ,$$

which is identical to (35). Similarly, the fluid cross-field heat flux in Section 2 was

$$Q_x = - \frac{1}{2} D_M \frac{\partial}{\partial x} (|Q''^R| + |Q''^L|) ,$$

which bears the same relation to (36), namely that the long-mean-free-path picture calls for the average of the absolute particle-energy fluxes rather than the absolute value of the average. Q''^R and Q''^L designate, in the fluid picture, heat flowing to the right and to the left, away from an initial point. If we again were to consider the right- and left-moving electrons as independent fluids, this equation for Q_x would be the same as (36).

The difference between the two results then distills down exactly to the difference in models -- in the first model, the plasma and its heat thread their way in fluid-fashion along the braided magnetic lines, while in the second model, the

particles are able to free-stream individually along $\vec{B}$,
the only important collective effect being $E_z \approx 0$.

While the difference between the two models can thus
be described quite briefly, the questions of identifying and
analyzing the transition regime remain, at the time of this
writing, unanswered.

14. APPLICATIONS

Discussing the various results, let us look first at Eqs. (4) and (35). These equations might be applied to the circumstance of sudden onset of local magnetic braiding in a tokamak plasma. The parallel current has not had time to readjust to the destruction of good magnetic surfaces, so $j_{||} = j_{||}(x)$ and (4) or (35) computes the current divergence, dj_x/dx , which accompanies the onset of stochastic current channels [5]. The accumulation of charge sets up plasma rotation which is, in turn, dissipated by ion magnetic pumping [6-9].

In this same vein, one might speculate about the nature of the internal $m = 0$ sawtooth oscillations [10]. The edge of the $q = 1$ region in the core of a tokamak plasma provides an excellent candidate for strong magnetic braiding. With $D = \langle (\Delta x)^2 \rangle / |\Delta z| \sim (0.5^2/600) \text{ cm}$, $v_{th} = 2 \times 10^9 \text{ cm/sec}$ for the electrons, and $d/dx = (20 \text{ cm})^{-1}$, one estimates from (27) a radial electron drift velocity around $2 \times 10^4 \text{ cm/sec}$. Ions would then be pulled out by the polarization process, which in turn relaxes via ion magnetic pumping. The radial depolarization current is balanced by a radial current due to ion viscosity -- it is this $\vec{j} \times \vec{B}$ force which slows down the plasma rotation -- the time required for relaxation being of the order of τ_{ii} or less. The proposed model for the rapid expansion phase of the sawtooth dynamics then is that an overlap of magnetic islands near the $q = 1$ surface induces sudden onset of magnetic braiding, followed by rapid expansion of the electrons which

free-stream along the new stochastic $\vec{B}$ lines. The divergence of the radial electron current accumulates space charge which produces ion displacement through polarization which is accompanied by plasma poloidal rotation. The rotation is damped in its turn by ion magnetic pumping, allowing the ions to remain in their displaced positions and maintain quasineutrality.

Second, we look at the two expressions for radial heat flow, Eqs. (8) and (36). Transport by electron heat conduction offers an interesting scaling law, $t \sim n_e a^4 / T^{5/2}$, but collision-free transport is much faster, $t \approx a^2 / D_M v_{th}$. With a rather modest coefficient for magnetic diffusion, say $D_M = (0.1)^2 / 600$ cm, and again using $a = 20$ and $v_{th} = 2 \times 10^9$ cm/sec, the characteristic electron energy confinement time would be 12 milliseconds.

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