REFERENCE PARAMETERS AND POWER-DEPENDENT VARIABLES
IN THE DYNAMIC SIMULATION OF A 500-MWe DEMONSTRATION PLANT

by

L. J. Habegger
Energy and Environmental Systems
and

W. C. Lipinski
Components Technology Division

U of C-AUA-USERDA

BASE TECHNOLOGY

March 1976

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

The facilities of Argonne National Laboratory are owned by the United States Government. Under the terms of a contract (W-31-109-Eng-38) between the U. S. Energy Research and Development Administration, Argonne Universities Association and The University of Chicago, the University employs the staff and operates the Laboratory in accordance with policies and programs formulated, approved and reviewed by the Association.

MEMBERS OF ARGONNE UNIVERSITIES ASSOCIATION

The University of Arizona	Kansas State University	The Ohio State University
Carnegie-Mellon University	The University of Kansas	Ohio University
Case Western Reserve University	Loyola University	The Pennsylvania State University
The University of Chicago	Marquette University	Purdue University
University of Cincinnati	Michigan State University	Saint Louis University
Illinois Institute of Technology	The University of Michigan	Southern Illinois University
University of Illinois	University of Minnesota	The University of Texas at Austin
Indiana University	University of Missouri	Washington University
Iowa State University	Northwestern University	Wayne State University
The University of Iowa	University of Notre Dame	The University of Wisconsin

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately-owned rights. Mention of commercial products, their manufacturers, or their suppliers in this publication does not imply or connote approval or disapproval of the product by Argonne National Laboratory or the U. S. Energy Research and Development Administration.

TECHNICAL MEMORANDUM

ANL-CT-76-28
LMFBR Instrumentation
and Control (UC-79m)

REFERENCE PARAMETERS AND POWER-DEPENDENT VARIABLES
IN THE DYNAMIC SIMULATION OF A 500-MWe DEMONSTRATION PLANT

by

L. J. Habegger
Energy and Environmental Systems
and

W. C. Lipinski
Components Technology Division

U of C-AUA-USERDA

NOTICE
This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

BASE TECHNOLOGY

March 1976

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

TABLE OF CONTENTS

	<u>PAGE</u>
NOMENCLATURE	11
I. INTRODUCTION	1
II. STEADY-STATE PLANT VARIABLES	1
III. REFERENCE PARAMETERS AND DESIGN ANALYSIS DESCRIPTION	6
A. Feedwater/Steam loop	6
1. Turbogenerator	6
2. Feedwater Pump and Turbine	10
3. Hydraulic Analysis	13
4. Thermal Analysis	15
a) Recirculating Steam Generator	15
b) Superheater	27
5. Partial Load Analysis	30
B. Primary Sodium Loop	34
1. Intermediate Heat Exchanger (IHx)	34
2. Reactor	37
3. Reactor and IHx Partial Load Analysis	41
APPENDIX A STEADY-STATE EVAPORATOR HEAT BALANCE	43
APPENDIX B AVERAGE BOILING-REGION VOID-FRACTION	47
REFERENCES	49

Nomenclature

A_x :	Cross-section area of region x (ft^2).
a_j :	Constant.
c_{px} :	Specific heat in region x ($\text{Btu/lb-}^\circ\text{F}$).
D :	Diameter (ft).
f :	Friction coefficients.
g_j :	Constant.
h_x :	Enthalpy at node point x (Btu/lb).
$\bar{h}_x$:	Average enthalph in region x (Btu/lb).
h_{fx} :	Enthalpy of saturated water at node point x (Btu/lb).
h_{gx} :	Enthalpy of saturated steam at node point x (Btu/lb).
h' :	Heat transfer coefficient ($\text{Btu/}^\circ\text{F-ft}^2\text{-sec}$).
k :	Constant.
L_x :	Length of region x (ft).
P_x :	Pressure at node point x (psia).
$\bar{P}_x$:	Average pressure in region x (psia).
q :	Overall heat-transfer rate (Btu/sec).
R_x :	Two-phase friction multiplier at node point x .
$\bar{R}_x$:	Average two-phase friction multiplier in region x .
S_{vj} :	Normalized stroke for valve j . (1 for full open, 0 for full closed).
t :	Time (sec).
T_x :	Temperature at node point x ($^\circ\text{F}$).
$\bar{T}_x$:	Average temperature in region x ($^\circ\text{F}$).
T_{sat} :	Saturation temperature ($^\circ\text{F}$).
V_x :	Volume in region x (ft^3).

W : Power (MW or Btu/sec or ft-lb/sec).
 w_x : Mass flowrate at node point x (lb/sec).
 y_x : Height of region x (ft).
 α_x : Steam void fraction at node point x .
 $\bar{\alpha}_x$: Average steam void fraction in region x .
 η : Efficiency.
 ρ_x : Mass density at node point x (lb/ft³).
 $\bar{\rho}_x$: Average mass density in region x (lb/ft³).
 ρ_f : Water mass density (lb/ft³).
 ρ_{gx} : Saturated steam mass density at node point x (lb/ft³).
 χ_x : Steam quality at node point x .
 ω : Normalized frequency.

I. Introduction

A reference design for a 500 MWe LMFBR demonstration plant has been developed for use in analytical plant dynamics and control studies (189a 02-528 Plant Dynamics and Control Analysis). The principal steady-state plant temperatures, flow rates, pressures, etc., which are obtained on the basis of this reference design are presented in the following section. Included are values of selected variables over the 40-100% load range (200-500 MWe).

The basic assumption relating to plant models, computation procedures, and parameter values which were used to determine these steady-state plant variables are described in Section III. Results in Section III are reported in sufficient detail to permit recomputation of steady-state plant variables following any changes in the design of the plant being simulated.

II. Steady-State Plant Variables

A flow diagram for the simulated Demonstration Plant is shown in Fig. 1. The regions and nodes used in the plant model are also indicated in Fig. 1.

Steady-state values of plant variables at the design power of 500 MWe are given in Table 1.

Sodium temperatures and sodium and feedwater flow rates as a function of power level are illustrated in Fig. 2 and 3 respectively. The steady-state enthalpies of the feedwater supply and steam at the turbine inlet are assumed constant for all power levels at the values given in Table 1.

The bases for the values in Table 1 and Fig. 2 and 3 are described in the following section.

Fig. 1 Flow Diagram for LMFBR Demonstration Plant with Recirculating Steam Generator.

TABLE I. FULL POWER (500 MWe) STEADY-STATE PLANT VARIABLES

$w_p = 10500. \text{ lb/sec}$	$T_{i7} = 892.4^\circ\text{F}$	$P_{s3} = 2016 \text{ psia}$
$w_i = 9380.6 \text{ lb/sec}$	$T_{sa} = 636.5^\circ\text{F}$	$P_{s4} = 2010 \text{ psia}$
$w_{s5} = 1047.4 \text{ lb/sec}$	$T_{s9} = 1000^\circ\text{F}$	$P_{sa} = 2010 \text{ psia}$
$w_{s8} = 1015.5 \text{ lb/sec}$	$h_{s5} = 394.6 \text{ Btu/lb}$	$P_{sc} = 1850 \text{ psia}$
$w_{s10} = 31.9 \text{ lb/sec}$	$h_{s8} = 1480.8 \text{ Btu/lb}$	$P_{s8} = 1800 \text{ psia}$
$w_{s1} = 3400.6 \text{ lb/sec}$	$x_{s4} = 0.308$	$P_{s9} = 1800 \text{ psia}$
$T_{p1} = 730^\circ\text{F}$	$P_s = 65 \text{ psia}$	$P_{s10} = 1800 \text{ psia}$
$T_{p4} = 1087.3^\circ\text{F}$	$P_{sd} = 2680 \text{ psia}$	$P_{s11} = 1800 \text{ psia}$
$T_{i1} = 620^\circ\text{F}$	$P_{s1} = 2060 \text{ psia}$	$P_{s12} = 1825 \text{ psia}$
$T_{i3} = 1020^\circ\text{F}$	$P_{s2} = 2049 \text{ psia}$

Fig. 2. Power-dependent plant temperatures.

Fig. 3. Power-dependent flow rates.

III. Reference Parameters and Design Analysis Description

This section provides a detailed description of the analysis and assumptions which resulted in the plant variables presented in the previous section.

Subscripts for variables refer to region or node points illustrated in Fig. 1 (see nomenclature). The subscript (o) refers to values at design power conditions.

A. Feedwater/Steam Loop

1. Turbogenerator.

For the reference design a gross electrical output of 500 MWe is assumed which includes auxiliary plant requirements and power supplied to the utility grid. The assumed turbine conditions at all power levels is 1800 psia, 1000°F at the inlet and 2 in Hg. at the exhaust.

The following heat-balance values are used at the design power. (Values are approximately 50% of those given for a 1000 MWe design in Ref.1):

- 480 MWe: Net electrical output
- 20 MWe: Auxiliary plant requirements
- 510.6 MWt: Power extracted from steam by turbine
- 37.4 MWt: Power extracted from steam by feedwater heaters
- 7.0 MW: Generator losses
- 3.6 MW: Turbine losses

The net plant efficiency is assumed to be 40%.

A multivalve turbine, such as the one assumed in the present model, in general has a number of its governing valves fully open with no pressure drop, a second group of valves are partially opened with a corresponding pressure drop, and the remaining valves are fully closed. Preceding the governing valves is a throttle valve which is assumed to be open in the model. Following each governing valve is a set of isentropic nozzles leading to the first stage blading. It will be assumed that the effect of partially opened valves is negligible and thus $P_{s8} = P_{s9}$, $T_{s8} = T_{s9}$, and $h_{s8} = h_{s9}$.

Flow through an individual nozzle following an open valve is (Ref. 6, p. 121)

$$w_{s9} = \Delta' [\rho_{s9} (P_{s9} - P_{T1})]^{1/2} \text{ (lb/sec)} \quad (1)$$

where P_{T1} is the pressure at the nozzle exit.

For the total flow through n open governing valves

$$w_{s9} = \left(\frac{n}{N} \Delta' \right) \left[\rho_{s9} (P_{s9} - P_{T1}) \right]^{1/2} \text{ (lb/sec)} \quad (2)$$

$\Delta = N\Delta'$, N is the total number of valves.

Assuming a large value for N, the discrete ratio n/N can be replaced by a continuous variable S_{v2} .

Equation (2) can then be written

$$P_{s9} - P_{T1} = \left(\frac{w_{s9}}{k_{v2,1} S_{v2}} \right)^2 \text{ (psia)} \quad (3)$$

where $S_{v2} = n/N$, $k_{v2,1} = \Delta \sqrt{\rho_{s9}}$, and ρ_{s9} is assumed constant.

Equation (1) is only valid for subcritical nozzle flow, i.e., $\frac{P_{T1}}{P_{s9}} > 0.55$.
For supercritical flow; i.e., $\frac{P_{T1}}{P_{s9}} < 0.55$,

$$P_{s9} = \frac{w_{s9}}{k_{v2,2} S_{v2}} \text{ (psia)} \quad (4)$$

The additional relation required to determine turbine flow is

$$P_{T1} = k_{T1} w_{s9} \text{ (psia)} \quad (5)$$

Power extracted from the steam is given by

$$W_{T1} = w_{s9} \eta_{T1} (h_{s9} - h_{exh}^*) \text{ (Btu/sec)} \quad (6)$$

where h_{exh}^* is the ideal enthalpy at the turbine exhaust which results from an isentropic expansion of the inlet steam to the turbine outlet pressure and η_{T1} is the efficiency given by

$$\eta_{T1} = \eta_r \eta_b \quad (7)$$

where η_r accounts for nonisentropic expansion and is assumed to be 0.85 and η_b accounts for steam extraction for feedwater heating and other turbine losses.

From the heat balance values given previously

$$\eta_b = 1 - \frac{37.4 + 3.6}{510.6 + 37.4}$$

$$= 0.925$$

$$\text{and } \eta_{T1} = 0.78625 \quad (8)$$

For inlet conditions of 1800 psia, 1000°F, and an exhaust pressure of 2 in. Hg.¹¹,

$$h_{\text{exh}}^* = 878.8 \text{ Btu/lb} \quad (9)$$

$$h_{s9} = 1480.8 \text{ Btu/lb}$$

The expression for generator losses is $k_{11} \omega_G^2$. Using a normalized shaft velocity, $\omega_{Go} = 1.0$, and generator losses of 7.0 MW (assumed constant at all power levels),

$$k_{11} = 7.0 \text{ MW} \quad (10)$$

The net turbine shaft power is thus

$$W_{T10} = 507 \text{ MW} \quad (11)$$

$$= 4.807 \times 10^5 \text{ Btu/sec}$$

Combining (6), (8), (9), and (11) gives the design turbine flow

$$w_{s90} = 1015.5 \text{ lb/sec} \quad (12)$$

By assuming that flow through the turbine nozzles becomes critical at 60% design flow, i.e., $P_{T1} = 0.55 P_{s9}$ at $w_{s9} = 0.6 w_{s90}$, then from (5)

$$k_{T1} = 1.6248 \text{ psia/(lb/sec)} \quad (13)$$

At full flow (5), (12), and (13) give

$$P_{T10} = 1650 \text{ psia} \quad (14)$$

A full power turbine valve position of

$$S_{v20} = 0.95 \quad (15)$$

is assumed, which from (3) and (14) gives

$$k_{v2,1} = 87.279 \text{ lb/sec-psia}^{1/2} \quad (16)$$

At critical flow the pressure difference given by (3) and (4) must be identical. Equation (3) can be solved for S_{v2} at critical flow, and S_{v2} is substituted into (4) to obtain

$$k_{v2,2} = 1.3800 \text{ lb/sec-psia} \quad (17)$$

2. Feedwater Pump and Turbine^{3,5}

The output power of the variable velocity turbine pump is

$$W_{T2} = \frac{w_{s11} - a_{53}}{a_{54} - a_{55} \omega_s} (\text{lb}_f\text{-ft/sec}) \quad (18)$$

which, in steady-state is equal to the pump power:

$$W_{ps} = \frac{144.(P_{sd} - P_s) w_{s5}}{\rho_f \eta_s} \text{ (lb}_f\text{-ft/sec)} \quad (19)$$

$$\text{where } w_{s5} = w_{s9} + w_{s11} \text{ (lb/sec)} \quad (20)$$

and the pump efficiency is⁷

$$\eta_s = \eta_{so} \left[2 \frac{w_{s5}}{w_{s5o}} \frac{\omega_{so}}{\omega_s} - \left(\frac{w_{s5}}{w_{s5o}} \frac{\omega_{so}}{\omega_s} \right)^2 \right] \quad (21)$$

The pressure drop across the pump is

$$P_{sd} - P_s = k_{sp,1} \omega_s^2 - k_{sp,2} w_{s5}^2 \text{ (psia)} \quad (22)$$

Pressure flow relations in the turbine are similar to those in the previous section:

$$P_{s11} - P_{T2} = \left(\frac{w_{s11}}{k_{v4,1} S_{v4}} \right)^2 \text{ (psia)}, \frac{P_{T2}}{P_{s11}} > 0.55 \quad (23a)$$

$$P_{s11} = \frac{w_{s11}}{k_{v4,2} S_{v4}} \text{ (psia)}, \frac{P_{T2}}{P_{s11}} < 0.55 \quad (23b)$$

$$P_{T2} = k_{T2} w_{s11} \text{ (psia)} \quad (23c)$$

To approximate the constants in (18) - (22), the following design values are assumed:

$$\omega_{so} = 1.0 \quad (24)$$

$$a_{53} = 0.05 \omega_{s110} \text{ lb/sec} \quad (25)$$

$$\frac{a_{55}}{a_{54}} = 0.485 \text{ (Ref. 5)} \quad (26)$$

$$\frac{1}{a_{54} - a_{55} \omega_{so}} = \eta_{T2} (h_{s11} - h_{exh}^*) J^\dagger (\text{lb}_f\text{-ft/lb}_m) \quad (27)$$

$$\eta_{T2} = 0.85 \quad (28)$$

$$\begin{aligned} h_{s11} &= h_{s9} \text{ (Btu/lb)} \\ &= 1480.8 \text{ Btu/lb} \end{aligned} \quad (29)$$

$$h_{exh}^* = 878.8 \text{ Btu/lb} \quad (30)$$

$$\eta_{so} = 0.85 \quad (31)$$

$$P_{sdo} - P_s = 0.85 k_{sp,1} \omega_{so}^2 \text{ (psia)} \quad (32)$$

$$S_{v40} = 0.95 \quad (33)$$

$$P_{T2} = 0.55 P_{s11} \text{ at } \omega_{s11} = 0.60 \omega_{s110} \quad (34)$$

Constant turbine inlet conditions of 1800 psia, 1000°F and an exhaust pressure of 2 in. Hg. are assumed as for the main steam turbine.

With the above assumptions and design values given in (55) - (62) the following parameters are obtained:

$$a_{53} = 1.5932 \text{ lb/sec} \quad (35)$$

$$a_{54} = 4.8775 \times 10^{-6} \text{ lbm/lb}_f\text{-ft} \quad (36)$$

$$a_{55} = 2.3656 \times 10^{-6} \text{ lbm/lb}_f\text{-ft} \quad (37)$$

$$\rho_f = 38.5 \text{ lb/ft}^3 \text{ (640°F)} \quad (38)$$

$$k_{sp,1} = 3076.5 \text{ psia} \quad (39)$$

$$k_{sp,2} = 4.2067 \times 10^{-4} \text{ psia/(lb/sec)}^2 \quad (40)$$

$$k_{v4,1} = 2.7282 \text{ lb/sec-psia}^{1/2} \quad (41)$$

$\dagger J = 778 \text{ ft-lb}_f/\text{Btu}$

$$k_{v4,2} = 0.043137 \text{ lb/sec-psia} \quad (42)$$

$$k_{T2} = 51.748 \text{ psia/(lb/sec)} \quad (43)$$

The resultant design-power turbine flow is

$$w_{s11o} = 31.863 \text{ lb/sec} \quad (44)$$

and

$$\begin{aligned} w_{s5o} &= 1015.5 + 31.863 \\ &= 1047.4 \text{ lb/sec} \end{aligned} \quad (45)$$

3. Hydraulic Analysis

The feedwater/steam pressure flow relations are

$$P_{s8} = P_{s9} \text{ (psia)} \quad (46)$$

$$P_{s12} = P_{s8} + \left(\frac{w_{s9}}{k_{s8}} \right)^2 \text{ (psia)} \quad (47)$$

$$P_{s10} = P_{s11} \text{ (psia)} \quad (48)$$

$$P_{s12} = P_{s10} + \left(\frac{w_{s11}}{k_{s10}} \right)^2 \text{ (psia)} \quad (49)$$

$$P_{sc} = P_{s12} + \left(\frac{w_{sc}}{k_{s12}} \right)^2 \text{ (psia)} \quad (50)$$

$$P_{sa} = P_{sc} + \left(\frac{w_{sc}}{k_{ss}} \right)^2 \text{ (psia)} \quad (51)$$

$$P_{s4} = P_{s5} = P_{s6} = P_{s7} = P_{sa} \quad (52)$$

$$P_{v3} = P_{s5} + \left(\frac{w_{s5}}{k_{v3} S_{v3}} \right)^2 \text{ (psia)} \quad (53)$$

$$P_{sd} = P_{v3} + \left(\frac{w_{s5}}{k_{aF}} \right)^2 \text{ (psia)} \quad (54)$$

In the steady-state: $w_{s5} = w_{s7} = w_{sa} = w_{sc}$

Pressure flow relations within the recirculating steam generator are discussed in Section III.A.4.a.

Constants in (46) - (54) are based on the following assumed design values which are similar to those in Ref. 1:

$$P_{s8o} = 1800 \text{ psia} \quad (55)$$

$$P_{s10o} = 1800 \text{ psia} \quad (56)$$

$$P_{s12o} = 1825 \text{ psia} \quad (57)$$

$$P_{sco} = 1850 \text{ psia} \quad (58)$$

$$P_{sao} = 2010 \text{ psia} \quad (59)$$

$$= P_{s4o}$$

$$P_{v3o} = 2310 \text{ psia} \quad (60)$$

$$P_{sdo} = 2680 \text{ psia} \quad (61)$$

$$P_s = 65 \text{ psia} \quad (62)$$

A steady-state feedwater valve position of $S_{v3} = 0.8$ is assumed for all power levels. Using the above

$$k_{s8} = 203.1 \text{ lb/sec-psia}^{1/2} \quad (63)$$

$$k_{s12} = 209.48 \text{ lb/sec-psia}^{1/2} \quad (64)$$

$$k_{s10} = 6.3726 \text{ lb/sec-psia}^{1/2} \quad (65)$$

$$k_{ss} = 82.804 \text{ lb/sec-psia}^{1/2} \quad (66)$$

$$k_{v3} = 75.590 \text{ lb/sec-psia}^{1/2} \quad (67)$$

$$k_{sF} = 54.452 \text{ lb/sec-psia}^{1/2} \quad (68)$$

4. Thermal Analysis

a) Recirculating Steam Generator

With a 40% plant efficiency and 480 MWe net output, the thermal output of the plant is 1200 MWt. The feedwater enthalpy at the steam drum input is given by (9), (45) and

$$w_{s5} (h_{s9} - h_{s5}) = 1200 \text{ MWt} \quad (69)$$

which gives

$$h_{s5} = 394.62 \text{ Btu/lb} \quad (70)$$

This value is assumed constant at all power levels.

The recirculating steam generator steady-state flow is

$$w_{s4} = \frac{w_{s5}}{x_{s4}} \text{ (lb/sec)} \quad (71)$$

Using the basic assumption that at full power the outlet quality is

$$x_{s4o} = 0.3 \quad (72)$$

gives, using (45)

$$w_{s4o} = 3491.3 \text{ lb/sec or } 0.51043 \text{ lb/sec/tube} \quad (73)$$

Using as reference points the steam drum pressures at 100% and 40% of full power, i.e. 2010 psia and 1834.4 psia, the following linearized equations-of-state are obtained at saturation¹¹:

$$T_{\text{sat}} (^{\circ}\text{F}) = 488.85 + 0.07346 P \text{ (psia)} \quad (74)$$

$$h_g \text{ (Btu/lb)} = 1279.7 - 0.072323 P \text{ (psia)} \quad (75)$$

$$h_f \text{ (Btu/lb)} = 438.25 + 0.11674 P \text{ (psia)} \quad (76)$$

$$T_{\text{sat}} (^{\circ}\text{F}) = 213.08 + 0.62926 h_f \text{ (Btu/lb)} \quad (77)$$

Eqn. (77) is also assumed to be a valid relation between temperature and enthalpy for subcooled liquid.

The following evaporator dimensions are assumed (see Ref. 1):

$$\text{Tube OD: } 0.625 \text{ in.} \quad (78)$$

$$\text{Pitch: } 1.25 \text{ in., square} \quad (79)$$

$$\text{Wall thickness: } 0.095 \text{ in.} \quad (80)$$

$$\text{Tubes/module: } 380 \quad (81)$$

$$\text{Modules: } 18 \quad (82)$$

The tube lengths are determined as follows. The riser cross-section for each module was assumed equal to the total tube cross-section for that module. The assumed riser height and tube length are, respectively,

$$y_{s3} = 8 \text{ ft} \quad (83)$$

$$L_{s3} = 14 \text{ ft} \quad (84)$$

The evaporator pressures which are assumed at full power are (see Ref. 1)

$$\begin{aligned} P_{s3o} &= P_{s4o} + 6 \\ &= 2016 \text{ psia} \end{aligned} \quad (85)$$

$$\begin{aligned} P_{s2o} &= P_{s3o} + 33 \\ &= 2049 \text{ psia} \end{aligned} \quad (86)$$

$$\begin{aligned} P_{s1o} &= P_{s2o} + 11 \\ &= 2060 \text{ psia} \end{aligned} \quad (87)$$

In the subcooled region of the evaporator the tube-to-water heat-transfer coefficient is (see Ref. 1 for definition of parameters)

$$h_w' = \frac{k_w}{D_i} Nu_w \text{ (Btu/}^\circ\text{F-ft}^2\text{-sec)} \quad (88)$$

where

$$Nu_w = 0.023 \left(\frac{w_{sl}' D_i}{A_i \mu_w} \right)^{0.8} \left(\frac{C_{pw} \mu_w}{k_w} \right)^{0.4} \quad (89)$$

and w_{sl}' is the flow per tube in lb/sec. Using ^{11*}

$$C_{pw} = 1.34 \text{ Btu/lb-}^\circ\text{F} \quad (90)$$

$$k_w = 8.97 \times 10^{-5} \text{ Btu/sec-}^\circ\text{F-ft} \quad (91)$$

$$\mu_w = 6.162 \times 10^{-5} \text{ lb/ft-sec} \quad (92)$$

gives

$$(k_w^{0.6}) (C_{pw}/\mu_w)^{0.4} \approx 0.20, \text{ and}$$

$$h_w' \text{ (Btu/sec-}^\circ\text{F-ft}^2\text{)} = 2.187 (w_{sl}')^{0.8} \quad (93)$$

On the shell side, the sodium-to-tube heat-transfer coefficient is (see Ref. 1 for definition of parameters)

*The temperature dependence of these parameters has little effect on the heat-transfer coefficient over the temperature range of interest.

$$h'_n = \frac{Nu \ k_n}{D_e} \text{ (Btu/°F-ft}^2\text{-sec)} \quad (94)$$

$$Nu_n = 1.3 \left[4.0 + 0.024 \left(\frac{w'_1 D_e C_{pn}}{A_c k_n} \right)^{0.8} \right] \quad (95)$$

where w'_1 is the sodium flow per tube in (lb/sec). Using*

$$C_{pn} = 0.3032 \text{ Btu/lb-°F} \quad (96)$$

$$k_n = 0.011 \text{ Btu/sec-°F-ft} \quad (97)$$

gives

$$h'_n = (\text{Btu/sec-°F-ft}^2) = 0.2683 + 0.2949 (w'_1)^{0.8} \quad (98)$$

The assumed tube thermal conductivity and water-side fouling-factor heat-transfer coefficient are, respectively ¹,

$$\begin{aligned} k_t &= 4.643 \times 10^{-3} \text{ Btu/sec-°F-ft} \\ h_{ff} &= 1.3169 \text{ Btu/sec-°F-ft}^2 \end{aligned} \quad (99)$$

The evaporator inlet enthalpy is

$$\begin{aligned} h_{s1} &= h_{s6} \text{ (Btu/lb)} \\ &= \frac{w_{s5} h_{s5} + w_{s6} (1 - \chi_{s4}) h_{f,s4}}{w_{s6}} \\ &= \chi_{s4} h_{s5} + (1 - \chi_{s4}) h_{f,s4} \end{aligned} \quad (100)$$

*The temperature dependence of these parameters has little effect on the heat-transfer coefficient over the temperature range of interest.

At full power, i.e. 2010 psia,¹¹

$$h_{f,s4o} = 672.96 \text{ Btu/lb} \quad (101)$$

and from (100)

$$h_{slo} = 589.46 \text{ Btu/lb} \quad (102)$$

From Ref. 11,

$$T_{slo} = 580.41^\circ\text{F} \quad (103)$$

At full power the heat-transfer rate per tube in the evaporator is

$$q = \frac{(h_{sa} - h_{s5}) w_{s5}}{380 \times 18} \text{ (Btu/sec-tube)} \quad (104)$$

Using

$$h_{sa} = h_{g,s4}, \text{ (Btu/lb)} \quad (105)$$

(59), and Ref. 11:

$$h_{sao} = 1134.3 \text{ Btu/lb} \quad (106)$$

From (45), (70), (104), and (106):

$$q = 113.27 \text{ Btu/sec-tube} \quad (107)$$

The assumed evaporator sodium outlet temperature is

$$T_{i9o} = 620^\circ\text{F} \quad (108)$$

and for the superheater inlet¹

$$\begin{aligned} T_{1ao} &= T_{19o} + 400 \\ &= 1020^{\circ}\text{F} \end{aligned} \quad (109)$$

The sodium flow rate at full power is obtained from

$$w_i = \frac{(h_{sc} - h_{s5}) w_{s5}}{c_{pi} (T_{1a} - T_{19})} \quad (\text{lb/sec}) \quad (110)$$

Using

$$\begin{aligned} h_{sc} &= h_{s9}, (9), (45), (70) \text{ and} \\ c_{pi} &= 0.3032 \text{ Btu/lb-}^{\circ}\text{F}, \end{aligned} \quad (111)$$

which is assumed constant at all power levels, gives

$$w_{io} = 9380.5 \text{ lb/sec} \quad (112)$$

At other power levels, the sodium flow rate is assumed proportional to the gross electrical output W_G :

$$w_i = w_{io} \frac{W_G}{W_{GO}} \quad (113)$$

The evaporator tube length which gives the required heat transfer rate can now be determined using the above full-power parameters and the method described in Appendix A. The computed tube length is

$$L_{s1} = 14.425 \text{ ft} \quad (114)$$

$$L_{s2} = 11.490 \text{ ft} \quad (115)$$

$$\begin{aligned} L_e &= L_{s1} + L_{s2} \\ &= 25.915 \text{ ft} \end{aligned} \quad (116)$$

Evaporator and Riser Pressure Drop Coefficients

The pressure drop across each region is the sum of frictional, hydrostatic, and momentum effects:³

$$\Delta P = \Delta P_f + \Delta P_h + \Delta P_m \quad (\text{psia}) \quad (117)$$

Where

$$\Delta P_f = (f_a \bar{R} L + f_b) (w')^2 \quad (\text{psia}) \quad (118)$$

$$\equiv \Delta P_{fa} + \Delta P_{fb} \quad (\text{psia})$$

$$\Delta P_h = \rho \Delta y \frac{g}{g_c (144)} \quad (\text{psia}) \quad (119)$$

Where $g = 32.2 \text{ ft/sec}^2$ and $g_c = 32.2 \text{ lb}_m\text{-ft/lb}_f\text{-sec}^2$

$$\Delta P_m = \left[\frac{1}{\rho_{\text{out}}} - \frac{1}{\rho_{\text{in}}} \right] \frac{(w')^2}{A^2 g_c (144)} \quad (120)$$

In (118)

$$f_a = \frac{4f}{2D A^2 \rho_f g_c} \quad (121)$$

where D is the tube inside diameter, A is the cross-section, ρ_f is given by (38), f is an empirical function of the Reynolds number (Fig. 6.19, Ref. 8), and w' is the flow per tube, e.g., 0.51042 lb/sec at full power.

The Reynolds number is

$$\text{Re} = \frac{D w'}{A \mu_w} \quad (122)$$

and at full power

$$\text{Re} = 2.91 \times 10^5$$

and the corresponding friction factor for average commercial pipe is

$$f = 4.7 \times 10^{-3}$$

where μ_w is given by (92). Using the full power parameters in the evaporator

$$Re = 2.91 \times 10^5$$

$$f = 4.7 \times 10^{-3} \text{ (average for commercial pipe from Ref. 8, Fig. 6.19)}$$

which gives

$$\begin{aligned} f_{as1} &= f_{as2} \\ &= 1.3636 \text{ psia/ft}-(\text{lb/sec})^2 \end{aligned} \quad (123)$$

This value is used for all power levels.

From Ref. 9, the friction multiplier, $\bar{R}$, can be computed from

$$\bar{R} = \frac{1}{3} \left[1 + \gamma_{\text{exit}} + (\gamma_{\text{exit}})^2 \right] \quad (124)$$

$$\begin{aligned} \gamma &= \frac{1 - \chi}{1 - \alpha} \\ &= 1 + \left(\frac{\rho_f}{S_\ell \rho_g} - 1 \right) \chi \end{aligned} \quad (125)$$

S_ℓ , the slip ratio, is assumed to be unity. Using reference pressures of 1850 psia and 2032.5 psia, the following linearized equation for ρ_g is obtained

$$\rho_g \text{ (lb/ft}^3\text{)} = -2.20316 + 3.7649 \times 10^{-3} P(\text{psia}) \quad (126)$$

Boiling Region. For the boiling region exit, s3, at full power from (85),

$\rho_{gs30} = 5.3869$, and using (38), (72), and (125) gives

$$\gamma_{s30} = 2.8441 \quad (127)$$

and

$$\bar{R}_{s20} = 3.977 \quad (128)$$

which gives, using (73), (115) and (118)

$$\Delta P_{fao} = -16.2 \text{ psia} \quad (129)$$

The average region void fraction, as described in Appendix B, is approximated by

$$\bar{\alpha}_{s2} = \frac{x_{s3} + 0.20534}{0.26987 + 3.6319 \times 10^{-4} \bar{P}_{s2} \text{ (psia)}} \quad (130)$$

where

$$\bar{P}_{s2} = \frac{P_{s2} + P_{s3}}{2} \text{ (psia)} \quad (131)$$

At full power, using

$$\begin{aligned} x_{s3o} &= x_{s4o} \\ &= 0.3 \end{aligned} \quad (132)$$

and

$$\bar{P}_{s2o} = 2032.5 \text{ psia} \quad (133)$$

gives

$$\bar{\alpha}_{s2} = 0.5013 \quad (134)$$

The average region density is

$$\bar{\rho}_{s2} = \bar{\alpha}_{s2} \bar{\rho}_{gs2} + (1 - \bar{\alpha}_{s2}) \rho_f \text{ (lb/ft}^3\text{)} \quad (135)$$

which, when used with (38) and (126) gives

$$\bar{\rho}_{s2o} = 21.932 \text{ lb/ft}^3 \quad (136)$$

The hydrostatic pressure drop in the boiling region at full power is thus, from (115) and (119)

$$\Delta P_h = -1.7 \text{ psia} \quad (137)$$

At the boiling region exit

$$\rho_{s3} = \alpha_{s3} \rho_{gs3} + (1 - \alpha_{s3}) \rho_f \quad (\text{lb/ft}^3) \quad (138)$$

where α_{s3} is computed from

$$\alpha_{s3} = \frac{\chi_{s3}}{S_L \rho_{gs3} \frac{\chi_{s3}}{(1 - \chi_{s3})/\rho_f + \chi_{s3}}} \quad (139)$$

Using the above and (85), (126), and (132), at full power

$$\alpha_{s3o} = 0.75388 \quad (140)$$

$$\rho_{s3o} = 13.537 \quad (\text{lb/ft}^3)$$

and thus from (120)

$$\Delta P_{mo} = -2.5 \text{ psia} \quad (141)$$

From (86) the assumed full power pressure drop in the boiling region is 33 psia, and thus from (117) and (118)

$$\begin{aligned}\Delta P_{fbo} &= 12.6 \\ &= (w_{s1})^2 f_{bs2} \text{ (psia)}\end{aligned}\tag{142}$$

or

$$f_{b,s2} = 48.361 \text{ psia}/(\text{lb/sec})^2\tag{143}$$

which is assumed constant at all power levels .

Preheat Region. In the preheat region of the evaporator using (114), (118) and

$$\bar{R}_{s1} = 1.0\tag{144}$$

gives

$$\Delta P_{fa} = -5.1 \text{ psia}\tag{145}$$

at full power. Since there is no density change

$$\Delta P_m = 0\tag{146}$$

From (38), (114) and (119),

$$\Delta P_h = -3.9 \text{ psia}\tag{147}$$

From (87), the total pressure drop in this region at full power is 11 psia, and thus from (117) and (118)

$$\begin{aligned}\Delta P_{fb} &= f_{b,s1} (w_{s1})^2 \\ &= -2.0 \text{ psia}\end{aligned}\tag{148}$$

and

$$f_{b,s1} = 7.6764 \text{ psia}/(\text{lb/sec})^2\tag{149}$$

which is assumed constant at all power levels.

In the riser region it is assumed that

$$\begin{aligned} P &\approx \Delta P_{fb} \\ &= f_{b,s3} (w'_{s3})^2 \quad (\text{psia}) \end{aligned} \quad (150)$$

where the flow per module is

$$w'_{s3} = w_{s1}/18 \quad (\text{lb/sec}) \quad (151)$$

For the pressure drop given by (85) and from (73),

$$f_{b,s3} = 1.5948 \times 10^{-4} \text{ psia}/(\text{lb/sec})^2 \quad (152)$$

b. Superheater

The superheater tube inside diameter, wall thickness, and pitch are assumed equal to evaporator dimensions given by (78), (79), and (80). These dimensions are also equal to superheater dimensions given in Ref. 1. Twelve modules with 380 tubes per module is assumed, which is equal to the number of tubes in the 1000 MWe plant in Ref. 1. The tube length is determined by the full power thermal analysis as described below.

The sodium-to-tube heat-transfer coefficient is as given by (98) for the evaporator. On the tube-side, the tube-to-steam heat-transfer coefficient is assumed to have the functional form as in (93). Using the Ref. 1 value of $h'_w = 0.45972 \text{ Btu/sec-}^\circ\text{F-ft}^2$ for $w'_{sa} = 0.41813 \text{ Btu/sec-tube}$, implies

$$h'_w (\text{Btu/sec-}^\circ\text{F-ft}^2) = 0.92352 (w'_{sa})^{0.8} \quad (153)$$

The assumed tube thermal conductivity and fouling-factor heat-transfer coefficient are¹

$$k_t = 3.7778 \times 10^{-3} \text{ Btu/sec-}^\circ\text{F-ft} \quad (154)$$

$$h'_{ff} = 1.3169 \text{ Btu/sec-}^\circ\text{F-ft}^2 \quad (155)$$

Using P_{sco} from (58) and $h_{sc} = h_{sg}$, where h_{sg} is given by (9), steam tables give¹¹

$$T_{sco} = 1002.3^\circ\text{F} \quad (156)$$

The steam inlet temperature is the saturation temperature given by (59) and (74)

$$T_{sao} = 636.5^{\circ}\text{F} \quad (157)$$

The sodium inlet temperature is given by (109) and the outlet temperature is computed from

$$T_{ic} = T_{ia} - \frac{w_{s5} (h_{sc} - h_{sa})}{C_{pi} w_i} \quad (^{\circ}\text{F}) \quad (158)$$

Using values given previously at full power

$$T_{ico} = 892.39^{\circ}\text{F} \quad (159)$$

The tube length which gives the required heat transfer rate at full power is given by

$$L_s = \frac{q}{U' \Delta T_{lm}} \quad (\text{ft}) \quad (160)$$

where

$$q \text{ (Btu/sec-tube)} = w'_{sa} (h_{sc} - h_{sa}) \quad (161)$$

$$\Delta T_{lm} (^{\circ}\text{F}) = \frac{(T_{ia} - T_{sc}) - (T_{ic} - T_{sa})}{\ln \left(\frac{T_{ia} - T_{sc}}{T_{ic} - T_{sa}} \right)} \quad (162)$$

$$U'(\text{Btu/sec-ft-}^\circ\text{F}) = \pi \left[\frac{1}{h'_n D_o} + \frac{\ln(D_o/D_i)}{2k_t} + \frac{1}{h'_{ff} D_i} + \frac{1}{h'_w D_i} \right]^{-1} (\text{Btu/}^\circ\text{F-ft-sec}) \quad (163)$$

D_o and D_i are outer and inner tube diameters. The above relations give

$$L_s = 53.99 \text{ ft} \quad (164)$$

The following additional relations are used to compute superheater variables as a function of power as described in the following section:

$$T_{ia} = \frac{T_{sc} - (\Delta T_i + T_{sa}) R}{1 - R} \quad (^\circ\text{F}) \quad (165)$$

$$R \equiv \exp \left[\frac{\Delta T_i - \Delta T_s}{q/U' L_s} \right] \quad (166)$$

$$\begin{aligned} \Delta T_i &\equiv T_{ia} - T_{ic} \\ &= \frac{w_{sa} C_{ps} \Delta T_s}{w_i C_{pi}} \quad (^\circ\text{F}) \end{aligned} \quad (167)$$

$$\Delta T_s \equiv T_{sc} - T_{sa} \quad (^\circ\text{F}) \quad (168)$$

$$T_{ic} = T_{ia} - \Delta T_i \quad (^\circ\text{F}) \quad (169)$$

$$C_{ps} = \frac{h_{sc} - h_{sa}}{T_{sc} - T_{sa}} \quad (\text{Btu/}^\circ\text{F-lb}) \quad (170)$$

In (170) for steady-state, h_{sc} and T_{sc} are constant at all power levels, and T_{sa} and h_{sa} are saturation values given by (74) and (75)

5. Partial Load Analysis

Steady-state variables in the feedwater/steam loop at various power levels are determined by the following computations.

1. Specify the gross electrical output, W_G (MW).
2. Main turbine power from

$$W_{T1} = k_{11} \omega_{Go}^2 + W_G \text{ (MW)}$$

3. Main turbine flow, w_{s9} , from (6).
4. Node pressure, P_{s12} , from (49).
5. Assume total flow $w_{s5} = w_{s9}$.

Iteration in steps 6-12:

6. Pump discharge pressure, P_{sd} , from (50) - (54).
7. Pump speed, ω_s , from (22).
8. Pump efficiency, η_s , from (21).
9. Feedwater pump and turbine power, W_{ps} and W_{T2} , from (19).
10. Feedwater turbine steam flow, w_{s11} , from (18).
11. New pump flow, w_{s5} , from (20).
12. If $\left| \frac{w_{s11, \text{new}} - w_{s11, \text{old}}}{w_{s11, \text{new}}} \right| > 10^{-4}$, set $w_{s11} = w_{s11, \text{new}}$ and return to step 6.
13. Node pressure, P_{sc} , P_{sa} , P_{v3} , from (50) - (53).
14. Main turbine pressure, P_{T1} , from (5).
15. Governing valve opening, S_{v2} , from (3) and (4).
16. Feedwater turbine pressure, P_{T2} , from (23c).
17. Feedwater turbine inlet pressure, P_{s11} , from (48) and (49).
18. Feedwater turbine control valve opening S_{v4} , from (23a, 23b).

19. Intermediate sodium flow, w_1 , from (113).
20. Inlet superheater steam temperature, T_{sa} , from (74).
21. Overall superheater heat-transfer coefficient, U' , from (98), (153) - (155) (163).
22. Heat-transfer rate, q from (161).
23. Inlet and outlet superheater sodium temperature T_{ia} and T_{ic} , from (165) - (170).
24. Enthalpy of saturated steam ($h_{g,s4}$) at riser exit and saturated liquid ($h_{f,s4}$) from (75) and (76).
25. Evaporator sodium outlet temperature from

$$T_{19} = T_{17} - \frac{w_{s5} (h_{sa} - h_{s5})}{C_{pi} w_i} \quad (^\circ F)$$

where $T_{17} = T_{ic}$.

26. Estimate χ_{s4} from previous power level, then

$$w_{s1} = \frac{w_{s5}}{\chi_{s4}} \quad (\text{lb/sec})$$

$$P_{s3} \approx P_{s5} + 6. \quad (\text{psia})$$

$$P_{s2} \approx P_{s3} + 33 \left(\frac{w_{s1}}{w_{s10}} \right)^2 \quad (\text{psia})$$

Outer iteration-Steps 27-42:

Inner iteration-Steps 27-41:

27. Flow per riser (flow/module) from

$$w'_{s3} = w_{s1}/18 \quad (\text{lb/sec})$$

28. Riser inlet pressure (evaporator outlet pressure), P_{s3} , from (150).
29. Evaporator inlet enthalpy, h_{s1} , from (100).
30. Evaporator inlet temperature, T_{s1} , from (77).
31. At node point s2, the saturated temperature, $T_{sat,s2}$ steam enthalpy, $h_{g,s2}$ and liquid enthalpy, $h_{f,s2}$, from (74), (75), and (76), respectively.
32. Average region pressure, $\bar{P}_{s2}$, from (131).
33. Evaporator heat-transfer coefficients on the shell-side, h'_n , and on the tube-side in the preheat region, h'_w , from (98) and (93), respectively.
34. Evaporator region lengths, L_{s1} , and L_{s2} , and overall heat-transfer rate, q (Btu/sec), using methods described in Appendix A.
35. Riser outlet enthalpy h_{s4} , from

$$h_{s4} = h_{s1} + q/w_{s1} \quad (\text{Btu/lb})$$

36. Evaporator outlet quality, χ_{s3} , from

$$\begin{aligned} \chi_{s3} &= \frac{h_{s4} - h_{f,s4}}{h_{g,s4} - h_{f,s4}} \\ &= \chi_{s4, \text{ new}} \end{aligned}$$

37. Void fraction in boiling region, $\bar{\alpha}_{s2}$, and at exit, α_{s3} , from (130) and (139), respectively.
38. Boiling region average and exit steam density, $\bar{\rho}_{s2}$ and ρ_{s3} , from (126) and (135).
39. Two-phase friction multiplier, $\bar{R}_{s2}$, from (124) and (125).
40. New estimate of node pressure, P_{s2} , from (117) - (120).

$$41. \text{ If } \left| \frac{P_{s2, \text{ new}} - P_{s2, \text{ old}}}{P_{s2, \text{ new}}} \right| > 10^{-4}, \text{ set } P_{s2} = P_{s2, \text{ new}}$$

and return to step 27.

$$42. \text{ If } \left| \frac{\chi_{s4, \text{ new}} - \chi_{s4, \text{ old}}}{\chi_{s4, \text{ new}}} \right| > 10^{-4}, \text{ set}$$

$$\chi_{s4} = 0.1 \chi_{s4, \text{ new}} + 0.9 \chi_{s4, \text{ old}}$$

$$w_{s1} = w_{s5} / \chi_{s4} \text{ (lb/sec)}$$

and return to step 27.

43. Evaporator inlet pressure, P_{s1} , from (117) - (120).

B. Primary Sodium Loop

1. Intermediate Heat Exchanger (IHX)

Single-channel cross-section dimensions in the IHX are based on Ref. 1 designs, i.e., 1.0 in. OD tubes with 0.058 in. wall thickness and 1.5 in. square pitch. The assumed number of tubes is 5100 which is one-half the number specified for the 1000 MWe design in Ref. 1. Using these dimensions and (94) - (97), the shell-side heat-transfer coefficient in the IHX is

$$h'_n \text{ (Btu/sec-}^\circ\text{F-ft}^2\text{)} = 0.36808 + 0.27776 (w'_p)^{0.8} \quad (171)$$

where w'_p is the shell-side flow per tube (lb/sec).

On the tube side (94) - (97) are also used with the exception that the factor 1.3 in. (95) is not included. The resulting tube-side heat-transfer coefficient is

$$h'_t \text{ (Btu/sec-}^\circ\text{F-ft}^2\text{)} = 0.59729 + 0.49741 (w'_i)^{0.8} \quad (172)$$

where w'_i is the tube-side flow per tube (lb/sec).

The tube thermal conductivity is¹

$$k_t = 0.003633 \text{ Btu/sec-}^\circ\text{F-ft} \quad (173)$$

and the equivalent sodium-to-sodium heat-transfer coefficient is

$$U' = \pi \left[\frac{1}{h'_n D_o} + \frac{1}{h'_t D_i} + \frac{\ln(D_o/D_i)}{2 k_t} \right]^{-1} \text{ (Btu/}^\circ\text{F-ft-sec)} \quad (174)$$

where D_o and D_i are the outer and inner tube diameters.

The full-power primary flow rate is assumed to be

$$w_{po} = 10,500 \text{ lb/sec} \quad (175)$$

which is 50% of the 1000 MWe design value in Ref. 1. The full-power intermediate flow rate was given previously as

$$w_{io} = 9380.6 \text{ lb/sec} \quad (111)$$

The intermediate sodium inlet and outlet temperatures are equal to the sodium temperatures at the evaporator outlet and superheater inlet, respectively. At full power these values are

$$T_{i1o} = 620^\circ\text{F}, T_{i3o} = 1020^\circ\text{F} \quad (176)$$

The assumed full-power reactor inlet temperature and IHX primary-sodium outlet temperature is

$$T_{p1o} = T_{p7o} = 730^\circ\text{F}.$$

The IHX primary sodium inlet temperature, T_{p5} , can then be computed from the heat balance

$$(T_{p5} - T_{p7}) = \frac{w_i C_{pi} (T_{i3} - T_{i1})}{w_p C_{pp}} \quad (^\circ\text{F}) \quad (177)$$

where C_{pi} is given by (111) and C_{pp} is assumed constant at

$$C_{pp} = 0.3032 \text{ Btu/lb-}^\circ\text{F} \quad (178)$$

At full power (177) gives

$$T_{p5} = 1087.3^\circ\text{F} \quad (179)$$

The tube length which gives the required heat-transfer rate at full power is

$$L_I = 33.091 \text{ ft} \quad (180)$$

which is computed with relations analogous to (160) - (162) for the superheater.

The primary sodium flow rate is assumed proportional to gross electrical output, W_G , according to

$$w_p = w_{po} \frac{W_G}{W_{Go}} \quad (\text{lb/sec}) \quad (181)$$

which is equivalent to the secondary sodium flow relation in (113).

The power-dependent IHX steady-state temperature computations make use of the following relations:

$$q \text{ (Btu/sec)} = w_i C_{pi} (T_{i3} - T_{i1}) \quad (182)$$

$$R \equiv \exp \left[\frac{(T_{p5} - T_{p7}) - (T_{i3} - T_{i1})}{q/U' L_I} \right] \quad (183)$$

$$T_{p5} = \frac{T_{i3} - [(T_{p5} - T_{p7}) + T_{i1}] R}{1 - R} \quad (^\circ\text{F}) \quad (184)$$

2. Reactor

From equations (53) - (57) in Ref. 10, the heat balance for each of the two axial regions in the reactor along an average fuel pin can be written as

$$w_p C_{pp} (T_{pi+1} - T_{pi}) = \frac{\pi D_a (L_c/2) h'}{\left(1 + \frac{D_a h'}{8 k_{fi}}\right)} (\bar{T}_{fi} - \bar{T}_{pi}), \quad i = 1, 2 \quad (\text{Btu/sec}) \quad (185)$$

where

w_p : Coolant flow per pin

$$w_p = \frac{w}{N_p} \quad (\text{lb/sec}) \quad (186)$$

N_p : No. of fuel pins

$$N_p = 29729 \quad (1/2 \text{ of no. in Ref. 1}) \quad (187)$$

D_a : Fuel diameter

$$D_a = 0.265 \text{ in.}^{(1)} \quad (188)$$

L_c : Active core height

$$L_c = 43 \text{ in.}^{(1)} \quad (189)$$

h' : Equivalent heat-transfer coefficient

$$h' = \frac{1}{D_a} \left[\frac{1}{D_a h'_e} + \frac{1}{D_b h'_c} + \frac{\ln(D_b/D_a)}{2 k_e} \right]^{-1} \quad (\text{Btu}/^\circ\text{F-ft}^2\text{-sec}) \quad (190)$$

D_b : Clad outside diameter

$$D_b = 0.3 \text{ in.}^{(1)} \quad (191)$$

h'_e : Heat-transfer coefficient across fuel-to-clad gap.

$$h'_e = 2.7778 \text{ Btu/sec-ft}^2\text{-}^\circ\text{F} \quad (192)$$

(Based on $\Delta T = 50^\circ\text{F}$ across gap at 10.15 kw/ft pin power⁽¹⁾)

h'_c : Clad-to-sodium heat-transfer coefficient.

Using (94) - (97) with a pitch/diameter ratio of 1.2 (triangular pitch) gives

$$h'_c (\text{Btu/sec-}^\circ\text{F-ft}^2) = 3.8925 + 7.6949 (w'_p)^{0.8} \quad (193)$$

k_e : Clad thermal conductivity

$$k_e = 0.0037222 \text{ Btu/sec-}^\circ\text{F-ft}^{(1)} \quad (194)$$

k_{fi} : Fuel thermal conductivity in axial region i.

$$k_{fi} (\text{Btu/sec-}^\circ\text{F-ft}) = 0.001111 - 3.0555 \times 10^{-7} \bar{T}_{fi} \quad (195)$$

(Based on linearization between values at 1000°F and 2000°F)⁽⁸⁾

$\bar{T}_{pi}$: Average region coolant temperature

$$\bar{T}_{pi} = (T_{pi} + T_{pi+1})/2 \quad (^\circ\text{F}) \quad (196)$$

The steady-state temperature feedback and control rod reactivities are based on the following coefficient values(1):

Fuel Temperature Reactivity Feedback

Doppler:

$$T_f \frac{dk_D}{dT_f} = -0.00795$$

or

$$\Delta k_D = \ln \left(\frac{T_{fo}}{T_f} \right)^{0.00795} \quad (197)$$

$$T_{fo} = 885^\circ R$$

Axial Expansion:

$$\frac{\Delta k_{fE}}{T_f} = -1.61 \times 10^{-6} \quad (^\circ F^{-1}) \quad (198)$$

Sodium Temperature Reactivity Feedback

Density Change:

$$\frac{\Delta k_v}{T_p} = -4.3 \times 10^{-6} \quad (^\circ F^{-1}) \quad (199)$$

Radial Core Expansion:

$$\frac{\Delta k_r}{T_p} = -4.81 \times 10^{-6} \quad (^\circ F^{-1}) \quad (200)$$

The total temperature reactivity feedback in dollars for two axial region is thus given by

$$\begin{aligned} \frac{\rho_T}{\beta} (\$) = \frac{1}{2\beta} \sum_{i=1}^2 \left[\ln \left(\frac{885}{460 + \bar{T}_{fi} \quad (^\circ F)} \right)^{0.00795} - 1.61 \times 10^{-6} \bar{T}_{fi} \right. \\ \left. + (4.3 - 4.81) \times 10^{-6} \bar{T}_{pi} \right] \quad (201) \end{aligned}$$

The delayed neutron fraction is assumed to be⁷

$$\beta = 0.00301958 \quad (202)$$

The assumed reactivity due to burnup is

$$\frac{\rho_B}{\beta} (\$) = -1.725 \quad (203)$$

which is 1/2 the maximum burnup given in Ref. 1.

The position dependent control rod reactivity is³

$$\frac{\rho_c}{\beta} (\$) = \frac{\rho_{c,o}}{\beta} \left[\frac{(X + N_{ex})}{L_{ex}} - \frac{1}{2\pi} \sin \left(\frac{2\pi (X + \Delta L_{ex})}{L_{ex}} \right) \right] \quad (204)$$

where

X: Rod position with X=0 at the active core bottom.

ΔL_{ex} : Extrapolation length

$$\Delta L_{ex} = 7.5 \text{ in.}^{(7)} \quad (205)$$

L_{ex} : Extrapolated core height

$$\begin{aligned} L_{ex} &= L_c + 2 \Delta L_{ex} \\ &= 58 \text{ in.} \end{aligned} \quad (206)$$

$\frac{\rho_{c,o}}{\beta}$: Total rod reactivity worth

$$\frac{\rho_{c,o}}{\beta} (\$) = 10.53^{(1)} \quad (207)$$

In the steady-state, the condition to be satisfied is

$$\frac{\rho_T}{\beta} + \frac{\rho_B}{\beta} + \frac{\rho_C}{\beta} = 0 \quad (208)$$

3. Reactor and IHX Partial Load Analysis

The following computations are used to determine steady-state IHX and reactor variables at various power levels.

1. Primary and secondary flow rates, w_p and w_i , from (181) and (113), respectively.
2. IHX secondary sodium inlet and outlet temperatures from

$$\begin{aligned} T_{i1} &= T_{i9} \quad (^\circ\text{F}) \\ T_{i3} &= T_{ia} \quad (^\circ\text{F}) \end{aligned} \quad (209)$$

where T_{i9} and T_{ia} are from the feedwater/steam loop analysis.

3. IHX primary sodium inlet and outlet temperatures, T_{p5} and T_{p7} , from (177), (182) - (184)
4. Reactor channel inlet and outlet and upper plenum outlet temperatures from

$$\begin{aligned} T_{p1} &= T_{p7} \quad (^\circ\text{F}) \\ T_{p3} &= T_{p4} = T_{p5} \quad (^\circ\text{F}) \end{aligned} \quad (210)$$

5. Reactor coolant centerline temperature from

$$T_{p2} = (T_{p1} + T_{p3})/2 \quad (^\circ\text{F}) \quad (211)$$

6. Average region coolant temperatures $\bar{T}_{pi}$, from (196).
7. Equivalent heat-transfer coefficient, h' , from (190) - (194).
8. Assume average fuel temperatures, $\bar{T}_{fi}$.

Iteration in steps 9-11:

9. Thermal conductivity of fuel, k_{fi} , from (195).
10. Compute new estimate of average fuel temperatures, $\bar{T}_{fi}$, from (185).
11. If $\left| \frac{\bar{T}_{fi, \text{ new}} - \bar{T}_{fi, \text{ old}}}{\bar{T}_{fi, \text{ new}}} \right| > 10^{-4}$, set $\bar{T}_{fi} = \bar{T}_{fi, \text{ new}}$ and return to step 9.
12. Temperature reactivity feedback, ρ_T/β , from (201).
13. Burnup reactivity, ρ_B/β , from (203).
14. Steady-state control rod reactivity, ρ_c/β , from (208).
15. Initial estimate of control rod position, X , from

$$\frac{X + \Delta L_{ex}}{L_{ex}} \approx \frac{\rho_c}{\rho_{c,o}} \quad (211)$$

Iteration in steps 16-17:

16. Estimate new rod position from

$$\frac{X_{\text{new}} + \Delta L_{ex}}{L_{ex}} = \frac{\rho_c}{\rho_{c,o}} + \frac{1}{2\pi} \sin \left[\frac{(X_{\text{old}} + \Delta L_{ex}) 2\pi}{L_{ex}} \right] \quad (212)$$

17. If $\left| \frac{X_{\text{new}} + X_{\text{old}}}{X_{\text{new}}} \right| > 10^{-4}$, set $X_{\text{old}} = X_{\text{new}}$ and return to step 16.

Appendix A - Steady-State Evaporator Heat Balance

The individual tubes of the evaporator contain an initial segment where the water is preheated and a second region where nucleate boiling occurs. A schematic illustration of a single tube is given in Fig. A-1.

Using the nomenclature of Fig. A-1, the heat-transfer rate per unit length, $q'(X)$ (Btu/sec-ft), in the preheat region is ($q'(X) \equiv dq(X)/dX$)

$$q'(X) = [T_1(X) - T_s(X)] U' \quad (\text{Btu/sec-ft}) \quad (\text{A-1})$$

where

$$U' = \left[\frac{1}{U_o'} + \frac{1}{U_i'} \right]^{-1} \quad (\text{Btu/ft-}^\circ\text{F-sec}) \quad (\text{A-2})$$

$$U_o' = \pi \left[\frac{1}{h_n' D_o} + \frac{\ln(D_o/D_i)}{2 k_t} + \frac{1}{h_{ff}' D_i} \right]^{-1} \quad (\text{Btu/ft-}^\circ\text{F-sec}) \quad (\text{A-3})$$

$$U_i' = \pi D_i h_w' \quad (\text{Btu/ft-}^\circ\text{F-sec}) \quad (\text{A-4})$$

where terms in (A-3) and (A-4) are defined in Section III. A.4.a.

Additional relations are

$$q'(X) = [U_o' (T_1(X) - T_a(X))] \quad (\text{Btu/sec-ft}) \quad (\text{A-5})$$

$$q'(X) = [U_i' (T_a(X) - T_s(X))] \quad (\text{Btu/sec-ft}) \quad (\text{A-6})$$

$$w_i C_{pi} [(T_1(X) - T_1(0))] = w_s C_{ps} [T_s(X) - T_s(0)]^* \quad (\text{Btu/sec}) \quad (\text{A-7})$$

In the nucleate boiling region (A-5) remains valid, however, (A-6) is replaced with

* $C_{ps} = \frac{\partial h}{\partial T}$, where h is given by (77).

Fig. A-1. Evaporator Tube Cross-section

$$\begin{aligned}
q'(X) &= \frac{\pi D_1}{(1.9)^4} \frac{e^{4P(X)/900}}{3600} [T_a(X) - T_{sat}]^4 \\
&= k(P) [T_a(X) - T_{sat}]^4 \quad (\text{Btu/sec-ft})
\end{aligned} \tag{A-8}$$

which is based on the Jens-Lottes correlation⁹.

Also, in the nucleate boiling region

$$\begin{aligned}
w_1 C_{pi} [T_i(X) - T_i(L_{s1})] &= \int_{L_{s1}}^X q'(X') dX' \\
&= q(X) - q(L_{s1}) \quad (\text{Btu/sec})
\end{aligned} \tag{A-9}$$

Combining (A-5), (A-8) and (A-9) gives the nonlinear differential equation for $q(X)$, the heat-transfer rate in the region from 0 to X ,

$$q'(X) = U_o \left\{ \left[T_i(L_{s1}) - T_{sat} \right] + \frac{[q(X) - q(L_{s1})]}{w_1 C_{pi}} - \left[\frac{q'(X)}{k(P)} \right]^{1/4} \right\} \quad (\text{Btu/sec-ft}) \tag{A-10}$$

The region boundary, $X=L_{s1}$, is assumed to be the location where the heat transfer rate per unit length, $q'(X)$, as given by (A-5) - (A-7) is equal to that given by (A-8). The following procedure is used for obtaining the numerical solutions of the above equations.

1. U_o , U_i , $w_1 C_{pi}$, $w_s C_{ps}$, $k(P)$, $T_i(0)$, $T_s(0)$, T_{sat} are given constant parameters

and initial conditions.

2. Estimate $T_a(L_{s1}) \approx T_{sat}$

Iteration in steps 3-6:

3. Primary and secondary node point temperatures, $T_i(L_{s1})$ and $T_s(L_{s1})$, from (A-5) - (A-7).

4. Node point heat transfer rate per unit length, $q'(L_{s1})$, from (A-1).

5. New estimate of node point wall temperature, $T_a(L_{s1})$, from (A-8).

6. If $\left| 1 - \frac{T_a(L_{s1})_{old}}{T_a(L_{s1})_{new}} \right| > 10^{-4}$, return to step 3 using new value of $T_a(L_{s1})$.

7. Total heat transfer rate in region s1 from $q(L_{s1}) = [T_s(L_{s1}) - T_s(0)] w_s C_{ps}$ (A-11)

8. Region s1 length from

$$L_{s1} = \frac{q(L_{s1})}{U' \Delta T_{lm}} \quad (\text{ft}) \quad (\text{A-12})$$

9. Total heat transfer rate, $q(X)$, in region s2 by numerical integration:

$$q(X + \Delta X) = q(X) + \Delta X q'(X) \quad (\text{Btu/sec}) \quad (\text{A-13})$$

where $q'(X)$ is determined at each step by the following iterative procedure:

a) Assume $q'(X)_0 = q'(X - \Delta X)$ (Btu/sec-ft)

b) At the n(th) iteration

$$q'(X)_n = a + b q(X) + c (q'(X)_{n-1})^{1/4} \quad (\text{Btu/sec-ft}) \quad (\text{A-14})$$

where the constants a, b, and c are obtained from (A-10).

c) The iteration terminates if $\left| \frac{q'_n - q'_{n-1}}{q'_n} \right| < 10^{-4}$.

10. In the full power design problem to determine tube length, the numerical integration, (A-13), terminates when $q(X)$ equals the required overall evaporator heat transfer rate. In partial load analysis studies, the termination condition is $X = L_{s1} + L_{s2} = L_E$, where L_E is fixed.

Appendix B - Average Boiling-Region Void Fraction.

In the boiling region of a counter-current flow steam generator, the assumption of constant heat flux to the two-phase mixture along the tube length implies a steam quality vs. distance relation¹²

$$\chi(z) = \chi(0) + [\chi(1) - \chi(0)]z \quad (B-1)$$

where χ is the steam quality and z is a normalization of the distance X :

$$z = (X - X_{in}) / (X_{out} - X_{in}) \quad (B-2)$$

Using boundary conditions $\chi(0) = 0$ and $\chi(1) = \chi_{out}$, (B-1) can be rewritten

$$\chi(z) = \chi_{out} z \quad (B-3)$$

The void fraction is given by

$$\alpha = \frac{\chi}{S_l \beta (1 - \chi) + \chi} \quad (B-4)$$

where $\beta = \rho_g / \rho_f$ and S_l is the slip ratio.

and the average void fraction in the two-phase region is

$$\bar{\alpha} = \int_0^1 \alpha(z) dz \quad (B-5)$$

Assuming constant values for β and S_l , the above relations give

$$\bar{\alpha} = \frac{1}{\chi_1 (1 - S_l \beta)^2} \left\{ [\chi_1 (1 - S_l \beta)] + S_l \beta \ln \left[\frac{S_l \beta}{S_l \beta + \chi_1 (1 - S_l \beta)} \right] \right\} \quad (B-6)$$

Eqn. (B-6) is approximated by

$$x_1 \approx C_1 + (C_2 + C_3 \bar{P}) \bar{\alpha} \quad (\text{B-7})$$

where the dependence on the average region pressure, $\bar{P}$, is included in (B-7) to account for the pressure dependence of β in (B-6). The constants C_1 , C_2 , and C_3 , are obtained by fitting (B-7) to the following data points. ($S_g = 1$)

a) $x_1 = 0.3$, $\bar{P} = 2032.5$ psia, $\bar{\alpha} = 0.5013$

b) $x_1 = 0.2$, $\bar{P} = 1850$ psia, $\bar{\alpha} = 0.4304$

c) $x_1 = 0.2$, $\bar{P} = 2032.5$, $\bar{\alpha} = 0.4021$

The resultant constant values are

$$C_1 = -0.20534$$

$$C_2 = 0.26987$$

$$C_3 = 3.6319 \times 10^{-4}$$

(B-8)

References

1. "1000 MWe LMFBR Follow-on Study. Conceptual Design Report," AI-AEC-12792 (Vol. IV), 1968.
2. L. J. Habegger, W. C. Lipinski, "The Dynamic Simulation of a 1000 MWe LMFBR Complete-Plant with Controllers--Reference Design Simulation," ANL/RAS 72-10, March 1972.
3. L. J. Habegger, W. C. Lipinski, "Revisions to the Dynamic Model of an LMFBR Complete Plant," ANL/RAS 72-21, July 1972.
4. J. P. McDonald, H. G. Kwatny, J. H. Spare, "A Nonlinear Model for Reheat Boiler-Turbine-Generator Systems; Part I and II," Paper No. 3-D4, 5, 1971, JACC.
5. C. D. Swanson, et al., "Analog Simulation of the Hanford N-Reactor Plant," HW-82589, Sept. 1964.
6. J. K. Salisbury, Steam Turbines and Their Cycles, Wiley, New York, N.Y. 1950.
7. W. L. Chase, FFTF Project, private communication (Nov 1970).
8. S. Glasstone, A. Sesonske, Nuclear Reactor Engineering, Van Nostrand, Princeton, N. J., 1963.
9. P. A. Lottes, "Nuclear Reactor Heat Transfer," ANL-6469, Dec. 1961.
10. L. J. Habegger, C. Hsu, "Space-Dependent Transfer-Function Analysis of Large Nuclear-Reactor Power Systems," ANL-7788, March 1971.
11. J. H. Keenan, F. G. Keyes, Thermodynamic Properties of Steam, Wiley, New York, N. Y., 1955.
12. L. J. Habegger, W. C. Lipinski, "Initial Simulation of Once-Through Steam Generator," ANL/RAS 70-18, Oct. 1970.