284
12/4/86 JS (2) (37)

DR 0044-2
UCLA ENG-86-38
PPG-1009
T-28847

UCLA

UCLA-ENG--86-38

DE87 002996

TOKOPS: TOKamak Reactor Operations Study

THE INFLUENCE OF REACTOR OPERATIONS
ON THE DESIGN AND PERFORMANCE OF
TOKAMAKS WITH SOLID-BREEDER BLANKETS

FINAL REPORT
September 1986

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

FUSION PHYSICS & ENGINEERING

MASTER

MECHANICAL, AEROSPACE AND NUCLEAR ENGINEERING DEPT.

AND

CENTER FOR PLASMA PHYSICS AND FUSION ENGINEERING
UNIVERSITY OF CALIFORNIA, LOS ANGELES

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

TOKOPS: TOKamak Reactor Operations Study

THE INFLUENCE OF REACTOR OPERATIONS
ON THE DESIGN AND PERFORMANCE OF
TOKAMAKS WITH SOLID-BREEDER BLANKETS:

FINAL REPORT

September 1986

Conn, R. W., Ghoniem, N. M. and
Firestone, M. A., Editors

Contributors

Amodeo, R.
Conn, R. W.
Blanchard, J. P.
Brown, D. I.* -
Firestone, M. A.
Ghoniem, N. M.
Grotz, S. P.
Ilgen, M. R.

Kessel, C. E.
Levin, P.**
Martin, R.
Mau, T. K.
Orient, G. E.
Sharafat, S.
Tillack, M.
Vold, E. L.

*Princeton Plasma Physics Laboratory

**On leave 1984-1985 from Physics Department, Nuclear Research Center - Negev.

Work supported by
U. S. Department of Energy,
Office of Fusion Energy,
Grant # DE-FG03-80ER52061,
with UCLA

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

Printed in the United States of America
Available from
National Technical Information Service
U. S. Department of Commerce
5285 Port Royal Road
Springfield, VA 22161

NTIS price codes
Printed copy: _____
Microfiche copy: A01

CONTENTS

Executive Summary	1-1
(N. M. Ghoniem, M. A. Firestone and R. W. Conn)	
Summary of Tokamak Machine Physics Parameters	2-1
(C. E. Kessel and M. A. Firestone)	
Fractional Power Operation of Tokamak Reactors	3-1
(T. K. Mau, E. L. Vold and R. W. Conn)	
Plasma Position and Shape Determination and Electromagnetics Calculations	4-1
(C. E. Kessel and M. A. Firestone)	
Eddy Current Models	5-1
(M. A. Firestone, D. I. Brown, C. E. Kessel and M. Tillack)	
Modeling of Lower Hybrid Current Drive in Tokamak Reactors	6-1
(T. K. Mau, M. R. Ilgen and R. W. Conn)	
Plasma Circuit Interaction Code	7-1
(M. A. Firestone)	
Neutronic Optimization of a LiAlO_2 Solid Breeder Blanket	8-1
(P. Levin and N. M. Ghoniem)	
Modeling of Tritium Transport in a Pin-Type Solid Breeder Blanket	9-1
(R. Martin and N. M. Ghoniem)	
Materials Data Base and Design Equations for the UCLA Solid Breeder Blanket	10-1
(S. Sharafat, R. Amodeo and N. M. Ghoniem)	
Thermal Response of a Pin-Type Fusion Reactor Blanket During Steady and Transient Reactor Operation	11-1
(S. P. Grotz and N. M. Ghoniem)	
Thermomechanical Analysis of Solid Breeders in Sphere-Pac, Plate, and Pellet Configurations	12-1
(J. P. Blanchard and N. M. Ghoniem)	
Bowing of Solid Breeder Fuel Pins and Multiplier Rods in a Pin-Type Fusion Blanket	13-1
(J. P. Blanchard and N. M. Ghoniem)	
Elastic Structural Analysis of the Pin-Type Solid Breeder Blanket First Wall	14-1
(G. E. Orient and N. M. Ghoniem)	
A Model for the Mechanical Pellet-Cladding Interaction During Power Cycles	15-1
(G. E. Orient and N. M. Ghoniem)	

CONTENTS

ABSTRACT	1-1
1.1. INTRODUCTION AND BACKGROUND	1-1
1.2. A REFERENCE SOLID-BREEDER TOKAMAK FUSION SYSTEM	1-4
1.3. PLASMA STARTUP, BURN ANALYSIS, AND OPERATION AT VARIOUS LEVELS OF OUTPUT POWER	1-7
1.3.1. Model for Reactor Electromagnetic Response	1-8
1.3.2. Tokamak Plasma Operation at Fractions of Rated Power	1-11
1.3.2.1. Reduction of Power from 1000 to 500 MW	1-13
1.3.2.2. Increase of Power from 500 to 1000 MW	1-14
1.4. ENGINEERING STARTUP ANALYSIS	1-15
1.4.1. Transient Thermal-Hydraulic Analysis	1-16
1.4.2. Breeder-Cladding Mechanical Interaction	1-20
1.4.2.1. Solid Breeder/Cladding Compatibility	1-22
1.4.2.2. Preheating	1-23
1.4.2.3. Temperature Ramp	1-24
1.4.2.4. Breeder Pellet Swelling: Integrated Analysis ...	1-25
1.4.3. Blanket Tritium Analysis During Transients	1-25
1.5. SUMMARY AND CONCLUSIONS	1-30
REFERENCES	1-33

CHAPTER 1

EXECUTIVE SUMMARY

**Nasr M. Ghoniem, Marc A. Firestone and
Robert W. Conn**

ABSTRACT

Reactor system operation and procedures have a profound impact on the conception and design of power plants. These issues are studied here using a model tokamak system employing a solid-breeder blanket. The model blanket is one which has evolved from the STARFIRE and BCSS studies. The reactor parameters are similar to those characterizing near-term fusion engineering reactors such as INTOR or NET (Next European Tokamak). Plasma startup, burn analysis, and methods for operation at various levels of output power are studied. A critical, and complicating, element is found to be the self-consistent electromagnetic response of the system, including the presence of the blanket and the resulting forces and loadings. Fractional power operation, and the strategy for burn control, is found to vary depending on the scaling law for energy confinement, and an extensive study is reported. Full-power reactor operation is at a neutron wall loading of 5 MW/m^2 and a surface heat flux of 1 MW/m^2 . The blanket is a pressurized steel module with bare beryllium rods and low-activation HT-9-(9-C-)clad LiAlO_2 rods. The helium coolant pressure is 5 MPa, entering the module at 297°C and exiting at 550°C . The system power output is rated at 1000 MW(e). In this report, we present our findings on various operational scenarios and their impact on system design. We first start with the salient aspects of operational physics. Time-dependent analyses of the blanket and balance of plant are then presented.

1.1. INTRODUCTION AND BACKGROUND

A fusion power station will be a complex machine made up of numerous subsystems and components. The physics and engineering aspects of these subsystems, together with their dynamic interaction, will pose various constraints on the transient operation of the power plant. Startup procedures, constrained in this fashion, should be devised such that transient operation does not result in any major degradation of the overall economics. This argument underlies the basic interaction of startup procedures and base-line design. Startup requirements should therefore be incorporated in the base-line design.

Startup constraints can be divided into three general categories. The first are those which are due to the design of plant components. These subsystems are generally constructed to operate most efficiently at full power. As such, the plant may encounter severe difficulties operating steadily at fractional power. For example, the turbines may not be operational below 20% of their rated design parameters. Careful base-line design of components reduces the scope of these constraints.

A second set of constraints arises due to the dynamic interaction of the plant's subsystems. Each system has its own response time and these times can vary greatly. Usually, the dynamic behavior of the reactor is dictated by the subsystem with the slowest response time. The fusion plasma has a relatively fast response time which is basically determined by the energy transport time and the rate of slowing down of α -particles in the system. Typical plasma response times are in the range of 0.5 to 3 sec. The engineering components such as blanket and thermal conversion systems have relatively long response times (minutes to hours or days). The rate of change in power is then dictated by the response time of these components. The fusion core should be programmed to follow this time path. Moreover, the time the reactor spends in each phase of the startup procedure is generally determined by the response time of the subsystems involved in that phase.

Finally, there are some systems which are solely needed for startup and which require special equipment. Clearly, startup procedures should be chosen which require a minimum of special equipment. The auxiliary heating systems are an example of a system required during startup but perhaps not at full power operation. For a tokamak, current drive is desirable at all power levels so there is an economic gain in combining the heating and current drive systems. If the startup input power is small enough, it can be tapped directly from the electric grid system, eliminating special startup power storage and supply systems. The operation of large-scale experimental machines and fusion power reactors is divided into five general phases which will be briefly discussed here. (See Ref. [1] for more detailed discussion.)

1. Phase 0: Initial commissioning. Initial commissioning occurs once the construction of the device is completed and tests of all components and subsystems are carried out to make the device ready for operation.

2. Phase I: Cold or hot shutdown phase. Two distinct shutdown phases are possible. Phase IA (cold shutdown) is expected to occur infrequently and would require that the blanket and other power related components be completely shut down. Phase IB (hot shutdown) is where the power conversion components and their coolants are maintained at near-ready operation conditions of temperature and pressure but there is no plasma operation.
3. Phase II: Plasma initiation and standby power phase. The operational specification of all systems, particularly those critical to the operation of the plasma, are tested in the absence of plasma in a component testing and checkout phase. During the first startup, plasma technology systems are tested simultaneously with plasma initiation and operation. Plasma auxiliary heating systems are used to achieve plasma parameters approaching those of full power operation, albeit without the use of deuterium-tritium (D-T) fuel. In this case, hydrogen may be preferred as the primary working gas to preclude neutron production and induced radioactivity.

Upon completion of component testing, the fusion core is deemed operational. A plasma using deuterium or deuterium and tritium can now be initiated and heated to fusion temperature, bringing the reactor to a standby condition. In the standby condition, the plasma produces thermal and fusion power between 1% and 20% of the rated power. In a reactor, the thermal power conversion system is unlikely to be operational during this phase so that no electricity can be produced by the thermal cycle.

4. Phase III: Stages or fractional power operation. The fusion system operates at fractions of full power while system tests are performed and licensing requirements are fulfilled.
5. Phase IV: Rated power operation. The fusion system power is produced continuously at its rated and licensed power level.

General constraints can be applied to the various startup phases. In order to minimize the startup equipment and input power, the plasma initiation phase should be implemented rapidly so the plasma can also be heated by fusion-born α -particles. On the other hand, the staged power increase phase to full power should be implemented slowly. This is because of the slow response time of the engineering subsystems.

The standby phase is especially important. The plasma is producing 1% to 20% of full power but the thermal conversion systems are not yet operational. Input power is needed to maintain the plasma in this phase. The plasma parameters in the standby phase should be carefully chosen to minimize this input power.

In the next section, we describe the main characteristics of a recent solid-breeder tokamak design by the UCLA group.²⁻⁸ Then results of plasma-burn dynamics and time-dependent electromagnetic modeling as it pertains to the various startup phases are discussed in Sect. 1.3. Engineering analyses of critical subsystem responses and interactions are given in Sect. 1.4. Finally, conclusions are presented in Sect. 1.5.

1.2. A REFERENCE SOLID-BREEDER TOKAMAK FUSION SYSTEM

A reference blanket design for this startup study is a helium-cooled solid-breeder concept. The breeder material is lithium aluminate (LiAlO_2) which is in the form of metal-clad rods. The cladding material is 9-C, a low activation ferritic steel similar to HT-9.⁹ A neutron multiplier zone of bare beryllium rods is included to enhance the tritium breeding. The blanket is divided toroidally into 24 sectors, each comprising 45 modules. While the sector configuration is that of a BCSS design,¹⁰ individual modules have recently been designed at UCLA.²⁻⁸ Figure 1 shows the configuration of a single module with neutron multiplier and breeder pins in three separate zones.

The coolant flows through the module shell and first wall (FW) and then enters the blanket. It then cools the neutron multiplier zone, followed by the tritium breeding zones. (Note that the breeding section is divided into two zones.) The larger rods, toward the back of the blanket, are sized to operate

Fig. 1. Single module configuration for the reference solid-breeder blanket.

at a maximum centerline temperature of 1100°C for the LiAlO_2 pellets. Selection of pin sizes, geometry, and operating conditions is based on several design iterations involving neutronics, thermal analysis, structural analysis, and tritium inventory calculations. Table I summarizes the major operating parameters of the blanket design. While a more detailed description of this design can be found in Refs. [2-8], a list of its salient features follow..

1. A pressurized lobular configuration is used to allow for good use of space in tight-fitting locations.
2. Helium coolant flow is mainly in the radial direction, helping to achieve a thermally homogeneous blanket.³
3. Helium cross flow, combined with small pin sizes, results in minimal temperature asymmetries around the circumference of individual pins. This is shown to result in minimal bowing and deflections within the blanket.⁶

TABLE I
MAJOR SYSTEM AND BLANKET PARAMETERS

Reactor type	Continuous burn D-T tokamak with He-cooled solid LiAlO_2 blanket
Major radius	5.7 m
Minor radius	1.56 m
Elongation	1.6
No. toroidal sectors	24
No. blanket modules/sector	45 (full poloidal coverage)
Blanket module type	He-cooled, pressurized module of 9-C structure (low activation martensitic steel)
Module poloidal height	0.28 m
Module mean toroidal length	1.50 m
Fusion power	2850 MW
Thermal power	3463 MW
FW 14 MeV neutron flux	5 MW/m^2
FW surface heat flux	1 MW/m^2
Coolant pressure	5 MPa
Coolant inlet temp	250°C
Coolant outlet temp	550°C
Coolant mass flow rate	$2.21 \times 10^3 \text{ kg/s}$
Thermal efficiency, gross	35%
Electric output, gross	1034 MW

4. Material allocations within the blanket are based upon neutronic optimization studies. The final blanket configuration shown in Fig. 1 is quite small, with total depth of 46.3 cm, including the manifolds and FW zones. Since the solid-breeder zone is only 21.8 cm deep, the amount of breeder material is small. The final breeding ratio (1-D) is calculated as 1.18.

1.3. PLASMA STARTUP, BURN ANALYSIS, AND OPERATION AT VARIOUS LEVELS OF OUTPUT POWER

The purpose of this part of the study is to develop the tools necessary to examine the transient behavior of tokamak reactors. To evaluate the physics performance during operation, the INTOR¹¹ design is chosen to perform a time-dependent system study. However, several plasma specifications have been altered to be consistent with recent experimental observations. The plasma edge safety factor, q_1 , is 2.1 while the burn average toroidal β is 3.4%, agreeing with the Troyon limit.¹² The energy confinement time, assuming an L-mode scaling of $\tau_E = 0.3 I_p(\text{MA}) a^2(\text{m})$, is 2.9 s. The average ion temperature remains at 10 keV and $n\tau_E$ becomes $3.66 \times 10^{20} \text{ s/m}^3$, which means the plasma is ignited. Table II indicates the original INTOR parameters along with those chosen for this study.

Since the study of startup and staged power operation is aimed at identifying key issues for power reactors, we have altered the discharge scenario to allow for lower-hybrid RF current-drive ramp-up and OH current maintenance. This "hybrid" mode of plasma current maintenance allows for fractional-power and long-pulse (>1000 s) operation.

TABLE II
PLASMA PARAMETERS FOR THIS STUDY AND THE ORIGINAL INTOR VALUES

	INTOR	UCLA/INTOR
Plasma major radius, m	5.3	5.3
Plasma minor radius, m	1.2	1.2
Plasma elongation	1.6	1.6
Plasma triangularity	0.2	0.2
Burn avg beta, %	5.6	3.4
Poloidal beta	2.6	1.6
Avg ion temp, keV	10.0	10.0
Avg ion density, m^{-3}	1.4×10^{20}	1.28×10^{20}
Energy confinement time, s	1.4	2.9
Plasma current, MA	6.4	6.4
Field on chamber axis, T	5.5	5.5
Safety factor at edge	2.1	2.1
Thermonuclear power, MW_{th}	620	510

1.3.1. MODEL FOR REACTOR ELECTROMAGNETIC RESPONSE

The plasma's changing electromagnetic environment is a key to the time-dependent system behavior affecting plasma equilibrium, stability, and energy confinement time. To this end we have modified a TFTR time-dependent electromagnetic and power balance code, PCIC,¹³ to incorporate plasma shape, eddy current models, α -particle heating, and lower-hybrid current drive.

The calculation of gross plasma equilibrium parameters is necessary in a dynamic system description in order to determine the effects of changes in the electromagnetic environment in the plasma. Normally this is done using a free-boundary MHD calculation. We have developed a computationally fast method for determining plasma shape parameters from magnetic multipoles of the external magnetic field produced by the poloidal field coils and eddy currents.¹⁴ These constitute a system of electric circuits which are solved together with the plasma and its power balance. Figure 2 shows a comparison of the free-boundary MHD results together with the plasma shape and position generated by the approximate multipole method. The agreement is quite satisfactory.

The plasma radial and vertical motion is determined from force balance, ignoring plasma mass. We include the time rate of change of the circuit currents and plasma parameters, in addition to the coupled radial and vertical motion.

We assume the vacuum vessel and blanket/shield are infinitesimally thin toroidal sheets¹⁵ located at the center of the volume. The actual thickness and resistivity is incorporated into the resistance calculation. In Fig. 3 we have used three sheets to simulate the vessel, blanket, and shield. The sheets are broken into toroidal strips corresponding to individual eddy current circuits. The concentric sheet model simulates the poloidal field diffusion through a thick inhomogeneous structure. As an example, the strips are toroidally continuous, but resistive breaks could be incorporated.¹⁶ INTOR has both divertor and pump limiter designs. For our example we have chosen the design with the limiter at the bottom. The pump limiters and other external structures are modeled as conducting strips in the appropriate locations. The strips are mutually coupled to each other, the plasma, and the active field coils. These currents contribute to the poloidal field at the plasma and hence influence its equilibrium properties.

Fig. 2. Comparison of the multipole method for gross plasma equilibrium and actual free boundary MHD results for a 1.5 elongation and 0.2 triangularity plasma. This is for the INTOR tokamak.
 (— = exact; --- = multipole)

Fig. 3. Illustration of the "sheet" eddy current model for INTOR.

To determine the evolution of the RF-driven current profile, a ray-tracing code¹⁸ is used which essentially follows the wave energy path, absorption, and current generation, given an incident lower-hybrid wave spectrum. For simplicity, a cold plasma wave description is used, the damping is assumed quasi-linear, and the current driven is obtained from the Fisch-Karney formula¹⁹ for the normalized efficiency J/P . The resultant "steady-state" current profile can then be treated as a distributed current source over the entire plasma circuit. It should be pointed out that in this treatment, the effect of runaway electrons is neglected.

In a separate current ramp-up study, the ray-tracing code is coupled to WHIST,²⁰ a time-dependent, 1-1/2-D transport/equilibrium code developed at Oak Ridge. The current profile evolution is governed by a poloidal field diffusion equation, with the steady-state RF current profile acting as sources in

each flux tube. Simple current ramp-up scenarios¹⁸ for INTOR at low densities were examined, with tearing mode instabilities invoked to model effectively inward current diffusion. A constant RF pulse of 10 MW was applied at a plasma density of $\langle n \rangle = 5 \times 10^{12} \text{ cm}^{-3}$ and temperature $\langle T \rangle = 1.5 \text{ keV}$, assuming a pre-formed current of 0.5 MA. For a plasma pulse of 300 s, the current rose to 4.4 MA, with an average L/R time of 60 s, and the ramp-up efficiency $LI(dI/dT)/P_{\text{rf}}$ starting at 5% and dropping to 3% towards the end of the pulse. A scenario involving current overdrive to shorten the ramp-up time and minimize energy input was also studied.

1.3.2. TOKAMAK PLASMA OPERATION AT FRACTIONS OF RATED POWER

A commercial fusion reactor must be able to operate at fractions of full power and to maintain thermal stability. We have used steady-state²¹ and time-dependent²² 0-D plasma-burn codes and determined that active feedback of ion fueling and auxiliary power is required to control power and maintain thermal stability. This is true even in the most strongly stable regions, i.e., at low plasma density and high plasma temperature, where reactor net power output is low. Burn control is found to be easiest if there is a soft β -limit, wherein the transport losses increase without plasma disruptions as β approaches the Troyon limit.¹² In this case the ignition region is accessible and operation is stable up to the Murakami density limit. A hard β -limit would require confinement degradation to operate at high output power.

Fractional power operation can be achieved by varying the ion fuel ratio, changing the plasma current, or altering confinement²³ from H-mode to L-mode, if the reactor has a divertor. The strategies for burn control and fractional power operation vary according to the assumed energy confinement scaling. Optimistic scaling laws with respect to ignition pose the greatest problems for burn control because their operating region is well above the ignition curve in the thermally unstable region. An example of such a τ_E scaling law is neo-Alcator scaling. Additionally we find that neoclassical ion transport leads to very optimistic ignition criteria when compared to anomalous ion loss for the global transport losses.

It is found experimentally that auxiliary heating of tokamak plasmas causes a degradation in confinement. This so-called L-mode scaling has been analyzed parametrically by Kaye and Goldston.²³ A major open question is

whether alpha heating will also cause confinement degradation. Also, operation with a divertor has led to the so-called H-mode in which the confinement time can be twice the value in the L-mode regime. We have therefore thoroughly examined whether and how tokamak reactors can be operated, depending on scaling law and physics regime.

We find thermally stable ignited tokamak operation for the following cases:

1. When the plasma is marginally ignited with the ion fuel source and auxiliary power synchronized to an active feedback system;
2. When the plasma has a soft β -limit;
3. When the scaling law is such that only a small fraction of α -particle power degrades confinement (if full degradation occurs, an INTOR-size tokamak will not ignite);
4. When auxiliary power degrades confinement but the region well above ignition is accessible and stable with auxiliary power feedback;
5. When plasma confinement degrades with temperature instead of power;
6. When plasma confinement shows a saturated power scaling, such as $\tau_E \sim (a + b/P)$, rather than the Kaye-Goldston scaling, $\tau_E \sim P^{-1/2}$. Here, P is the non-ohmic heating power.

Maintaining thermal equilibrium while varying the output power requires marginally ignited operation attained by moving along the ignition curve in n - T space. A number of mechanisms are available for changing the ignition point, depending upon the assumed energy confinement scaling. Primarily we have examined variations of plasma current, density, fuel mixture ratio, auxiliary power, and plasma size and/or shape.

The connection between the electromagnetics and a burning plasma can best be seen from the following three equations for the energy confinement time, safety factor, and critical β -limit:

$$\tau_E \propto f(R, a, \kappa) I_p \quad , \quad (1)$$

$$q_I = \frac{5 B_o(T) a^2(m)}{R(m) I_p(MA)} \left(\frac{1 + \kappa^2}{2} \right) \quad , \quad (2)$$

$$\beta_c = C(\kappa, \delta, i) \frac{I_p(MA)}{a(m) B_o(T)} \quad . \quad (3)$$

The energy confinement time and critical β -limit are proportional to plasma current. The edge safety factor, energy confinement, and β -limit depend upon plasma size and shape. Since the electromagnetic environment is responsible for plasma current, size, and shape, it therefore influences the burning plasma energetics. As discussed, the assumed τ_E scaling and β -limit influence how burn control and fractional power operation are accomplished.

As an example of the interrelationships, consider moving between two output power levels, 1000 MW and 500 MW, in the density-temperature plot of Fig. 4., assuming modified Kaye-Goldston scaling. For simplicity, let us assume there exists a soft β -limit and that the power is changed by modifying either density or temperature, but not both. This case is illustrated in the figure as moving from point A to point B.

1.3.2.1. Reduction of Power from 1000 to 500 MW

Starting at point A in Fig. 4, we wish to reduce the power from 1000 to 500 MW. Since the most efficient reactor operation is at the β -limit, β_c , one should aim to decrease β_c such that its curve crosses the 500 MW curve near the desired operating point. If the power reduction is to be achieved by decreasing the density at constant temperature (A $\rightarrow$ B), the plasma triangularity should be decreased along with the density. If β_c had been lowered by decreasing I_p , τ_E would be proportionally lowered [Eqs. (1) and (3)] resulting in a temperature decrease. Reducing the power at constant density (A $\rightarrow$ C), requires a spoiling of confinement to lower the plasma temperature. Here the logical method is to reduce the plasma current, thereby reducing confinement and β_c together.

Fig. 4. Fusion power and critical beta Goldstone scaling without alpha degradation.

1.3.2.2. Increase of Power from 500 to 1000 MW

If the power is to be increased by changing density ($B \rightarrow A$), the plasma triangularity should be increased such that β_c crosses the 1000 MW curve at point A. If the plasma current were increased, $\langle T \rangle$ would increase because of the enhanced confinement. A more serious objection is that q_I might fall below the limit required for stability or for soft β -limited operation.²⁴ An increase in power with temperature ($C \rightarrow A$) can most easily be performed by increasing plasma current. This naturally increases both temperature through enhanced confinement and β_c . Again, there could be problems if q_I decreases. In that case, a combination of increases in plasma current and elongation should maintain q_I while increasing $\langle T \rangle$ and β_c .

These two examples are oversimplified but useful for illustration. The confinement time and critical β -limit are not linear functions of shape. Also, the interplay between confinement time and temperature is more complex

than assumed above. In reality, an appropriate path may include both density and temperature changes (e.g., $A \rightarrow D$ or $D \rightarrow A$ in Fig. 4). By assuming a soft β -limit, the question of active burn control has been ignored. Experimentally, most evidence to date indicates the β -limit is hard and that active burn control will be required to avoid disruptions in tokamaks. Burn control can be considered as very small changes around a nominal power level, and therefore the same techniques described above could be applied. Finally, if the confinement time is inversely proportional to the square root of the auxiliary power (Kaye-Goldston scaling), the auxiliary power could be used for burn control or power changes. Since the aim is to operate near the β -limit, changes in plasma current, size, or shape would still be required in conjunction with auxiliary power modification.

These results tempt comparison with an earlier study of tandem mirror reactors, START(TM).¹ The primary finding was that if radial transport is negligible, the input power can be simply controlled by varying the center cell volume. This is accomplished by controlling the magnetic field at the direct converter. Additionally, the direct converter allows zero net power operation even for small fusion power. The tokamak reactor, on the other hand, allows no straightforward technique for power modification. First, both the design and operation of the tokamak depend upon its assumed energy confinement scaling. For example, if it is required to operate well above ignition for useful net power, the reactor has been overbuilt. Second, the actual techniques for burn control and power modification depend strongly on the assumed scaling. Finally, the coupling between the energy confinement time and the MHD quantities, q_1 and β_0 , [Eqs. (1)-(3)], further complicate burn control and power modification. The operations which depend upon energy confinement changes must not adversely affect plasma equilibrium and stability. These issues all make burn control and fractional power operation in a tokamak a much more significant and difficult problem.

1.4. ENGINEERING STARTUP ANALYSIS

A serious concern relating to fractional power operation is the time rate-of-change from one power plateau to the next. The time rate of power production, which is controlled by plasma collisional and transport mechanisms, is quite fast and on the order of few seconds, as shown in Sect. 1.3.

If the power loading on the FW and blanket are increased on this time scale, severe transient thermal fields could develop. These transients result from the coolant-flow response time in the primary loop. The coolant-flow time constants can range from 10s to 100s of sec. The longest time constant is the time required to reach thermal equilibrium after a change in the level of power production. We will first discuss the thermal response of the blanket during power variations. This is followed by a thermo-mechanical analysis of the solid-breeder blanket during various operational scenarios. Finally, tritium transport is modeled to prescribe operational procedures that lead to minimum tritium inventory.

1.4.1. TRANSIENT THERMAL-HYDRAULIC ANALYSIS

Using the time-dependent option of the finite-element code (TACO-2D),²⁵ a range of power ramp conditions was investigated and the temperature histories for the FW structure were analyzed. Both power ramp and coolant flow rates were varied during these studies. The power in each case started at the 50% level and was ramped up to full power. The entire ramping cycle lasts from 5 sec to 50 min, depending on the specific case.

The calculation results are shown in Fig. 5. Case A is for a fast power increase rate (10%/sec) and a slow flow increase rate (1%/min). Case B is for a fast power increase rate (10%/sec) and a moderate flow increase rate (10%/min). Case C is for a fast power increase rate (10%/sec) and a fast flow increase rate (10%/sec). Finally, Case D is for a moderate power increase rate (10%/min) and a fast flow increase rate (10%/sec). The most severe conditions exist when the change in coolant flow rate is slower than the change in power production (Case A). The FW structure reaches a maximum temperature of 702°C and then slowly cools down. It is desirable to ramp power and coolant flow at the same rate, or even to increase the coolant flow rate faster than the rate of power increase. Plasma power ramp rate will therefore be dictated by the available flow ramp rate. Much slower flow and power ramp rates must be used during initial startup.

The time constant for the FW structure is generally on the order of 10s of sec and is quite dependent on the power ramp rate/flow ramp rate ratio. However, the multilayer breeder pins respond to thermal transients on a longer time scale because of their inherently slow thermal diffusivity. The

Fig. 5. Thermal response of the FW surface temperature under various power and coolant flow ramps. (A) Power = $10\% \text{ sec}^{-1}$, flow = $1\% \text{ min}^{-1}$; (B) Power = $10\% \text{ s}^{-1}$, flow = $10\% \text{ min}^{-1}$; (C) Power = $10\% \text{ sec}^{-1}$, flow = $10\% \text{ s}^{-1}$; (D) Power = $10\% \text{ min}^{-1}$, flow = $10\% \text{ s}^{-1}$.

breeder rods studied were the first row of 3-cm-diam pins (row 6). These pins were chosen because they have a longer thermal time constant as compared with pins in rows 1-5. The ramp rate for both power and flow was chosen as 10%/min. The time response of the centerline temperature closely follows the power ramp and reaches equilibrium at 600 sec. It is interesting to note that the inherently slow thermal response characteristics of breeder pins determines the overall heat transfer rate from inside the pins. It is concluded therefore that flow rate control is extremely important to the reduction of first wall thermal transients. The breeder material has an inherent capacity to reduce temperature fluctuations resulting from rapid power increase rates.

The situation described above is more applicable to rapid power variations during hot startups. As will be shown later, however, the initial startup of the fusion system requires much slower rates of power variations.

Because of the development of high stresses between the cladding and solid breeder material, a matched power and flow ramp rate of $\approx 1\%/hr$ is to be used.

An important consideration of thermal transient analysis is the ability to determine the necessary requirements and procedures for the primary loop preheat. It is desirable from the viewpoint of reactor operations to perform hot functional tests prior to power generation. This ensures the functionality of components under operating temperatures and pressures. To perform hot functional testing, the primary loop must be heated with an auxiliary heat source. It has been previously shown¹ that helium gas preheat is more desirable than other gas systems.

Taghavi and Ghoniem²⁶ developed a model for fusion reactor preheat analysis. The analysis presented here is an extension of their work to account for the physical differences in the specific reactor design. In a lumped capacitance model, the time constant for each component is estimated by

$$\tau_i = \left(\frac{\rho C_p V}{Ah} \right)_i, \quad (4)$$

where ρ is the density, C_p is the heat capacity at constant pressure, V is the volume, A is an effective surface area, and h is an effective heat transfer coefficient. Various component time constants are given in Table III.

TABLE III
TIME CONSTANTS FOR VARIOUS SYSTEM COMPONENTS

Component, i	τ_i (s)	Total No. Components with τ_i
1/2 primary piping	750	2
FW plus module shell	57	1,080
Be rods	24	64,800
LiAlO ₂ rods		
1.22 cm o.d.	45	86,400
3.0 cm o.d.	150	45,360

All of the primary loop components are initially at ambient temperatures ($\sim 20^{\circ}\text{C}$). During the hot functional testing phase, we will operate all components at 250°C . We also assume here for simplicity that no major heat loss mechanisms take place during this phase. The energy required is simply calculated by summing up energy contributions required for all components to reach a temperature of 250°C . This is estimated to be ~ 25 MW-hr.

We will assume that the heater can continually produce hot helium with an outlet temperature of 400°C . Figure 6 illustrates the temperature history of the primary loop components during preheat. It is seen that the first half of the primary piping heats up quite rapidly, reaching the desired temperature of 250°C in ~ 20 min. It is suggested though that the primary loop be heated in reverse order to normal coolant circulation. Thus, the hot leg of the primary piping is heated up first, then the blanket, and finally the cold leg of the primary. Now the hot leg, which normally operates at 550°C , will be at 400°C for the majority of the preheat cycle. The average thermal power required for this cycle is estimated to be 8.4 MW.

Fig. 6. Temperature histories for various primary loop components during the pre-heating phase.

1.4.2. BREEDER-CLADDING MECHANICAL INTERACTION

Finding the smallest allowable ramp rate for startup is important for the economy of the operation, since overly conservative startup can amount to substantial percentage of the power cycle. In the stage of preliminary design the main concern is failure of the cladding due to high stresses or large total strains. The following analysis provides a simplified tool for design based on macroscopic mechanical interaction between the breeder and the cladding.

Since the thermal field³ exhibits only a slight azimuthal asymmetry, the problem is formulated based on an axisymmetric analytical solution for the radial temperature distribution. Interaction between the changing geometry and the thermal field has been neglected during the course of the analysis. The size of the developed gap is on the order of hundredth of a millimeter, which justifies this simplification.

Material models for the breeder pellet include linear elastic behavior, linear thermal expansion, and temperature-dependent swelling. The cladding material was assumed to exhibit linear elastic behavior, linear thermal expansion, and temperature-dependent creep. All linear material properties are taken as constants, although temperature dependence, especially in the pellet, may have an influence on the results. The details of the thermostructural breeder pellet clad interaction model are given in Ref. [4].

The time-dependent temperature field is also represented in a quasi-static manner as follows:

$$T(r,t) = T_{\text{preheat}} + [R(r) - T_{\text{preheat}}]T(t) \quad , \quad (5)$$

where $R(r)$ is the steady-state temperature profile. Swelling is assumed to act only at full power, that is, when the temperature profile is fully developed. The field equations corresponding to the above specified assumptions are as follows:

Strain-displacement relations:

$$\epsilon_r = u/r \quad , \quad (6)$$

$$\epsilon_t = du/dr \quad . \quad (7)$$

Strain-stress relations:

$$\epsilon_r = (1/E)[(1+\nu)\sigma_r - \nu\sigma_{kk}] + \alpha T + \epsilon^{sw}/3 \quad , \quad (8)$$

$$\epsilon_t = (1/E)[(1+\nu)\sigma_t - \nu\sigma_{kk}] + \alpha T + \epsilon^{sw}/3 \quad , \quad (9)$$

$$\sigma_{kk} = \sigma_{rr} + \sigma_{tt} \quad , \quad (10)$$

where r and t subscripts denote radial and tangential quantities respectively, ϵ^{sw} is the swelling strain, and the temperature measured from some base value, T_0 . The $1/3$ factor gives dilatational strains from the volumetric strain when infinitesimal strains are assumed.

Equilibrium equations for an axisymmetric stress state reduce to one single equation:

$$d\sigma_r/dr + (\sigma_r - \sigma_t)/r = 0 \quad . \quad (11)$$

Substituting the strain-displacement relations (6,7) into the constitutive equations (8)-(10) then the resulting stresses into the equilibrium equation (11), we arrive at the displacement form of the equilibrium condition

$$\frac{d}{dr} \left[\frac{1}{r} \frac{d(ru)}{dr} \right] = \frac{1+\nu}{1-\nu} (T\alpha + \epsilon^{sw}/3) \quad . \quad (12)$$

The solution of this equation for an axisymmetric solid cylinder is:

$$u_r = \frac{(1+\nu)}{4} \int_a^r \left[\alpha T(r',t) + \frac{\epsilon^{sw}(r',t)}{3} \right] r' dr' + Cr \quad , \quad (13)$$

$$\sigma_r = -\frac{E}{2} \int_a^r \left[\alpha T(r',t) + \frac{\epsilon^{sw}(r',t)}{3} \right] r' dr' + C[E/(1-\nu)] \quad , \quad (14)$$

$$\sigma_t = \sigma_r - E[\alpha T(r,t) + \epsilon^{sw}/3] \quad . \quad (15)$$

Equation (13) represents the radial displacement of the breeder pellet at any given position r while Eqs. (14,15) give the 2-D stress field within the cross section of the breeder pellet.

Equations (13)-(15) are then used in continuity boundary conditions in both tractions and displacements. This leads to an integral equation in the unknown function $C(t)$, given by

$$C(t) = F[T(t)] + \int_0^t K[C(\xi), T(\xi), \xi] d\xi \quad , \quad (16)$$

where $F[T(t)]$ is a forcing function related to the temperature history and $K[C(t), T(t), t]$ is a kernel function related to the creep strain rate.⁴ The method of numerical solution is explained in Ref. [4].

To simulate real operational conditions, a typical time history is used. One cycle consists of a power ramp, operation at full power, shutdown, and radiation swelling time at preheat temperature. If creep deformation occurs during the cycle, then there will be a residual plastic strain in the clad after it separates from the breeder pellet. This effect is equivalent to a permanent increase in the clad diameter and causes it to reach contact with the breeder pellet in the subsequent startup at a later time within the cycle.

Following are the effects of several important variables on the pellet-cladding mechanical interaction problem.

1.4.2.1. Solid Breeder/Cladding Compatibility

The stresses for different designs under the same thermal operating conditions are shown in Fig. 7 for LiAlO_2 and 9-C cladding. Solid lines represent the thermoelastic stresses had plastic strains not developed; broken lines correspond to the modeled interaction. Swelling of the breeder pellet has been neglected for these runs. Since the coefficients of thermal expansion for the solid breeder is close to that of the cladding displacement of the pellet surface and the cladding is nearly matched. Consequently, the same elastic stress levels occur at significantly smaller gap sizes--an advantage for thermohydraulics and operational purposes. In fact, zero initial gap is safe for this design as will be shown in the integrated analysis.

Fig. 7. Cladding stress evolution for the first row of pins (Zone 2). No initial gap is assumed and the pre-heating temperature is 550°C. 1 = elastic (1/2 day to full power), 3 elastic (2 days to full power), 5 = elastic (5 days to full power); 2, 4, and 6 = elastic.

1.4.2.2. Preheating

Figure 8 shows the results for the same material choice, steady-state temperature profile, and geometry as Fig. 7. This time, the system is not preheated to 550°C. Because the coefficient of the thermal expansion for the cladding is slightly higher than that for the breeder pellet, there is no initial contact when the pin is preheated. Compressive stress because of coolant pressure develops in the cladding. Its value is about one third of the critical buckling stress which represents adequate margin of safety. From a structural point of view, the advantage of preheating is the interaction occurring at cladding temperature which permits creep. Therefore during startup, stress relaxation can significantly relieve peak stress.

Fig. 8. Cladding stress histories for the first row (Zone 2), with zero initial gap. No pre-heating is assumed. 1 = elastic (2 days to full power), 3 = elastic (5 days to full power); 2 and 4 = elastic.

1.4.2.3. Temperature Ramp

Figures 7 and 8 illustrate the sensitivity of the results to the temperature ramp rate. When the slope of the temperature ramp is low, the cladding can creep more before steady-state profile is attained. The resulting displacement helps to accommodate the thermal expansion of the pellet. A power ramp rate of 20%/day will not result in excessive peak stresses.

1.4.2.4. Breeder Pellet Swelling: Integrated Analysis

Figure 9 shows the interaction under cyclic operation when swelling of the breeder pellet is included. Swelling gradually builds up and, instead of the decreasing stresses, a slight increase and eventually higher stress levels occur. Accumulated plastic strain also limits pin lifetime. The figure depicts the plastic strain histories over four cycles for the $\text{LiAlO}_2/9\text{-C}$ design. Reflecting the stress relaxation process, elastic strains decrease while plastic strains increase. The plastic strains stay below 0.5%. The same stress and strain components are shown for the $\text{Li}_2\text{O}/2.25\text{Cr-1Mo}$ design over one period. In this case, swelling of the breeder pellet is stronger but the creep rate is also higher so stresses remain low. On the other hand, the accumulated plastic strain exceeds the 1% value by the end of the first power cycle, which indicates failure of the breeder pin. Because of this last finding and the sensitivity of the peak stress to the gap size, the LiAlO_2 design is recommended. It is also found that the peak stress occurs only during the first cycle. In subsequent startup cycles, contact between the pellet and cladding is delayed, and the developing stress is almost totally relaxed to the threshold creep stress. This means that subsequent hot startups can be much faster as discussed in the earlier section.

1.4.3. BLANKET TRITIUM ANALYSIS DURING TRANSIENTS

Tritium inventory is controlled by diffusive and solubility properties. Tritium diffusion plays a major role in determining tritium inventory; the slower the diffusion, the longer the residence time within the blanket, and the greater the diffusive inventory. For LiAlO_2 , there is a great variation in the experimentally determined diffusion coefficient.^{27,28} The entire tritium inventory within the blanket solid breeder can be represented by:

$$I_b = I_d + I_s + I_a + I_g + I_p \quad ,$$

where I_d is the diffusive inventory, I_s is the inventory associated with solubility within the blanket, I_a is the adsorbed inventory, I_g is the grain boundary inventory, and I_p is the porosity/percolation/purge stream convection inventory. The total inventory, I_{tot} , is composed of I_b plus contributions from coolant contamination and metal structure permeation. Analysis of the

Fig. 9. Accumulated cladding plastic strain during operational cycles for 9-C(HT-9)/LiAlO₂ and 2-1/4Cr-1Mo/LiO₂.

order of magnitude of these inventory contributions and the available experimental data is given in Ref. [8]. For the LiAlO_2 use, the diffusive inventory is identified as the major contribution. The DIFFUSE²⁹ code is used to model tritium inventories during transients (see Ref. [8]).

Reactor startup was modeled for two different situations: (1) helium coolant flow at 100% of full power status and (2) significantly reduced helium flow during the transients to achieve higher operating temperatures and lower transient tritium inventories.

The transients modeled were at 10%, 20%, 50%, and 80% of full power, each lasting 3.5 days, followed by 100% power. The percentage of full power was used to scale the volumetric heating rates for ceramic and cladding accordingly, and likewise for tritium generation rate. The rise in coolant temperature from module entrance (constant at 250°C) to each row of pins was taken as a corresponding percentage of the temperature rise at full power.

The diffusive inventory can be obtained from the DIFFUSE output at any point during the two weeks prior to full power. After two weeks, the input parameters were set at full power values and the inventory soon returned to its steady-state values. The total transient inventories were obtained as function of time for each row, and summed for the total blanket inventory.

The transient results involving 100% helium flow gave total blanket inventories of 226 g after 3.5 days at 10% power, 621 g after 3.5 days at 20% power, 429 g at 50% power, 61 g at 80%, and a short time at 100% returned the total inventory to the steady-state value of 12.6 g. These results are plotted in Fig. 10. The inventories are obviously much higher than steady-state during 10%, 20%, and 50% power levels, reaching several hundred grams with the largest contribution from the first row and its dominant tritium generation rate. The characteristic temperatures ranged from 277° to 317°C at 10% power, 317° to 377°C at 20%, 407° to 577°C at 50%, 507° to 737°C at 80%, and 567° to 857°C at full power. The temperatures fall within the regime of very slow tritium diffusion for 10% to 50% power, leading to rapid buildup of the bred tritium within the pins.

These significant inventories could lead to safety-related concerns of large blanket inventory, especially for protracted startup times at these temperatures. For this reason, the same startup regime was re-analyzed under conditions of restricted helium flow, reducing the rate of cooling and

Fig. 10. Transient blanket tritium inventory for a typical startup scenario.

allowing higher temperatures to be achieved within the breeder. For the 10% and 20% power transients, the helium flow rate was set at 20% of that at full power. Thus, at 20% of full power the same coolant temperature should be obtained for each row as at 100% full power. Likewise, at 50% and 80% of full power the helium flow rate was set at 50% and 80% of full power, again giving the same coolant temperatures. For 10% of full power, with helium flow set at 20%, the rise in temperature between blanket inlet and the row in question was taken to be one half that at full power. The volumetric heating rate and tritium generation rate were scaled down from full power according to the transient power level as mentioned previously.

Because of considerably higher ceramic temperatures under these conditions of controlled coolant flow, the total blanket inventories were greatly reduced from the full power flow transients. The results give total inventories of 153 g at 10% after 3.5 days, 40.2 g at 20%, 25.6 g at 50%, and 16.5 g at 80%. These results are also shown in Fig. 10.

The results for these two cases clearly show the need for tailoring the helium flow rate to the transient power levels. An extended startup could

lead to massive inventories if full power flow conditions are used. Even with controlled flow, a gradual inventory buildup might be a concern for low power levels over an extended period of testing. In this case, supplemental preheating of the coolant would alleviate such concerns. Further study of the minimum practical helium flow rates is also warranted: the minimum of 20% of full power flow is a conservative estimate, to avoid problems with flow instabilities.

For shutdown, two different cases were analyzed: (1) coolant flow at 20% of full power and (2) a combination of coolant flow reduction and heating such that the coolant temperature ranged from 470° to 500°C across the pins. This second scenario can be thought of as "baking" the blanket to speed its purging of tritium inventory. Both scenarios assume afterheat volumetric generation of 1% of full power and zero tritium generation.

For the first case, that of simply reducing the helium flow, the temperature of the coolant entering the blanket was again 250°C, and the temperature rise at each row of pins was taken to be 1/20 that of full power. These new coolant temperatures, 1% volumetric heating, and a corrected heat transfer coefficient were used to calculate new temperature profiles across the pins as before, and diffusion-averaged temperatures were again calculated for each pin. These temperatures were used as input to *DIFFUSE* along with zero generation rate as an additional step following the analysis for the three-year run at 100% power. Thus, the solid breeder grains start with their steady-state inventories, then instantaneous shutdown occurs at which no more tritium is generated but the steady-state concentration profiles within each grain are now subjected to the much lower temperatures at shutdown. As can be expected, tritium diffusion from the grains slows down drastically. The diffusive inventories are modeled for a year after shutdown, with total blanket inventory calculated as before.

Figure 11 shows that, after two weeks, the total inventory is reduced from the steady-state full-power inventory of 12.6 g to 12.4 g, only a 1.6% decrease in total inventory. At the end of six months at these shutdown conditions, an inventory of 12.1 g still remains and after one year an inventory of 11.7 g, indicative of the almost negligible rate of diffusion at these cold shutdown temperatures. In comparison, the characteristic grains are then modeled for hot shutdown, with the coolant temperature of 500°C. As

Fig. 11. Shutdown tritium inventories for a characteristic blanket temperature of 270°C and a bake-out temperature of 500°C.

could be expected, the result was a significantly accelerated tritium purge from the blanket. Figure 11 shows that, after two weeks, the inventory was reduced to an insignificant 0.02 g. Such auxiliary heating should alleviate inventory concerns in a post-shutdown environment.

1.5. SUMMARY AND CONCLUSIONS

Design and performance analysis of fusion power systems must take serious consideration of their operational characteristics. In this report, a comprehensive assessment of the physics and engineering operational design constraints is given for a model tokamak with a solid-breeder blanket. The following are conclusions of the study:

1. Startup/shutdown operational design constraints are due to:
 - (a) special component design requirements, (b) dynamic plant subsystem interactions, and (c) special startup components.

2. Operation of fusion power systems can be divided into five phases: (a) initial commissioning, (b) cold or hot shutdown, (c) plasma initiation and standby power, (d) fractional power operation, and (e) rated power.
3. The longest time scales are dictated by engineering constraints rather than physics performance. The rate of fusion power variation must be slow enough to meet the following limitations: (a) stress and material compatibility limitations (e.g., mechanical interaction between the breeder and structure system); (b) auxiliary power limitations. A total energy of 25 MW-hr is required to preheat the primary loop and blanket to 250°C; (c) mechanical limits. Pump and valve inertia is responsible for the slow response of the primary coolant to rapid power variations.
4. From simplified analysis, it is evident that the passive electromagnetic environment (eddy currents) could significantly affect the plasma equilibrium and control. This is the impetus behind our electromagnetic modeling described in this paper. We built our models (eddy current, plasma position and shape, lower hybrid current drive) such that they would all be consistent and could be brought together in a total systems code. This part of the effort is still in progress.
5. Fractional power operation studies show a strong dependence on the energy confinement scaling assumed, both for reactor design and operation. Burn control and transitions between power levels are seen to be closely related with appropriate control levels including auxiliary power, fueling rate (density), and confinement control. This last knob ties back into the electromagnetic work since confinement is dependent upon such MHD quantities as plasma current, size, and shape. It is noted that changes in confinement must occur in a manner conducive to plasma stability and equilibrium. Finally, we demonstrated lower hybrid current ramp-up with our ray-tracing code coupled to a transport/equilibrium code.

6. The time constants for the FW structure are generally on the order of 10s of s, while those for the solid breeder are on the order of 10s of min. The ramp rates must be matched for both power and flow, and slow enough ($<10\%/min$) so as not to result in excessive FW transient temperature rise.
7. The preheating energy required for the nominal 1000 MW(e) plant is on the order of 25 MW-hr. It is estimated that less than three hours are required to bring the system to its operating temperatures. The controlling factor is the response time of the blanket system.
8. Mechanical interaction between the solid breeder and cladding materials is driven by intermittent system operation. A fast startup results in high transient cladding peak stresses which eventually relax to the threshold stress for creep deformation. Prior preheating to 550°C for the $\text{LiAlO}_2/\text{9-C}$ design allows thermal creep to mitigate the developing high stresses. An initial startup rate on the order of $1\%/hr$ is recommended to avoid the development of high transient stresses.
9. Cladding plastic strains accumulate during preheat cycles, but do not exceed 0.5% for one year of operation.
10. Unrestricted coolant flow during startup ramping will lead to hundreds of grams of tritium inventory. Reduction of coolant flow can keep inventory at acceptable levels.
11. Intermittent blanket preheating to 550°C during shutdown can lead to rapid purging of residual tritium.

REFERENCES

1. "START/TM: A Study of Start-up and Fractional Power Operation of Tandem Mirror Fusion Reactors," R. W. Conn, N. M. Ghoniem and F. Najmabadi, Eds., University of California Los Angeles Report PPG-764 (NTIS, Springfield, VA, 1984).
2. P. Levin and N. M. Ghoniem, "Neutronic Optimization of a LiAlO_2 Solid Breeder Blanket," presented at 7th Top. Mtg. on Technol. of Fusion Energy, Amer. Nucl. Soc., Reno, NV, June 15-19, 1986; to be pub. Fusion Technol. See also University of California Los Angeles Report ENG-8607/PPG-933 (1986).
3. S. P. Grotz and N. M. Ghoniem, "Thermal Analysis of a Pin-Type Blanket for Tokamak Reactors," presented at 7th Top. Mtg. on Technol. of Fusion Energy, Amer. Nucl. Soc., Reno, NV, June 15-19, 1986; to be pub. Fusion Technol. See also University of California Los Angeles Report ENG-8612/PPG-938 (1986).
4. G. E. Orient and N. M. Ghoniem, "A Model for the Mechanical Interaction Between Solid Breeder and Cladding Materials," presented at 7th Top. Mtg. on Technol. of Fusion Energy, Amer. Nucl. Soc., Reno, NV, June 15-19, 1986; to be pub. Fusion Technol. See also University of California Los Angeles Report ENG-8610/PPG-936 (1986).
5. Ibid., "Elastic Structural Analysis of the Pin-Type Solid Breeder Blanket First Wall." See also University of California Los Angeles Report ENG-8609/PPG-935 (1986).
6. J. P. Blanchard and N. M. Ghoniem, "The Bowing of Solid Breeder Rods in a Pin-type Fusion Blanket," presented at 7th Top. Mtg. on Technol. of Fusion Energy, Amer. Nucl. Soc., Reno, NV, June 15-19, 1986; to be pub. Fusion Technol. See also University of California Los Angeles Report ENG-8605/PPG-931 (1986).
7. Ibid., "Thermomechanical Analysis of Solid Breeders in Sphere-Pac, Plate, and Pellet Configurations," see also University of California Los Angeles Report ENG-8606/PPG-932 (1986).

8. R. Martin and N. M. Ghoniem "Modeling of Tritium Transport in a Fusion Reactor Pin-Type Solid Breeder Blanket Using the DIFFUSE Code," presented at ICFRM-2, Chicago, March 1986, to be pub. J. Nucl. Mater. See also University of California Los Angeles Report ENG-8608/PPG-934 (1986).
9. D. S. Gelles and M. L. Hamilton, "Effects of Irradiation on Low Activation Ferritic Alloys," Dept. of Energy Report DOE/ER-0045/13 (Sept. 1984).
10. D. L. Smith et al., "Blanket Comparison and Selection Study, Final Report," Argonne National Lab. Report ANL/FPP-84-1 (Sep. 1984).
11. "U.S. INTOR, The U.S. Contribution to the International Tokamak Reactor Phase-I Workshop," USA INTOR/81-1 (1981).
12. F. Troyon et al., Plasma Phys. Contr. Fusion, 26 1A (1984) 209.
13. R. J. Hawryluk, M. A. Firestone, U. R. Christensen, H. Fishman and J. A. Schmidt, "Plasma-Circuit Interaction in TFTR," Princeton Plasma Physics Lab., TFTR Physics Group Report 29 (1980).
14. C. E. Kessel and M. A. Firestone, "Determination of Gross Plasma Equilibrium from Magnetic Multipoles," presented at 7th Top. Mtg. on Technol. of Fusion Energy, Amer. Nucl. Soc., Reno, NV, June 15-19, 1986; to be pub. Fusion Technol.
15. M. A. Firestone, D. I. Brown, C. Kessel and M. Tillack, "Eddy Current Models for a Tokamak Reactor Vessel and Blanket/Shield," in Proc. 11th Symp. Fusion Eng., Austin, TX, 1985.
16. D. I. Brown, M. G. Bell and J. Coonrod, Proc. 10th Symp. Fusion Eng., IEEE 83CH1916-6-NPS (IEEE, 1983) p. 1233.
17. T. K. Mau, M. R. Ilgen and R. W. Conn, "One-Dimensional Modeling of Lower Hybrid Current Ramping in Tokamaks," in Trans. Am. Nucl. Soc., 52 (Reno, NV, June 1986) p. 229; see also Chapter 6 this report.
18. N. J. Fisch and C. F. F. Karnet, Phys. Fluids, 24 (1981) 27.
19. W. A. Houlberg, S. E. Attenberger, and L. L. Lao, "Developments in Tokamak Transport Modeling," in Advances in Mathematical Methods for the Solution of Nuclear Engineering Problems (Proc. Am. Nucl. Soc./Europ. Nucl. Soc. Int. Top. Mtg. Munich, 1981) Vol. 2, p. 426.
20. T. K. Mau, E. L. Vold, and R. W. Conn, "Fractional Power Operation of Tokamak Reactors: Issues and Prospects," University of California Los Angeles Report PPG-964 (1986).

21. E. L. Vold, T. K. Mau, and R. W. Conn, "Ignition and Time-Dependent Fractional Power Operation of Tokamak Reactors," presented at 7th Top. Mtg. on Technol. of Fusion Energy, Amer. Nucl. Soc., Reno, NV, June 15-19, 1986; to be pub. Fusion Technol.
22. S. M. Kaye, Phys. Fluids, **28** (1985) 2327.
23. M. Keilhacker, "Confinement and Beta-Limit Studies in ASDEX H-Mode Discharges," in Plasma Physics and Controlled Nuclear Fusion Research, (Proc. 10th Int. Conf. London, 1984) Vol. 1, IAEA, Vienna (1985) p. 71.
24. P. J. Burns, "TACO2D - A Finite Element Heat Transfer Code," Lawrence Livermore National Lab. Report UCID-17980 (1982).
25. K. Taghavi and N. M. Ghoniem, "Primary Loop Conditioning and Design Constraints of Li-Pb Cooled Tandem Mirror Reactors During Start-up/Shutdown Operations," Nucl. Eng. Des./Fusion, **1** (1984) 375.
26. Bruning, D. Guggi and H. R. Ihle, "The Diffusivity of Tritium in the System $\text{Li}_2\text{O}-\text{Al}_2\text{O}_3$," in Proc. 12th Symp. on Fusion Technol., EUR 7983EN, (Jülich, FRG, 1982) pp. 543-548.
27. R. G. Clemmer et al., "The TRIO Experiment," Argonne National Lab. Report ANL-84-55, (1984).
28. M. I. Baskes, "DIFFUSE-83," Sandia National Lab. Report SAND-83-8231 (1983).

CHAPTER 2

SUMMARY OF TOKAMAK MACHINE PHYSICS PARAMETERS

Charles E. Kessel and Marc A. Firestone

CONTENTS

2. SUMMARY	2-1
REFERENCE	2-5

2. SUMMARY

The purpose of this study is to develop the tools necessary to examine the transient behavior of tokamak fusion reactors in order to gain a better understanding of the system and to arrive at implications for design. Since the purpose was not to design a machine, one existing conceptual design was chosen, the INTOR [1] design. This particular machine reflects the trends for future tokamak reactors, but has enough previous analysis to be well-defined. The primary reason for this choice is the extensive magnet design and optimization. The INTOR machine has a blanket configuration, although smaller than those predicted for reactors, which is sufficient to examine eddy current effects. The design can be used with a divertor or limiter, although only the limiter option is examined. The machine design, although in an advanced stage, is quite flexible for analysis. A schematic of the INTOR tokamak is given in Fig. 1.

The ohmic heating solenoid and poloidal field coils are located outside of the toroidal field coils. The locations and dimensions are listed in Table I. We allow for both normal and superconducting OH and PF coil sets.

The blanket is basically preserved with the breeder located in the outboard and upper regions. The remainder of the blanket is shield which surrounds the entire plasma chamber. This geometry is used in the eddy current model of the blanket. Both solid, Li_2O , and liquid lithium breeders are considered. A schematic showing the dimensions throughout a width of the machine is shown in Fig. 2.

A few of the design specifications for the plasma have been altered to be more congruent with experimental observations. The plasma edge safety factor is preserved at about 2.1, while the burn average toroidal beta is set at 3.4%, agreeing with the Troyon limit. This correspondingly drops the poloidal beta to 1.6. In addition, the energy confinement time, assuming an L-mode scaling of

$$\tau_E = 0.31 I_p \text{ (MA)} a^2 \text{ (m)} ,$$

Fig. 1. FED/INTOR elevation view.

Fig. 2. FED/INTOR radial build.

TABLE I
LOCATIONS AND DIMENSIONS OF OHMIC HEATING AND POLOIDAL FIELD COILS

Coil #	R(m)	Z(m)	$\Delta R(m)$	$\Delta Z(m)$	Turns
1	1.35	4.60	0.56	0.56	93
2	1.35	-4.60	0.56	0.56	93
3	2.25	4.45	0.26	0.26	20
4	3.20	-6.85	0.22	0.22	14
5	3.55	5.45	0.62	0.62	26
6	6.00	-6.85	0.82	0.82	202
7	6.45	5.45	0.62	0.62	114
8	10.00	4.80	0.74	0.74	166
9	12.35	-5.50	0.98	0.98	286
10	1.35	0.00	8.56	0.45	1200

yields about 2.9 s. The reduction in beta was chosen so as to maintain the ion average temperature at 10 keV, thus lowering the average ion density. However, the nT_E for this is $3.66 \times 10^{20} \text{ s/m}^3$, which is ignited. The thermonuclear power is also decreased to 510 MW. A table of the INTOR [1] plasma parameters and those used in this study are given in Table II.

All other parameters are preserved, although numerous energy confinement time scalings are examined in the fractional power studies and burn control discussions. It should also be emphasized that the engineering work has been done on an INTOR-like machine with different plasma characteristics to simulate a power producing reactor, and the blanket characteristics do not correspond to INTOR.

The discharge scenario is altered so that the plasma current is not just driven by the ohmic current, but also by RF sources. This sort of hybrid current drive is considered necessary to achieve fractional power level operation and long discharge times approaching continuous operation. The burn times will then be well over the 100 to 200 seconds prescribed in INTOR.

TABLE II
PLASMA PARAMETERS FOR THIS STUDY AND THE ORIGINAL INTOR VALUES

	INTOR	UCLA/INTOR
Plasma major radius, m	5.3	5.3
Plasma minor radius, m	1.2	1.2
Plasma elongation	1.6	1.6
Plasma triangularity	0.2	0.2
Burn average beta, %	5.6	3.4
Poloidal beta	2.6	1.6
Average ion temperature, keV	10.0	10.0
Average ion density, m ⁻³	1.4 x 10 ²⁰	1.28 x 10 ²⁰
Energy confinement times, s	1.4	2.9
Plasma current, MA	6.4	6.4
Field on chamber axis, T	5.5	5.5
Safety factor at edge	2.1	2.1
Thermonuclear power, MW(t)	620	510

REFERENCE

1. U.S. FED-INTOR and U.S. Contribution to the INTOR Phase 2A Workshop (Oct. 1982).

CHAPTER 3

FRACTIONAL POWER OPERATION OF TOKAMAK REACTORS

Tak Kuen Mau, Eric L. Vold and Robert W. Conn

CONTENTS

3.1.	INTRODUCTION	3-1
3.2.	PHYSICS BASIS	3-2
3.2.1.	Energy Confinement	3-3
3.2.2.	Alpha Particle Effects	3-6
3.2.3.	Plasma Beta Limit	3-7
3.2.4.	Density Limit	3-9
3.2.5.	Thermal Stability	3-9
3.2.6.	Operating Mode	3-10
3.3.	CALCULATIONAL MODEL	3-11
3.3.1.	Basic Equations and Solution Method	3-11
3.3.2.	Power Terms	3-14
3.3.3.	Reference Device	3-18
3.4.	STEADY STATE ANALYSIS	3-20
3.4.1.	Neo-Alcator Scaling	3-20
3.4.2.	Goldston Scaling	3-22
3.4.3.	Ohkawa Scaling	3-27
3.4.4.	Doublet-III Scaling	3-30
3.4.5.	Reactor Power Control	3-30
3.5.	TIME DEPENDENT ANALYSIS	3-32
3.5.1.	Soft Beta Limit	3-32
3.5.2.	Active Feedback Control for Thermal Stability (in the Low Density High Temperature Region)	3-35
3.5.3.	Active Feedback Control for Thermal Stability (in the High Density, Low Temperature Region)	3-40
3.5.4.	Confinement Degraded With Temperature)	3-45
3.6.	FRACTIONAL POWER OPERATION SCENARIOS	3-48
3.6.1.	Confinement and Auxiliary Power	3-48
3.6.2.	Temperature Dependent Confinement	3-50
3.6.3.	Optimum Confinement Conditions	3-51
3.7.	DISCUSSIONS AND CONCLUSIONS	3-54
	REFERENCES	3-56

3.1. INTRODUCTION

Conceptual designs of fusion reactors have been based primarily on achieving rated power output by maintaining the plasma parameters at their optimum operating levels. However, as with fossil and fission power plants, the commissioning of fusion power reactors will entail a series of plasma and system component performance checks at low power (<5% of rated power) and subsequent testing at progressive power stages. In addition, we can envision normal start-up/shutdowns of the plant for scheduled maintenance and when system components malfunction. In these procedures, the fusion power can vary from below 5% to 100% of the plant rated power, and the rate of change must be controlled to satisfy physics and engineering constraints on the plasma and its support systems. What this implies is that, in addition to operating at full load, the plant should have the capability of operating at fractions of its full power and of making controlled transitions between power levels.

In this chapter, we explore methods to operate a tokamak fusion reactor at fractions of its rated power, identify the more effective control "knobs" and assess the impact of the requirements of fractional power operation on full power reactor design. In particular, we study the role of burn control in maintaining the plasma at thermal equilibrium throughout these operations. As a prerequisite to this task, we carefully examine the critical physics issues relevant to reactor performance predictions and offer some insight into their impact on fractional power operation. The basic tool of analysis consists of a zero-dimensional (0-D) time-dependent plasma power balance code which incorporates the most advanced data base and models in transport and burn plasma physics relevant to tokamaks.

Because the plasma power balance is dominated by the transport loss and given the large uncertainty in the confinement model, we have studied the problem for a wide range of energy confinement scalings. These scalings for global confinement time τ are most often empirical and can be divided into three groups: (A) τ independent of heating power, (B) τ degraded with auxiliary power, and (C) τ degraded with both auxiliary and alpha power. As such, a detailed comparison of ignition criteria for those scaling laws was

also carried out with the use of steady-state analysis. The results of this analysis form the basis for studying the temporal behavior of the plasma under various thermal control mechanisms. Scenarios of thermally stable full and fractional power operations have been determined for a variety of transport models, with either passive or active feedback burn control. Important power control parameters, such as gas fueling rate, auxiliary power and other plasma quantities that affect transport losses, have also been identified. The results of these studies vary with the individual transport scaling used and, in particular, with respect to the effect of alpha heating power on confinement.

The contents are outlined as follows. Section 3.2 gives a brief description of the physics basis upon which this work depends. In addition, a number of critical issues are identified and an assessment of how they impact reactor operation in general is given. Section 3.3 describes the calculational model and techniques used in this study. This is followed by section 3.4 in which results of the steady-state analysis are presented. Section 3.5 gives the results of time-dependent analyses for various thermal control scenarios under a wide range of operation conditions. In section 3.6, scenarios of fractional power operation are explored. Discussion of results and their impact on reactor power operation, and a list of conclusions are given in section 3.7.

3.2. PHYSICS BASIS

The level of thermonuclear power output from a fusion reactor depends on the operating plasma density $\langle n \rangle$ and temperature $\langle T \rangle$. ($\langle \rangle$ denotes volume average.) At thermal equilibrium, the plasma heating power, including ohmic, auxiliary and alpha, is balanced against the plasma losses, which consist of transport and radiation. The properties of each of these power components are, in addition to $\langle n \rangle$ and $\langle T \rangle$, functions of the plasma discharge parameters, namely: the plasma current I_p , size (R_0 = major radius, a = minor radius), shape (K = elongation, etc.) and the toroidal magnetic field B_0 . The profiles of density, temperature and auxiliary power deposition are also important factors in determining the power balance. Externally induced plasma losses, such as transport due to magnetic field ripples and radiation arising from impurity injection can also play an important role.

In the following subsections, we will focus on the more important physics issues related to reactor-grade plasmas and briefly review their experimental and theoretical status. We will also describe the various physics models to be employed in our analyses and assess the uncertainties and implications that come with these models.

3.2.1. ENERGY CONFINEMENT

The current status of understanding of energy transport in tokamak plasmas was well documented in a recent review article by S.M. Kaye. [1] It has long been established that transport in a tokamak is anomalous and governed primarily by plasma turbulence [2] as a result of the existence of free energy sources within the plasma. These sources can be in the form of current and pressure gradients, or a distorted or nonisotropic particle distribution function, a likely consequence of externally applied auxiliary heating. Experiments to date also confirmed that the main energy loss channel is through the electrons. Because of the complexity of the problem, there are relatively few reliable theoretical models that can adequately describe the parameter dependence of energy confinement obtained from these experiments.

Under these circumstances, tokamak transport has been studied through the so-called empirical scalings, which express the global energy confinement time τ_E as a function of plasma discharge parameters such as I_p , $\langle n_e \rangle$, R_0 , a , etc. These empirical scalings are obtained by curve-fitting to a large collection of data over a wide range of parameters using standard multiple linear regression techniques. [3] In low-density ohmic discharges, it was found that [4]

$$\tau_{E,NA}(\text{sec}) = 7.1 \times 10^{-22} \langle n_e \rangle (\text{cm}^{-3}) a^{1.04} (\text{cm}) R_0^{2.04} (\text{cm}) q^{0.5} \quad (1)$$

where q is the safety factor at the plasma boundary. This is the well-known neo-Alcator scaling. For auxiliary-heated tokamaks, particularly neutral beam injection heating, the empirical scaling is given by [4]

$$\tau_{E,aux}(\text{sec}) = 6.4 \times 10^{-8} K^{0.5} I_p(A) P_{tot}^{-0.5}(W) a^{-0.37} (\text{cm}) R_0^{1.75} (\text{cm}) \quad (2)$$

where P_{tot} is the sum of ohmic power P_{OH} and auxiliary heating power P_{aux} . Comparing Eqs. (1) and (2), we note that for ohmic discharges, τ_E is strongly dependent on the average density $\langle n_e \rangle$ while for $P_{aux} > P_{OH}$, this density dependence disappears. Instead, for these discharges, τ_E increases linearly with I_p and decreases monotonically with P_{aux} . Noting that τ_E levels off above a critical density for ohmic discharges on Alcator-C [5] and D-III, [6] Goldston proposed an inverse quadrature relation of τ_E to $\tau_{E,NA}$ and $\tau_{E,aux}$ to model the confinement [4]:

$$\tau_{E,G} = (\tau_{E,NA}^{-2} + \tau_{E,aux}^{-2})^{-1/2} \quad (3)$$

and was able to fit the experimental data very well. Equations (1), (2), (3) represent what we call Goldston scaling. Clearly, above a critical density, given by the equality $\tau_{E,NA} = \tau_{E,aux}$, τ_E is dominated by $\tau_{E,aux}$. Conversely, for constant density and auxiliary power, τ_E becomes neo-Alcator like above a critical current. The latter has been observed on D-III [7] and is well modeled by Eq. (3).

Recently, an extension of Goldston scaling, the Kaye-Goldston scaling, [1] into discharges with $P_{aux} \gtrsim 2 P_{OH}$ has resulted in a slightly more optimistic model for $\tau_{E,aux}$, given by

$$\begin{aligned} \tau_{K-G} \text{ (sec)} = & 6.02 \times 10^{-8} K^{0.28} B^{-0.09} (T) I_p^{1.24} (MA) \langle n_e \rangle^{0.26} (cm^{-3}) \\ & \times P_{TOT}^{-0.58} (MW) a^{-0.49} (cm) R_o^{1.65} (cm) \end{aligned} \quad (4)$$

We note that τ_{K-G} has a stronger dependence on I_p and $\langle n_e \rangle$ than $\tau_{E,aux}$ and, according to Eq. (4), ignition operation tends to favor devices with high current and high toroidal field, a fact that is borne out by recent studies of the Compact Ignition Torus. [8]

With regard to beam-heated tokamaks, experimental data on D-III with $P_{tot} \approx 8$ MW reveal that τ_E may equally well be approximated by [9]

$$\tau_{E,DIII} \text{ (sec)} = I_p (MA) K^{1/2} [0.022 + 0.061/P_{tot} (MW)] \quad (5)$$

for limiter discharges. Here we note that τ_E decreases but eventually levels off with increasing P_{tot} . With respect to scaling with P_{aux} , $\tau_{E,DIII}$ is much more optimistic than $\tau_{E,aux}$. It is clear that data with high auxiliary power (>15 MW) are needed to clarify this point.

Based on the phenomenological free energy argument, Ohkawa [2] derived a scaling law for τ_E given by

$$\tau_{E,OK}(sec) = 6.32 \times 10^{-22} \langle n_e \rangle (m^{-3}) R_o^2(m) a(m) q \langle T_e \rangle^{-1/2} (keV) \quad (6)$$

$$\times [1 + P_{aux}(MW)/P_{OH}(MW)]^{-1/2}$$

which has the same functional dependence on P_{aux} as Goldston scaling for $P_{aux} \gg P_{OH}$. For a pure ohmic discharge, we note that $\tau_{E,OK}$ reduces to a Merezhsin-like scaling [10] with the characteristic $\langle T_e \rangle^{-1/2}$ -dependence.

There are several other theoretical transport scalings which appear to fit the experimental data from present-day tokamaks over a wide range of parameters. One of these is due to Perkins, [11] who postulates that the electron thermal conduction is governed mainly by the strong drift wave turbulence between the $q=1$ and $q=2$ surfaces. He arrived at an empirical scaling of the form [11]

$$\tau_{E,P} \propto I_p^{0.9} B_o^{-0.1} \bar{n}_e^{0.6} P_{tot}^{-0.6} a^{0.4} R_o^{1.7} \quad (7)$$

which is remarkably close to the form of $\tau_{E,aux}$. Thus it can be said that much progress has been made toward the theoretical understanding of the physical processes governing transport in toroidal plasmas.

Although a large data base has been accumulated in neutral beam heated discharges, there are as yet very little systematic studies on the effect of RF heating on confinement. Ion cyclotron radio frequency (ICRF) heating experiments on PLT [12] and TEXTOR [13] have recorded τ_E degradation with RF power in a manner very close to that found for beam heated experiments. Very similar behavior of τ_E with P_{aux} was also observed on D-III for electron cyclotron heating. [7] Definitive scaling laws are not available because of the limited RF power levels involved. Thus, for the time being, we will assume that RF-heated plasmas follow the same confinement scalings as for beam-heated plasmas.

The empirical scalings in Eqs. (2)-(7) are derived for the so-called L-mode neutral beam injection (NBI) discharges, in which a substantial drop in τ_E from that in ohmic discharges is observed. However, "good" confinement in these NBI discharges can be restored by essentially reducing the neutral recycling near the edge of the plasma. This so-called H-mode operation can be obtained by a variety of means, for example, by a "closed" divertor configuration. The H-mode plasma is characterized by a radial T_e profile with a pedestal of several hundred eV and a relatively flat density profile. However, detailed processes involved in the transition from the L- to the H-mode are not well understood, except that H-mode confinement seems to exhibit the same parameter dependence as for L-mode plasmas, and the confinement time for H-mode is roughly twice that of L-mode. There are other methods to improve energy confinement, such as pellet fueling, trace impurity injection and ungettered operation. But these techniques do not yet have sufficiently large data bases to warrant serious consideration for reactors.

In summary, we point out that there are no experimental data on confinement properties for either reactor-sized or D-T burning plasmas. On the other hand, a large collection of data has accumulated over the years from a variety of tokamak experiments in which τ_E appears to exhibit a consistent functional dependence on machine and discharge parameters. These empirical scalings seem to be supported by recent experimental results on TFTR, [14] JET, [15] and by transport theory [11] on tokamaks. Extrapolation to near-term large non-burning tokamaks using the empirical transport scaling is justified. However, for burning tokamaks, the issue of alpha particle effects on confinement needs to be addressed.

3.2.2. ALPHA PARTICLE EFFECTS

From a phenomenological point of view, Ohkawa [2] argued that the beam ions heat the background plasma, creating a nonisotropic electron distribution function. Part of the beam power then ends up as a source of free energy for plasma turbulence or fluctuations, which can lead to enhanced energy transport. Using an argument along this line, an approximate transport scaling is derived that yields a $P_{aux}^{-1/2}$ dependence for τ_E .

It is tempting to invoke the same reasoning and infer that $\tau_E = P_{\alpha}^{-1/2}$,

where P_α is the alpha heating power. We note that a variety of physical processes exist through which alpha particles can give rise to plasma turbulence. For example, energetic trapped alpha particles may interact through their precessional drift with high-mode-number ballooning modes in tokamaks. [16] The alpha particles may also destabilize the shear Alfvén mode and the fast magnetosonic wave via their anisotropic distribution, or the drift Alfvén spectrum by virtue of plasma inhomogeneities. [17] Indeed, that the influence of alpha particles on confinement is analogous to that of neutral beam ions may be a very reasonable assumption, which we will consider in our work. However, there exists the possibility that the alpha particles may transfer most of their energy to the background plasma via anomalous absorption through the kinetic shear Alfvén instabilities. [18] In that case, the deleterious effect on confinement may be alleviated.

3.2.3. PLASMA BETA LIMIT

The fusion power P_{fus} from a tokamak reactor has the following parameter dependence:

$$P_{fus} \propto \beta_c^2 B_0^4 a^2 R_0 K \langle \sigma v \rangle_{DT} / T^2 \quad (8)$$

where β_c is the toroidal beta limit for mhd plasma stability. Small β_c can severely limit the range of fusion power output for a reactor of given size. Indeed, if additional constraints such as passive thermal stability and ignited operation are imposed, the range of power output may be further reduced. Thus, reactor concepts that raise the beta limit are a subject of vital interest to the fusion community.

There are basically two regimes of stability for tokamaks in terms of the plasma beta. The first stability regime is characterized by plasmas with elongated Dee shapes, and $\langle \beta_c \rangle \lesssim 10\%$. Stability against external kinks without wall stabilization imposes a beta limit given by Troyon et al.: [19]

$$\langle \beta_c \rangle (\%) = C_0 I_p (MA) / a(m) B_0 (T) \quad (9)$$

where C_0 is a constant ($C_0=2.7$). Sykes et al. [20] found a similar beta limit for ideal ballooning modes with $C_0=4.4$. Consideration of resistive tearing modes will somewhat lower this beta limit. Recently neutral beam experiments on D-III [21] indicated the same parameter dependence of $\langle\beta_c\rangle$ but with $C_0=3.5$. In our analyses, we will adopt the D-III beta limit scaling.

In recent years, numerical and theoretical studies [22,23] showed that a marked improvement to the stability of pressure-driven ideal mhd modes can be achieved by creating an indentation on the inboard side of a conventional Dee-shaped plasma. Access to the so-called second stability regime, characterized by beta limits in excess of 20%, is further facilitated by the presence of a nearby conducting wall. However, stability in this regime requires precise control of the local plasma current profile which may impose severe requirements on the current drive system. The design of the pusher coil, together with its shielding, used for indenting the plasma could also pose a major technological challenge when extrapolating to the reactor regime. Due to the uncertainty of this mode of operation and the lack of an extensive experimental data base, we will limit our analysis to first stability discharges.

An equally important consideration in fractional power operation concerns the plasma transport as $\langle\beta\rangle$ approaches the mhd limit. In the D-III experiments, [21] it was found that the β -limit is generally disruptive, caused by destabilization of the $m=2$, $n=1$ external kink mode. In contrast, "soft" beta limits were observed on ASDEX [24] as the neutral beam power was increased except for $q < 3$ and/or high power discharges. This saturated beta is a direct consequence of enhanced transport caused by turbulence as the plasma pressure approaches the mhd limit. Very similar confinement properties were also observed on ISX-B experiments [25] a few years ago, in which the plasma transport is well modeled by a thermal diffusivity K_e proportional to $(\epsilon\beta_p)^{3/2}$, [26] where β_p is the poloidal beta and ϵ is the inverse aspect ratio. It is at present not precisely known as to what leads to a "soft" or "disruptive" beta limit. If the "disruptive" beta limit is assumed, a maximum beta of perhaps $0.9\langle\beta_c\rangle$ will be the practical limit to avoid disruption, thus further reducing the region of operation in the $\langle n \rangle$ - $\langle T \rangle$ space. On the other hand, a "soft" beta limit permits full exploitation of the plasma pressure and

can potentially lead to an excellent scheme for power control with inherent thermal stability.

3.2.4. DENSITY LIMIT

It was observed in tokamak discharges that there is a maximum density limit to mhd stable operation, known as the Murakami density limit, [27] given by

$$n_{\max} (10^{14} \text{ cm}^{-3}) = k_m B_0 / q R_0 \quad (10)$$

where $k_m \sim 1.5$ for ohmic discharges and $k_m \sim 2-3$ for auxiliary heated plasmas. This density limit can potentially narrow the region of operation for reactors if for some reason, low-temperature operation is desired.

3.2.5. THERMAL STABILITY

From a design point of view, it is desirable to operate the reactor over a region of thermal equilibria in $\langle n \rangle - \langle T \rangle$ space in which the system is passively stable. Operation outside of this region probably requires a feedback burn control scheme that may add to the complexity of the discharge scenario and the related engineering subsystems. The issue of reactor thermal stability may best be understood by examining the plasma energy balance near ignition:

$$\frac{dW}{dt} = P_\alpha + P_{\text{aux}} - P'_L \equiv F(\langle n \rangle, \langle T \rangle) \quad (11)$$

where W , P_α , P_{aux} and P'_L are the energy, alpha power, auxiliary power and loss power (including transport and radiation) respectively. Thermal equilibrium is defined by

$$F(\langle n \rangle, \langle T \rangle) = 0 \quad (12)$$

and a necessary and sufficient criterion for thermal stability is

$$\frac{\partial}{\partial T} F(\langle n \rangle, \langle T \rangle) < 0 \quad (13)$$

For a constant P_{aux} , Eq. (12) defines a contour in the $\langle n \rangle$ - $\langle T \rangle$ space, along which

$$\frac{d\langle n \rangle}{d\langle T \rangle} = \frac{\partial F(\langle n \rangle, \langle T \rangle) / \partial T}{\partial F(\langle n \rangle, \langle T \rangle) / \partial n} \quad (14)$$

The part of the contour that satisfies

$$\frac{\partial F(\langle n \rangle, \langle T \rangle)}{\partial n} \cdot \frac{d\langle n \rangle}{d\langle T \rangle} > 0 \quad (15)$$

is then thermally stable, and will depend on the empirical scaling governing P_L . As an example, for neo-Alcator scaling (Eq. (1)) and a 50-50 DT plasma, it is found that $\partial F / \partial n > 0$ for $\langle T \rangle \gtrsim 5$ keV such that in that range of temperatures, the part of the contour with $d\langle n \rangle / d\langle T \rangle > 0$ is thermally stable.

3.2.6. OPERATING MODE

It has long been established that steady state operation is critical to the future of the tokamak configuration as an economically viable reactor concept. Noninductive current drive by lower hybrid waves has been shown to be effective in recent experiments, [28] although only at relatively low densities ($\langle n_e \rangle \lesssim 5 \times 10^{13} \text{ cm}^{-3}$). Theory predicts [29] that the current drive efficiency scales as $I_p / P_{rf} \propto n_e^{-1}$ such that within the reactor range of parameters, the efficiency will extrapolate to unattractively low values. To exploit the relatively good lower hybrid current drive efficiency at low densities, the "hybrid transformer" mode of operation was proposed. [30] In this mode, the plasma current is alternatively driven by the OH coil during the burn phase and by lower hybrid waves during the low-density, non-burning and coil re-charging phase. What emerges is a plasma discharge scenario in which the burn pulse is long ($\gg 1000$ s), depending on the transformer size, while the current can be kept constant, thus minimizing the occurrence of major disruption.

In the analyses here, we will treat the "hybrid transformer" as the reference mode of operation at all levels of fusion power output. The study will be focused on the OH-driven burn pulse, and the problem of plasma

transients between the low density dwell phase and the fusion operation phase will be the subject of Chapter 6. We also note that by adopting this hybrid mode of operation, transition between power plateaus is naturally achieved while starting up from a dwell phase. In the case of genuine steady-state operation with RF current drive, direct transition from one power stage to another may have to be considered. Even then, the choice between direct transition and fresh start-up remains to be determined.

3.3. CALCULATIONAL MODEL

We use a relatively simple, zero-dimensional plasma model to analyze tokamak operation at various power levels. There are two reasons for this simple approach: first, detailed local transport in a tokamak is not well understood, making it difficult to model, especially in reactors where auxiliary and alpha heating power dominate ohmic heating; secondly, a more sophisticated one-dimensional plasma model leads to a much more time-consuming computer code which is unnecessary for reactor parametric analyses.

3.3.1. BASIC EQUATIONS AND SOLUTION METHOD

The set of equations for the time dependent analyses include the particle conservation equations for the mean fuel ion density $\langle n_i \rangle$ and for alphas, $\langle n_\alpha \rangle$ and energy conservation equations for mean particle energies, $\langle T_i \rangle$, $\langle T_e \rangle$, $\langle T_\alpha \rangle$ for ions, electrons, and alphas, respectively. Electron density is obtained by assuming charge neutrality. The system of equations is written in the form:

$$\frac{\partial \langle n_i \rangle}{\partial t} = (S_i - [\int n_i^2 f_D (1 - f_D) \langle \sigma v \rangle dr] - \frac{\langle n_i \rangle \cdot V_p}{\tau_{pi}}) / V_p \quad (16)$$

$$\frac{\partial \langle n_\alpha \rangle}{\partial t} = ([\int n_i^2 f_D (1 - f_D) \langle \sigma v \rangle dr] - \frac{\langle n_\alpha \rangle \cdot V_p}{\tau_{p\alpha}}) / V_p \quad (17)$$

$$\frac{\partial \langle T_i \rangle}{\partial t} = [\frac{\Sigma P_i}{1.5 V_p} - \langle T_i \rangle \frac{\partial n_i}{\partial t}] / \langle n_i \rangle \quad (18)$$

$$\frac{\partial \langle T_e \rangle}{\partial t} = \left[\frac{\Sigma P_e}{1.5 V_p} - \langle T_e \rangle \frac{\partial n_e}{\partial t} \right] / \langle n_e \rangle \quad (19)$$

$$\frac{\partial \langle T_\alpha \rangle}{\partial t} = \left[\frac{\Sigma P_\alpha}{1.5 V_p} - \langle T_\alpha \rangle \frac{\partial n_\alpha}{\partial t} \right] / \langle n_\alpha \rangle \quad (20)$$

$$\text{where: } \langle n_e \rangle = \langle n_i \rangle + 2 \cdot \langle n_\alpha \rangle \quad (21)$$

$$\frac{\partial \langle n_e \rangle}{\partial t} = \frac{\partial \langle n_i \rangle}{\partial t} + 2 \cdot \frac{\partial \langle n_\alpha \rangle}{\partial t} \quad (22)$$

S_i = volume integrated ion fuel source

$\int () dr$ = plasma volume integral

V_p = total plasma volume

τ_{pj} = particle confinement time for species j

ΣP_j = sum of the volume integrated power terms for species j

The power terms considered in this model include:

$$\Sigma P_i = P_{\alpha i} + P_{aux,i} + P_{ie} - P_{tr,i} \quad (23)$$

$$\Sigma P_e = P_{\alpha e} + P_{aux,e} - P_{ie} - P_{tr,e} + P_{OH} - P_{rad} \quad (24)$$

$$\Sigma P_\alpha = P_{fus} - P_{\alpha i} - P_{\alpha e} \quad (25)$$

Each of these terms are volume integrals (MW total), where:

$P_{\alpha j}$ = alpha thermalization power to species, j

$P_{aux,j}$ = auxiliary power to species, j

P_{ie} = ion-electron rethermalization

$P_{tr,j}$ = total transport loss (including energy conduction and convection) for species, j

P_{OH} = ohmic heating power

P_{rad} = total radiation loss including bremsstrahlung and synchrotron radiation

P_{fus} = alpha power released in fusion

The radial (r) profiles for the density and temperature of each species, j , are computed in the form:

$$n_j(r) = n_{j0}(1 - f_j r^2/a^2)^{pf_j} \quad (26)$$

where:

n_{j0} = the peak value of density (or temperature) for species j
 f_j = the fraction of the peak value at the plasma edge (≈ 0.02)
 pf_j = the profile factor
 a = plasma minor radius

The power terms [in Eqs. (23), (24) and (25)] are integrated over the plasma volume using the profiles in Eq. (26) expressed in the general form:

$$P_k \text{ (MW)} = 4\pi^2 R_0 K C_k \int_0^a p_k(w/m^3) r dr \quad (27)$$

where:

C_k = energy conversion factor for the power term, k
 $p_k(w/m^3)$ = power density as a function of radius, r

The density can be fixed during a time-dependent run and the required ion-fuel source is computed. Multiple time-dependent sweeps ramping up the temperature at constant density can be combined while successively incrementing the density for each sweep. Operating parameters can then be plotted in the resulting density-temperature space as in the POPCON plots generated by the ORNL Tokamak Code, WHIST. [31] For a sufficiently small temperature ramp rate, these results approach the equilibrium values ($dT/dt \approx 0$), and the resulting "pseudo-equilibrium" contours give valuable information on reactor steady-state operation.

3.3.2. POWER TERMS

3.3.2.1. Alpha Thermalization

For simplicity, alpha thermalization can be assumed to be instantaneous, and thus, Eq. (20) can be dropped from the system of equations. $P_{\alpha i}$ and $P_{\alpha e}$ in Eqs. (23) and (24) are replaced by $(f_{\alpha j} \cdot P_{fus})$ where $f_{\alpha j}$ is the fraction of alpha power going to species, j , at the present plasma conditions. Instantaneous alpha thermalization is a good approximation under steady-state conditions. Since it is extremely fast computationally, this assumption is used in some cases, for instance, in generating the pseudo-equilibrium contour plots. For a more exact treatment by the code, the alpha thermalization terms are retained explicitly in Eqs. (23), (24) and (25) as $P_{\alpha i}$ and $P_{\alpha e}$. These are computed by numerical integration over all alpha energies and their sources contributing to the thermalization power in the current integration time interval, following a similar model developed by Houlberg and Conn. [32]

3.3.2.2. Auxiliary Power

It was assumed that over the range of plasma parameters examined, the auxiliary RF power has a constant fraction going to the ions (usually 0.8) and a constant net absorption efficiency, defined as power absorbed by the plasma per unit of power delivered to the RF generator (usually set at 0.55). These assumptions are sufficiently accurate for the present purposes of examining global power balances.

The RF power can be fixed at a constant level, or varied with a programmed time dependence, or it can include a feedback scheme. The constant temperature ramp rate is fixed by the linear feedback scheme with feedback power, P_{fdbk} , based on the equation:

$$P_{fdbk} = -P_{net} + P_{ramp} + \frac{E}{\tau_{cfn}} - \frac{(E - E_{ramp})}{\tau_{fdbk}} \quad (28)$$

where $P_{net} \sim P_{fus} + P_{oh} - P_{rad}$

P_{ramp} = power required for desired temperature ramp rate

= $(k_r \langle n \rangle V_p)$ and k_r is the desired ramp rate input

E = actual plasma energy present.

$$= 3\langle nT \rangle V_p \text{ for: } \langle n \rangle = (\langle n_e \rangle + \langle n_i \rangle)/2$$

$$\langle T \rangle = (\langle T_e \rangle + \langle T_i \rangle)/2$$

E_{ramp} = present plasma energy for desired ramp rate

$$= (P_{\text{ramp}} \cdot t) \text{ for } t = \text{current time}$$

τ_{cfn} = global confinement time

$$= 3\langle nT \rangle / P_{\text{tr}}$$

τ_{fdbk} = feedback scheme response time
(fixed input, default = 0.4 sec)

3.3.2.3. Plasma Transport Power Loss

3.3.2.3.1. Conduction and Convection. The transport power loss contains the largest uncertainty in the plasma power balance due to uncertainty in the energy and particle confinement scaling laws. This is especially true when the results of derived scaling laws are extrapolated to the regime of an ignited power reactor. As such, our calculational model has allowed for a range of scaling laws and simple relations between ion and electron scalings and between particle and energy confinement in order to easily interpret the differences in results between the scaling laws examined here. Since the scaling laws are often in the form of an empirical relation involving global plasma parameters, this code utilizes global averages of density and temperature when computing specific confinement times.

We assume that total transport loss, P_{tr} , can be written for electrons, e, and for an average fuel ion species, i, simplified for global averages as:

$$P_{\text{tr}} = \frac{3}{2} \frac{\langle n_e T_e \rangle}{\tau_{\text{pe}}} + \frac{3}{2} \frac{\langle n_e T_e \rangle}{\tau_{\text{Ee}}} + \frac{3}{2} \frac{\langle n_i T_i \rangle}{\tau_i} + \frac{3}{2} \frac{\langle n_i T_i \rangle}{\tau_{\text{Ei}}} \quad (29)$$

Each of these confinement times can be simply related to a given experimentally (or theoretically) determined confinement time from a global scaling law, τ_{gs} , which is generally evaluated from the equation:

$$P_{\text{tr}} \sim P_{\text{INJECTION}} = \frac{3 \langle nT \rangle}{\tau_{\text{gs}}} \quad (30)$$

For the typical case where $n_e \sim n_i$ and $T_e \sim T_i$, comparing Eqs. (29) and (30), the following relation must hold:

$$\frac{1}{\tau_{gs}} = \frac{1}{2} \left(\frac{1}{\tau_{pe}} + \frac{1}{\tau_{Ee}} + \frac{1}{\tau_{pi}} + \frac{1}{\tau_{Ei}} \right) = \frac{1}{2\tau_{Eff}} \quad (31)$$

which says that the global confinement time, τ_{gs} , will be twice the effective confinement time, τ_{Eff} . Thus, one can vary each of the confinement times in Eq. (29) as a fraction of the global confinement, such that Eq. (31) is satisfied, and while simultaneously, the experimentally determined constraints regarding the relative magnitude of τ_{pi} , τ_{pe} , τ_{Ei} , and τ_{Ee} are satisfied. Since these relative magnitudes are only roughly known from experiments and there is considerable uncertainty here, we selected the following relations as appropriate to test the relative merits of the confinement scaling laws examined in this study.

We let $\tau_{gs} = \tau_{Ee}$ since the electron energy transport is considered the major transport loss channel. Some experiments indicate that energy loss times tend to be 10-100% of particle loss times, and we allow electron and ion particle loss rates to be similar. We have taken $\tau_{pi} \sim \tau_{pe} \sim 5\tau_{gs}$. As the τ_p values increase, the fuel source required at a given operating point decreases. In order to satisfy Eq. (31), we must then let $\tau_{Ei} \sim 1.67\tau_{gs}$ for the above values of τ_{Ee} , τ_{pi} and τ_{pe} . In this case, the ion energy loss is nearly as large as the electron energy, which corresponds to an anomalous ion energy loss. This is consistent with recent experimental observations on Doublet-III. [33] If we had selected τ_{Ee} to be smaller, i.e., less than τ_{gs} , then τ_{Ei} could be relatively larger corresponding to smaller (neoclassical) ion energy losses.

3.3.2.3.2. Ripple Loss. Early studies showed that the ion energy transport power loss due to ions trapped in the ripples of the toroidal magnetic field, and to the banana trapped ions, has a strong temperature dependence. [34] More recently, the ion ripple loss has been found to exhibit the $T^{3/2}$ dependence only up to about 5 keV and to roll off and even decrease above that temperature. [35] Thus, in the reactor operating temperature range the net ripple transport losses are likely to be orders of magnitude less than

previously predicted and so, much less than the anomalous energy confinement losses. For this reason, ripple loss was omitted from our time dependent model. However, since ripple loss varies as $\delta^{9/2}$, where δ is the percent of field ripple, it may still be useful as an active thermal control mechanism, in which some of the saddle coil currents can be varied to induce larger ripple loss. [35]

3.3.2.4. Radiation

Radiation power loss is integrated over the plasma profile using Eq. (27) where:

$$P_{rad} = P_{brem} + P_{sync} \quad (32)$$

and

$$P_{brem} = 4.8 \times 10^{-43} \cdot n_e(r) n_i(r) Z_{eff} \cdot T_e^{0.5} \quad (33a)$$

$$P_{sync} = 6.2 \times 10^{-23} \cdot B \cdot r^2 \cdot K_a \cdot R_f \cdot n_e(r) T_e(r) \times \left(1 + \frac{T_e(r)}{146} + \dots\right) \quad (33b)$$

where the synchrotron radiation parameters include:

$$K_a = \text{plasma absorption coefficient} \\ = 5 \times 10^{-3} \cdot T_e(r)^{1.5} / (6.025e-17 \cdot n_e(r) \cdot a/B_r)^{0.5}$$

and:

$$R_f = (1 - R)^{0.5} = 0.22 \text{ (for reflectivity, } R = 0.95)$$

3.3.2.5. Ohmic Heating

Ohmic heating power density is taken to be:

$$P_{OH}(\text{MW/m}^3) = 1. \times 10^{-6} n_i J^2 \quad (34)$$

The parallel resistivity, $\eta_{\parallel}$, for a tokamak is calculated from the Spitzer resistivity, η_{sp} :

$$\eta_{sp} = 2.4 \times 10^{-9} \cdot Z_{eff}^2 \ln \Lambda_e / T_e^{1.5} \quad , \quad (35)$$

and using the neoclassical correction factor:

$$\eta_{\parallel} = \eta_{sp} / (1 - 1.95(a/R_o) + 0.95 \cdot (a/R_o)) \quad (36)$$

To evaluate the total ohmic heating power, Eq. (35) for resistivity is evaluated at the plasma averaged temperature and density, and the average current density is used in evaluating the ohmic heating power density [Eq. (34)].

3.3.2.6. Rethermalization

Electron-ion power exchange or rethermalization power is integrated using Eq. (27), where:

$$p_{ie} (MW/m^3) = \frac{3}{2} \frac{(n_e(r)T_e(r) - n_i(r)T_i(r))}{\tau_{ie}} \quad (37)$$

and

$$\tau_{ie} = 1.0 \times 10^{19} \cdot (M_j / M_{proton}) \cdot T^{3/2} / (n \cdot \ln \Lambda) \quad (38)$$

for S.I. units, except $T(r)$ in KeV.

3.3.3. REFERENCE DEVICE

For purposes of illustration, we use INTOR, the International Tokamak Reactor, [35] as a reference device, which is designed to meet steady-state performance criteria based on its objectives. Since the main objectives of INTOR are to demonstrate net power production beyond the present generation of tokamaks and to assess critical physics and engineering issues related to the tokamak reactor, the device design is not optimized. As such, the parameters

of INTOR are listed in Table I. We emphasize here that one of the aims of this work is to assess the impact of startup and fractional power operation requirements on a fusion reactor design based on delivering its rated full-power performance parameters. We fully expect that the results of our analyses could unfold critical physics issues that imply modifications and further optimizations of conventional reactor designs.

TABLE I
BASIC DEVICE PARAMETERS

	<u>INTOR/UCLA</u>
MAJOR RADIUS (M), R_0	5.3
MINOR RADIUS (M), a	1.2
TOROIDAL FIELD, B_0 (T) (ON AXIS)	5.5
STEADY STATE CURRENT (MA)	6.4
ELONGATION, K	1.6
CYLINDRICAL SAFETY FACTOR, Q	2.1
CRITICAL BETA (%)	3.5
MAXIMUM DENSITY (10^{20} m^{-3})	1.5
THERMAL CYCLE EFFICIENCY, η_{TH}	0.35
AUXILIARY POWER SYSTEM EFFICIENCY, η_{AUX}	0.55
OHMIC POWER EFFICIENCY, η_{OH}	0.95
ADDITIONAL BASE-LINE POWER REQUIREMENTS, P_{BAS}	150 MW
BLANKET MULTIPLICATION, M	1.2

3.4. STEADY STATE ANALYSIS

Due to the uncertainty in tokamak confinement scaling and how it extrapolates to reactors, we have studied the problem of fractional power operation using four plausible scaling laws, namely: (i) neo-Alcator, (ii) Goldston, (iii) Ohkawa and (iv) D-III. For each scaling law, we present the POPCON plot using the standard parameters of INTOR (Table I) and discuss the list of possibilities for varying the fusion power output. A simplified steady-state version of the code described in section 3.3 is used, in which $\langle T_i \rangle = \langle T_e \rangle = \langle T \rangle$, $\langle n_i \rangle = \langle n_e \rangle = \langle n \rangle$, $\langle n_\alpha \rangle = 0$ and instantaneous α -thermalization is assumed.

3.4.1. NEO-ALCATOR SCALING

Shown in Fig. 1 is the POPCON plot for INTOR with neo-Alcator confinement scaling, given by Eq. (1). Contours of P_{aux} , P_{fus} and $\langle \beta_c \rangle$ are plotted. We observe that full power (500 MW) operation brings the plasma way above ignition and is limited by $\langle \beta \rangle = 3.4\%$, spanning a parameter range of $\langle T \rangle = 5-15$ keV and $\langle n \rangle = 0.65-2 \times 10^{20} m^{-3}$, between points A and B in Fig. 1. It is also evident that a burn control mechanism is necessary to keep the plasma in equilibrium along the full power contour, particularly if the $\langle \beta \rangle$ limit is disruptive. Operation at $\langle \beta \rangle < 0.9 \langle \beta_c \rangle$ may be necessary. On the other hand, if the $\langle \beta \rangle$ limit is soft, τ_E is expected to deteriorate as $\langle \beta \rangle$ approaches $\langle \beta_c \rangle$ until a stable equilibrium is reached. It is speculated, and not entirely impossible, that the soft $\langle \beta \rangle$ limit may be used as a passive thermal stability control mechanism, and with programmed density and auxiliary power, may serve as a "knob" for fusion power control.

Referring to Fig. 1, reactor operation at 50% and 25% of rated power can be achieved at points C and D respectively, along the ignition boundary. We note that point D is unstable while point C is only marginally stable. To ensure thermal stability, points C' and D' are suggested, requiring 5 and 15 MW of auxiliary driving power respectively. Apparently there is a tradeoff between driven operation (finite $Q = P_{fus}/P_{aux}$) and the need for burn control. However, if full power operation requires burn control anyway, the obvious choice will be points D and C for 25% and 50% power operation. Likewise, if passive control via "soft" beta limit is available for full power

Figure 1 POPCON plot of auxiliary power P_{aux} , fusion power P_{fus} and critical beta $\langle \beta \rangle$ over $\langle n \rangle$ - $\langle T \rangle$ space, for an INTOR-like tokamak with parameters listed in Table I. Neo-Alcator scaling is used. Solid lines: P_{aux} (MW), dashed lines: P_{fus} (MW) and dotted line: $\langle \beta \rangle = 3.4\%$. Arrowed dash-dotted lines indicate possible trajectory for transition from 25% to 50% rated power.

operation, points D' and C' will be the preferred equilibria for maintaining 25% and 50% fractional power.

Based on the hybrid transformer mode of operation, we can envision a likely scenario for transiting between power plateaus. For example, let us take the case of transiting from 25% to 50% of rated power and we are interested in going from point D to point C. From point D, we may turn off the fueling to bring the plasma down from ignition and both $\langle n \rangle$ and $\langle T \rangle$ will drop with a characteristic time of τ_E . At the same time, the current I_p will decrease with a resistive skin time of $\tau_s = L/R_p$, where L is the plasma

inductance, R_p is the plasma resistance and $\tau_E \ll \tau_s$ usually. When $\langle n \rangle$ decays to below $3 \times 10^{19} \text{ m}^{-3}$, 10 MW of lower hybrid power for current drive is switched on. The rate of density drop will be controlled to slowly bring the plasma to point E, when the plasma current will be totally driven by RF. The point E is determined by the empirical lower hybrid current drive efficiency formula: [36]

$$\frac{I_p}{P_{RF}} (A/W) \approx \frac{0.15}{R_o (m) \langle n_{20} \rangle} \quad (39)$$

where $\langle n_{20} \rangle$ is in units of 10^{20} m^{-3} , and by global power balance, assuming all the RF power is dissipated in the plasma. Substituting $I_p = 6.4 \text{ MA}$ and $P_{RF} = 10 \text{ MW}$ into Eq. (39), we obtain $\langle n \rangle = 4.5 \times 10^{18} \text{ m}^{-3}$ and $\langle T \rangle = 3.2 \text{ keV}$ at point E from power balance. To reach point C after recharging the transformer primary, we increase the ion fueling rate and turn on the auxiliary heating power while removing the RF current drive power. In a highly programmed fashion, the plasma can be brought to the equilibrium point C by increasing the density to and maintaining it at $5 \times 10^{19} \text{ m}^{-3}$. The trajectory for the whole procedure is shown by the dashed curve DEC in Fig. 1 with the arrows pointing in the appropriate direction. Clearly, a more detailed, time-dependent analysis is required to determine the exact path of this operation. Note also that during this process, at most only 10 MW of P_{aux} for heating and 10 MW of P_{RF} for current drive are involved, which are well within the power requirements for the full power steady state design.

3.4.2. GOLDSTON SCALING

The main feature of Goldston empirical confinement scaling, as given by Eqs. (1)-(3), is that it describes correctly two experimentally observed phenomena: (i) for an ohmically heated tokamak, τ_E rises with density but saturates above a critical value; (ii) for tokamaks where $P_{aux}/P_{OH} > 1$, τ_E drops with P_{aux} . When extrapolating to a reactor plasma, the issue of how the energetic alphas may affect global confinement becomes critical. Using a phenomenological argument, it can be inferred that the effect of alpha power may be similar to that of auxiliary heating power, in which case very pessimistic predictions may result. On the other hand, anomalous absorption by the background ions may reduce the turbulence level to a minimum with the

result that overall confinement is unaffected. In the absence of any experimental data in this regard, we will examine two extreme cases: (i) confinement without alpha degradation, and (ii) confinement with alpha degradation.

3.4.2.1. Confinement without Alpha Degradation

Using Eqs. (2) and (3), we can rewrite the plasma power balance equation as

$$\frac{W}{\tau_{NA}} (1 + \nu P_{tot})^{1/2} = P_{tot} + P_{\alpha}' \quad (40)$$

where $W = 3\langle nT \rangle V_p$ is the total plasma energy, $P_{tot} = P_{aux} + P_{OH}$, $\nu = (\tau_{NA}/\tau_{aux})^2/P_{tot}$ and $P_{\alpha}' = P_{\alpha} - P_{rad}$. We note here that since τ_E is dependent on P_{aux} , it opens up an avenue for controlling τ_E via an external means. To illustrate this notion, we first point out that, with Goldston scaling, the ignition curve is obtained by substituting $P_{aux}=0$ in Eq. (40), given explicitly by

$$P_{\alpha}' + P_{OH} = \frac{W}{\tau_{NA}}, \quad (41)$$

with $\tau_{NA}' = \tau_{NA}(1 + \nu P_{OH})^{-1/2}$. For nonzero P_{aux} , the solution to Eq. (40) given $\langle n \rangle$ and $\langle T \rangle$ is divided into two regions: subignition and ignition, as illustrated in Figure 2. Below ignition, the plasma equilibria are given solely by the solution $P_{aux}=P_2$, as shown in Fig. 2(a). The corresponding POPCON plot is depicted in Fig. 3, using the standard INTOR parameters in Table I. Comparison with Fig. 1 indicates that the ignition boundary is only slightly modified from that for neo-Alcator transport, since $\tau_{NA} = \tau_{NA}'$ near ignition. However, due to the deleterious effect on confinement by auxiliary heating, the power required for driven equilibrium is increased by approximately tenfold. Practically, all these equilibria are inherently thermally stable, since P_{aux} is comparable to P_{α} and dominates the overall heating power. Figure 2(a) also shows that $P_{aux} > P_2$ has to be satisfied at all points along the path to ignition. This implies that within this plasma model, at least 100 MW of auxiliary power is required to ignite an INTOR-like device, as shown in Fig. 3. It is proposed that the finite thermalization

Case A. $P_{\alpha}' + P_{OH} < W / \tau_{NA}'$ Sub-ignition

Case B. $P_{\alpha}' + P_{OH} < W / \tau_{NA}'$ Ignition

Figure 2 Typical diagram of power balance terms as a function of auxiliary power P_{aux} , with a Golston transport model without alpha degradation for (A) sub-ignition and (B) ignition, for a given $\langle n \rangle$ and $\langle T \rangle$. $P_{aux} = P_1$ and $P_{aux} = P_2$ are solutions to the power balance equation (22).

Figure 3 Same as in Fig. 1, except that Goldston scaling without alpha degradation is used, i.e., $P_{\text{tot}} = P_{\text{aux}} + P_{\text{OH}}$.

time τ_α of the alpha particles can be utilized to somewhat reduce this power requirement. This scenario will be studied in the following section.

The conditions and requirements for full and fractional power operation with Goldston scaling are similar to those with neo-Alcator confinement, except that the auxiliary power required for driven equilibrium are much higher. Therefore, there is much incentive in locating the operating points on the ignition boundary, even at the expense of a costly feedback stabilization system.

With Goldston scaling, it may be possible to use a modest amount of injection power to control the burn well above ignition. Shown in Fig. 2(b) are the solutions P_1 and P_2 ($P_2 > P_1$) for P_{aux} in Eq. (22) for $\langle n \rangle$, $\langle T \rangle$ which satisfies the ignition condition: $P_\alpha' + P_{\text{OH}} > W/\tau_{\text{NA}}'$. Contours of constant $P_{\text{aux}} - P_1$ are shown as dashed curves in the POPCON plot of Fig. 4. It is evident that full power operation can be maintained with auxiliary power not

more than 15 MW. For $\langle T \rangle > 12.5$ keV above ignition, these "driven" high Q equilibrium points may actually be thermally stable. Unfortunately this stable region is limited by the mhd critical beta. However, an important prerequisite for this scheme to work is that the effect of the alpha power on confinement is small compared to that of the driving power which is usually less than 10% of P_α . Concrete experimental data is needed to determine the validity of the scheme.

Figure 4 Same as in Fig. 3, except that contours of modest auxiliary power above the ohmic ignition boundary, given by $P_{aux} = P_l$ in Fig. 2(b). These contours are shown as dashed-dot lines and the labels are in MW.

3.4.2.2. Confinement with Alpha Degradation

Full account of alpha degradation of confinement can be taken by solving the equation

$$\frac{W}{\tau_{NA}} [1 + \nu(P_{tot} + P_\alpha)]^{1/2} = P_{tot} + P_\alpha \quad (42)$$

for P_{aux} given $\langle n \rangle$ and $\langle T \rangle$. The result is given by the POPCON plot in Fig. 5, where we note that (i) ignited operation is not available, and (ii) even for driven reactor mode, the auxiliary power is considered too high to make the INTOR-like machine economically viable. This highly pessimistic extrapolation to reactors underscores the importance of the need to understand the behavior of alpha particles in a burning plasma and, in particular, how they affect confinement.

Figure 5 Same as in Fig. 1, except that Goldston scaling with alpha degradation is used, i.e. $P_{tot} = P_{aux} + P_{OH} + P_{\alpha}$.

3.4.3. OHKAWA SCALING

The Ohkawa [2] empirical scaling law, as given in Eq. (6), can be rewritten as

$$\tau_{E,OK} = \tau_{E,OH} (1 + P_{aux}/P_{OH})^{-1/2} \quad (43)$$

where $\tau_{E,OH}(s.) = 6.32 \times 10^{-22} \langle n_e \rangle (m^{-3}) R_o^2(m) a(m) q / \langle T_e \rangle^{-1/2} (keV)$. We note that $\tau_{E,OK}$ and $\tau_{E,G}$ have roughly the same form for $P_{aux} \gg P_{OH}$. However, for an ohmically heated plasma, $\tau_{E,OK} = \tau_{E,OH}$ which is neo-Alcator like except for a $\langle T_e \rangle^{-1/2}$ dependence. As a result, the ignition boundary is markedly different from that for Goldston scaling, as depicted in Fig. 6, where no confinement degradation due to alphas is assumed. Because of the $\langle T_e \rangle^{-1/2}$ variation, unacceptably high levels of auxiliary power are necessary to drive the reactor near or below ignition, particularly for electron temperatures above 5 keV. Above ignition, as in the case of Goldston scaling, thermal equilibrium, whether or not stable, can be maintained by low levels of auxiliary power of the order of a few megawatts. Because of the nature of the confinement scaling, marginally viable regimes of operation are limited to the low-temperature, high-density portion of the $\langle n \rangle$ - $\langle T \rangle$ space. Full power (500 MW) can be maintained at the point $\langle n \rangle = 1 \times 10^{14} \text{ cm}^{-3}$ and $\langle T \rangle \approx 8 \text{ keV}$ without any auxiliary power. To minimize the auxiliary power requirement, access to this point has to be achieved via a high-density path, e.g., for $\langle n \rangle = 3 \times 10^{14} \text{ cm}^{-3}$, (above Murakami's limit) a minimum of 200 MW is required [see Fig. 6(b)]. It is evident by now that the basic problem with reactor operation under this scaling law is that the access to ignited scenario requires a very large amount of auxiliary power and an active feedback control scheme, which may make this concept economically unattractive. Even if the above difficulty is overcome, ignited operation regimes are also extremely limited, while the alternative will be the low-Q driven mode. By the same argument, fractional power operation, particularly at lower levels (<50%), appears to be problematical.

Inclusion of the fact that alpha power could degrade confinement much like auxiliary heating leads to extremely unattractive reactor power balance for the INTOR-like machine. However, substituting for $P_{aux} \approx P_\alpha$ in Eq. (43), we can define an ignition condition

$$\frac{P_\alpha}{P_L} = \frac{\tau_{E,OH} P_\alpha^{1/2} P_{OH}^{1/2}}{W} > 1 \quad (44)$$

Noting that $P_\alpha/P_L \propto \langle n \rangle \langle T \rangle^{\lambda/2-9/4} B_o^2 R_o$, where $\langle \sigma v \rangle \propto \langle T \rangle^{\lambda(T)}$, we conclude that, even with alpha confinement degradation, we can build an ignition tokamak by choosing parameters cautiously. According to Eq. (44), we need a reactor with

Figure 6 (a) Same as in Fig. 1, except that Ohkawa scaling without alpha degradation is used.

(b) POPCON plot for parameters used in Fig. 6(a), in the high density: $\langle n \rangle = 1.5\text{--}3.5 \times 10^{20} \text{ m}^{-3}$ and low temperature: $\langle T \rangle = 0\text{--}10$ keV region.

high density, high current, high magnetic field and relatively large size to achieve ignition.

3.4.4. DOUBLET-III SCALING

An empirical scaling can be used to fit the Doublet-III confinement experimental data, [9] in which τ_E saturates to a constant value if the auxiliary power is increased indefinitely. Unfortunately there is no size scaling corresponding to this set of data. We assume, as an example, an Alcator-like size scaling and, normalizing to a typical D-III parameter set, obtain the following confinement law:

$$\tau_{E,G}(s) = 0.156 I_p(\text{MA}) a^2(\text{m}) K^{1/2} [1 + 2.32/P_{\text{tot}}(\text{MW})] \quad (45)$$

where we conservatively take the coefficient obtained from L-mode data. For this case, we will include the fact that the alpha power is deleterious to confinement the same way auxiliary power is, such that $P_{\text{tot}} = P_\alpha + P_{\text{aux}} + P_{\text{OH}}$. The POPCON plot for the INTOR-like device (Table I) under the D-III confinement model is shown in Fig. 7, where the $P_{\text{fus}} = 500$ MW and $\langle \beta_c \rangle = 3.4\%$ contours are also depicted. Full power operation has to be run in the ignited mode with burn control required in the event that the beta-limit is disruptive. However, fractional power operation can be carried out in the thermally stable portion of the $\langle n \rangle$ - $\langle T \rangle$ space with $\langle n \rangle < 6 \times 10^{19} \text{m}^{-3}$ and $\langle T \rangle \geq 7.5$ keV, with plasma Q in excess of 10. The minimum auxiliary power requirement for ignition is 15 MW as long as the density is kept below $6 \times 10^{19} \text{m}^{-3}$ during the heating phase. It should be pointed out that the D-III data can be fitted equally well by a $\tau_E \propto P_{\text{aux}}^{-1/2}$ scaling. Thus, higher power experiments should be carried out to determine the true scaling with P_{aux} .

3.4.5. REACTOR POWER CONTROL

The ideal reactor would produce the range of desirable power output over a region in $\langle n \rangle$ - $\langle T \rangle$ space in an ignited, passively thermally stable mode. However, as evidenced in the previous subsections, if the first stability beta

Figure 7 Same as in Fig. 1, except that D-III scaling with alpha degradation is used.

limit is imposed, such a region will be severely narrowed and, for some range of power output, driven and/or burn control will be required. To increase the region of thermal stability, we may externally induce enhanced power loss that scales as T^N ($N > 1$) such that the thermally stable part of the ignition contour is raised in $\langle n \rangle$ - $\langle T \rangle$ space. Mechanisms that may achieve this include enhanced transport due to toroidal field ripples [34] and radiation loss via injection of a high-Z impurity into the plasma. [37] The strong dependence of ripple loss on ion temperature above 5 keV is now called into question. [35] Even if it exists, the induced losses could impose an unacceptably large heat load on the edge particle removal system. On the other hand, high-Z impurity injection may substantially dilute the plasma beta and cause a reduction in fusion power output. A judicious choice of impurity, such as iodine in the STARFIRE study, [37] could presumably alleviate this concern. However, a recent INTOR study [35] found it impractical because of the short impurity

confinement time (compared to the fuel ions) required and the excessive demands on the injection systems for diagnostics and feedback control. Thus the effectiveness of this mechanism is unclear at the present time.

Turning to the case of driven operation and/or burn control, there are a variety of control elements that can be used. Burn control requires "knobs" that induce a fast response from the plasma, particularly if it is inside the ignited, thermal runaway regime. The control elements then comprise all parameters that can affect the energy balance, such as density, auxiliary power, fuel mixture ratio, current, plasma shape and size, radial profiles and divertor/limiter operation. The viability of some of these control "knobs" has already been mentioned. In conjunction with burn control, direct transition between power plateaus, if needed, may be achieved in a highly controlled manner. We will explore scenarios of fractional power operation using a combination of these control schemes in a time dependent analysis in the following section.

3.5. TIME DEPENDENT ANALYSIS

We now consider the dynamics of moving a reactor plasma through a trajectory in $\langle n \rangle$ - $\langle T \rangle$ space to reach the point of desired output power. Since power operation is limited to a region near the ignition curve there are two distinct ways to approach the fuel power: 1) at lower density with increasing temperature, or 2) at lower temperature with increasing density. The first approach works in a more thermally stable regime and is thus favorable from a reactor control viewpoint, but the second approach permits greater net power for a given beta value (i.e., greater maximum power at the beta limit). This is more favorable economically. Time evolutions of plasma parameters along these optional paths were studied with the time dependent code. The results are summarized in the following sub-sections, starting with the simple case in which we assume a 'soft' beta limit.

3.5.1. SOFT BETA LIMIT

This case was analyzed assuming Goldston scaling with auxiliary power degradation. A 'soft' beta or non-disruptive beta limit was modeled by

making τ dependent upon β so that it degrades rapidly as the β approaches the critical beta limit β_c , ($\tau(\beta, n, T) = \tau(n, T) \cdot \exp(-1 (\beta/\beta_{crit})^{**10})$). Near the beta limit, transport losses increase until thermal equilibrium is achieved with saturation in beta, and thus restricts further increase in temperature. The time dependent analysis confirmed that this case is thermally stable at the beta limit even at points well above ignition. The $\langle n \rangle - \langle T \rangle$ trajectory for such a case is shown in Fig. 8 followed by the time variations of the major operating parameters, shown in Figs. 9 (a-c).

At times, $t = 0 - 11.5$ s, the ion source and auxiliary power are varied to achieve linear density and temperature ramps in time until ignition is reached. [Note that this scenario is not optimized, and that a slower ramp rate to ignition would greatly reduce that peak auxiliary power from the level shown in Fig. 9(b).] At $t = 11.5$ s, the auxiliary power is turned off, transport losses decay rapidly with half-life of the energy confinement time. At $t = 12.5$ s, the temperature falls briefly until the alphas thermalize sufficiently to restore ignition. At 13.5 s, the temperature increases rapidly through the ignited $\langle n \rangle - \langle T \rangle$ space until the beta limit of 3.5% is reached.

Figure 8 Density-temperature trajectory for the "soft-beta" case, assuming Goldston scaling and using INTOR parameters listed in Table I.

a

b

c

Figure 9 (a-c) Time dependence of several plasma and operating parameters during the trajectory shown in Fig. 8. (Note: Net electron power does not include the 150 MW base line power.)

In the interval, $t = 13.5 - 18$ s, the density continues to increase at a constant rate, and the operating point moves up in density along the isobaric contour at the beta limit. This isobaric movement is stable, and thermal equilibrium exists at all points. At $t = 18.0$ s, a preprogrammed control sequence causes the steady-state plasma current to drop by 10%, which forces the beta limit and thus the temperature to decrease at constant density accordingly to the new lower Troyon beta limit. In the last interval, $t = 18.2 - 20$ s, the density continues to increase isobarically along the contour at the new beta limit of about 3.2%.

This operating point remains thermally stable at all times for any specified density up to the Murakami density limit. It is important to recall that as we increase the $\langle n \rangle - \langle T \rangle$ point isobarically in density we are also increasing the net reactor electric power output.

Trajectories in $\langle n \rangle - \langle T \rangle$ space for this case can be controlled by varying the ion fuel source, and thus moving up or down in density along the isobaric contours. The contour for the desired trajectory can be varied by changing the beta limit. Assuming a Troyon beta limit, a lower value can be achieved by lowering the steady state plasma toroidal current, which will then in turn shift the beta-limited operating point to a lower temperature at constant density. It is, therefore, clear that a 'soft' beta limit, if it exists, can provide an excellent scheme for passive feedback burn control.

3.5.2. ACTIVE FEEDBACK CONTROL FOR THERMAL STABILITY (IN THE LOW DENSITY HIGH TEMPERATURE REGION)

Two trajectories are shown for Goldston scaling; the first, in which thermal equilibrium is not achieved, is shown in Fig. 10, and the second, in which equilibrium is achieved, is shown in Fig. 11. The time evolutions of the key parameters for the second case are then shown in Figs. 12 (a-g).

In both cases, once ignition is reached, density is controlled so the operating point remains on the ignition curve. This point is thermally unstable and so the temperature drifts, slowly increasing. When 10 keV is exceeded, the feedback system is programmed to decrease density further by simply cutting off the ion fuel source until the operating point falls out of

Figure 10 Density-temperature trajectory for simple feedback scheme showing unstable operating point leading to thermal runaway (Goldston ohmic-only scaling).

Figure 11 Density-temperature trajectory for feedback scheme maintaining plasma at marginally subignited densities, and reaching equilibrium (Goldston ohmic-only scaling).

Figure 12 (a-d) Time dependence of several plasma and operating parameters during the trajectory shown in Fig. 11. (Note: Net electron power does not include the 150 MW base line power.)

e

f

g

Figure 12 (e-g) Time dependence of several plasma and operating parameters during the trajectory shown in Fig. 11. (Note: Net electron power does not include the 150 MW base line power.)

ignition and the temperature is allowed to swing back down below the 10 keV point. Once below 10 keV, the ion fuel source is turned on again so that the density is increased, and auxiliary power is added to return to the ignition curve. This is apparent in the first trajectory, shown in Fig. 10. Using this simple feedback scheme, and owing to the slow thermal response time of the plasma in this regime, the $\langle n \rangle - \langle T \rangle$ trajectory does not equilibrate quickly. The first trajectory returns to the ignition curve at a higher density value and well below the 10 keV trigger point (indicated as point A in the figure). From here, ignition leads to thermal runaway. Even with no ion fuel source, the density does not drop fast enough during the temperature excursion to bring the operating point back out of ignition again, before the plasma beta limit is exceeded. This example illustrates the extreme thermal instability of the ignited region at low temperatures and high density.

In the second trajectory (Fig. 11), the density is dropped only slightly below the ignition contour at the 10 keV trigger point. The slow temperature excursion falls below 10 keV while a constant density is maintained in the marginally subignited region. Then, when the density and auxiliary power are increased again, the trajectory returns to the ignition curve at an adjacent $\langle n \rangle - \langle T \rangle$ value. Repeating this cycle, the operating point oscillates about $T = 10$ keV in an actively controlled thermal equilibrium, as seen clearly in the temperature verses time plot in Fig. 12(a). If the plasma temperature ramp rate was maintained well below this thermal response time, the oscillations about the $\langle n \rangle - \langle T \rangle$ equilibrium point could be entirely avoided.

The analysis confirmed that active feedback can be used to maintain a thermal equilibrium point just below the ignition curve. Modulating the ion fuel source in order to maintain the density at a point exactly on the ignition curve is insufficient to fix the operating point because equilibrium points on the ignition curve in this case are thermally unstable, and the temperature will slowly increase while the operating point remains on the ignition curve. Thus, thermal equilibrium requires an operating point just below the ignition curve which must be maintained by a synchronized feedback of both the ion source and the auxiliary power.

The thermal response time of the plasma near ignition is slow (several seconds) owing to the small net power. The simple feedback scheme used here resulted in large fluctuations in the net power output due to the pulsed

auxiliary power. It is believed that a more sophisticated feedback scheme with proportional control could be used to reduce the output power fluctuations to acceptable levels (and this should be investigated in future work).

If we had assumed that confinement time degrades with auxiliary power the feedback scheme is expected to be similar but with larger amounts of auxiliary power required as the plasma drops marginally out of the ignited mode. In that case, a good feedback scheme would be even more essential in reducing output power fluctuations, and it may be difficult to reduce the power fluctuations in that case to acceptable levels.

In these cases, thermal equilibrium can be maintained at any point near ignition up to the beta limit which determines the maximum power output value. Transition between fractional power levels is relatively simple. By shutting off the feedback scheme when marginally subignited, the temperature and the net output power level will drop. By shutting off the feedback when marginally ignited, or by maintaining the density exactly at ignition, then the temperature will rise and with it the reactor power will increase in a controlled manner.

3.5.3. ACTIVE FEEDBACK CONTROL FOR THERMAL STABILITY (IN THE HIGH DENSITY, LOW TEMPERATURE REGION)

For the case where confinement is independent of heating power, we were unable to maintain thermal equilibrium in this $\langle n \rangle - \langle T \rangle$ region even with active feedback control working to keep the operating point marginally sub-ignited. This confirms expectations that the region is unconditionally thermally unstable. For thermal stability one would have to move to a point on the low power side of the hump in the sub-ignited auxiliary power contours, and would thus end up with a driven device operating in a region far from the desired net power output contours.

One may operate in this ignited region, however, if confinement time degrades with increasing auxiliary power. Time dependent studies confirm that thermal equilibrium can then be maintained by supplying the appropriate amount of auxiliary power. Adding auxiliary power can increase the transport losses

so automatically that the temperature of the plasma decreases, while cutting back on the auxiliary power will improve transport losses so that an ignited regime is again reached and the plasma temperature increases. Large fluctuations in output power again result from the simple active feedback scheme, but a more sophisticated scheme should alleviate this problem. The operating point will be bounded by the Murakami density and by the Troyon beta limit.

The $\langle n \rangle$ - $\langle T \rangle$ trajectory is shown for such a case in Fig. 13, followed by the time dependent variations of operating parameters in Figs. 14 (a-g). The density and temperature are ramped linearly as in the previous case, but much larger values of auxiliary power are now required. These values could be reduced significantly if allowing for smaller temperature and density ramp rates in time. Once the operating point is above the ohmic ignition curve (at about $t = 5$ s), the auxiliary power is modulated to maintain thermal equilibrium. Density increases until $t = 10$ seconds, then the source is adjusted to maintain a constant density at $n \sim 1.5 \times 10^{20} \text{ m}^{-3}$.

Reactor power output fluctuations, in Fig. 14(g), would be large in this simple scheme, because of the large fluctuating power requirement on the auxiliary power feedback system. These power fluctuations cannot be reduced when operating at a point well above ignition to the extent shown in the previous case. Still, to some extent not yet determined, the observed power fluctuations can be reduced by refinement of the feedback scheme.

Fractional power operation in this mode is varied by changing density possibly along a contour of constant plasma equilibrium power. This mode requires additional work to clarify the conditions required to avoid thermal runaway. An important consideration for reactor operation in this mode is plasma control following a loss of auxiliary power. Such a loss would lead to an uncontrolled temperature excursion which might lead to a disruption as the plasma beta approaches the beta limit, unless a 'soft beta' limit prevailed. A counter argument might be that only about 20-30 MW of auxiliary power are required to maintain thermal control, but much more power, (>100 MW) may be required to reach ignition. Thus, a "back-up" system of auxiliary power would be available to arrest a thermal runaway following the failure of the primary thermal control system. Thermal runaway times are on the order of seconds, so this back-up system could be implemented practically. Overall, however, the

Figure 13 Density-temperature trajectory using auxiliary power to degrade confinement time and thus maintain thermal stability in the ignited region (Goldston only).

Figure 14 (a-d) Time dependence of several plasma and operating parameters during the trajectory shown in Fig. 13. (Note: Net electron power does not include the 150 MW base line power.)

Figure 14 (e-g) Time dependence of several plasma and operating parameters during the trajectory shown in Fig. 13. (Note: Net electron power does not include the 150 MW base line power.)

large magnitude of plasma power required to be removed for thermal equilibrium in this region makes it very thermally unstable, and thus undesirable as an operating region.

3.5.4. CONFINEMENT DEGRADED WITH TEMPERATURE

Confinement time may be scaled to degrade with increasing temperature or with increasing auxiliary power. Since these are intimately related, it is probably not appropriate to assume confinement time degradation with both simultaneously. The ohmic confinement time of Ohkawa [$\tau_{E,OH}$ in Eq. (43)] shows a dependence of $T^{-0.5}$, which can be seen in Fig. 6(a) to provide an ignition curve with a slight positive slope with temperature above about 10 KeV. This assures passive thermal stability, provided that the density can be held constant in that narrow range ($\sim 1.1 - 1.2 \times 10^{20} \text{ m}^{-3}$) where the desired positive slope of the ignition curve exists.

This passively thermal stable region was examined in a series of time dependent runs at constant density. Temperature versus time results are shown in Figs. 15 and 16. Figure 15 shows $\langle T \rangle(t)$ for three sweeps, at different values of constant density. Figure 16 shows $\langle T \rangle(t)$ for $\langle n \rangle = 1.1 \times 10^{20} \text{ m}^{-3}$, which was the minimum density required for ignition. The three runs here differ after $t = 30$ seconds, when the fueling ratio, deuterium atoms per tritium plus deuterium atoms, was reduced to decrease fusion power.

Examining first Fig. 15, runs at average density less than $1.1 \times 10^{20} \text{ m}^{-3}$ did not reach ignition. The constant temperature ramp at early time in each case is the subignited region driven by feedback control to auxiliary power. The time dependence in this case of the ohmic and required auxiliary power are the same as that superimposed in Fig. 16, with $P_{OH}(t)$ and $P_{aux}(t)$ drawn qualitatively only. For higher density values in Fig. 15, ignition is achieved at lower temperatures. This ignition point is where the linear temperature ramp stops when auxiliary power is shut-off, and the plasma drifts to its own thermal equilibrium. As density increases from 1.1 to $1.2 \times 10^{20} \text{ m}^{-3}$, the plasma equilibrium temperature increases from about 17 to 24 keV (assuming we are not restricted by plasma beta for this point). This seems to imply fairly accurate density control would be required to control plasma temperature relying on passive control in this regime. Also, the minimum stable

Figure 15 Temperature versus time for cases of different constant density (Ohkawa scaling - ohmic term only).

Figure 16 Temperature versus time for cases of different f_D , the deuterium fraction of the ion fuel, at constant average density of $1.1 (10^{20} \text{ m}^{-3})$. The time dependence of the ohmic power and the auxiliary power curves are superimposed in a qualitative manner. Ignition is at $t \sim 20 \text{ s}$ (Ohkawa scaling, ohmic term only).

ignited temperature possible in this manner is about 15 keV. Equilibrium points at lower temperatures are in a sub-ignited and thermally unstable region.

Figure 16 shows that at the minimum ignition density of $\sim 1.1 \times 10^{20} \text{ m}^{-3}$ we can reduce the equilibrium temperature to less than 15 keV, about 12-13 keV, by changing the fueling fraction from its value for optimal power density, 50%D/50%T, to about 40%D/60%T, reducing P_q by about 5%. Further reducing the fueling ratio, e.g., to 30%D/70%T causes the reactor plasma to fall out of ignition as shown. The consequences of these findings to power operating scenario are discussed further in section 3.6.

Without the non-disruptive beta limit, passively thermal stable operating regimes can be obtained by a driven sub-ignited reactor or by a temperature dependent confinement time (e.g., Ohkawa's scaling). Recent studies indicate that ripple-trapped particle banana modes do not offer the strong temperature dependence, as previously thought; thus, ripple losses do not guarantee a passive thermal stability. The mechanisms which decrease confinement with increased temperature need additional study since they could be used to effect passive thermal stability control. Since temperature dependent confinement is not assured and the driven reactor may prove less economically attractive, we investigated active control points near the ignition curve in the representative confinement scaling given by Goldston.

Overall, the consequences of several possible confinement scaling laws on reactor operations at full and fractional power were investigated by examining relevant operating parameters in a steady-state region of density-temperature space and by following several time dependent analyses. The results suggest a range of methods for operating at different power levels and for transiting between adjacent power levels depending upon the confinement scaling in effect. Thermally stable operating points are limited to the sub-ignited regions and the marginally ignited regions along the low density side of the ignition contour. Unless a soft beta limit applies or confinement has a significant temperature dependence, a marginally ignited tokamak will need an active feedback control of auxiliary power and fuel ion injection in order to maintain thermal equilibrium at a point corresponding to the desired fractional power output of the tokamak reactor.

3.6. FRACTIONAL POWER OPERATION SCENARIOS

We choose to present possible operating scenarios as trajectories in $\langle n \rangle$ - $\langle T \rangle$ space superimposed upon contours of net electric power, P_{net} . The assumed values used to calculate P_{net} are listed in Table I, and P_{net} is defined as:

$$P_{\text{net}} = \eta_{\text{th}}(P_{\alpha} + 4 \cdot M \cdot P_{\alpha} + P_{\text{aux}} + P_{\text{OH}}) - P_{\text{aux}}/\eta_{\text{aux}} - P_{\text{OH}}/\eta_{\text{OH}} - P_{\text{bas}} \quad (46)$$

The value of $P_{\text{bas}} = 150$ MW is consistent with the INTOR (1982) value of 200 MW, which also includes $P_{\text{aux}}/\eta_{\text{aux}}$ and $P_{\text{OH}}/\eta_{\text{OH}}$. The plasma heating power and the blanket multiplied neutron power, $4 \cdot M \cdot P_{\alpha}$, are recovered at a single thermal efficiency, η_{th} .

3.6.1. CONFINEMENT AND AUXILIARY POWER

The trajectories for the two cases, namely confinement scalings independent of and dependent upon auxiliary power, are taken directly from the results in section 5 and are shown together in Fig. 17. Net electric power (positive and negative value) contours are included as well as beta values of $\beta = 3.5\%$, and $\beta = 5\%$ and the ignition curve, $P_{\text{aux}} = 0$, for Goldston scaling.

This illustrates the dilemma of an optimistic ignition curve. Ignition is achieved when the reactor has a net electric power output of about -100 MW, or 100 MW are needed to sustain operation. This is primarily due to the 150 MW assumed as a base-line or constant load requirement, thus the reactor is actually putting out about 50 MW of electric power near ignition.

For confinement time independent of auxiliary power (trajectory A), thermal stability restricts operation to points along the lower right side of the ignition curve. In this case, the full ignited reactor range of power is from about -100 MW to -60 MW. These points are stable only with active feedback of the ion fuel source and auxiliary power. Regions of greater power output are only accessible for stable operation if the ignition curve can be raised by increasing transport or radiation losses.

Figure 17 Operating point trajectories in $\langle n \rangle$ - $\langle T \rangle$ space for two cases superimposed on the net electric power contours. The Goldston (ohmic-only) scaling ignition curve and two beta value contours are also shown.

Trajectory B depicts the case for confinement time degrading with auxiliary power. In this case, feedback of the auxiliary power allows for active thermal control above the ignition curve. All $\langle n \rangle$ - $\langle T \rangle$ points can be made stable with feedback up to the beta limit, thus all positive net power levels up to that limit are accessible. For the assumed $\beta_{\max} = 3.5\%$ and $\langle n \rangle_{\max} \sim 1.5 \times 10^{20} \text{ m}^{-3}$ this reactor can operate along the 200 MW contour. However, this contour is for the ignited case and does not consider the auxiliary power needed to maintain the thermal equilibrium. It was found in the previous section that about 25 MW of auxiliary power was needed at the equilibrium position, $\langle T \rangle \sim 6 \text{ keV}$, $\langle n \rangle \sim 1.5 \times 10^{20} \text{ m}^{-3}$. At this density and the temperature corresponding to the beta limit of 3.5% it is estimated about 50 MW are needed for thermal stability so the 200 MW contour there would be effectively reduced to about 150 MW. If the reactor were assumed to have larger stable beta, e.g., $\beta = 5\%$, then correspondingly larger net output power

could be achieved but proportionately larger auxiliary power would also be required to maintain thermal stability.

3.6.2. TEMPERATURE DEPENDENT CONFINEMENT

The time dependent studies described in section 3.5.4 can be summarized on a net electric power contour plot as shown in Fig. 18. Contours of beta, $\beta = 3.5\%$ and $\beta = 5\%$ are shown as well as the thermally stable operating points along the ignition curve, marked with the heavier line in the figure.

Figure 18 Operating point trajectories on $\langle n \rangle$ - $\langle T \rangle$ space superimposed on the net electric power contours for the case of an ignition curve with marginal thermal stability. (Ohkawa scaling, ohmic term only).

The time dependent study showed that the region labeled ($\Delta \langle n \rangle$) is thermally stable by varying the density against the effects of the temperature dependent confinement. The region labeled (Δf_d) is thermally stable by varying the deuterium fuel fraction at constant total fuel ion density.

First, to operate a reactor for this scenario in an ignited, thermally stable region we must raise a beta limit to 5%, since at $\beta < 3.5\%$ there is no stable ignited region accessible. Then, we find there is a minimum power point for ignited thermally stable operation (at the left end of the heavier line, $\langle T \rangle \sim 14$ keV, $\langle n \rangle \sim 1.1 \times 10^{20} \text{ m}^{-3}$). Thus, in this scenario, (A, at constant density equal to desired operating density) the reactor must power up rapidly to this point, with a net electric power output of about 300 MW. Alternatively, one could approach this stable operating point from lower densities at approximately constant temperature, in which case the reactor would be subignited and therefore thermally stable all the way to the ignited operating point (labeled B in the figure).

A middle road may be ideal (C), increasing density along the ohmic power equilibrium contour until the operating point reaches the minimum density at which $P_{\text{net}} = 0$ for some temperature. At this point, $\langle n \rangle \sim 0.85 \times 10^{20} \text{ m}^{-3}$, auxiliary power is added as needed to increase temperature at constant density until $\langle T \rangle \sim 14\text{--}16$ keV. At this point, operation is subignited, therefore stable, and net electric power output is zero, so there is no major penalty in staying at that point during start-up procedures or testing. Increasing the density now while proportionately cutting back on auxiliary power will bring the operating point up to ignition, while increasing net reactor power output.

3.6.3 OPTIMUM CONFINEMENT CONDITIONS

Three cases of "optimum" or desirable confinement conditions are shown together in Fig. 19, including: an ignition curve with a strong temperature dependence (A), an ignition curve which crosses the contours of desirable net electric power output (B), and the 'soft' beta case (C).

A temperature dependent ignition curve (like A in Fig. 19) is ideal in that net electric power levels from zero up to the maximum at the governing beta limit are thermally stable. This curve might be achieved with $\tau \propto aT^{-b}$, where comparison to the mild temperature dependence discussed for the Ohkawa scaling law indicates that $b > 0.5$ is needed. Ignition curves similar to this can also be generated by varying the coefficient and the exponent values (within reasonable experimental agreement) of a $\tau \sim aP_{\text{IN}}^{-b}$ type of scaling. As discussed in an earlier section, if a small fraction of the alpha power

Figure 19 Operating point trajectories in $\langle n \rangle$ - $\langle T \rangle$ space superimposed on net electric power contours for the cases of more desirable ignition curves (A,B). This also includes a trajectory, C, for the desirable "soft-beta" case.

degrades confinement time, then thermal stability is achieved, due to an ignition curve of this type. This underscores the need to understand the dependence of these values on controllable plasma parameters. The scaling dependence on temperature or auxiliary power, and the extent that alpha power degrades confinement can vary the ignition curve from the worst case of non-ignition to the optimal case with passive thermal stability at ignition and in regions of desirable net power output.

The best ignition curve (B in the figure) that may result if an inverse temperature dependence of confinement cannot be achieved, would be an ignition curve that coincides with $\langle n \rangle$ - $\langle T \rangle$ points of zero net power up to the maximum desired power output. Any point along this ignition curve could be maintained in equilibrium by active feedback as shown in the time dependent studies. Active feedback of the fuel source and auxiliary power would be required to achieve desirable power output levels.

The trajectory depicted in Fig. 8 for the case of a 'soft' beta limit is shown as curve C in the Fig. 19. That trajectory is extended here to points of greater net reactor power along the beta limit contour of 3.5%. Higher net reactor power would be achieved if a larger beta limit could be obtained.

The trajectory jogs slightly as it crosses the -100 MW contour, which is the transient effect of alpha thermalization at the point ignition is reached and auxiliary power is turned off. From there, the ignited plasma goes into a thermal runaway, but this is slow because the density is being continuously increased, and so too, the plasma heat capacity is increasing. The temperature increase continues until the beta limit is reached, where the increased transport losses result in thermal equilibrium, arresting the runaway at a temperature of $\langle T \rangle \sim 19$ keV in this case. The ion fuel source is kept on and the operating point moves to higher density, being constrained to the beta limit contour. The plasma current was then arbitrarily decreased at $\langle T \rangle \sim 17.5$ keV. The Troyon beta limit decreases proportionately and the temperature drops rapidly to the temperature of the new beta limit. The density then continues to increase isobarically on this new beta limit until the assumed Murakami limit on density is reached.

The operating point can then be varied with temperature at constant density simply by changing the beta limit through increasing or decreasing the plasma current. It is important to note that all ignited points within the density and beta limits are inherently thermally stable, thus eliminating the need for burn control. Operating close to the beta limit will allow higher levels of net reactor power as we move isobarically to the low temperature, high density region; thus, the maximum power will generally be set by the density limit. In this case, at $\langle n \rangle_{\max} \sim 1.5 \times 10^{20} \text{ m}^{-3}$ the reactor could produce about 200 MW net electric power.

The problems of thermal stability in reactor power operation disappear if a soft beta limit is attainable. The plasma increases its transport losses as its beta value approaches a critical beta limit in such a way that thermal equilibrium is maintained. A passive mechanism for obtaining a 'soft' beta limit based on better understanding of plasma transport properties would be an ideal method for power control. On the other hand, an active feedback scheme; for instance, increasing ripple losses as the plasma beta approaches its critical value, may also be considered. Such a scheme has been examined

previously [38] and was found to require large engineering modifications to the TF coil systems. However, both schemes involve large amounts of heat flux on the outer edge of the plasma that may impact the required heat removal capabilities of the divertor/limiter system. A detailed trade-off study between the plasma and engineering constraints is needed to address this issue.

3.7. DISCUSSIONS AND CONCLUSIONS

It has been recognized that fractional power operation is an important consideration in the design of power reactors, whose commissioning requires that their power output be brought in stages to their rated level for equipment testing and safety checks. This work attempts to assess the impact of this additional procedural requirement on tokamak fusion reactor design. The critical physics issues related to fractional power operation are carefully reviewed within the framework of experimental data obtained in non-burning tokamaks. Due to the lack of data on burning plasma experiments, both "bold" and conservative extrapolations of the plasma confinement model have been used in evaluating the reactor performance in the fusion regime. In particular, several well-known confinement scalings, such as neo-Alcator, Goldston, Ohkawa and Doublet-III, have been employed, resulting in varying conclusions.

Since the plasma transport loss dominates the power balance in the reactor regime of parameters, the requirements for fractional power operation are sensitive to the empirical transport model used in the analysis. In general, optimistic performance, characterized by high Q or ignited operation, is predicted for scaling laws for which τ_E does not depend on or degrade indefinitely with heating power, such as neo-Alcator and D-III scalings. However, beta limits in the first stability regime often dictate that the full power operating point be located well above the ignition boundary. This implies that, if the beta limit is disruptive, an elaborate feedback scheme, involving both programmed ion fueling and confinement control, will be needed to stabilize the output power. Programmed ion fueling may include changing the gas puffing or pellet injection rate, and varying the fuel n_D/n_T ratio to modify the power balance. Control of energy confinement, on the other hand,

may be achieved by enhanced radiation induced by injecting appropriate amounts of impurities into the plasma, or by simply varying the confinement sensitive parameters, such as plasma size and shape, and current. If the beta limit turns out to be "soft," we may be able to utilize this inherent "confinement control" to maintain a stable operating point at full power, depending on how τ_E degrades with $\langle\beta\rangle$. Operation at fractions of full power can be accomplished in the low-density, high-temperature region, with inherent passive thermal stability, and either ignited or high-Q operation is possible. For driven modes, plasma equilibrium points can be found where the required auxiliary heating power does not exceed the minimum power needed to heat the plasma to ignition. As for transition between power plateaus, a convenient spot at which to achieve this is during the normal dwell phase of the hybrid transformer reactor mode. As a result, only the start-up and heating procedures to reach these various power stages need to be determined individually, once the target operating points are determined. Based on these optimistic transport scalings, the physics impact of fractional power operation on full power reactor designs appears to be minimal.

For transport models where τ_E degrades with P_{aux} , such as Goldston and Ohkawa scalings, ignition can still be reached provided the auxiliary power is turned off at a programmed fashion as the plasma approaches the ignition boundary. Above ignition, only moderate levels of driving power are needed to sustain equilibrium below the critical $\langle\beta\rangle$. However, these equilibrium points are thermally unstable thus requiring burn control with programmed ion fueling and auxiliary power. Generally speaking, fractional power can be maintained in the low-density, high-temperature, thermally stable regime, as is the case for neo-Alcator scaling, except that higher auxiliary power is required to sustain equilibrium. Consequently, there may be stronger incentive to operate in the ignited mode even for lower fractional power output, in which case complex burn control schemes may have to be installed. As with neo-Alcator scaling, the impact of fractional power consideration on reactor designs based on full power does not appear to be significant.

If we consider the possibility that alpha heating power degrades confinement in the same manner as does auxiliary power for either Goldston or Ohkawa scaling, highly pessimistic performance predictions for reactors with conventional size and plasma parameters will result. These reactors will

definitely not be viable options and are excluded from further consideration. However, there is a class of tokamak reactors that may exhibit desirable features under these highly conservative transport assumptions. These reactors are characterized by high densities ($\langle n \rangle \sim 10^{21} \text{ m}^{-3}$), high magnetic field ($B_0 \sim 15\text{T}$), high current ($I_p \sim 10 \text{ MA}$), and ignition is possible in these devices. The recently proposed CIT (Compact Ignition Torus) [8] is a prime example. Amongst all these analyses, we should keep in mind the fact that the effect of alpha particles on energy confinement remains a major open question in plasma physics. The answer to this question may significantly impact the direction of fusion reactor design and the methodology to operate the tokamak at different power levels. The sensitivity of reactor operation on alpha particle dynamics also underlines the urgent need for a near-term burning plasma tokamak experiment.

Finally, a series of ignited, feedback stabilized operating scenarios of the tokamak reactor has been identified using a time-dependent power balance analysis with different transport models. Again, the ion fueling rate and auxiliary power are the two main control "knobs." Maintaining thermal equilibrium while varying the reactor power output is often synonymous with moving the ignition contour in $\langle n \rangle$ - $\langle T \rangle$ space, or simply moving along a fixed ignition contour. In most cases, we found that burn control is an integral part of the operational procedure for transition between power plateaus. We must point out, however, that maintaining thermal stability by burn control is itself a non-trivial problem particularly for operation deep into the ignited region. What is clear from this study is that if a reactor is designed with burn control required for full-power operation, the impact of fractional power operation on the design appears to be insignificant.

REFERENCES

1. S. M. Kaye, Phys. Fluids, **28** (1985) 2327.
2. T. Ohkawa, Comments Plasma Phys. Controlled Fusion, **9** (1985) 127.
3. S. M. Kaye and R. J. Goldston, Nucl. Fusion, **25** (1985) 65.
4. R. J. Goldston, Plasma Phys. Contr. Fusion, **26** 1A (1984) 87 (Invited papers of 11th Europ. Conf. Aachen, 1983).

5. S. Fairfax, A. Gondhelekar, R. Granetz et al., in Plasma Physics and Controlled Nuclear Fusion Research (Proc. 8th Int. Conf. Brussels, 1980), Vol. I, IAEA, Vienna (1981) p. 439.
6. S. Ejima, T. W. Petrie, A. C. Riviere et al., Nucl. Fusion, 22 (1982) 1627.
7. M. Nagami et al., Nucl. Fusion, 24 (1984) 183.
8. W. A. Houlberg, Bull. Am. Phys. Soc., 30 (1985) 1362.
9. K. H. Burrell et al., in Plasma Physics and Controlled Nuclear Fusion Research (Proc. 10th Int. Conf. London, 1984), Vol. 1, IAEA, Vienna (1985) p. 131.
10. V. G. Merezhkin and V. S. Mukhovatov, Sov. JETP Lett., 33 (1981) 446.
11. F. W. Perkins, in Heating in Toroidal Plasmas (Proc. 4th Int. Symp. Rome, 1984), Vol. 2, ENEA, Frascati (1984) p. 977. See also F. W. Perkins and Y. C. Sun, "On Confinement Scaling and Ignition in Tokamaks," Princeton Plasma Physics Lab. Report PPPL-2261 (1985).
12. E. Mazzucato et al., in Plasma Physics and Controlled Nuclear Fusion Research (Proc. 10th Int. Conf. London, 1984), Vol. 1, IAEA, Vienna (1985) p. 433.
13. A. M. Messiaen et al., invited paper in Controlled Fusion and Plasma Physics (Proc. 12th Europ. Conf. Budapest, 1985).
14. P. C. Efthimion et al., in Plasma Physics and Controlled Nuclear Fusion Research (Proc. 10th Int. Conf. London, 1984), Vol. 1, IAEA, Vienna (1985) p. 29.
15. J. G. Cordey et al., *ibid.*, p. 167.
16. D. A. Spong et al., "Effects of Trapped Alpha Particles on Ballooning Modes in Tokamaks," Oak Ridge National Lab. Report ORNL/TM-9598 (1985). To be published in Phys. Fluids.
17. A. B. Mikhailovskii, in Physics of Plasmas in Thermonuclear Regimes (Proc. 1979 Workshop, Int. School of Plasma Physics, Varenna, Italy, 1979) p. 49.
18. B. Bornatici and F. Engelmann, *ibid.*, p. 61.
19. F. Troyon et al., Plasma Phys. Contr. Fusion, 26, 1A (1984) 209 (Invited papers of 11th Europ. Conf. Aachen, 1983).
20. A. Sykes, M. F. Turner and S. Patel, in Controlled Fusion and Plasma Physics (Proc. 11th Europ. Conf. Aachen, 1983), Vol. 7D, Part 2, EPS (1983) p. 363.

21. R. D. Stambaugh et al., in Plasma Physics and Controlled Nuclear Fusion Research (Proc. 10th Int. Conf. London, 1984), Vol. 1, IAEA, Vienna (1985) p. 217.
22. J. Manickam, R. C. Grimm and M. Okabayashi, Phys. Rev. Lett. 51 (1983) 1959.
23. M. S. Chance, S. G. Jardin and T. H. Stix, *ibid*, p. 1963.
24. M. Keilhacker et al., in Plasma Physics and Controlled Nuclear Fusion Research (Proc. 10th Int. Conf. London, 1984), Vol. 1, IAEA, Vienna (1985) p. 71.
25. D. W. Swain et al., Nucl. Fusion, 21 (1981) 1409.
26. B. D. Carreras et al., Phys. Rev. Lett., 50 (1983) 503.
27. M. Murakami et al., Nucl. Fusion, 16 (1976) 347.
28. W. Hooke, Plasma Phys. Contr. Fusion, 26, 1A (1984) 133 (Invited papers of 11th Europ. Conf. Aachen, 1983).
29. N. J. Fisch, Phys. Rev. Lett., 41 (1978) 873.
30. F. Engelmann et al., in Plasma Physics and Controlled Nuclear Fusion Research (Proc. 10th Int. Conf. London, 1984), Vol. 3, IAEA, Vienna (1985) p. 221.
31. W. A. Houlberg, S. E. Attenberger and L.L. Lao, in Advances in Mathematical Methods for the Solution of Nuclear Engineering Problems (Proc. Am. Nucl. Soc./Europ. Nucl. Soc. Int. Top. Mtg. Munich, 1981) Vol. 2, p. 425.
32. W. A. Houlberg and R. W. Conn, Nucl. Fusion, 19 (1979) 81.
33. R. J. Groebner et al., Nucl. Fusion, 26 (1986) 543.
34. T. K. Mau and R. W. Conn, J. Fusion Energy, 2 (1982) 207.
35. INTOR Group, "International Tokamak Reactor: Phase Two A Part 1" (Report Int. Tokamak Reactor Wksp. Vienna 1981-83), IAEA, Vienna (1983).
36. J. G. Cordey, Plasma Phys. Contr. Fusion, 26, 1A (1984) 123 (Invited papers of 11th Europ. Conf. Aachen, 1983).
37. C. Baker et al., "Starfire," Argonne National Laboratory Report ANL/FFP-80-1 (1980).
38. INTOR Group, "U.S. Contribution to the International Tokamak Reactor, Phase Two A, Part 2 Workshop," Rep. INTRO/85-1, Georgia Inst. of Technol., Atlanta, Georgia (July 1985).

CHAPTER 4

PLASMA POSITION AND SHAPE DETERMINATION AND ELECTROMAGNETICS CALCULATIONS

Charles E. Kessel and Marc A. Firestone

CONTENTS

4.1. PLASMA POSITION AND SHAPE DETERMINATION	4-1
4.2. ELECTROMAGNETIC CALCULATIONS	4-10
REFERENCES	4-13

4.1. PLASMA POSITION AND SHAPE DETERMINATION

The rapid calculation of gross plasma equilibrium parameters is necessary in a dynamic system description in order to determine the effects of changes in the electromagnetic environment on the plasma. Since this would involve a free boundary MHD calculation at each time step, and there are a large number of time steps in a discharge simulation, the use of conventional equilibrium calculations is excluded. It is thus required that a new approximate technique be developed that can yield the necessary plasma parameters, such as major radius, minor radius, elongation and triangularity, in an acceptable computation time.

A method has been developed to perform this task. The shape parameters are determined from magnetic multipoles of the externally applied magnetic field from the poloidal field coils and eddy currents. The poloidal magnetic field along the midplane for an up-down symmetric plasma is expanded in a Taylor series [1,2],

$$B_p(R, Z = 0) = B_z(R_0, 0) + (R - R_0) \frac{\partial B_z}{\partial R} \bigg|_{R_0} + \frac{(R - R_0)^2}{2} \frac{\partial^2 B_z}{\partial R^2} \bigg|_{R_0} + \dots \quad (1)$$

Each term is then related to a multipole by the following expansion,

$$B_p(R, Z = 0) = B_D + \left(\frac{R - R_0}{a}\right) B_Q + \left(\frac{R - R_0}{a}\right)^2 B_H + \dots \quad (2)$$

where R_0 is the plasma major radius and a is the minor radius. B_D , B_Q , and B_H are the dipole, quadrupole, and hexapole fields, respectively. The multipoles are then defined as

$$B_D = B_z(R_0, Z = 0) \quad ,$$

$$B_Q = a \frac{\partial B_z}{\partial R} \bigg|_{R_0} \quad ,$$

$$B_H \approx \frac{a^2}{2} \frac{\partial^2 B_z}{\partial R^2} \Big|_{R_0} .$$

The dipole field is the usual vertical field required to restrain the plasma from expanding radially. The quadrupole field is related to plasma elongation and the field decay index for vertical and radial stability. The hexapole field is related to both triangularity and elongation. A discussion of these field multipoles and their effects on plasma shape is in Refs. [3-5].

In order to relate the multipoles to plasma position and shape parameters, several fixed/free boundary MHD equilibrium calculations have been done using NEQ [6] developed at ORNL/FEDC. The plasma shape and position are specified and the required coil currents determined. These coil currents are calculated interactively with the plasma internal conditions, such as current distribution and pressure profile. With the coil currents the poloidal magnetic field along the midplane can be calculated and from it the multipoles can be determined. These multipoles are then related to the plasma elongation and triangularity.

The results of this procedure are displayed in Figs. 1 and 2. The quadrupole field shows a cubic dependence on the plasma elongation and a linear shifting in poloidal beta. The hexapole field is found to have a linear dependence on the ratio of triangularity to the square of elongation. These are also shifted linearly in poloidal beta. Fitting to these data points yields the following multipole to shape correlations, for INTOR parameters,

$$\begin{aligned} B_Q &= -\frac{\mu_0 I_p}{2\pi a} [-1.488 + 2.543\kappa - 1.338\kappa^2 + .235\kappa^3 + .006\beta_p] \\ B_H &= -\frac{\mu_0 I_p}{2\pi a} \left(-\frac{\delta}{\kappa^2} + 0.0185\beta_p - 0.01378 \right) \end{aligned} \quad (3)$$

where κ is the plasma elongation, δ is the triangularity, β_p the poloidal beta, I_p the plasma current, and a the plasma minor radius. With these correlations one can determine the plasma shape by knowing the currents in the external coils and eddy currents. At present the dependences of these multipoles on aspect ratio and plasma internal self-inductance per unit length have not been found.

QUADRUPOLE FIELD VERSUS PLASMA ELONGATION TRIANGULARITY=0.0-0.2 ELONGATION=1.0-1.8

Fig. 1. Normalized quadrupole field as a function of plasma elongation for poloidal beta at 0.0, 0.5, 1.0, 1.5, and 2.0. The lowest curve is for poloidal beta and increases upward.

HEXAPOLE FIELD VERSUS TRIANGULARITY/ELONGATION**2 TRIANGULARITY=0.0-0.2 ELONGATION=1.0-1.8

Fig. 2. Normalized hexapole field as a function of triangularity divided by elongation squared for poloidal beta at 0.0, 0.5, 1.0, 1.5, and 2.0. The lowest curve is for zero poloidal beta and increases upward.

The plasma major radius is related to the external vertical field by a modified Shafranov formula, valid for noncircular and high beta plasmas. The representation is similar to that used for analysis on ISX-B [7].

$$B_v = B_D = - \frac{\mu_0 I_p}{4\pi R \left(\frac{1+\kappa^2}{2}\right)^{1/4}} \left[\ln \frac{8R}{a} + \beta_p + \frac{l_1}{2} - \frac{3}{2} - \frac{1}{2} \frac{a^2}{R^2} \left(\beta_p + \frac{l_1}{2}\right)^2 \right], \quad (4)$$

where l_1 is the plasma internal self-inductance per unit length. Shown in Fig. 3 is the relationship between the vertical field and the quantity $(\beta_p + l_1/2)$. The low beta circular plasma result of Shafranov [8] would indicate a linear dependence; however, from equilibrium calculations it proves to be quadratic at higher beta. It is also true that for an elongated plasma of the same major and minor radius, less vertical field is required compared to a circular plasma. The dependence is weak and the quarter power appears to show good agreement.

The minor radius is determined using a knowledge of the safety factor at the plasma edge (the engineering safety factor). This quantity should be minimized without inducing instability. Thus in a discharge, one would like to control the safety factor. The minor radius is then determined from

$$a \text{ (m)} = \left[\frac{q_I R \text{ (m)} I_p \text{ (MA)}}{5 B_t \text{ (T)} \left(\frac{1+\kappa^2}{2}\right)^{1/2}} \right]^{1/2}, \quad (5)$$

where q_I is the safety factor and B_t is the toroidal magnetic field at the plasma center.

With a knowledge of the plasma poloidal beta, internal self-inductance per unit length, coil and eddy current, plasma current, toroidal field, and plasma edge safety factor, the system of multipole/shape, major radius, and minor radius equations can be solved iteratively. However, the iteration technique is susceptible to divergence if the vertical field is decreasing in major radius, which is the case for circular plasmas. Thus, it is necessary, and actually more desirable, to calculate the major radius by integrating the plasma radial velocity. The radial velocity is derived from a balance of the above vertical field formula and the actual applied vertical field. The

EQUILIBRIUM VERTICAL FIELD VERSUS (POL. BETA + INT. SELF IND.)
 TRIANGULARITY=0.0-0.2 ELONGATION=1.0-1.8

Fig. 3. Comparison of (S) - the linear Shafranov equilibrium vertical field expression, (triangle) - the modified quadratic equilibrium expression, and (diamond) actual MHD results averaged for elongations between 1.0 and 1.8 and triangularities between 0.0 and 0.2.

Alfvén time scale motion is neglected and it is assumed that the plasma instantaneously equilibrates with the applied field. Dynamically, this assumes the plasma moves through a series of equilibria when it moves radially. The same is done for the plasma vertical position. The formulation of the radial and vertical plasma velocities will be given in the next section.

The modeling relies on the premise that the plasma does not have large excursions radially or from the midplane. For this to be true, feedback control is required to regulate the radial and vertical plasma positions. In addition, it becomes necessary to regulate the plasma current in order to achieve good position control. All three use a voltage regulator with both proportional and derivative control. The feedback voltages for the plasma position and current are given by

$$\begin{aligned} V_R &= k_R \{ (R - R_0) + \tau_R \dot{R} \} \quad , \\ V_Z &= k_Z \{ (Z - Z_0) + \tau_Z \dot{Z} \} \quad , \\ V_I &= k_I \{ (I - I_0) + \tau_I (\dot{I} - \dot{I}_0) \} \quad , \end{aligned} \tag{6}$$

where k_R , k_Z and k_I are gains, and τ_R , τ_Z , and τ_I are derivative weighting terms. The feedback must be sufficient to keep the excursions within some tolerance so that the up-down symmetric assumption is a valid representation.

Examples of the iterative technique for noncircular plasmas are compared with free boundary MHD results for INTOR parameters, and are shown in Figs. 4 and 5. The accuracy is considered reasonable for examining the dynamic electromagnetic effects on plasma shape and position. Shown in Table I are a series of gross equilibrium parameters in time when using the $\dot{R}$ and $\dot{Z}$ method. Control coils were introduced to maintain positional control. The plasma vertical position was kept to less than 0.01 millimeter from the midplane. These also show good agreement with free boundary MHD.

MHD EQUILIBRIUM PLASMA SHAPE

DIAMONDS=EXACT MHD

STARS=MULTIPOLE MHD

Fig. 4. Comparison of the multipole method for gross plasma equilibrium and actual free boundary MHD results for a 1.3 elongation and 0.2 tri-angularity plasma. This is for the INTOR tokamak.

MHD EQUILIBRIUM PLASMA SHAPE

DIAMONDS-EXACT MHD

STARS-MULTIPOLE MHD

Fig. 5. Comparison of the multipole method for gross plasma equilibrium and actual free boundary MHD results for a 1.5 elongation and 0.2 tri-angularity plasma. This is for the INTOR tokamak.

TABLE I
THE EQUILIBRIUM EVOLUTION FOR A DISCHARGE SIMULATION ON INTOR

Time (sec)	R (m)	a (m)	κ	δ	I_p (MA)	B_p	Decay Index
1.0	5.3	1.2	1.00	-0.028	0.5	0.001	0.74
2.0	5.29	1.2	0.97	-0.027	1.0	0.005	1.02
3.0	5.3	1.19	1.03	-0.027	3.0	0.01	0.50
4.0	5.29	1.20	1.30	-0.01	4.0	0.05	-1.26
5.0	5.30	1.20	1.49	-0.002	5.0	0.10	-1.81
6.0	5.31	1.19	1.51	-0.004	6.0	0.50	-1.63
7.0	5.30	1.19	1.48	0.200	6.0	0.97	-1.40
8.0	5.31	1.19	1.48	0.198	6.0	1.47	-1.29
9.0	5.31	1.19	1.57	0.193	6.0	1.77	-1.35

4.2. ELECTROMAGNETIC CALCULATIONS

The circuit equations describing the external poloidal field coils, eddy currents, and plasma are solved in a circuit interaction code. This requires that the mutual inductances, their spatial derivatives, and the radial and vertical plasma velocities be known. These quantities are discussed in this section.

The electromagnetic environment consists of the ohmic heating coils, the poloidal field coils (providing the vertical, radial, and shaping fields), the eddy current paths in the structure (first wall, breeder region, and shield), and the plasma. The description of the eddy current paths is discussed in Chapter 5.

The mutual inductances between the OH coils, PF coils, and eddy current paths are constant, while those between these components and the plasma are variable since the plasma can move. The plasma is modeled as a filament located at the magnetic axis (since this is generally very close to the current centroid). This will produce very accurate mutual inductances comparable to the actual distributed plasma current values. The coils and eddy current paths are modeled as strips which have incorporated their finite size.

In order to determine the magnetic axis location, an MHD equilibrium calculation must be made; however, a semi-empirical relationship has been developed for up-down symmetric plasmas,

$$R_m = R_o + \frac{a^2}{2R_o} \left(\beta_p + \frac{l_1}{2} \right) - \frac{a^3}{3R_o^2} \left(\beta_p + \frac{l_1}{2} \right)^2, \quad (7)$$

where R_o is the plasma center, a is the minor radius, β_p the poloidal beta, and l_1 is the plasma internal self-inductance per unit length.

The derivatives of the mutual inductances are determined by using the plasma filament approximation and a filament located at the coil or eddy current central location. Thus, one can use the analytic expression for the mutual inductance between two filaments and differentiate it to arrive at analytic expressions for $\partial M / \partial R$ and $\partial M / \partial Z$.

The plasma radial and vertical velocities are determined from the force balances in those directions. The radial force balance can be written, assuming a filament plasma, as

$$m\ddot{R} = 2\pi R I_p (B_v - B_z) \quad (8)$$

where m is the plasma mass, R the plasma radial position, I_p the plasma current, B_v the vertical field required to hold the plasma at R , and B_z is the actual vertical field present. The inclusion of the plasma mass will yield Alfvén time scale oscillations, which are not the important motions. The plasma mass is set to zero yielding the slower "circuit" time scale motion. This assumption requires that there be some material surrounding the plasma so that eddy currents are created to resist motion for times longer than the Alfvén time scale. Since the Alfvén time scale is on the order of microseconds while the eddy current decay times are around milliseconds, the assumption should be valid.

In order to determine the slow time scale radial velocity the derivative with respect to time is taken of the force balance equation. The vertical fields are given by

$$B_v = - \frac{\mu_0 I_p}{4\pi R \left(\frac{1+\kappa^2}{2}\right)^{1/4}} \left[\ln \frac{8R}{a} + \beta_p + \frac{\ell_1}{2} - \frac{3}{2} - \frac{1}{2} \frac{a^2}{R} \left(\beta_p + \frac{\ell_1}{2}\right)^2 \right] \quad (9)$$

$$B_z = \frac{\mu_0}{4\pi} \sum_{i=1}^N f_i(R_1, Z_1; R, Z) I_i \quad (10)$$

where κ is the plasma elongation, f_i is the geometric function relating the vertical magnetic field at (R, Z) to the unit current at (R_1, Z_1) , and N is the total number of coils and eddy currents. The time derivative of the force balance

$$\frac{dB_v}{dt} = \frac{dB_z}{dt} \quad ,$$

yields the radial velocity

$$\frac{\partial R}{\partial t} = \frac{\frac{\partial B_v}{\partial I_p} \frac{\partial I_p}{\partial t} + \frac{\partial B_v}{\partial \kappa} \frac{\partial \kappa}{\partial t} + \frac{\partial B_v}{\partial a} \frac{\partial a}{\partial t} + \frac{\partial B_v}{\partial \beta_p} \frac{\partial \beta_p}{\partial t} + \frac{\partial B_v}{\partial \ell_1} \frac{\partial \ell_1}{\partial t} - \frac{\mu_0}{4\pi} \sum_{i=1}^N f_i \frac{\partial I_i}{\partial t} - \frac{\mu_0}{4\pi} \sum_{i=1}^N \frac{\partial f_i}{\partial Z} \frac{\partial Z}{\partial t} I_i}{\frac{\mu_0}{4\pi} \sum_{i=1}^N \frac{\partial f_i}{\partial R} I_i - \frac{\partial B_v}{\partial R}} \quad (11)$$

It should be noticed in Eq. (11) that the vertical plasma velocity is a component of the radial velocity. This will also be seen in the vertical velocity equation. The time rates of change of the plasma current, elongation, minor radius, poloidal beta, and internal self-inductance per unit length can be determined from previous time step information.

The vertical plasma velocity is also derived from a force balance, neglecting Alfvén time scale motions. The radial field is given by

$$B_R = \frac{\mu_0}{4\pi} \sum_{i=1}^N g_i(R_1, Z_1; R, Z) I_i \quad (12)$$

where force balance dictates $B_R = 0$. In Eq. (12) g_i is similar to f_i in Eq. (10) except that it is the radial field being related to the unit current. Taking the time derivative yields

$$\frac{\partial Z}{\partial t} = - \frac{\frac{\mu_0}{4\pi} \sum_{i=1}^N g_i \frac{\partial I_i}{\partial t} - \frac{\mu_0}{4\pi} \sum_{i=1}^N \frac{\partial g_i}{\partial R} \frac{\partial R}{\partial t} I_i}{\frac{\mu_0}{4\pi} \sum_{i=1}^N \frac{\partial g_i}{\partial Z} I_i} \quad (13)$$

These derivative expressions are integrated to yield the plasma radial and vertical positions.

REFERENCES

1. H. Ninomiya, K. Shinya and A. Kameari, in Proc. 8th Symp. Eng. Problems of Fusion Research, IEEE No. 79CH1441-5NPS (Institute of Electrical and Electronics Engineers, 1979) p. 75.
2. K. Shinya, et al., in Proc. 9th Symp. Eng. Problems of Fusion Research, IEEE No. 81CH1715-2NPS (Institute of Electrical and Electronics Engineers, 1981) p. 2039.
3. V. S. Mukhovatov and V. D. Shafranov, Nucl. Fusion, **11** (1971).
4. C. E. Kessel, M. A. Firestone and R. W. Conn, Fusion Technol., **8** (1985) 680.
5. C. E. Kessel, "Determination of Gross Plasma Equilibrium from Magnetic Multipoles," University of California Los Angeles Report PPG-941 (1986).
6. D. J. Strickler, J. B. Miller, K. E. Rothe and Y.-K. M. Peng, "Equilibrium Modelling of the TFCX Poloidal Field Coil System," Oak Ridge National Lab. Report ORNL/FEDC-83/10 (April 1984).
7. D. Swain and G. Neilson, Nucl. Fusion, **22**, 8 (1982).
8. V. D. Shafranov, in Reviews of Plasma Physics, M. A. Leontovich, Ed., Vol. 2 (Consultants Bureau, New York, 1966).

CHAPTER 5

EDDY CURRENT MODELS

Marc A. Firestone, David I. Brown,* Charles E. Kessel and Mark Tillack

*** Princeton Plasma Physics Laboratory**

CONTENTS

5.1. INTRODUCTION	5-1
5.2. EDDY CURRENT MODEL	5-1
REFERENCES	5-4

5.1. INTRODUCTION

Electromagnetic effects on plasma behavior pose important design constraints for tokamak reactors. While the poloidal field coils are carefully studied in reactor designs, the passive poloidal field system, i.e., eddy currents, are not. The coupling between the plasma, poloidal field coils, and eddy currents in the vessel and surrounding structure is complex and dependent on the plasma discharge and burn cycle time history. Additionally, reactors have a thick blanket/shield structure between the plasma and poloidal field coils which enhances the importance of this coupling. We propose a simple, but realistic eddy current model of the vacuum vessel and blanket/shield which can be utilized in an accurate assessment of the electromagnetic influences of a given design [1].

5.2. EDDY CURRENT MODEL

Chapter 4 showed the importance of including eddy currents directly into the electromagnetic model. To accomplish this, we assume the vacuum vessel and blanket/shield are infinitesimally thin toroidal sheets [2] located at the center of the volume being modeled. The actual thickness of the structure as well as its resistivity is incorporated in the resistance calculation. An example is shown in Fig. 1 for an INTOR-like device. Here we have used three sheets to simulate the vessel, blanket, and shield. These sheets are broken into toroidal strips which correspond to the individual eddy current circuits. The strips in our example are toroidally continuous, but resistive breaks could be incorporated [2]. The concentric sheet model simulates the poloidal field diffusion through a thick inhomogeneous structure, i.e., blanket and shield. This complication is not present in current tokamak experiments, but should be an important factor in electromagnetic design and control considerations for reactors. Figure 1 also indicates the pump limiter modeled as a toroidally continuous strip at the plasma bottom edge. Other external structures can similarly be incorporated by this method. The pump duct is shown as the absence of conducting strips at the appropriate location. This, along with the limiter, introduces an up-down asymmetry in the passive electromagnetic environment.

Fig. 1. Illustration of the "sheet" eddy current model for INTOR.

The fixed mutual inductances are computed between each of the eddy current strips and between the strips and the poloidal field coils. The only mutuals that change are those involving the plasma, since it can move and change shape and current distribution. These are modeled by placing filaments at the plasma magnetic axis and center of the coil or eddy current strip. The mutual inductance is then

$$M_{ip} = \mu_0 (r_i R_p)^{1/2} \left\{ \left(\frac{2}{\gamma} - \gamma \right) K(\gamma) - \frac{2}{\gamma} E(\gamma) \right\} , \quad (1)$$

where

$$\gamma^2 = \frac{4R_i R_p}{(R_i + R_p)^2 + (Z_p + Z_i)^2} ,$$

and $K(\gamma)$ and $E(\gamma)$ are the elliptic integrals of the first and second kind, respectively. Equation (1) is used as a semi-empirical formula for the magnetic axis R_p valid at high beta. This eddy current model has been applied to two different blanket/shield designs. The first is a liquid metal blanket, and the second is a solid breeder. These were chosen as representative of a highly conducting blanket and very resistive one. Both designs are modeled with the scheme shown in Fig. 1. The dimensions and conductivities of the various sections are listed in Table I for both blankets.

The model is easily incorporated into the framework of a plasma kinetic and circuits code for determining plasma size, shape, position, and current. The method can be made as accurate as needed by simulating more conducting structures and/or finer resolution through an increase in the number of shells and/or eddy current strips. This allows specific conducting structures and toroidal breaks to be included within the overall plasma-circuits interaction model.

TABLE I
BLANKET/SHIELD CHARACTERISTICS

Region	Material	Resistivity ($\Omega\text{-m}$)	Thickness (m)	No. Strips
Shield	316SS	9.6×10^{-7}	0.8 to 1.15	24
Breeder				
Liquid Metal	Li	3.0×10^{-7}	0.5	8
Solid	Li ₂ O	2.8×10^3	0.5	8
First Wall	316SS	9.6×10^{-7}	0.0125	16
Limiter	316SS	9.6×10^{-7}	0.05	1

REFERENCES

1. M. A. Firestone, D. I. Brown, C. Kessel and M. Tillack, in Proc. 11th Symp. Fusion Engineering, Austin (1985).
2. D. I. Brown, M. G. Bell and J. Coonrod, Proc. 10th Symp. Fusion Engineering, IEEE No. 83CH1916-6-NPS, (Institute for Electrical and Electronic Engineers, 1983) p. 1233.

CHAPTER 6

MODELING OF LOWER HYBRID CURRENT DRIVE IN TOKAMAK REACTORS

Tak Kuen Mau, Mark R. Ilgen and Robert W. Conn

CONTENTS

6.1.	INTRODUCTION	6-1
6.2.	PHYSICS OF LH CURRENT RAMP-UP	6-3
6.2.1.	Bulk Electron Heating	6-3
6.2.2.	Electron Tail Conductivity	6-4
6.2.3.	Current Profile Evolution	6-5
6.2.4.	Runaway Electrons	6-6
6.3.	DESCRIPTION OF THE MODEL	6-6
6.3.1.	RAYCO, A LH Ray Tracing Code	6-8
6.3.2.	Current (Poloidal Field) Spatial Diffusion	6-10
6.3.3.	Double Tearing Mode Model	6-14
6.3.4.	Power Balance	6-15
6.4.	RESULTS OF RAMP-UP SIMULATIONS	6-17
6.4.1.	The Base Scenario	6-17
6.4.2.	The Case With Tail Conductivity Correction	6-21
6.4.3.	The "Overdrive" Case	6-23
6.4.4.	Sensitivity to the Transport Model	6-26
6.5.	CONCLUSION AND DISCUSSION	6-27
	REFERENCES	6-28

6.1 INTRODUCTION

For the tokamak to succeed as a viable reactor concept, efficient steady state operation is one of the main goals that needs to be achieved. If the plasma current in a tokamak can be sustained continuously, deleterious thermal cycling effects associated with transformer driven tokamaks can be eliminated, thus resulting in much longer lifetimes for the reactor components. A plethora of methods [1] has been proposed and used to drive currents in experimental tokamak plasmas. Among them, the lower hybrid (LH) wave scheme has been demonstrated to be the most promising driver, although its efficiency at reactor density levels ($n_e \gtrsim 10^{14} \text{ cm}^{-3}$) is called into question. [2] As such, the relatively good current drive efficiency ($I/P \sim 0.2 \text{ A/W}$) at low densities can be fully exploited in the ramp-up of the plasma current during the start-up phase of reactor operation. The result is that the bulk of the volt-seconds in the OH coils will be expended at the flat top of the discharge, giving rise to a longer burn pulse. Conversely, since the amount of volt-seconds depends on the size of the coils, lower hybrid current ramp-up could potentially lead to a more compact machine with the same pulse length.

While the economic feasibility of sustaining a steady state tokamak reactor with radiofrequency (RF) waves remains an open question, one can consider operating the reactor in the so-called "hybrid transformer" mode. [3] In this mode of operation, the plasma current is alternately maintained by the OH primary during the high density, burn phase and by LH waves during the low density, non-burning, transformer recharging phase. What emerges is a quasi-steady state plasma discharge scenario in which the burn pulse can be made very long (10^3 - 10^4 s), depending on the transformer size, while the current is kept relatively constant. These features translate into long lifetime, high availability, and perhaps disruption-free operation in a tokamak reactor.

In chapter 3, fractional power operation was considered mainly for burn pulses driven by OH coils. However, it was also pointed out that operating the reactor in the hybrid transformer mode results in substantial simplifications in the procedure of transiting between power stages. Variation of the

plasma current can also affect the plasma beta limit and its confinement properties, making it an effective power and burn control "knob." The control should become even more effective for an RF-sustained steady state burn. Thus the need to study the feasibility and efficiency of lower hybrid current drive, and ramp-up in particular, in a reactor environment is clearly established.

In this chapter, we examine the physics and engineering issues related to time-dependent lower hybrid current drive in a reactor, and their impact on reactor start-up and, indirectly, on fractional power operation. The kinetic theory of current ramp-up by lower hybrid waves has been studied quite extensively. [4,5] On the other hand, experiments on the PLT tokamak have indicated [6] a ramp-up efficiency as high as 20%, where efficiency is defined as the ratio of the electric power flowing into the poloidal field to the RF power absorbed by the electrons resonating with the wave. Modeling of current ramp-up with lower hybrid waves has also been attempted on experiments in PLT [7] and Alcator-C. [8] In these simulations, emphasis was placed on the wave propagation and absorption calculations based on a kinetic description of the plasma, which are coupled to a relatively simple radial transport model. In this work, we use basically a one-dimensional model to describe the physical processes during current ramping. The RF analysis is simplified in that wave damping is calculated analytically using a quasilinear approximation [9] while the local driven current is obtained by applying the Fisch-Karney formula. [10] The 3-D, LH ray tracing code is then coupled to WHIST, [11] a $1\frac{1}{2}$ -D transport, magnetohydrodynamic (mhd) equilibrium code developed at Oak Ridge. Self-consistent flux surface geometry, plasma current and pressure profiles can then be used to study issues related to the power balance and mhd stability during the transient current drive phase of reactor operation. Methods to optimize the ramp-up scenario under various criteria, e.g., minimum energy input, have been explored. The work presented in this chapter can also form the basis for detailed modeling of the transition phase between ohmic and LH current drive for the hybrid transformer mode of operation.

The content of this chapter is organized as follows. Section 2 briefly describes some of the more important physics issues related to LH current ramp-up, such as electron tail induced conductivity, stability of current profile and runaway electrons. In section 3, we present our calculational

model, with special emphasis on LH ray tracing and absorption, current diffusion model, power balance and the double tearing mode instabilities. Section 4 contains results of a set of current ramp-up calculations using the INTOR reactor [3] as a reference device. This is followed by an attempt at optimizing the ramp-up scenario using the "overdrive" technique. Finally, in section 5, we give a list of conclusions based on the present calculations and discuss their impact on reactor start-up and staged power operation as appropriate.

6.2. PHYSICS OF LH CURRENT RAMP-UP

The current generated by LH waves is carried by a high energy population of electrons which manifests itself in the form of a plateau in the velocity distribution in the direction parallel to the background magnetic field. The time scale for setting up the plateau is the quasilinear diffusion time which is very short compared to the resistive skin time of the bulk plasma. As such, Lenz's law dictates that, as the RF current drive power is turned on, a back emf will be induced that drives an opposing current such that the continuity of the global current is preserved. This back current is carried by thermal electrons and decays in a resistive skin time, as the global plasma current reaches its steady state value. Thus the theory of LH current ramp-up is essentially that of current drive in the presence of a dc electric field. A substantial amount of theoretical work has already been carried out on this topic. [12,13] In this section, we review some of the issues relevant to current ramp-up in a reactor.

6.2.1. BULK ELECTRON HEATING

When RF power is injected to drive current, there are two channels through which the bulk electrons will be heated. The majority of the incident wave power is absorbed by the current carrying tail electrons via linear Landau damping. These tail electrons then thermalize with the background electrons and ions, in the process transferring most of their energy to the electrons. In the presence of a back emf, part of the tail electron energy will be transferred to the electrons carrying the back current, which in turn

is dissipated in the background plasma. In our analysis, we make the approximation that all of the tail electron energy is dissipated in the background electrons, except for that part which ends up in the poloidal field. The bulk electron dissipated power is entered as a heating term in the electron energy transport equation. Depending on the electron transport scaling, the electron temperature may vary thus changing the skin time ($\tau_s = L_p/R_p$ and $R_p = T_e^{-3/2}$, with L_p , R_p being the plasma inductance and resistivity) and the LH wave power deposition profile.

6.2.2. ELECTRON TAIL CONDUCTIVITY

The Spitzer conductivity σ_{sp} of a fully ionized plasma is determined under the usually valid assumption that the velocity distribution of electrons is Maxwellian. However, in the presence of intense RF power density, resonant diffusion in the velocity space may substantially distort the electron distribution from a Maxwellian, and the dc conductivity may be modified. For a toroidal plasma current $J_\parallel$ driven by an RF source, but under the influence of a toroidal electric field E (either OH or back emf, or both), we can write

$$J_\parallel = J_{rf} + (\sigma_{sp} + \sigma_1)E \quad (1)$$

where J_{rf} is the current carried by the resonant tail electrons and σ_1 is the RF-induced tail conductivity. Assuming that $\sigma_1 < \sigma_{sp}$, it can be shown [12] that

$$\frac{\sigma_1}{\sigma_{sp}} = \frac{(Z_1 + 0.72)}{32(2\pi)^{1/2}} \cdot \frac{(4 + Z_1)}{(3 + Z_1)} \hat{j} w^2 \quad (2)$$

where $\hat{j} = J_{rf}/(n_e e v_{Te})$ is the normalized RF-driven current and $w = v_\parallel/v_{Te}$, with $v_\parallel$ (v_{Te}) being the resonant (electron thermal) speed. Of course, in the case of Landau absorption, $v_\parallel = \omega/k_\parallel$. Equation (2) implies that the effective conductivity is increased for a plasma with an RF-induced tail, resulting in a longer current ramp-up time. Depending on the plasma and wave parameters, this effect may be substantial. It was suggested [12] that the "overdrive" technique may be used to shorten the effective current rise time, and this will be investigated in a later section.

6.2.3. CURRENT PROFILE EVOLUTION

The time evolution of the plasma current profile can be influenced by the local RF power deposition, minor disruptions associated with the double tearing mode and anomalous radial diffusion due to saturated mhd instabilities. Because of the strong dependence of Landau damping on temperature, it is possible that a hollow current profile develops as a result of off-axis power deposition. The corresponding non-monotonic q-profile destabilizes the double tearing mode ($n = 1$, $m > 2$) which destroys the hollow current profile. [14] The consequence is that the plasma current, density and energy within the affected region are redistributed as magnetic reconnection to the plasma center occurs, in a manner similar to the Kadomtsev model [15] for the sawtooth oscillations. The redistribution process can repeat itself and result in effective inward current penetration, a process [16] that has been proposed to explain the absence of a skin current in the early stages of an ohmic discharge. The above process can also be described in terms of enhanced current penetration caused by an anomalous shear viscosity for parallel electron flow due to the instability-driven turbulence. [17] In fact, any saturated instabilities, e.g., that due to the pressure-driven ballooning mode, can potentially give rise to enhanced current diffusion. In our analysis, we will only focus on the double tearing mode and model its effects on current diffusion in the form of a series of minor disruptions.

To complete this discussion, we point out that soft x-ray measurements on LH current drive experiments [17] indicate that the energetic electron population is consistently centered about the plasma axis, even when theoretical calculations predict a hollow current profile for the incident wave spectrum. It has been suggested [18] that these suprathermal electrons diffuse inward in a time scale short compared to the collisional rethermalization time. Within a thermal diffusion time scale, we can practically assume a centrally peaked RF-driven current profile even though the RF power is deposited off-axis. To model this effect requires a kinetic description of the electrons, which is beyond the scope of the present work.

6.2.4. RUNAWAY ELECTRONS

In the presence of a back emf, the current carrying electrons are decelerated by collisions with the background plasma and by the electric field E . If $E/E_0 \gtrsim 1\%$, a significant number of runaway electrons are produced [5] by the electric field with $v_1 > v_R = (E_0/E)^{1/2} v_{Te}$ where $E_0 = m v_{Te}^2 / e \approx n_e / T_e$ is the Dreicer field. These runaway electrons greatly increase the magnitude of the back current and because of their low collisionality can significantly lengthen the decay time of the inductive field. A nonlinear inductive effect also sets in, for example, when $E/E_0 \gtrsim 4\%$ for $c/v_{Te} = 30$, [5] which can lower the magnitude of the back emf and thus reduce the ramp-up efficiency.

In this work, we do not take into account the effect of runaway electrons in our calculations. We note that in our analyses, the inductive electric field at the point of maximum RF power deposition can be as high as 10% of the Dreicer field. Therefore, the results of our simulations should be somewhat on the optimistic side, particularly in cases of low density or high RF power.

6.3. DESCRIPTION OF THE MODEL

The calculational model of our lower hybrid current ramp-up studies can be divided into three parts, namely: (i) RF wave propagation and absorption, (ii) spatial current diffusion, and (iii) plasma transport and equilibrium. The basic tool of analysis consists of a LH wave ray tracing code RAYCO [19] coupled to a $1\frac{1}{2}$ -D tokamak transport and equilibrium code WHIST. [11] The framework of the calculations is shown in Fig. 1, where the transport and electromagnetic equations are solved as an initial, boundary-value problem. The ray tracing code mainly provides input to the current diffusion equation in the form of a LH source current profile while bulk electron heating is incorporated into the transport equations as the difference between the absorbed RF power and the time rate of change of the poloidal field energy. The LH power deposition profile and the flux-surface geometry are recalculated as the plasma pressure and current profiles evolve. The end result of the code calculation is the self-consistent time-history of the plasma density, temperature and current profiles, together with the current ramp-up efficiency. In the following subsections, we briefly describe the ray tracing code RAYCO, the current diffusion equation and the double tearing mode model.

Figure 1 Flowchart for lower hybrid current ramp-up modeling

6.3.1. RAYCO, A LH RAY TRACING CODE

Using a cylindrical coordinate system about the major axis, the ray equations can be written in the form [20]

$$\frac{d}{d\tau} s_j = \left(\frac{\partial D}{\partial k_j} - 2k_j \frac{\partial D}{\partial k_j^2} \right) b_j + \frac{\partial D}{\partial k_j^2} k_j \quad (3)$$

$$\begin{aligned} \frac{d}{d\tau} k_j = & \sum_i \sum_l A_{ijl} k_l \frac{\partial s_l}{\partial \tau} - \left(\frac{\partial}{\partial s_j} D \right) \Big|_{\vec{k}, \vec{b}} + \\ & \left(\frac{\partial D}{\partial k_j} - 2k_j \frac{\partial D}{\partial k_j^2} \right) \left[\sum_i \sum_l b_l A_{ijl} k_l + \sum_i k_i \frac{\partial b_i}{\partial s_j} \right] \end{aligned} \quad (4)$$

where $\vec{r} = (s_1, s_2, s_3) = (R, \phi, z)$, $\vec{k} = (k_1, k_2, k_3) = (k_R, k_\phi, k_z)$ and $\vec{b} = \vec{E}/|\vec{E}| = (b_1, b_2, b_3) = (B_R/B, B_\phi/B, B_z/B)$. In Eqs. (3) and (4), τ is a parameter proportional to the arclength along the ray and A_{ijk} 's are the Christoffel symbols commonly used in tensor calculus. For the coordinate system we used, $A_{ijk} \equiv 0$ except for $A_{122} = -A_{221} = R^{-1}$. We also note that

$$D(\omega, k_j, k_j^2, \vec{r}) = 0 \quad (5)$$

is the local wave dispersion relation with $k_j = \vec{k} \cdot \vec{b}$ and $k_j^2 = |\vec{k} \times \vec{b}|^2$. Equation (3) describes the trajectory in an inhomogeneous plasma, along which the wave energy is transported while the wave dispersion characteristic is governed by Eq. (4).

For the LH wave, the cold plasma electromagnetic wave dispersion relation is given by [21]

$$Rn_\perp^4 + Sn_\perp^2 + T = 0 \quad (6)$$

where $R = \epsilon_\perp$, $S = (\epsilon_\perp + \epsilon_\parallel)(n_\perp^2 - \epsilon_\perp) + \epsilon_x^2$, $T = \epsilon_\parallel[(n_\perp^2 - \epsilon_\perp)^2 - \epsilon_x^2]$, $n_\perp = k_\perp c/\omega$ and $n_\parallel = k_\parallel c/\omega$. The plasma dielectric tensor elements are given by:

$$\epsilon_\perp = 1 + (\omega_{pe}/\omega_{ce})^2 - (\omega_{pi}/\omega)^2 \quad (7a)$$

$$\epsilon_x = \omega_{pe}^2/(\omega\omega_{ce}) \quad (7b)$$

$$\epsilon_1 = 1 - \omega_{pe}^2 / \omega^2 \quad (7c)$$

where ω_{pe} , ω_{ce} and ω_{pi} are respectively the electron plasma and cyclotron frequencies, and the ion plasma frequency. The LH slow mode corresponds to the smaller root for $n_{\perp}^2$ of the bi-quadratic equation in Eq. (6) and is the mode we follow in the ray equations.

The primary wave absorption mechanism along the trajectory is that due to electron Landau damping. An approximate quasilinear spatial damping decrement α has been derived [9] and is given by

$$\alpha \approx 4\pi^{3/2} (m_i/m_e)^{1/2} (\omega/2\pi c) n_i x_{oe}^3 \exp(-x_{oe}^2) \quad (8)$$

where $x_{oe} = \omega/(\sqrt{2} k_{\perp} v_{Te})$. This expression for α is derived with the assumption that the ratio of $D_{rf}/D_c = \frac{1}{2}$, where D_{rf} and D_c are the velocity diffusion coefficients due to RF and collisions, and should be sufficiently accurate for modeling current drive in a reactor plasma.

The wave power along the trajectory is then given simply by

$$P(s) = P_0 \exp\left[-\int_0^s \alpha(s') ds'\right] \quad (9)$$

where s is the arclength and P_0 is the incident power. To calculate the incremental power ΔP deposited in a flux tube bounded by the flux surfaces ψ_1 and ψ_2 in one transit of the ray, we write

$$\Delta P = P(s_1) - P(s_2) \quad (10)$$

where s_1 , s_2 are the arclengths of intersection with ψ_1 and ψ_2 . The incremental power density Δp is simply given by ΔP divided by the volume of the flux tube.

To determine the incremental current generated ΔI_{rf} for each transit of the ray through a flux tube, we invoke the Fisch-Karney formula [10] for the normalized current drive efficiency. For the lower hybrid wave, this is given by

$$(J/P)_{FK} = 2 + 1.4w^2 + 8/w \quad (11)$$

for a plasma of $Z_1 = 1$ and in the absence of neoclassical effects. The incremental RF-driven current density is calculated by

$$\Delta J_{rf} \text{ (A/cm}^2\text{)} = 10^{14} \cdot \frac{T_e \text{ (keV)}}{n_e \text{ (cm}^{-3}\text{)}} \left(\frac{J}{P}\right)_{FK} \cdot \Delta p \text{ (w/cm}^3\text{)} \quad (12)$$

The RF-driven current profile can be obtained by summing, for each flux tube, the incremental current for each traversal of the ray through the tube. Typically, in our calculations, a ray trajectory is followed until 95% of the power has been absorbed.

In Fig. 2, we show the typical results of a lower hybrid ray tracing calculation using RAYCO alone. We use INTOR [3] as the reference device, with its machine parameters shown in Table I. In this example, we assume parabolic profiles for both the density and temperature with peak $n_0 = 10^{12} \text{ cm}^{-3}$ and $T_0 = 2 \text{ keV}$, and the safety factor at the edge is 2.1. To ensure wave penetration to the center, a wave with frequency $f = 1 \text{ GHz}$ and initial toroidal wavenumber $k_\phi = 0.5 \text{ cm}^{-1}$ is launched from the low field side along the equatorial plane. The ray trajectory, as shown in Fig. 2(a), is mainly determined by the density profile through refraction while the damping along the ray is highly dependent on T_e and $k_\parallel$. In this case, reflection at the wall drastically increases $k_\parallel$ and the wave power is rapidly absorbed on the subsequent radial inward transit. In Figs. 2(b),(c), the flux-surface-averaged RF power deposition $\tilde{P}_{rf}(\rho)$ and the corresponding generated current profile $\tilde{J}_{rf}(\rho)$ are shown. It is evident that a hollow RF current profile is developed because most of the power is deposited away from the plasma center.

6.3.2. CURRENT (POLOIDAL FIELD) SPATIAL DIFFUSION

The evolution of the plasma current profile is governed by the poloidal field (B_p) diffusion equation in the presence of an RF source current profile derived in the previous subsection. Since the EM equations are to be solved in conjunction with the transport equations, they have to be expressed in flux-surface coordinates (ρ, θ, ϕ) , where ρ is the flux surface label and θ, ϕ are respectively the poloidal and toroidal angles. Following an earlier work,

Figure 2 (a) Typical ray trajectory for the lower hybrid wave using RAYCO alone. The INTOR tokamak parameters are used with $n_{eo} = 1 \times 10^{12} \text{ cm}^{-3}$, $T_{eo} = 2 \text{ keV}$, $k_{\perp} = 0.5 \text{ cm}^{-1}$ and $f = 1 \text{ GHz}$.
 (b) Flux-surface-averaged LH power deposition profile $\tilde{p}_{RF}(\rho)$ corresponding to (a).
 (c) Flux-surface-averaged LH current profile $\tilde{j}_{RF}(\rho)$ corresponding to (a).

TABLE I
PARAMETERS FOR CURRENT RAMP-UP SIMULATIONS

• INTOR Machine Parameters:

Major radius $R_0 = 520$ cm
 Minor radius $a = 120$ cm
 Elongation $K = 1.6$
 Triangularity $\Delta = 0.3$

• Initial Plasma Parameters:

Peak density $n_{eo} = 10^{13}$ cm⁻³
 Edge density $n_{ea} = 10^{12}$ cm⁻³
 Peak temperature $T_{eo} = 2$ keV
 Edge temperature $T_{ea} = 500$ eV
 Plasma current $I_p = 0.5$ MA

• Lower Hybrid Wave Parameters:

Frequency $f = 2$ GHz
 No. of rays followed = 3
 Initial ray position = $(R, z) = (638 \text{ cm}, 0.5 \text{ cm})$
 Initial toroidal wavenumber $k_\phi = 1.0 \quad 1.2 \quad 1.4 \quad (\text{cm}^{-1})$
 Initial poloidal wavenumber $k_\theta = 0 \quad 0 \quad 0 \quad (\text{cm}^{-1})$

[11] we write down Ampere's law:

$$\tilde{J}_\parallel = \frac{F}{\mu_0 G_\phi} \frac{\partial}{\partial \rho} \left(\frac{G_\theta \tilde{B}_p}{F} \right) \quad (13)$$

where $F(\rho)$ is the poloidal current outside of the flux surface ρ , and $G_\phi(\rho)$ and $G_\theta(\rho)$ are dimensionless flux-surface-averaged geometric factors defined in Ref. 11. The superscript " $\sim$ " denotes effective flux-surface-averaged quantities. Faraday's law is expressed as

$$\frac{\partial \tilde{B}_p}{\partial t} = \frac{\partial}{\partial \rho} (v_\phi \tilde{Y}_p) + \frac{\partial}{\partial \rho} \tilde{E}_\parallel \quad (14)$$

where $\tilde{E}_\parallel$ is the inductive electric field and v_ϕ is the velocity of a surface of constant toroidal flux relative to the surface labeled p . Ohm's law can be written as

$$\tilde{E}_\parallel = \eta (\tilde{J}_\parallel - \tilde{J}_{RF}) \quad (15)$$

where $\eta = (\sigma_{SP} + \sigma_1)^{-1}$, with the tail conductivity included. Combining Eqs. (13), (14) and (15), we arrive at the poloidal field diffusion equation:

$$\frac{\partial \tilde{B}_p}{\partial t} = \frac{\partial}{\partial \rho} (v_\phi \tilde{B}_p) + \frac{\partial}{\partial \rho} \left\{ \frac{\eta F}{\mu_0 G_\phi} \frac{\partial}{\partial \rho} \left(\frac{G_\theta \tilde{B}_p}{F} \right) \right\} - \frac{\partial}{\partial \rho} (\eta \tilde{J}_{RF}) \quad (16)$$

Given $\tilde{J}_{RF}$, Eq. (16) can be solved as an initial, boundary-value problem. The initial condition is set by assuming a current profile at $t = 0$, namely,

$$\tilde{B}_p(\rho, t=0) = \frac{\mu_0 F(\rho)}{G_\theta(\rho)} \int_0^\rho d\rho' \left[\frac{G_\phi(\rho') \tilde{J}_\parallel(\rho', t=0)}{F(\rho')} \right] \quad (17)$$

There are two boundary conditions, the first of which is given by

$$\tilde{B}_p(\rho=0, t) = 0 \quad (18)$$

The other condition is set at the plasma edge $\rho = a$, given by

$$\tilde{B}_p(\rho=a, t) = \frac{\mu_0 I_p(t)}{2\pi R_0 G_\theta(\rho=a)} \quad (19)$$

where I_p is the global plasma current and R_0 is the major radius. To calculate $I_p(t)$, we adopt a simple plasma model in the form of a parallel R_p - L_p circuit connected to an ideal current source I_{RF} . I_p is then given simply by

$$I_p(t) = (I_{RF} - I_p(0))(1 - e^{-R_p t/L_p}) + I_p(0) \quad (20)$$

We note that: $I_{RF} = 4\pi \int_0^a d\rho \langle \tau_1 \rangle \tilde{J}_{RF}$, $R_p = 4\pi^2 \int_0^a d\rho \langle \sqrt{g} \rangle \eta$ and $L_p = \mu_0 R_0 a_1(\epsilon)(1 - \epsilon)/[1 - \epsilon + a_2(\epsilon)K] + L_i$, [21] where τ_1 and g are geometric factors defined in Ref. 11, $\epsilon = a/R_0$, a_1, a_2 are analytic functions of ϵ , K is the plasma elongation and L_i is the plasma internal inductance. These quantities themselves evolve with time. For a time step Δt short compared to the time scale of change for I_{RF} , R_p and L_p , Eq. (20) can be advanced in time according to

$$I_p(t + \Delta t) = [I_{RF}(t) - I_p(t)] \left(1 - e^{-\frac{R_p(t)\Delta t}{L_p(t)}}\right) + I_p(t) \quad (21)$$

6.3.3. DOUBLE TEARING MODE MODEL

Depending on the electron temperature and the incident wave spectrum, the LH wave may deposit its power away from the plasma center such that the plasma current evolves toward a hollow profile. If that occurs, the double tearing mode is destabilized leading to a field line reconnection and subsequent flattening of the current and plasma profiles within the affected region. We describe an empirical model of the double tearing mode in the WHIST code. [22]

As the plasma current density evolves toward a hollow profile, the q -profile becomes non-monotonic. Suppose that at some stage, $q = m$ at $\rho = \rho_s$ and $\rho = \bar{\rho}_s$, where m is an integer and ρ is the flux-surface label, with $\bar{\rho}_s > \rho_s$. We now define a helical flux function $\psi_{m1}^*(\rho)$ corresponding to the $(n=1, m)$ mode as

$$\psi_{m1}^*(\rho) = \int_\rho^a d\rho' \mu_0 F(\rho') G_\phi(\rho') \left[\frac{1}{q(\rho')} - \frac{1}{m} \right] \quad (22)$$

where $\rho = a$ is the plasma edge, with $\psi_{m1}^*(a) = 0$. A typical plot of ψ_{m1}^* as a function of ρ is shown in Fig. 3, where we observe that ψ_{m1}^* has a maximum at $\rho = \bar{\rho}_s$ and a minimum at $\rho = \rho_s$. Magnetic reconnection to the center occurs when $\psi_{m1}^*(\bar{\rho}_s) = \psi_{m1}^*(0)$ and the redistribution of current, density and energy takes place within the region: $0 < \rho < \rho_2$, with $\psi_{m1}^*(\rho_2) = \psi_{m1}^*(\rho_s)$ and $\rho_2 > \bar{\rho}_s$. The new $\psi_{m1}^*(\rho)$ profile is shown as a dashed line in Fig. 3. This internal disruptive process could lead to anomalous radial diffusion of the current toward the center of the plasma, such as that observed for the ohmic current in tokamak discharges.

Figure 3 Profile of typical helical flux function $\psi_{m1}^*(\rho)$ before (solid line) and after (dashed line) magnetic field line reconnection.

6.3.4. POWER BALANCE

To clarify the power balance picture between the LH waves and the plasma during current ramp-up, we utilize a power flow diagram [23] as shown in Fig. 4. A fraction η' of the incident RF power P_{rf} will be absorbed by the resonant hot electrons via Landau damping. Part of the absorbed power, P_H , is directly dissipated in the bulk electrons through collisions, while the rest, P_{el} , ends up as poloidal field energy $\frac{1}{2} L_p I_p^2$. If an external voltage V_{ext} is applied, e.g., from the OH or PF coils, additional power P_{ext} is added to the poloidal field. However, part of the poloidal field energy is dissipated in the back current carried by the bulk electrons at the rate V_B^2/R_p where V_B is the back emf and R_p is the plasma resistance.

With a full kinetic analysis, P_{el} can be expressed as a function of P_{rf} in a constitutive relation. [4] In the work presented here, we make a few simplifications. We set $\eta' = 1$, corresponding to the ideal case of total absorption of the RF power, and we assume no externally applied voltage, thus $V_{ext} = 0$. In the absence of a full kinetic calculation of transient current

Figure 4 Power flow diagram for LH current ramp-up. For our model, $V_{\text{ext}} = 0$ and $\eta' = 1$.

drive, we determine the LH current driven at steady state making use of Eqs. (11) and (12) obtained from kinetic considerations. By solving Eq. (16) together with the transport equations, we find the evolution of the inductive field $\tilde{E}_\parallel$ and the plasma current $\tilde{J}_\parallel$ as a function of ρ . The rate of change of the poloidal field energy density $\tilde{W}(\rho)$ is given by $\dot{\tilde{W}}(\rho) = -\tilde{E}_\parallel(\rho)\tilde{J}_\parallel(\rho)$ where we note that $\tilde{E}_\parallel(\rho) < 0$ in the region of positive current ramp-up. The bulk electron dissipation $\tilde{p}_d^e(\rho)$ from the RF power is then given simply by

$$\tilde{p}_d^e(\rho) = \tilde{P}_{\text{rf}}(\rho) + \tilde{E}_\parallel(\rho)\tilde{J}_\parallel(\rho) \quad (23)$$

where $\tilde{P}_{rf}(\rho)$ is the RF power deposition profile. This term then appears as an additional heating power in the electron energy transport equation.

6.4. RESULTS OF RAMP-UP SIMULATIONS

Lower hybrid current ramp-up simulations have been performed for the INTOR tokamak [13] in the absence of an externally applied voltage. In Table I, we list the parameters for the INTOR machine, the initial density and temperature profiles and the lower hybrid waves in all our calculations. A total number of three rays are being used to model the spectrum of waves ($1 \text{ cm}^{-1} < k_{\phi} < 1.4 \text{ cm}^{-1}$) incident from the outboard side of the device along the equatorial plane. Plasma fueling by gas puffing at the edge is used to keep the average density $\langle n_e \rangle$ constant during ramp-up. For simplicity, we use the Alcator transport scaling to model the anomalous electron loss in our base case.

A series of current ramp-up scenarios are studied by the combined RAYCO/WHIST code with various physics assumptions so as to evaluate the significance of each of them. Five cases have been tested and they are described in Table II. Case (A) will be our base scenario which involves a constant RF pulse at 10 MW power, Alcator transport scaling and Spitzer conductivity. We will focus our attention on the results for this base case.

6.4.1. THE BASE SCENARIO

In this case, a constant RF pulse of 10 MW is applied to a target plasma and the current ramp-up is followed for 300 s, during which time it rose from 0.5 MA to 4.4 MA, as shown in Fig. 5. We note that, from Fig. 5, I_p rises smoothly to its steady state value with an average L/R time of 60 s, while the RF generated current I_{RF} fluctuates "periodically" even after steady state is reached. This is mainly a consequence of the "periodic" onset of the double tearing mode which abruptly changes the plasma profiles, wave power deposition and the resultant RF current. In this respect, we should note that the LH current drive efficiency is highly sensitive to where the bulk of the current is driven in the plasma. As an example, when k_{ϕ} is large or T_e is high, we

TABLE II
LOWER HYBRID CURRENT RAMP-UP SCENARIOS

A.	Constant RF pulse of 10 MW Alcator transport scaling $\sigma = \sigma_{SP}$
B.	Constant RF pulse of 10 MW Alcator transport scaling $\sigma = \sigma_{SP} + \sigma_1$ (includes tail conductivity)
C.	"Overdrive" - Programmed RF pulse Alcator transport scaling $\sigma = \sigma_{SP}$
D.	Constant RF pulse of 10 MW PLT transport scaling $\sigma = \sigma_{SP}$
E.	Constant RF pulse of 10 MW Ohkawa transport scaling $\sigma = \sigma_{SP}$

would expect the wave to be damped near the plasma periphery where the density is low, thus generating a large amount of current, since current drive efficiency scales inversely with density [see Eq. (12)]. It is exactly this scenario which leads to the build-up of the hollow current profile as the plasma heats up initially from wave power dissipation in the core. After the onset of the mhd mode, the temperature profile flattens toward the plasma axis and, depending on the density profile which determines the evolution of $k_{\parallel}$, a more centrally peaked RF power deposition profile can be restored. Thus the process of plasma current evolving toward a hollow profile repeats itself "periodically" while the bulk current is being built up slowly. In fact, this phenomenon will persist even after the current has attained its steady state value, at which point a current drive efficiency of 0.44 A/W is obtained, as depicted in Fig. 5.

To illustrate this point further, we show in Fig. 6(a), (b) and (c) the radial profiles of the plasma current $\tilde{J}_z$, RF-induced current $\tilde{J}_{RF}$ and electron temperature T_e at selected time intervals during the 300 s discharge for scenario A. We note that in the absence of tearing modes, the plasma current should evolve toward the RF-driven current, both in magnitude and profile.

Figure 5 Time evolution of plasma and RF-driven current (I_p and I_{RF}) for the base scenario (A) of LH current ramp-up. Machine and wave parameters are as shown in Table I.

Figure 6(a) Radial profile of plasma current density $J_p(\rho)$ at selected time intervals for scenario A.

Figure 6(b) Radial profile of RF-driven current density $\bar{J}_{RF}(\rho)$ at selected time intervals for scenario A.

Figure 6(c) Radial profile of electron temperature $T_e(\rho)$ at selected time intervals for scenario A.

This is evident for $t = 12$ s and $t = 24$ s in Fig. 6(a), (b). In particular, at $t = 24$ s, a small hump starts to form in the $\tilde{J}_1$ profile. We also point out in Fig. 6(c), $T_e(0)$ rises from 2 keV to 3.5 keV in the first 12 s due mainly to ohmic dissipation of the back current. At $t = 60$ s, we have evidence of the first onset of the double tearing mode, when $\tilde{J}_1$ becomes flattened over the bulk of the plasma. The spatial extent of the profile flattening may be the result of a series of destabilization of the $n = 1$ and $m = 2-6$ modes over different regions of the plasma. From then on, both the $\tilde{J}_1$ and $\tilde{J}_{rf}$ profiles evolve under the influence of the tearing mode activities while T_e rises slowly with an increasingly broad profile.

An important measure of the effectiveness with which the lower hybrid wave energy is converted to poloidal field energy is the ramp-up efficiency η_B , defined as

$$\eta_B = \frac{L_p I_p \frac{dI_p}{dt}}{P_{RF}} \quad (24)$$

where the numerator is the time rate of increase of the poloidal field energy $\frac{1}{2} L_p I_p^2$. A plot of η_B versus time is given in Fig. 7 for scenario A. We note that the value of η_B decreases to zero as the plasma current rises toward the LH source current, as it should. Averaging over the tearing mode fluctuations, the value of η_B is found to be quite low, on the order of 5% for the initial 120 s and only about 2 to 3% after that. This means that about 96% of the wave power ends up as heat to the bulk electrons rather than as poloidal field energy. This scenario is clearly not optimized in terms of ramp-up efficiency as η_B as high as 20% has been recorded in PLT experiments. [24]

6.4.2. THE CASE WITH TAIL CONDUCTIVITY CORRECTION

This is the case for a constant LH power of 10 MW with a correction to the plasma conductivity due to the fast electron plateau. Only a slight increase in the ramp-up time has been observed when compared to the base scenario, indicating that the effect of tail correction to the plasma resistivity is small in this case. In Fig. 8, the spatially averaged Spitzer conductivity σ_{sp} and that due to the electron tail σ_1 are plotted over the 300 s discharge. The slow fluctuations in σ_{sp} are again a result of tearing mode

Figure 7 Time evolution of current ramp-up efficiency η_B for scenario A.

Figure 8 Time history of spatially averaged Spitzer conductivity σ_{SP} and tail conductivity σ_1 for scenario B.

activities. It is interesting to observe that $\sigma_1 < \sigma_{SP}$ during the entire discharge.

6.4.3. THE "OVERDRIVE" CASE

The current ramp-up efficiency, as calculated for scenario A, is very low (~4%) resulting in a long rise time (≥ 100 sec) for the plasma current. A long ramp-up time may be tolerable for a tokamak reactor that is capable of steady state operation. However, for a hybrid transformer reactor, it may result in a long dwell time thus lowering the reactor duty cycle. An obvious remedy is to raise the RF power above the level needed to drive the current at steady state and increase the initial ramp-up rate. As the current evolves toward the level for steady state operation, the RF power can be dropped systematically to the desired value. This is the so-called "overdrive" technique [12] to minimize the ramp-up time. We must keep in mind that a larger current drive power means a higher back emf and thus higher ohmic dissipated power, besides the effect of increased runaway electrons which we do not consider.

In the "overdrive" scenario we examined, a programmed RF pulse is applied as shown in Fig. 9. This pulse shape is based on results obtained from an extensive study of the optimal scenarios for reactor plasma start-up. [25] In Fig. 10, we show the time evolution of both I_p and I_{RF} for the 80 s discharge. The plasma current reaches 3.8 MA in the first 75 s of the discharge. The LH current, on the other hand, follows the general form of the injected wave power (Fig. 9), although mhd activities tend to cause slow fluctuations. At $t = 50$ s, the RF power is dropped back to 10 MW at which point the plasma current practically reaches equilibrium except for the tearing mode instabilities. Figure 11 depicts the time dependence of η_B for this scenario, a notable feature of which is that η_B reaches a maximum value of 17% at $t = 20$ s, and then decreases as the RF power is dropped. We also note that throughout the "overdrive" scenario, the tearing mode induced fluctuations decrease substantially from those found in the base case, probably because of a different current profile evolution.

A comparison between the base case and the "overdrive" case of current ramp-up scenarios has been made and their relevant parameters are shown in Table III. In both cases, the RF pulse power eventually reduces to 10 MW so

Figure 9 Programmed RF pulse for current "overdrive" in scenario C.

Figure 10 Time evolution of plasma current I_P and RF-driven current I_{RF} for scenario C.

Figure 11 Time evolution of current ramp-up efficiency η_B for scenario C.

TABLE III
COMPARISON OF RESULTS BETWEEN CASE A AND CASE C

	<u>Case A</u>	<u>Case C</u>
RF pulse shape	10 MW constant	Programmed
Ramp-up time (s)	122	75
Energy input (MJ)	1220	1050
Peak RF power (MW)	10	20
Peak ramp-up efficiency (%)	8	17
Steady state current (MA)	4.4	4.4

that the steady state current should be 4.4 MA. The ramp-up time as listed in Table III is the time it takes for the current to rise to a value of 3.8 MA.

In general, the "overdrive" case exhibits more optimistic parameters in terms of shorter ramp-up time, smaller energy input and higher peak ramp-up efficiency, but at the expense of a higher required power from the RF system. For a reactor operated in the hybrid mode, a trade-off between high availability and low RF power requirement needs to be studied from the point of cost analysis.

6.4.4. SENSITIVITY TO THE TRANSPORT MODEL

The first three scenarios listed in Table II are all based on the Alcator transport scaling, in which the electron thermal diffusivity χ_e is inversely proportional to the local density $\tilde{n}_e(\rho)$. If a more optimistic scaling is used, such as the PLT scaling, in which χ_e is small on axis but increases rapidly toward the edge, a markedly different current ramp-up scenario results. This scenario corresponds to case D in Table II. The energy confinement initially is very good and the central T_e rises to over 5 keV within the first 0.2 s of the discharge. The resultant sharply peaked T_e profile triggers a series of tearing mode activities, at the end of which T_e drops to less than 200 eV. At this low temperature, wave absorption becomes entirely negligible and wall dissipation of the RF power dominates, resulting in practical termination of the RF-driven discharge.

In case E, we repeat the ramp-up simulation using the Ohkawa transport scaling, [26] which stipulates that χ_e is proportional to $(1 + P_{rf}/P_{OH})^{1/2}$, where P_{rf} and P_{OH} are the local RF and OH absorbed power respectively. In this scenario, the absorbed power density is very large compared to the Joule heating such that χ_e can become large locally. As a result, the plasma energy is lost very rapidly and T_e drops to less than 100 eV only 0.15 s after the LH power is turned on.

It is thus clear that current ramp-up simulations are extremely sensitive to the local power balance, particularly since the LH wave damping is highly dependent on the electron temperature. Even though we characterize the results of scenarios D and E as preliminary in nature, the sensitivity of LH current ramp-up to the transport model is well demonstrated. Perhaps this problem may be somewhat mitigated by smoothing the wave absorption profile using a larger number of rays or a wider $k_{\parallel}$ -spectrum. Nevertheless, more

research efforts need to be devoted to this area in the future to better understand the issue.

6.5. CONCLUSION AND DISCUSSION

A 3-D, LH ray tracing code has been successfully coupled to a transport/equilibrium code to simulate plasma current ramp-up in a tokamak reactor. This current ramp-up is studied in the absence of an externally imposed toroidal electric field, assuming Alcator transport scaling in the INTOR tokamak. To account for possible inward current diffusion, a model involving the double tearing mode destabilized by a hollow current profile is invoked. This results in "periodic" redistribution and flattening of the plasma profiles even after equilibrium condition is achieved. The effect of the electron tail conductivity on the current rise time appears to be minimal in the cases we study due to the low power level involved.

Preliminary results are obtained for the case of start-up optimization by the "overdrive" technique. Using a programmed RF pulse, the ramp-up time and total input energy are lowered at the expense of a higher power requirement when compared to the base scenario of constant power input. Within the physics assumptions used in the model, current ramp-up is also found to be sensitive to the transport scaling used.

Two relatively important physical processes have been neglected in our analysis of LH current ramp-up. The first is the experimentally observed phenomenon of the inward diffusion of the current carrying tail electrons in a time scale short compared to the thermalization time, which results in a centrally peaked RF current profile. The second is the effect of runaway electrons in the presence of a strong back emf, which can potentially lower the plasma resistivity and ramp-up efficiency. The implications of these processes to the results of our present work need to be addressed in the future.

As stated previously, we have developed a powerful tool of modeling LH current ramp-up in a tokamak reactor, conceivably an important element of the start-up phase. We have also demonstrated the possibility of using programmed RF pulses to optimize the current ramp-up scenario by minimizing the current rise time and energy input. However, to model the dwell phase operation of

the hybrid transformer reactor, further steps need to be taken. Here, the region of interest is the transition between an ohmically driven current in the burn phase and an RF-generated current in the dwell phase. What needs to be deployed is to couple the primary OH circuit, and any other external inductive coils, such as the vertical field coil, into our model. The ohmic voltage diffuses in from the plasma edge and therefore must be included in the poloidal field diffusion equation as a boundary condition. Therefore, as stated in Chapter 7, the reactor will have to be modeled as a network of mutually interacting circuits, rather than as a simple RL circuit with an ideal current source. Furthermore, the circuit analysis must include the eddy currents generated in the conducting structures surrounding the plasma. None of these is presently taken into account in our work.

REFERENCES

1. N. A. Uckan, "Noninductive Current Drive in Tokamaks," Oak Ridge National Lab. Report ORNL/TM-9296 (1985).
2. G. Tonon, Plasma Phys. Contr. Fusion, 26 1A (1984) 145 (Invited papers of 11th Europ. Conf. Aachen, 1983).
3. INTOR Group, "International Tokamak Reactor: Phase II A, Part 1" (Report Int. Tokamak Reactor Wksp. Vienna 1981-83), IAEA, Vienna (1983).
4. N. J. Fisch and C. F. F. Karney, Phys. Rev. Lett., 54 (1985) 897.
5. C. S. Liu, V. S. Chan and Y. C. Lee, *ibid.*, 55 (1985) 2583.
6. F. C. Jobes et al., *ibid.*, 55 (1985) 1295.
7. E. J. Valeo and D. C. Eder, "Numerical Modelling of Lower Hybrid Heating and Current Drive," Princeton Plasma Physics Lab. Report PPPL-2272 (1986).
8. R. Engade and P. T. Bonoli, "Computer Modeling of Lower Hybrid Current Ramp-Up," in Radiofrequency Plasma Heating, D. W. Swain, Ed. (American Institute of Physics, 1985).
9. P. Bonoli, "Linear Theory of Lower Hybrid Heating," in Wave Heating and Current Drive in Plasmas, V. L. Granatstein, P. L. Colestock, Eds. (Gordon and Breach Science Pub., 1985).
10. N. J. Fisch and C. F. F. Karney, Phys. Fluids, 24 (1981) 27.
11. W. A. Houlberg, S. E. Attenberger and L. L. Lao, "Developments in Tokamak Transport Modeling," in Advances in Mathematical Methods for the Solution

- of Nuclear Engineering Problems (Proc. Am. Nucl. Soc./Europ. Nucl. Soc. Int. Top. Mtg. Munich, 1981) Vol. 2, p. 425.
12. N. J. Fisch, Phys. Fluids, 28 (1985) 245. K. Borass and A. Nocentini, Plasma Phys. Contr. Fusion, 26 1A (1984) 1299.
 13. C. F. F. Karney and N. J. Fisch, Phys. Fluids, 29 (1986) 180.
 14. B. Carreras, H. R. Hicks and B. V. Waddell, Nucl. Fusion, 19 (1979) 583.
 15. B. B. Kadomtsev, Sov. J. Plasma Phys., 1 (1975) 389.
 16. H. Furth, P. Rutherford and H. Selberg, Phys. Fluids, 16 (1973) 1054.
 17. W. Hooke, Plasma Phys. Contr. Fusion, 26 1A (1984) 133.
 18. P. T. Bonoli, private communication (1986).
 19. T. K. Mau, M. R. Ilgen and R. W. Conn, "One-Dimensional Modelling of Lower Hybrid Current Ramping in Tokamaks," Trans. Am. Nucl. Soc., 52 (Proc. Ann. Mtg. Reno, NV, June 1986) p. 229.
 20. T. K. Mau and R. W. Conn, Bull. Am. Phys. Soc., 28 (1983) 1091.
 21. S. P. Hirshman and G. H. Nielson, Phys. Fluids, 29 (1986) 790.
 22. W. A. Houlberg and J. T. Hogan, Nucl. Technol./Fusion, 3 (1983) 244.
 23. N. J. Fisch, "Theory of Current-Drive in Plasmas," to be published.
 24. F. C. Jobes et al., Phys. Rev. Lett., 55 (1985) 1295.
 25. T. Okazaki, S. Masayoshi, S. Nishio and N. Fujisawa, Nucl. Engrg. Des./Fusion, 3 (1985) 73.
 26. T. Ohkawa, Comments Plasma Phys. Controlled Fusion, 9 (1985) 127.

CHAPTER 7

PLASMA CIRCUIT INTERACTION CODE

Marc A. Firestone

CONTENTS

7.1. INTRODUCTION	7-1
7.2. PLASMA EVOLUTION	7-1
7.3. CIRCUIT MODEL	7-2
7.4. PARTICLE AND ENERGY BALANCE	7-3
7.5. LOW Z IMPURITY MODEL	7-4
7.6. ALPHA PARTICLE AND NEUTRAL BEAM HEATING	7-6
REFERENCE	7-7

7.1. INTRODUCTION

Our objective has been to develop a computer code which combines plasma particle and energy balance together with the electromagnetic coupling between the plasma and the engineering systems. The latter include both active poloidal field coils and the fields due to passive eddy currents in the surrounding structures. To this end we have modified a TFLN code, PCIC[1], to include features expected in a power producing tokamak. This modeling effort has been previously described in the report and consists of plasma position and shape techniques, eddy currents in a thick inhomogeneous structure, and lower hybrid current drive. Additionally, an alpha particle heating model has also been incorporated in the new code.

7.2. PLASMA EVOLUTION

Initially, the plasma is at a fixed major, R_0 , and minor, a_0 , radius. The current increases at this position until a certain q_I value is reached. Then, the current increases further while the major radius decreases maintaining a constant q_I at the limiter. The plasma current I_p is then related to R , a , and the elongation K by

$$I_p \text{ (MA)} = \frac{5a^2 \text{ (m)} B_T \text{ (T)}}{R \text{ (m)} q_I} \left(\frac{1 + k^2}{2} \right) \quad , \quad (1)$$

$$R + a = R_0 + a \quad .$$

After startup the major radius is determined by integrating the radial equation of motion from force balance and the minor radius from assumed knowledge of q_I . The vertical position is likewise determined by integrating the vertical force balance equation. The plasma shape parameters are obtained from the multipole method described in Chapter 4.

7.3. CIRCUIT MODEL

The plasma is treated as if it were a conductor with conductivity $\sigma(r,t)$. The poloidal magnetic field diffusion equation is

$$\frac{\partial B}{\partial t}(r,t) = \frac{\partial}{\partial r} \left[\frac{1}{\mu_0 r \sigma(r,t)} \frac{\partial}{\partial r} r B_\theta(r,t) \right] \quad (2)$$

The evolution of the plasma current is solved simultaneously with the current in the external circuits by rewriting Eq. (2) as

$$\frac{\partial}{\partial t} \frac{\mu_0 I_p(r)}{r} = \frac{\partial}{\partial r} \left[\frac{1}{\sigma(r,t)r} \frac{\partial}{\partial r} I_p(r) \right] \quad (3)$$

Equation (3) can be finite differenced by subdividing the minor radius into K circular shells of width Δ . The final result is an LR ladder network with

$$L_K = \mu_0 R \frac{\Delta}{r_K} \quad , \quad R_{K+(1/2)} = \frac{R}{\sigma[r_{K+(1/2)}, t] r_{K+(1/2)} \Delta} \quad .$$

The external inductance is given by

$$L_p^{\text{ext}} = \mu_0 R \left\{ \left[1 + \frac{1}{8} \left(\frac{a}{R} \right)^2 \right] \ln \left(\frac{8R}{a} \right) + \frac{1}{24} \left(\frac{a}{R} \right)^2 - 2 \right\} \quad (4)$$

The plasma circuit model including the lower hybrid current drive is shown in Fig. 1. The neoclassical conductivity is calculated assuming a temperature profile of the form

$$T_e(r) = T_{e0} [1 - (r/a)^2]^m + T_{\text{edge}} \quad .$$

The average temperature and Z_{eff} are determined from the zero-dimensional particle and energy balance section of the code.

Fig. 1. Plasma zone model with RF current drive.

7.4. PARTICLE AND ENERGY BALANCE

The hydrogenic density, electron and ion thermal energy, and the neutral density evolution are described by the following differential equations:

$$\begin{aligned} \frac{d}{dt}(n_H^{I,II}) = & n_e n_o^{I,II} S_H - \frac{n_H^{I,II}}{\tau_p} - n_H^{I,II} \frac{1}{V} \frac{dV}{dt} + S_b^{I,II} \\ & + n_{th} n_o^{I,II} (\langle \sigma v \rangle_{cx} + \langle \sigma v \rangle_{im}) + n_o^{I,II} n_a (\langle \sigma v \rangle_{cx} + \langle \sigma v \rangle_{im}), \end{aligned} \quad (5)$$

$$\begin{aligned} \frac{3}{2} \frac{d}{dt}(V_p n_e k T_e) = & P_{OH} - P_{ion} - P_{ion}^{impur} - Q_{ie} - P_{rad} - P_{tran}^e + P_{comp}^e \\ & + P_b^e + P_{icw}^e - P_{brem} - P_{cycl} + P_{lhw} + P_a^e - Q_{ae}, \end{aligned} \quad (6)$$

$$\begin{aligned} \frac{3}{2} \frac{d}{dt}(V_p n_i k T_i) = & Q_{ie} + P_{comp}^i + P_b^i - P_{tran}^i - P_{cx} + P_a^i \\ & + P_b^{thmi} - P_a^{rxn} + P_a^{thm}, \end{aligned} \quad (7)$$

$$V_v \frac{d}{dt} n_o^{I,II} = - \frac{d}{dt}(V_p n_p^{I,II}) + \Gamma - \frac{dn}{dt} (cx + imp)_a. \quad (8)$$

In Eq. (5) S_H is the hydrogen ionization rate coefficient and τ_p an average particle confinement time. The superscripts I,II refer to the two hydrogenic species present. $S_b^{I,II}$ is the source due to beam ions slowing down and thermalizing. The alpha particle densities n_α and n_α^{th} refer to thermalized and fast populations respectively. The reaction rates $\langle\sigma v\rangle_{cx}$ and $\langle\sigma v\rangle_{im}$ are charge exchange and impact rates. In Eq. (6), P_{OH} is the ohmic heating power; P_{comp}^e is the power due to compression,

$$P_{comp}^e = -\frac{4}{3} W_e \frac{1}{R} \frac{dR}{dt},$$

where W_e is the electron thermal energy; P_b^e is the power input to the electrons by neutral beam heating; P_{ion}^e is the power loss due to ionization; Q_{ie} is the electron-ion power transfer; P_{tran}^e is the power loss by electron transport; P_{rad} is the power radiated by impurities; P_{ion}^{impur} is the power loss due to ionization of the impurities; P_{icw}^e is the breakdown RF power to the plasma; P_{brem} and P_{cycl} are the power lost by bremsstrahlung and cyclotron radiation; P_{lhw} is the heating power input by lower hybrid current drive; P_α^e is the power to the electrons from the fast alpha particles; and Q_{ae} is electron-thermal alpha power transfer. In Eq. (7), P_{comp}^i is the power due to compression; P_b^i is the power to the ions from neutral beam heating; P_{tran}^i is the power lost by ion transport; P_{cx} is the charge-exchange power loss; P_α^i is the power transfer from the fast alphas; P_α^{thm} is the power increase due to the addition of the thermal alphas; P_b^{thm} is the power due to the addition of the thermalized neutral beam ions; and P_α^{rxn} is the power lost by the ions to the fusion reaction. In Eq. (8) the recycling coefficient is assumed to be unity; Γ represents the fueling rate; and $(dn/dt)(cx + imp)_\alpha$ is the charge exchange and impact due to alpha particle density rate loss.

The principal uncertainty in these calculations is the transport model where the power losses are:

$$P_{tran}^e = \frac{3}{2} \frac{n_e k T_e V}{\tau_e}, \text{ and } P_{tran}^i = \frac{3}{2} \frac{n_i k T_i V}{\tau_i}.$$

The code utilizes a number of empirical and theoretical energy confinement time models as input devices. In this way the system sensitivity to confinement scalings can be investigated. In all of the scaling laws, τ_p is set to $3\tau_e$.

7.5. LOW Z IMPURITY MODEL

The effects of low Z impurities in the startup phase are calculated by solving the rate equations for carbon and oxygen:

$$\frac{d}{dt} O_1 = r_o + \frac{v_p}{v} - \{n_e O_1 S_1 + n_e O_2 (\alpha_1 + \gamma_1 n_e) + \sum_{k=2}^9 \frac{O_k}{\tau_p}\} , \quad (9)$$

$$\begin{aligned} \frac{d}{dt} (v_p O_k) = & n_e O_{k-1} S_{k-1} - n_e O_k (S_k + \alpha_{k-1} + \gamma_{k-1} n_e) \\ & + n_e O_{k+1} (\alpha_k + \gamma_k n_e) - \frac{O_k}{\tau_p} , \quad (k = 2 \dots 8) , \end{aligned} \quad (10)$$

$$\frac{d}{dt} (v_p O_9) = n_e O_8 S_8 - n_e O_9 (\alpha_8 + \gamma_8 n_e) - \frac{O_9}{\tau_p} . \quad (11)$$

A similar set of equations is solved for carbon. S_k is the ionization rate; α_k is the photo and dielectric recombination rate; r_o is the influx of neutral oxygen; and τ_p is the average particle confinement time of oxygen.

From the population in each ionization state the energy loss by line radiation and ionization and recombination is

$$P_{rad} = \sum_{j=1}^9 \Delta E_j^l Q_j^l (T_e) O_j , \quad (12)$$

and

$$P_{ion} = \sum_{j=1}^9 \chi_j n_e (S_j O_j - \gamma_j n_e O_{j+1}) + \frac{3}{2} k T_e \alpha_j n_e O_{j+1} , \quad (13)$$

where ΔE_j is the transition energy of line l of ionization state j ; Q_j^l is the transition rate; and χ_j is the ionization energy.

At $T_e > 100$ eV line radiation is not important in the plasma core and a simple treatment is used:

$$\frac{d}{dt} v_p O_k = 0 , \quad (k = 2 \dots 8) , \quad (14)$$

$$\frac{d}{dt} (v_p O_1 + \sum_{k=2}^8 v_p O_k) = r_o . \quad (15)$$

For $\tau_p = \infty$, this corresponds to coronal equilibrium. Also, an approximate treatment is used for the neutral hydrogenic density

$$\frac{d}{dt}(v_p n_p^{I,II}) = r \quad , \quad (16)$$

$$n_o^{I,II} = \frac{n_p^{I,II}}{\tau_p n_{e \text{ ion}}} + \frac{r}{v_p n_{e \text{ ion}}} \quad . \quad (17)$$

These calculations then yield Z_{eff} .

7.6. ALPHA PARTICLE AND NEUTRAL BEAM HEATING

The energy deposition and particle source due to alpha particle production and neutral beam heating is calculated by a simple slug model. Several approximations are employed:

1. No loss due to bad orbits,
2. $v_{\perp} = 0$,
3. Neglecting pitch angle scattering,
4. No account for neutral beam penetration and deposition.

The change in beam energy of a slug of beam particles with charge Z_k and atomic weight A_k is

$$\begin{aligned} \frac{dE_k}{dt} = & - \frac{2\pi Z_k^2 e^4}{\sqrt{m_p/2}} \left[\ln A_1 \left(\frac{A_k}{E_k} \right)^{1/2} \sum_i \frac{n_i Z_i^2}{A_i} \right. \\ & \left. + \frac{4}{3\sqrt{\pi}} \ln A_e \frac{n_e}{(kT_e)^{3/2}} \left(\frac{E_k}{A_k} \right) \left(\frac{m_e}{m_p} \right)^{1/2} G\left(\frac{v}{v_{\text{the}}} \right) - \frac{2E_k}{R} \frac{dR}{dt} \right] \quad , \quad (18) \end{aligned}$$

where

$$G(y) = \frac{1}{2} \frac{d}{dy} \left(\frac{1}{y} \frac{2}{\sqrt{\pi}} \int_0^y e^{-x^2} dx \right) \quad .$$

The first term is the energy deposited in the ions, the second is the energy deposited to the electrons, and the third term is due to compressional heating

of the beam ions. When the alpha particles or neutral beam ion velocity becomes less than $(3kT_i/m_i)^{1/2}$, the slug of particles is treated as part of the ion energy resulting in both a source of particles as well as energy. The neutral beam is composed of full, half and third energy particles, which are handled separately in this calculation.

REFERENCE

1. R. J. Hawryluk, M. A. Firestone, U. R. Christensen, H. Fishman, J. A. Schmidt, "Plasma-Circuit Interaction in TFTR," TFTR Physics Group Report #29, Princeton Plasma Physics Laboratory, 1980.

CHAPTER 8

NEUTRONIC OPTIMIZATION OF A LiAlO₂ SOLID BREEDER BLANKET

Peretz Levin* and Nasr M. Ghoniem

*** On leave 1984-1985 from Physics Department, Nuclear Research Center - Negev.**

CONTENTS

ABSTRACT	8-1
8.1. INTRODUCTION	8-1
8.2. SYSTEM CONFIGURATION AND MODELING	8-4
8.3. THE SWAN OPTIMIZATION CODE	8-5
8.4. OPTIMIZATION RESULTS	8-7
8.4.1. Preliminary Calculations	8-7
8.4.2. Stage I Optimization: Maximum TBR	8-8
8.4.3. Stage II: Two-Zone Constrained Blanket	8-10
8.4.4. Stage III: Reference Design	8-12
8.5. CONCLUSIONS AND RECOMMENDATIONS	8-17
REFERENCES	8-17

ABSTRACT

In this report, a pressurized lobular blanket configuration is neutronically optimized. The blanket configuration features the use of beryllium and LiAlO_2 solid breeder pins in a helium-cooled cross-flow pattern. One-dimensional (1-D) neutronic optimization calculations are performed to maximize the tritium breeding ratio (TBR). The procedure involves spatial allocations of Be, LiAlO_2 , 9-C (ferritic steel), and He, in such a way as to maximize the TBR subject to several material, engineering, and geometrical constraints. Consistent with all imposed engineering constraints, a TBR of 1.17 is achieved for a relatively thin blanket (≈ 43 cm depth).

8.1. INTRODUCTION

The development of blanket concepts based upon individual pods or canisters has been pursued since the early days of conceptual fusion designs [1,2]. Several perceived advantages have been considered for these designs. Results from a recent blanket comparison and selection study (BCSS) [3] have indicated that a combination of helium as a coolant and ceramic Li-bearing solid breeder can satisfy necessary neutronic and thermomechanical performance criteria. In a version of a CA blanket design [4], the Li_2O ceramic breeder has been proposed in the form of clad plates. The helium coolant temperature increases from the front to the back side of the blanket. Even though the energy desposition profile in the solid breeder decreases rapidly with depth, helium flow can be controlled to achieve thermal homogeneity within the blanket. However, several critical areas remain unaddressed in the previous studies:

1. The plate cladding structure, with its thin dimensions, may not be tolerant to thermal and irradiation inelastic strains resulting in reduced lifetimes.
2. Box-type structures are generally prone to stress concentrations at corners, leading to early failure.
3. Even though Li_2O has a higher lithium atom density as compared with LiAlO_2 , its swelling rate is about an order of magnitude larger.

4. With a greatly reduced structure-to-breeder ratio, the breeding ratio was found to be marginal [4].

Recent efforts undertaken at CEA Saclay (France) have concentrated on clad breeder elements. This has been dictated by the desirability of maintaining a reasonable breeder geometrical integrity and a well-controlled working temperature. Their studies of canister blankets with beryllium multiplier and solid breeder indicated that better tritium breeding is achieved when beryllium is mixed with the solid breeder [5]. However, the chemical compatibility problems, which are crucial for the viability of such a design, remain to be resolved.

In the present study, we adopt the following:

1. A pressurized lobular configuration is used rather than the canister configuration. This allows for flat side plates as shown in Fig. 1. The configuration is well suited for tight-fitting locations (i.e., the in-board blanket). A high volume fraction can thus be achieved.
2. Pressurized helium flows in the radial direction, achieving thermal homogeneity, as described in Ref. [6].

Fig. 1. Preliminary pressurized lobular blanket configuration

3. The use of beryllium in the front zone of the blanket is consistent with its thermo-physical properties (i.e., high conductivity). The lower conductivity breeder material is used deep inside the blanket where the nuclear heat generation is lower.
4. Beryllium and solid-breeder pins are arranged such that helium cross-flow conditions are achieved. The small size of the pins ensures minimum temperature asymmetries. This is shown to result in minimal bowing and deflections within the blanket [7].
5. The side plates are tapered from the back to the front in order to accommodate the high internal pressure in a stand-alone, hypothetical accident scenario (see Ref. [8]).

We present here the results of neutronic optimization calculations of a solid-breeder blanket. The objective is to optimize the spatial material allocations in the blanket, consistent with a number of engineering constraints. The materials used in the present analysis are given below:

1. Structural Material: Low activation material 9-C. This material, which is structurally equivalent to HT-9, has the following composition [9]:

Cr = 11.81%	C = 0.097%	V = 0.28%
W = 0.89%	Mn = 6.47%	Si = 0.11%
N = 0.003%	P < 0.005%	S < 0.005%
Fe = remainder		

The 9-C structural material is used for the lobe shell as well as for cladding the solid breeder material.

2. Solid Breeder Material: LiAlO_2 , with variable enrichment of Li to be determined by the optimization study.
3. Neutron Multiplier: Beryllium.
4. Coolant: Helium.
5. Shield-1: Water and ferritic steel Fe-1422.
6. Shield-2: Water, ferritic steel Fe-1422, and B_4C .

8.2. SYSTEM CONFIGURATION AND MODELING

One-dimensional neutron transport calculations have been typically based on a cylindrical approximation to the toroidal geometry. The ANISN neutron transport code [10], which uses the S-N method of solution to the transport problem, provides for approximate 1-D solutions for infinite slabs or cylinders. The actual blanket configuration (Fig. 1) reveals more geometric details than can actually be modeled in 1-D calculations. For example, the solid-breeder pins are also clad with the 9-C ferritic material. A cross section of a typical pin is shown in Fig. 2. To determine such details, many engineering factors have to be considered. The final blanket configuration must result in acceptable neutronic and thermostructural performance. A basic consideration, however, is how to maximize the tritium breeding ratio by an appropriate spatial distribution of materials without exceeding or conflicting with other engineering constraints.

A 1-D model is obtained by treating the various zones as shells of infinite height. Materials are smeared within each zone according to their densities. This is consistent with a flat flux approximation where self-shielding effects are neglected.

Fig. 2. Cross section of a typical breeder pin

The neutron transport and gamma production cross sections were obtained as a coupled set of 46 neutron groups and 21 gamma groups produced by the AMPX modular code system [11] from the nuclear data in ENDF/B-IV. The cross sections were weighted with a $1/E$ spectrum for $E_n > 0.345$ eV, and with a Maxwellian distribution for $E_n < 0.345$ eV. The gamma interaction cross sections were uniformly weighted. Because of cost limits, the 46 neutron-group cross sections were collapsed to a smaller number of groups.

In the next section, we describe the methodology adopted in this optimization study, outlining the basic principles of the technique. This is followed by the results of optimization studies, starting with modeling an idealistic system without any constraints. This is then followed by modeling a system that is more realistic with appropriate engineering constraints.

8.3. THE SWAN OPTIMIZATION CODE

SWAN [12] is a code developed for the analysis and optimization of the nucleonic characteristics of CTR blankets. Any nuclear system that is described by the inhomogeneous linear transport equation, can be analyzed by the SWAN code.

SWAN is composed of two modules, SWIF and ANISN. ANISN [10] is a 1-D discrete ordinates transport code. SWIF is a code developed for perturbation calculations and optimization studies.

The type of optimization problems that can be handled by SWAN can be characterized as follows. Given the external source distribution

$$S(\underline{x}) \equiv S(\underline{r}, E, \underline{\Omega}) \quad , \quad (1)$$

and the atomic density distribution $N_i(\underline{r})$ for all I materials, the density of which are variables of the optimization, find the material density distribution that will extremize the functional $F_e(N_1, N_2, \dots, N_I)$ subject to:

1. The constraints imposed by the density limits

$$N_i^{\min}(\underline{r}) < N_i(\underline{r}) < N_i^{\max}(\underline{r}) < N_i^0(\underline{r}) \quad , \quad (2)$$

where these densities are assumed constant in each zone. $N_i^{\max}$ is the maximum allowable material number density and $N_i^0(\underline{r})$ is the natural material density at location $\underline{r}$.

2. The constraint on the total volume fraction available

$$\sum_{i=1}^I \frac{N_i(\underline{r})}{N_i^0(\underline{r})} = \text{const} < 1.0 \quad ; \quad (3)$$

3. Preservation of the value of the constraints [denoted by the functional $F_c(N_1, N_2, \dots, N_I)$] imposed on the problem.

The characteristic to be extremized (F_c) can be either of two general categories: weight-type or nucleonic. A weight-type characteristic is any characteristic expressible in the form

$$F_w(N_1, N_2, \dots, N_I) = \sum_{i=1}^I \int d\underline{r} C_{w,i}(\underline{r}) N_i(\underline{r}) \quad , \quad (4)$$

to be referred to as a weight functional. A nucleonic characteristic is one expressible in the form of a bilinear functional:

$$F_b(N_1, N_2, \dots, N_I) = \int d\underline{r} [\langle \phi, s_b^+ \rangle + \langle \phi_b^+, S \rangle - \langle \phi_b^+, H\phi \rangle] \quad , \quad (5)$$

(using the notation $\langle, \rangle$ for $\iint d\underline{E} d\underline{\Omega}$) where the flux $\phi \equiv \phi(\underline{r}, E, \underline{\Omega})$ and adjoint $\phi_b^+ \equiv \phi_b^+(\underline{r}, E, \underline{\Omega})$ are the solution of, respectively, the linear Boltzmann equation and its adjoint

$$\text{and} \quad \left. \begin{aligned} H\phi(\underline{r}, E, \underline{\Omega}) &= S(\underline{r}, E, \underline{\Omega}) \\ H^+\phi_b^+(\underline{r}, E, \underline{\Omega}) &= S_b^+(\underline{r}, E, \underline{\Omega}) \end{aligned} \right\} \quad . \quad (6)$$

Consequently the functional has the value

$$F_b = \int d\underline{r} \langle \phi, S_b^+ \rangle = \int d\underline{r} \langle \phi_b^+, S \rangle \quad , \quad (7)$$

provided the condition $\delta(\underline{r} - \underline{r}^s) \phi_b^+(\underline{r}, E, \underline{\Omega}) = 0$ is satisfied at the system surface boundary $\underline{r}^s$ for the outer directions, $(n \cdot \underline{\Omega}) > 0$. The adjoint source term S_b^+ is to be selected so as to give the adjoint function, and consequently the functional, an adequate physical meaning.

An effectiveness function (EF) may be defined for a bilinear function as:

$$e_{b,i}(\underline{r}) \equiv \left\langle \phi, \frac{\delta S_b^+}{\delta N_i} \right\rangle - \left\langle \phi_b^+, \frac{\delta H}{\delta N_i} \phi \right\rangle \quad , \quad (8)$$

and for a weight function as:

$$e_{w,i}(\underline{r}) \equiv C_{w,i}(\underline{r}) \quad . \quad (9)$$

The substitution effectiveness function (SEF) is defined as follows:

$$Q_i(\underline{r}) \equiv e_i(\underline{r}) - e_I(\underline{r}) \frac{N_I^0(\underline{r})}{N_i^0(\underline{r})} \quad , \quad (10)$$

so that

$$\delta F = \sum_{i=1}^I \int d\underline{r} e_i(\underline{r}) \delta N_i = \sum_{i=1}^{I-1} \int d\underline{r} Q_i(\underline{r}) \delta N_i \quad . \quad (11)$$

Equations (10) and (11) hold for both bilinear and weight functionals.

The material densities for the n th iteration are obtained from the parameters in the $(n-1)$ iteration from the relation:

$$n_i^n(\underline{r}) = N_i^{n-1}(\underline{r}) + A_i^1 Q_{e,i}^{n-1}(\underline{r}) + B_i^n Q_{c,i}^{n-1}(\underline{r}) \quad . \quad (12)$$

The procedures for calculating the A_i^n and B_i^m are described in detail in Ref. [12].

8.4. OPTIMIZATION RESULTS

8.4.1. PRELIMINARY CALCULATIONS

Preliminary calculations were first performed with slab geometry as shown in Fig. 3. In this case, we assumed 60% enrichment of the Li^6 in $LiAlO_2$ at 95% of the theoretical density. Volume fractions were taken as 0.4 for $LiAlO_2$, 0.18 for 9-C, and 0.48 for He. The first shield was assumed to be 95% Fe-1422 and 5% H_2O , while the second shield was assumed to contain 45% Fe-1422, 5% H_2O , and 50% B_4C . Reference calculations were performed using the S16-P3 approximation.

Fig. 3. A 1-D model for preliminary calculations

Several runs were performed with plane and cylindrical geometry options in the code. It was found that the results for both the TBR and various reaction rates are within 0.01%. Because of the cost of optimization procedures, a smaller set of groups was used. A small library of 9 neutron groups and 3 gamma groups was created by collapsing the basic library of 46 neutron groups and 21 gamma groups, using flux weighting from preliminary blanket calculations. It was found that the TBR is within 10%. Moreover, the relative spatial allocation of materials was relatively unaffected.

8.4.2. STAGE I OPTIMIZATION: MAXIMUM TBR

The objective of this phase of the study is to allocate various materials in fixed geometry in order to obtain the maximum tritium breeding ratio. The choice of materials is limited to LiAlO₂, 9-C, He, and Be within the blanket. The composition of the two shields was not changed, however. The arrangement of materials is shown in Fig. 4.

Fig. 4. A 1-D blanket model for maximum TBR

Starting with a uniform distribution of materials in the blanket, the SWAN code calculates the effectiveness of each material at each spatial point. New material distributions are then calculated using the method of steepest descent. No constraint is imposed on the distributions except that the volume fractions must be positive and must add up to unity. The new material distributions are then used to calculate a new value of tritium breeding ratio and to generate new effectiveness functions. This iterative scheme is repeated until convergence is achieved.

First, it was found that the TBR was increased from 0.94 to 1.54, an increase of 64% due to the optimized allocations. Figure 5 indicates that the TBR is very sensitive to the location of Be. The material distributions (Fig. 5) display similar trends as the effectiveness functions. Beryllium is shown to occupy the front zone of the blanket, with small contributions from other materials. Except for the front section, where Be is dominant, all other materials maintain a uniform distribution with the following ratios: (1) Breeder to structure ratio ≈ 4 , (2) Li^6 enrichment of 60%. Nuclear heating, like most other reaction rates, was found to be high in the front and to decrease rapidly with depth. In LiAlO_2 , the heating rate was found to be high (126 W/cm^3).

Fig. 5. Results of optimum material distributions for maximum TBR for He-cooled design

A comparative study was then performed for a water-cooled system. All other materials, as well as the geometry, were adopted from the previous helium case. Figures 6 and 7 show the TBR effectiveness and material allocations, respectively. The effectiveness is shown for the first iteration, while the allocations are given after convergence was achieved (21st iteration). It can be observed that the material distribution is similar to the He-cooled blanket case. However, the Be volume fraction reached a maximum of 1.0, while it was only 0.78 for the He case. Most of the other features of the results are very similar to the helium case. After 21 iterations, the TBR was found to increase from 0.94 to 1.7, an increase of 79%. The heating of LiAlO_2 was found to be exceptionally high in Zone 1 (870 W/cm^3), dropping to 180 W/cm^3 in the second interval. It is to be noted that these high rates are only for few atomic layers and should not be considered indicative of the average value over rod dimensions.

8.4.3. STAGE II: TWO-ZONE CONSTRAINED BLANKET

Even though very high TBR values were obtained in the previous case (1.54 for He and 1.7 for H_2O), the spatial allocation of materials was not subject to geometrical or engineering constraints. We then attempted to impose realistic constraints on the optimization. A two-zone blanket was first subjected to analysis and optimization, where a beryllium zone was introduced behind the first wall. This zone comprised 6 rows of 1.71-cm-thick unclad Be rods (0.3 cm rod spacing). The composition of this zone was not changed during subsequent optimization. The back zone of the blanket was a mixture of LiAlO_2 , 9-C, and He, with a fixed Li^6 enrichment of 60%. The structure to breeder ratio was varied and its effect on the TBR was studied. With a collapsed library of 12 neutron groups, the TBR was found to be reasonably insensitive to this ratio (Fig. 8). Calculations with the full 46-group library resulted in a TBR of 1.42, which is 8% lower than the 12-group results because of spectral differences.

The nuclear heating was found to be high in the front of the second zone, with a cell average value of 65 W/cm^3 . However, averaging over separate materials showed even higher values reaching 98.8 W/cm^3 in LiAlO_2 alone. These high heating rates have led to temperatures well above the 1100°C limit

Fig. 6. Dependence of effectiveness on distance for water-cooled design for the zeroth iteration

Fig. 7. Optimum material distributions for water-cooled design for the 21st iteration

Fig. 8. Dependence of breeding ratio on the structure to breeder volume fraction

set for LiAlO_2 . This, in turn, led to the next and final optimization step of including temperature limits as well as density constraints.

8.4.4. STAGE III OPTIMIZATION: REFERENCE DESIGN

First, nuclear heating and rod temperature limits were considered as drivers for the blanket arrangements. Based upon the results of previous stages a 3-zone arrangement was set:

- Zone 1: 6 rows of bare Be rods (1.7 cm o.d.)
- Zone 2: 5 rows of LiAlO_2 /9-C rods (1.22 cm o.d.)
- Zone 3: 6 rows of LiAlO_2 /3-C rods (3.22 cm o.d.)

The cladding thickness was chosen as 0.1 cm, with a nominal gap thickness of 0.01 cm. The distance between adjacent pins was taken as 0.3 cm. Thermal and mechanical considerations for these choices can be reviewed in Refs. [6] and [7]. The final blanket configuration is shown in Fig. 9.

Fig. 9. Final configuration of optimized reference blanket

Throughout all previous optimization studies, the breeder density was taken as 95%. However, modeling of tritium transport indicated that such a high density may result in unacceptable tritium inventory [13]. Consequently, the density of LiAlO_2 was reduced to 85% of the theoretical density. Furthermore, the total blanket depth was reduced to about 0.5 m, including helium gas manifolds, so that modules can fit either in the out- or in-board sections of the tokamak.

It was found that with all of the previous constraints and the reasonably short blanket modules, the tritium breeding ratio is 1.175.

For a 14.1 MeV neutron incident on the first wall, the integrated values of nuclear heating were as follows:

First wall:	2.45 MeV
Beryllium	5.25 MeV
LiAlO ₂ :	8.85 MeV
9-C cladding:	1.52 MeV
Manifold section:	0.54 MeV
Shield 1:	2.20 MeV
Shield 2:	0.34 MeV

TOTAL: 21.15 MeV

As can be seen, the heat generated in the first wall, Be, LiAlO₂/9-C, and manifold section is 18.61 MeV, yielding an energy multiplication factor of 1.32. The actual volumetric heat distribution in the blanket is shown in Fig. 10. It is to be noted that the maximum values for LiAlO₂ heat generation rate is 179 W/cm³, while that for Be is 35 W/cm³. However, when averaging is performed over rod dimensions, the following maximum values are obtained:

Beryllium:	35 W/cm ³
LiAlO ₂ (1.22 cm o.d.):	127 W/cm ³
LiAlO ₂ (3.22 cm o.d.):	21 W/cm ³

These values were found to result in acceptable temperature distributions within the breeder and Be rods [6].

The distribution of tritium production in the LiAlO₂ throughout the blanket is shown in Fig. 11. It was found that the average tritium production rate is 4.31×10^{-5} wppm/s (1359 wppm/hr). However, 76% is produced in the first breeder zone and only 24% is produced in the back breeder zone. Nearly 99% of produced tritium was in Li⁶ reactions. Tritium was also found to be produced in beryllium, with an average rate of 122.8 appm/y and a maximum value of 204.5 appm/y.

Helium was found to be produced in all blanket materials (Fig. 12). However, the production rate of helium in LiAlO₂ was found to be exceptionally high, reaching a maximum of 85,000 appm/y. Displacement damage rates in the first wall, LiAlO₂ breeder, and the 9-C cladding, are shown in Fig. 13. The maximum first wall value is 64 dpa/y.

Fig. 10. Distribution of volumetric heat generation rate within the reference blanket

Fig. 11. Distribution of tritium production within the reference blanket

Fig. 12. Distribution of helium production within the reference blanket

Fig. 13. Distribution of displacement damage rate within the reference blanket

8.5. CONCLUSIONS AND RECOMMENDATIONS

The present calculations have shown the viability of a pin-type solid breeder blanket for tritium breeding. Engineering materials and geometrical constraints were all considered self-consistently, without a sacrifice of tritium self-sufficiency. Moreover, the final reference blanket is quite thin, with a total depth of 46.3 cm including the manifolds and first wall zones. The amount of breeder material is small, since the solid breeder zone is only 21.8 cm. The reference blanket can therefore be used in in- or out-board configurations. It was also found that all reaction rates display a steep dependence on distance into the blanket. Thus, across the breeder zone only, the tritium production rate decreases by a factor of 25, the heating rate by a factor of 16, and helium production by a factor of 56. As in many other blanket designs, this behavior is a result of the nature of the fusion reaction, producing neutrons that impinge on one side of the blanket. The volume utilization of the blanket is less than optimal, and consequently, the design can be further improved.

REFERENCES

1. B. Badger et al., "UMWAK-II, A Conceptual Tokamak Reactor Design," University of Wisconsin Report UWFDM-112 (1975).
2. J. T. D. Mitchell, "Wall Loading Limitations for a Helium Cooled Blanket Design," Culham Laboratory Report CLM-R126 (1974).
3. D. L. Smith et al., "Blanket Comparison and Selection Study, Final Report," Argonne National Laboratory Report ANL/FPP-84-1 (1984).
4. C. Wong et al., "Helium-Cooled Blanket Design," Fusion Technol., 8, 1 (1985) 114-132.
5. F. Carre, G. Chevereau, F. Gervaise and E. Proust, "Status of Fusion Blanket Evaluation Studies in France," Fusion Technol., 8 (1985) 491.
6. S. Grotz and N. Ghoniem, "Thermal Analysis of a Pin-Type Solid Breeder Blanket for Tokamak Reactors," Chapter 11 this report; also University of California Los Angeles Report ENG-8612/PPG-938 (1986).
7. J. Blanchard and N. Ghoniem, "Flowing of Breeder Pins in a Solid Breeder Fusion Blanket," Chapter 13 this report; also *ibid.* ENG-8605/PPG-931 (1986).

8. G. Orient and N. Ghoniem, "Elastic Structural Analysis of the Pin-Type Solid Breeder Blanket First Wall," Chapter 14 this report; also *ibid.* ENG-0609/PPG-935 (1986).
9. D. S. Gelles and M. L. Hamilton, "Effects of Irradiation on Low Activation Ferritic Alloys," Department of Energy Report DOE/ER-0045/13 (Sept. 1984).
10. W. W. Engle, Jr., "A User's Manual for ANISN, a One-Dimensional Discrete Ordinates Transport Code with Anisotropic Scattering," Oak Ridge National Laboratory Report K-1693 (1967); also Radiation Shielding Information Center Computer Code Collection CCC-248.
11. Y. Gohar and M. Abdou, "MACK-LIB-IV: A Library of Nuclear Response Functions Generated with the MACK-IV Computer Program from ENDF/B-IV," Argonne National Laboratory Report ANL/FFP/TM-106 (1978).
12. E. Greenspan, W. G. Price, Jr. and H. Fishman, "SWAN: A Code for the Analysis and Optimization of Fusion Reactor Nucleonic Characteristics," Plasma Physics Laboratory, Princeton University Report MATT-1008 (1973); also Radiation Shielding Information Center Computer Code Collection CCC-248.
13. R. Martin and N. Ghoniem, "Modeling of Tritium Transport in a Fusion Reactor Pin-Type Solid Breeder Blanket Using the DIFFUSE Code," in Proc. Interntl. Conf. on Fusion Reactor Mater. (Chicago, IL, April 1986); also University of California Los Angeles Report ENG-8626/PPG-966 (1986).

CHAPTER 9

MODELING OF TRITIUM TRANSPORT IN A PIN-TYPE SOLID BREEDER BLANKET

Rodger Martin and Nasr M. Ghoniem

CONTENTS

9.1. INTRODUCTION	9-1
9.2. PREVIOUS EFFORTS IN TRITIUM TRANSPORT MODELING	9-6
9.3. MODELING OF TRITIUM TRANSPORT FOR THE CURRENT BLANKET DESIGN	9-17
9.4. COMPARISON OF DIFFUSE RESULTS TO OTHER MODELS	9-22
9.5. STEADY-STATE TRITIUM INVENTORY	9-24
9.6. TRANSIENT TRITIUM INVENTORY	9-34
9.6.1. Startup	9-34
9.6.2. Shutdown	9-39
9.7. ADDITIONAL DIFFUSE MODELING	9-41
9.7.1. Completely Dense Pins	9-41
9.7.2. Other Inventory Phenomena	9-45
9.7.3. Radiation Trapping	9-47
9.7. SUMMARY AND RECOMMENDATIONS	9-48
REFERENCES	9-50

9.1. INTRODUCTION

This study supplements a larger study of a solid breeder blanket design featuring lithium ceramic pins. This aspect of the study looks at tritium transport, release, and inventory within this blanket design. Li_2O and $\gamma\text{-LiAlO}_2$ are the two primary candidates for ceramic solid breeders. $\gamma\text{-LiAlO}_2$ was chosen for this blanket design due to its higher structural stability. Analysis of tritium behavior in solid breeder blankets is of great importance due to its impact on several critical issues: the generation of an adequate amount of fusion fuel, the safety-related issue of keeping radioactive blanket inventories as low as possible, and the release, purge, and economical processing of the bred tritium without undue contamination of the coolant and other reactor structures.

Tritium inventory is related to both transport (diffusive) and solubility (thermodynamic) properties [1]. Tritium recovery is achieved by passing helium through the purge channels and porosity of the ceramic to sweep out the tritium. Tritium diffusion plays a major role in determining tritium inventory: the slower the diffusion, the longer the residence time within the blanket, and the greater the diffusive inventory. This phenomenon is very temperature sensitive, with the diffusion coefficient generally varying as:

$$D = D_0 \exp (-Q/RT) \quad (1)$$

where Q is the activation energy for diffusion in kcal/mol, R is the ideal gas constant equal to 1.987×10^{-3} kcal/mol/ $^{\circ}\text{K}$, D_0 is a pre-exponential factor, and T is in units of degrees Kelvin. Both D and D_0 are in units of cm^2/sec . Experimental determination of diffusion coefficients is very important in

estimating blanket inventories. Experiments on both Li_2O and LiAlO_2 show wide variations in the experimental coefficients, likely due to both variations in experimental conditions and in the grain sizes used in each experiment.

Figure 1 shows this situation for Li_2O .

For Li_2O , the recent BCSS and FINESSE studies [2,3] have used the diffusion coefficient obtained by Guggi *et al.* [4] because of the well characterized grain sizes used and the controlled experimental conditions:

$$\ln (D/\text{cm}^2\text{s}^{-1}) = -(5.93 \pm 0.48) - (19.52 \pm 1.01 \text{ kcal/mol/RT}) \quad (2)$$

measured between 850 and 1200°K.

LiAlO_2 shows even greater variation in experimental coefficients. Brüning, Guggi, and Ihle [5] determined a value of:

$$\ln (D/\text{cm}^2\text{s}^{-1}) = -5.78 - (19.3 \text{ kcal/mol/RT}) \quad (3)$$

between 878 and 1178°K. However, the TRIO-01 experiment [6] determined a value of:

$$\ln (D/\text{cm}^2\text{s}^{-1}) = (-13.7 \pm 1.8) - (35.8 \pm 3.9 \text{ kcal/mol/RT}) \quad (4)$$

which has been taken as the reference value in the BCSS and FINESSE reports. Choice of D can change the calculated tritium inventory by several orders of magnitude; this mandates an accurate characterization of the coefficient for the relevant blanket conditions.

The solubility effect is most significant for Li_2O , with increased tritium inventory from $\text{HTO}/\text{T}_2\text{O}$ dissolution and LiOT formation:

Fig. 1. Temperature dependence of diffusion coefficients of tritium, oxygen, lithium, hydrogen, and deuterium in Li_2O .

One experiment was reported [1] which measured the solubility of H_2O in Li_2O as a function of temperature from 973 to 1273°K to give an analytical expression for the concentration of dissolved LiOH :

$$\begin{aligned} \log (X_{\text{LiOH}}) = & (0.427 + 1.700 \times 10^{-4} T) \log (P_{\text{H}_2\text{O}}) \\ & + (-17.667 + 2.502 \times 10^{-2} T - 9.62 \times 10^{-6} T^2) \end{aligned} \quad (6)$$

where X_{LiOH} is the mole fraction of LiOH , $P_{\text{H}_2\text{O}}$ is the gas phase partial pressure of H_2O in atmospheres, and the temperature is in degrees Kelvin. The chemical properties of hydrogen and tritium are virtually identical.

Above a threshold pressure of H_2O a separate LiOH/T phase will form, tying up tritium in a stable form difficult to remove from the bulk of the Li_2O ceramic. Experimental values for this critical pressure were determined in the range of 573 to 894°K [7]:

$$\begin{aligned} \log P_{\text{H}_2\text{O}} = & -6718/T + 6.45 , & T < 723^\circ\text{K} \\ \log P_{\text{H}_2\text{O}} = & -4406/T + 3.27 , & T > 723^\circ\text{K} \end{aligned} \quad (7)$$

with 723°K the melting point of LiOH . These results are given in Fig. 2 (from Ref. [8]). Higher temperature allows a higher critical partial pressure of H_2O before onset of LiOH/T phase formation, but it also increases the LiOH/T solubility (eq. 6).

Experimental values of activity coefficient data indicate thermodynamic non-ideality for Li_2O [9]. A positive deviation from ideality was determined indicating a lower LiOH solubility than for an ideal solution, a favorable

Fig. 1. Partial pressure of H_2O above the diphasic Li_2O (solid)- $LiOH$ (solid, liquid) system as a function of temperature.

result for decreased tritium inventory. The thermodynamic data used for ideal solution calculations was taken from the JANAF tables [10].

The greater LiOH solubility at higher temperatures and partial pressures of H₂O has another unfavorable result: greater LiOH vapor pressure outside the Li₂O ceramic causing lithium transport from the ceramic and the structural corrosion problems associated with LiOH.

9.2. PREVIOUS EFFORTS IN TRITIUM TRANSPORT MODELING

One of the earliest attempts to model tritium blanket inventory was the STARFIRE [11] reactor study. The model was relatively crude, using macroscopic variables in a percolation model to approximate T₂O partial pressures within the large pores, and using an assumed average partial pressure of one torr in the LiAlO₂/Al₂O₃ equilibrium to obtain a very approximate value of 13 wppm tritium or 8.1 kg of blanket inventory at 1000°K and 1% lithium burn-up.

Far more refined models were devised for the BCSS and FINESSE studies to approximate blanket inventories. All models begin with the simple analytical solution of tritium diffusion in a spherical ceramic grain. By assuming zero tritium concentration at the grain boundary, a simple equation can be established for the inventory within the grain:

$$I_g = \tau \dot{g} V_g \quad (8)$$

with the mean residence time within the grain given by:

$$\tau = r_g^2 / 15D \quad (9)$$

D is the previously defined diffusion coefficient, r_g the grain radius, $\dot{g}$ the tritium generation rate in grams/sec/cm³, and V_g the grain volume. After applying various summations and correction factors for the temperature and/or generation profiles over unit cells and the blanket as a whole (described below), a value is obtained for the tritium diffusive inventory (I_d) throughout the blanket. Additional inventory can be associated with tritium solubility within the ceramic (I_s), adsorption of tritium on the surfaces of the grains and the larger pellets (I_a), tritium diffusing through the grain boundaries (I_g), and tritium passing out through the purge gas within the porosity/percolation/purge stream convection (I_p). The entire inventory within the breeder can then be represented by:

$$I_b = I_d + I_s + I_a + I_g + I_p . \quad (10)$$

These interrelated physical phenomena can be visualized using the schematic in Fig. 3. If the entire inventory throughout the blanket is considered, then contributions due to contamination of the coolant (I_c) and the metal structure from permeation (I_m) can be added:

$$I_{\text{total}} = I_b + I_c + I_m . \quad (11)$$

Models have been devised to approximate all inventory contributions within the breeder except for grain boundary diffusion. Each model has assumed that, analogous to diffusion within metals, the transport of tritium within the grain boundaries is so rapid relative to the bulk diffusion rates that it could be neglected as a significant contribution to total blanket inventory.

granular structure

traps

grain boundary diffusion

diffusive and solubility inventory

surface recombination effects

spherical grain

Fig. 3. Schematic of tritium inventory phenomena.

From the basic equation for diffusive inventories within a sphere, an equation can be established to represent the diffusive inventory within a characteristic unit cell of the blanket:

$$I_{\text{cell}} = \int_{V_{\text{cell}}} \tau \dot{g} (1 - \epsilon) dV. \quad (12)$$

with ϵ the fraction of porosity within the breeder. The diffusive inventory is calculated by summing over all unit cells.

For plate-type solid breeders, with the length of the plate and the coolant path positioned radially from the plasma, the above equation is converted to:

$$(I_d)_{\text{plate}} = \dot{G}_p \int_{-1/2}^{1/2} h(\xi) \left[\int_0^1 \tau(\eta, \xi) d\eta \right] d\xi \quad (13)$$

with $\dot{G}_p$ the total generation rate for the plate in grams/sec, ξ the normalized position along the coolant path, $h(\xi) = \dot{g}(\xi)/\bar{\dot{g}}$ the normalized tritium production rate along the ξ direction, η the normalized position along the thickness of the plate, and τ defined by eqs. 1 and 9 but with $T = T(\xi, \eta)$. The only published results from this model [12] used the simplifying assumption of both blanket heating rate and tritium generation rate varying with the same exponential dependence along the coolant path, i.e.:

$$\dot{g}(\xi) = \dot{g}_{\text{max}} \exp [-\alpha(\xi + 1/2)] . \quad (14)$$

Different values of α were then used for purposes of comparison. For detailed calculations, this study found equation 14 to be not applicable over the entire blanket. The results of a preliminary neutronics calculation were

plotted in Figs. 4 and 5 to test this approximation. Although the exponential dependence with radial position is true on average, the greater neutron populations just beyond the beryllium zone and reflected by the shield cause significant deviation from the exponential approximation.

Models of cylindrical unit cells employ the simpler equation:

$$(I_d)_{cyl} = f \dot{G}_{cyl} r_g^2 / 15D(\bar{T}) \quad (15)$$

where

$$f = (\beta - 1)^{-1} \int_1^\beta \exp \left[\left(\frac{Q}{RT} \right) \left(\frac{\bar{T}}{T} - 1 \right) \right] d\eta \quad (16)$$

with $\beta = r_o^2/r_i^2$ and $\eta = r^2/r_i^2$ for r_i the unit cell inner radius, r_o the unit cell outer radius, and r and T the local position and temperature at any point between r_i and r_o . This equation requires evaluation of the diffusion coefficient at only one value, the volume-averaged temperature $\bar{T}$, for each unit cell. Equations can be used for T and $\bar{T}$, given T_{max} and T_{min} , or actual values can be used for a known or given temperature profile.

For the solubility inventory, BCSS [13] first makes a calculation for an assumed situation for Li_2O to make a qualitative comparison with I_d ; the result was a solubility inventory of about ten times the diffusive. A more realistic evaluation was then employed by BCSS [14] using temperature profiles over a blanket-averaged breeder pellet and calculated pressure and concentration distributions of T_2O within the breeder pores to obtain a solubility inventory about 3300 times that of the diffusive inventory for a Li_2O blanket.

Estimates of low to negligible tritium solubility inventories for $LiAlO_2$ were thermodynamically evaluated or assumed by several studies: STARFIRE, the GA Technologies Inc. model in BCSS [14], and the TRIO experiment.

Fig. 4. Volumetric heating for (a) first zone; (b) second zone.

Fig. 5. Tritium generation rate for (a) first zone; (b) second zone.

Inventory based on surface adsorption is a big question due to limited experimental data and problems of interpretation of results. The only experiment attempting to interpret surface effects is the TRIO experiment. Thirty-three irradiation runs of γ - LiAlO_2 pellets were made under varying conditions. Those runs with 0.1% hydrogen in the helium purge gas were interpreted to exhibit bulk diffusion as the rate-limiting step; tritium release was assumed to be uninhibited by surface adsorption, and the surface inventory was assumed to be negligible. Any changes in tritium release due to alteration of purge stream composition was attributed to surface effects and a non-negligible surface inventory. The conclusions drawn were $I_s = 3 \pm 2$ wppm for LiAlO_2 blankets with pure helium purge and $I_s = 0$ for 0.1% H_2 within the purge gas. Both ECSS and FINESSE assumed 0.1% H_2 in the purge gas and used the $I_s = 0$ assumption in modeling LiAlO_2 blankets.

The inventory attributed to porosity/percolation/purge stream transport can be approximated by applying mass balance and ideal gas law considerations to an equation from ECSS [15]:

$$P_A = (P_A)_i + \frac{(1/\epsilon - 1) R \bar{T} \dot{g}_A r_i^2 f(\beta)}{4 W_A D_e(\bar{T})} \quad (17)$$

where A is the tritium species, $(P_A)_i$ the partial pressure of A in the purge stream, ϵ the interconnected (as opposed to closed) porosity fraction, $\bar{T}$ the average temperature within the porosity, $\dot{g}_A$ the generation rate of A per unit of 100% dense ceramic volume, W_A the molecular weight of A, $f(\beta) = \ln \beta - \beta + 1$ for maximum P_A or $f(\beta) = (1 - \beta^{-1})^{-1} \ln \beta - 1$ for average P_A , and all else defined as before. These equations assume a cylindrical unit cell with the purge tube located next to the central coolant tube. The ratio of the mean free path for molecular collisions to the average pore radius λ/r_p determines

what equation to use for the calculation of $D_g(\bar{T})$. Different values of $D_g(\bar{T})$ are obtained if the transport of the tritium species within the porosity is dominated by Knudsen diffusion, ordinary molecular diffusion, or some intermediate regime. FINESSE [16] reports an error of $\pm 20\%$ with this model and its assumption of an isothermal unit cell, and calculates a typical rise in tritium partial pressure across a unit cell of less than one pascal.

The GA Technologies model of tritium inventories for the BCSS helium-cooled blanket design is the only one to approximate porous inventories in a blanket design. For all BCSS blanket designs, a value of 0.04 g tritium within the porosity was calculated. This GA model, based on the TRIT4 computer code, is the most comprehensive to date in attempting to simultaneously model the diffusive, solubility, and porous contributions to tritium inventory. However, it still suffers from the weaknesses of the assumptions it employs, and the limited experimental basis upon which those assumptions were made. The assumptions of both the GA and other models which require greater scrutiny are discussed below.

No model has yet been compared to a numerical analysis of inventory over the blanket based on calculated temperature and tritium generation profiles. The only model using radial temperature and generation profiles assumed idealized exponential spatial dependencies identical for both profiles, and then analyzed the results for three different examples of exponential decrease. All other models simply applied average volumetric heating and generation rates to a blanket-averaged unit cell, and scaled up to an approximate tritium inventory. The overall accuracy of this approach has not been evaluated.

All models are based on experimental diffusion coefficients which vary significantly from one experiment to another. This uncertainty is especially

true for LiAlO_2 . For example, the Brining at al. values [5] would substantially reduce the calculated diffusive inventory and remove perhaps the greatest argument against the use of LiAlO_2 with respect to Li_2O .

Only one transport mode has been considered significant in any of the models, that of bulk diffusion and release. All models assume the surface inventory and grain boundary effects to be negligible, and only limited attempts have been made to investigate their roles. A real question remains over the relevance of surface effects. As mentioned previously, surface effects and inventories were assumed significant for LiAlO_2 in a pure helium purge stream in the TRIO experiment.

The simplest possible boundary condition has been used in all models, i.e. zero concentration of all tritium species on the spherical grain surfaces. Any non-zero boundary value assumptions would alter the analytical solutions for the diffusive inventories.

Solubility effects are poorly modeled at present. The GA model made solubility calculations for Li_2O , but it appears an average T_2O partial pressure within the porosity was calculated using equation 17, and the thermodynamic equilibrium equations for LiOH solubility were employed for an overall solubility inventory. Lack of background for the TRIT4 calculations precludes detailed analysis. A model more detailed than average equilibrium calculations may be in order.

Effects due to chemical reactions, particularly oxidation and reduction effects within the breeder and purge stream, are very poorly understood. The large majority of experiments prior to TRIO found T_2O to be the predominant tritium species released from the breeder. However, TRIO determined 94% of the release to be in the form of HT and T_2 . The mass of the released species may affect the calculations for I_p (although this is insignificant compared to

$I_d + I_g$ values), but more important are the problems of tritium permeation through the metal structures and coolant which are far more severe for HT and T₂ than for HTO or T₂O. Any oxidation effects within the breeder itself could have profound effects on tritium inventory and release, as TRIO discovered when 0.2% O₂ was added to the purge stream and release kinetics were disturbed well after the purge was returned to the H₂/He combination. Experimental evidence [17] of globular elemental lithium, a potent reducing agent, has been found after irradiation. Some thermodynamic calculations indicate preferential release of HT or HTO under different oxygen activity regimes [18]. None of these effects has been incorporated into a comprehensive model of tritium inventory.

A major weakness of all inventory calculations is the total lack of any radiation effects models. No firm conclusion of radiation effects can be drawn from the experiments [6,19], most of which were based on small lithium burn-ups relative to blanket lifetimes. Significant radiation trapping and detrapping would have profound effects on tritium release calculations.

No significant modeling of the change of ceramic properties over time has been done, e.g. grain growth, pore closure due to sintering, possible formation of release pathways by internal gas generation, restructuring of the solid and pores over time (as with uranium oxide fuels), effects this restructuring will have on thermal conductivity, temperature, and diffusive profiles, and the effects extended lithium burn-up will have on tritium generation and inventory profiles across the blanket. The discussion of the GA model mentions the use of reduced thermal conductivities for Li₂O and LiAlO₂ under high neutron fluence, but no discussion is made of any time-dependent effects.

Additional modeling is also needed for change of inventory and release profiles under start-up, shutdown, and non-continuous operation. Little work has been done in this area: limited modeling of a few TRIO transients and some calculations of tritium release and inventory fractions over time for one of the BCSS LiAlO₂ blankets.

9.3. MODELING OF TRITIUM TRANSPORT FOR THE CURRENT BLANKET DESIGN

Blanket tritium release and inventory for this pin-type solid breeder design is based on the DIFFUSE code developed by M. Baskes [20,21]. This code provides numerical solutions of the diffusion equation for one or two diffusing species in up to five different materials. Although primarily designed to study hydrogen permeation effects in metals, it can also model tritium generation, inventory, and release in solid breeders by a convenient choice of input parameters.

DIFFUSE calculates the one dimensional diffusion of atoms within a material of specified properties for a variety of initial, source, and boundary conditions. The diffusive equations used are given by Fick's laws:

$$J_j(x,t) = -D_j(T(x,t)) \left[\nabla c_j(x,t) + \frac{c_j(x,t)Q^*}{kT(x,t)^2} \nabla T(x,t) \right] \quad (18)$$

and

$$\partial c_j(x,t)/\partial t = -\nabla J_j(x,t) + G_j(x) - \sum_i \partial c_T^{ij}(x,t)/\partial t \quad (19)$$

where $c_j(x,t)$ is the concentration of the j^{th} diffusing species, $c_T^{ij}(x,t)$ represents the concentration of the j^{th} species in the i^{th} trap, Q^* is the heat of transport, $G_j(x)$ is the implant (generation) source term for the j^{th} species, and all other terms have the standard meanings.

Solutions can be obtained for planar, spherical, and cylindrical geometries, where the spheres and cylinders can be hollow or solid. Trapping of the diffusing species can be modeled by DIFFUSE, a very convenient feature for modeling radiation trapping effects. For the i^{th} type of trap, the governing equations are:

$$\begin{aligned} c_{T^{ij}}(x,t)/\partial t = D_j(T(x,t)) \, c_j(x,t) \, C_{T_e}^i(x,t)/\lambda^2 \\ - c_{T^{ij}}(x,t) \, \nu_0 \exp[-(E_t^i + E_d)/kT(x,t)] \end{aligned} \quad (20)$$

and

$$C_{T_e}^i(x,t) = C_{T_0}^i(x) - \sum_j f^i c_{T^{ij}}(x,t) \quad (21)$$

where $C_{T_e}^i(x,t)$ is the effective concentration of the i^{th} trap, $C_{T_0}^i(x)$ is the actual concentration of the i^{th} trap, λ is the jump distance, ν_0 is the detrapping attempt frequency, E_t^i is the i^{th} trap energy, E_d is the activation energy for diffusion, and f^i is the inverse trap saturability for the i^{th} trap.

Tritium decay to helium can also be modeled as a trap:

$$\partial c_{T^{i1}}(x,t)/\partial t = \gamma c_1(x,t) \quad (22)$$

with γ the tritium decay constant and the resultant helium considered to be the first trap for diffusing tritium.

The source of the diffusing species can be modeled as an implantation, a diatomic gas permeation of surfaces, or an initial concentration. Two traps and/or two implantation profiles are allowed simultaneously. By using both the implantation and initial concentration provisions, homogeneous steady-state tritium generation can be modeled within the material. The

initial conditions for diffusing and trapped species are given by:

$$c_j(x,0) = c_{j,0}(x) \quad (23)$$

and $c_T^{ij}(x,0) = c_{T0}^{ij}(x) . \quad (24)$

Five types of boundary conditions are available in DIFFUSE:

(1) a perfectly reflecting boundary:

$$J_j(x_s, t) = 0 , \quad (25)$$

(2) zero concentration at the boundary:

$$c_j(x_s, t) = 0 , \quad (26)$$

(3) Sievert's law which models bulk solubility due to gas pressure at the surface:

$$c_j(x_s, t) = S[T(x_s, t), P(t)] X_j(t) , \quad (27)$$

with $S(T,P)$ the solubility, $P(t)$ the total pressure, and $X_j(t)$ the mole fraction of the j^{th} species in the gas phase;

(4) recombination of single atoms into a diatomic molecule at the surface prior to release:

$$J_j(x_s, t) = \sum_{j'} K_{jj'} [T(x_s, t)] [c_j(x_s, t) c_{j'}(x_s, t) - X_j(t) X_{j'}(t) S[T(x_s, t), P(t)]^2] \quad (28)$$

where

$$K_{jj'}(T) = \frac{4 \alpha c_1 \exp [(2E_s - E_x)/kT]}{\rho \sqrt{(M_j + M_{j'})} T^1 S_0^2} \quad (29)$$

and

$$E_x = \text{MAX} (E_s + E_d, 0) \quad (30)$$

- with $K_{jj'}(T)$ is the recombination constant for the j^{th} and j'^{th} species, α is a sticking coefficient, c_1 is a kinetic theory constant, E_s is the heat of solution, E_x is the surface barrier energy, ρ is the matrix density, and M_j is the mass of species j ;
- (5) a simple interface between two materials which mandates continuity of flux and chemical potential of the diffusing species:

$$J_j(x_i^-, t) = J_j(x_i^+, t) \quad (31)$$

and

$$\frac{c_j(x_i^-, t)}{S^-(T(x_i, t), P)} = \frac{c_j(x_i^+, t)}{S^+(T(x_i, t), P)} \quad (32)$$

Both diffusivity and solubility of the material are assumed to exhibit Arrhenius behavior, i.e.

$$D_j(T) = D_0 M_j^{-1/2} \exp (-E_d/kT) \quad (33)$$

and

$$S(T, P) = S_0 P^{1/2} \exp (-E_s/kT) \quad (34)$$

with D_0 and S_0 the pre-exponential terms.

Additional features of DIFFUSE are provisions for different temperatures at each interface (and thus a linear temperature profile across each

material), different partial pressures of the diffusing gas at the outer surfaces, and linear heating ramps at the interfaces and surfaces and linear pressure ramps at the surfaces.

The value of applying DIFFUSE to blanket tritium phenomena is its ability to analyze a variety of geometries, boundary conditions, and material parameters under different blanket conditions. This versatility gives both consistency in modeling and a convenient tool to approximate blanket inventories with a minimum of analytical modeling. DIFFUSE can approximate both experimental results and the inventory values of some ECSS and FINESEE models. Good agreement between DIFFUSE and the other models would encourage use of DIFFUSE for calculations of blanket inventory and release. The provisions for surface effects (recombination and solubility), trapping, and a variety of material and boundary conditions in DIFFUSE permit relatively straightforward analysis of these effects, unlike the analytical models presently employed which require a piecemeal approach to model each individual phenomenon.

Another significant advantage of DIFFUSE is the use of results from one time/temperature/generation step as the input to a new step using different input parameters. This feature makes DIFFUSE ideal for analyzing transient behavior in a straightforward and self-consistent manner, e.g. for the analysis of reactor start-up and shutdown effects on tritium behavior. The provision for temperature and pressure ramps over time is another major aid in transient modeling.

9.4. COMPARISON OF DIFFUSE RESULTS TO OTHER MODELS

Only limited numerical results are available in BCSS and FINESSE for given input parameters with which DIFFUSE outputs can be compared. Two analyses are given in FINESSE [22] directly amenable to DIFFUSE analysis. The first is an analysis of a Li_2O plate design, for input parameters of 10 micron grain radius, tritium generation rate of 1.85×10^{-5} wppm/s, 510°C minimum and 795°C maximum temperature, and the diffusion coefficient given by eq. (2). FINESSE applied a value $\alpha = 0$ to eq. (14) and converted eq. (13) to the form:

$$I_d = [r_g^2 \dot{G}/15D_0] f \exp(Q/RT) \quad (35)$$

where

$$f = \int_0^1 \exp[(Q/RT)(T/T - 1)] d\xi \quad (36)$$

For $\alpha = 0$, both temperature and generation profiles are constant across the blanket, making the above equations amenable to a DIFFUSE analysis of a characteristic Li_2O grain for the given parameters.

The above equations were converted to the form:

$$I_d = \frac{r_g^2 \dot{G}}{15 D_0 \exp(-Q/RT)} \exp(-Q/RT) \int_0^1 \exp(Q/RT) d\xi \quad (37)$$

$$\text{for } T = T_{\min} + (T_{\max} - T_{\min})(1 - \xi^2) \quad (38)$$

Note that the first term on the right hand side of eq. (37) is identical to the spherical grain model of eqs. (8) and (9). This term should correspond to the DIFFUSE output, and the following two terms can be evaluated numerically. The average temperature was calculated to be 700°C. FINESSE

reported a result of 2.09×10^{-5} wppm inventory. The DIFFUSE-based evaluation gave an inventory of 2.05×10^{-5} wppm, differing 1.9% from the FINESSE value.

FINESSE also evaluated a LiAlO_2 plate using the same equations, a diffusion coefficient given by eq. (14), and input values of 0.1 micron grain radius, a generation rate of 2.41×10^{-5} wppm/s, and 520°C minimum and 1000°C maximum temperatures. The average temperature was calculated to be 840°C. FINESSE reported a value of 0.0460 wppm inventory. The DIFFUSE-based result gave a value of 0.0440 wppm, a difference of 4.3% from FINESSE.

As another comparison, the same input values for Li_2O and LiAlO_2 were applied to the spherical grain model of eqs. (8) and (9). The DIFFUSE inventory results for a single grain were compared directly to calculated values obtained using:

$$I_d = r_g^2 \dot{G} / [15 D_0 \exp (-Q/RT)] . \quad (39)$$

For Li_2O , eq. (39) gave an inventory of 1.13×10^{-5} wppm while DIFFUSE gave an inventory of 1.11×10^{-5} wppm, a difference of only 1.8%. For LiAlO_2 , eq. (39) gave a value of 1.58×10^{-3} wppm and DIFFUSE gave a result of 1.51×10^{-3} , a difference of 4.4%.

For all comparisons, DIFFUSE results agreed with the models well within 5%. These results are very compatible and support the conclusion that the DIFFUSE code can be used to provide valid results for diffusive tritium inventories within spherical grains comparable to those of other commonly used models.

9.5. STEADY-STATE TRITIUM INVENTORY

Having verified the DIFFUSE results to be comparable to other models, the code was then used to explore the effects of several of the relevant parameters on diffusive inventory. Having chosen LiAlO_2 as the candidate solid breeder, grains of this material were modeled using the diffusion coefficient obtained from TRIO and the arbitrary tritium generation rate of 2.41×10^{-5} wppm/s.

The effects of temperature and grain size on diffusive inventory were analyzed. The results are shown in Fig. 6. The temperature range relevant to this blanket design, approximately 800 to 1400 °K, was used in these figures. Figure 6a indicates substantial inventory build-up as the temperature decreases below 800°K, encouraging design of higher temperature blankets. Figure 6b shows a rapid increase in inventory as the grain sizes become larger; this figure supports the BCSS choice of 0.1 micron grain radii, the same size as chosen for this blanket design.

A parameter relevant to transient effects is the time to steady-state diffusive inventories as a function of temperature and grain size, shown in Fig. 7. This steady-state condition represents a tritium release rate equal to the generation rate. For a 0.1 micron LiAlO_2 grain, Fig. 7a shows time to steady-state varying from a few seconds at high temperatures to days at low temperatures. Figure 7b shows a similar increase in time to steady-state as the grain size increases.

In agreement with the results of early modeling efforts, the only component of ceramic tritium inventory used in this evaluation of steady-state inventory was the diffusive inventory within the grains. The other modes of transport, grain boundary diffusion, solubility and surface effects, and

Fig. 6(a). Diffusive inventory in LiAlO_2 vs temperature.

Fig. 6(b). LiAlO_2 tritium inventory vs grain size (1300K).

Fig. 7(a). $LiAlO_2$ time to steady state vs temperature.

Fig. 7(b). TC time to steady state vs grain size ($T=1300K$).

porosity/purge stream transport were all assumed negligible compared with the magnitude of the diffusive inventory. By considering only diffusive inventory within the grain, we can model the entire blanket inventory by a judicious choice of characteristic grains to represent different portions of each rod. By scaling up the steady-state inventories of these grains, we can obtain approximations for the entire blanket diffusive inventory.

To analyze the effect of diffusion coefficient on steady-state inventory, the two reported values for LiAlO_2 in eqs. (3) and (4) were used as input to DIFFUSE in modeling the diffusive inventory as a function of temperature for a typical grain. A major variation in results is obvious in Fig. 8 across the entire temperature range, up to seven or eight orders of magnitude. Use of the larger diffusion coefficient reported by Brüning et al. would lead to insignificant inventories and remove one of the major advantages claimed for Li_2O over LiAlO_2 as a tritium breeder. However, the TRIO value was obtained from an in-situ reactor experiment and may more accurately reflect actual operating conditions. For this reason, the TRIO value was chosen over the Brüning et al. value for modeling LiAlO_2 in BCSS, FINESSE, and in this study.

The input values for tritium generation rate were obtained from the results of neutronics analysis. The generation rate for each pin was taken to be its axial value in graphs of generation rate (in wppm/s) vs. distance into the module. Asymmetries in generation rate across the pin should largely average out from front to back. The largest error might come in the first row, where the generation profile is very steep. Previous thermal hydraulic analyses provided the average coolant temperature at each row of pins. Application of a standard heat transfer equation to the row by row volumetric heating rates gave the temperature profile within each pin. For steady-state conditions, the maximum central pin temperature approached 1150°C and the

Fig. 8. Effect of diffusion coefficient on inventory.

minimum outer temperature was slightly above 530°C. These relatively high temperatures played a major role in keeping total blanket inventory to a minimum.

Some question exists of the behavior of LiAlO_2 at the maximum temperatures, since this exceeds the 900°C experimental temperatures used in the detailed FUBR-1A experiments with LiAlO_2 [19]. Concerns over grain growth at high temperature can be alleviated by extrapolation of FUBR results: no significant grain growth was observed over the range 500 to 900°C. Likewise, extrapolation of both FUBR results and thermodynamic calculations [18] indicate no significant ceramic vapor phase transport at high temperatures, unlike the case for Li_2O . Another concern might be the effect of high lithium burnup rates in the first row pins: all lithium-6 would likely be converted to tritium over the expected three year blanket lifetime. Although unexpected ceramic properties might become evident, no serious structural transformations can be foreseen. The final ceramic might be closer to Al_2O_3 in composition than to LiAlO_2 . If this severe burnup poses a problem, a lower percentage of lithium-6 enrichment for the first row would reduce both the burnup rate and the steep heat generation profile for this row, but at a cost of a greater parasitic neutron capture by the structural materials in this region of the module, and thus a lower overall tritium breeding ratio.

As mentioned before, the method of characteristic grains is used to model inventory within each rod. As a first approximation, the ceramic portion of each rod was divided into five concentric zones of equal width. Within each zone, the volumetrically-averaged position was determined and the temperature at that position calculated from the temperature profile across the pin. This temperature was used to represent a characteristic grain for each zone. DIFFUSE gave the diffusive steady-state inventory for each of these grains in

$$\bar{T}_{gr} = \frac{2 r_g^2 \dot{G}}{15 D_o (r_o^2 - r_i^2)} \int_{r_i}^{r_o} \exp (Q/RT) r dr . \quad (41)$$

After insertion of the temperature $T=T(r)$ obtained from the calculated profile across the ceramic pin, this equation provided the average grain inventories for each row of pins. A total blanket inventory of 13.2 g tritium was directly obtained from these results, independent of but in very good agreement with the DIFFUSE-based results. Figure 9 gives the total row-by-row inventories obtained with this numerical method, showing the predominance of inventory within the cooler small pins of rows 2 through 5. The inventories of rows 2 through 5 approach 0.6 wppm while rows 6 through 11 have less than 0.02 wppm and row 1 about 0.03 wppm tritium. If inventories are a major concern, this figure suggests tailoring the pin diameters in increasing size away from the plasma, to maintain consistently high ceramic temperatures for each row of pins.

The results of these three methods are plotted, row by row, in Fig. 10. Significant agreement exists between all three methods, further supporting the validity of the DIFFUSE-based inventories. These results compare favorably with the GA model total of 38.3 g for a 522,000 kg $LiAlO_2$ blanket [14] vs. 129,000 kg for this blanket design. But the GA method (using the TRIT4 code) analyzes only one unit cell, with the uncertainties it implies for tritium diffusive inventories. The exact temperatures and other input parameters are never stated, preventing more detailed comparison.

Because of the large number of DIFFUSE runs needed to characterize the total inventory in the above methods (five to six runs per rod, eleven rows of rods over the blanket), a simpler approach was required to model each rod during transient and start-up/shutdown operations. Again, eq. (39) was used

Fig. 9. Total blanket inventory, row by row.

Fig. 10. Comparison of tritium inventory models.

as the starting point, but now an equation for an average temperature representative of the total pin inventory was desired:

$$I(\bar{T}) = \frac{r_g^2 \dot{G}}{15 D_o \exp(-Q/RT)} = \frac{\int_V I_d dV}{\int_V dV} \quad (42)$$

Manipulation of this equation gives:

$$\bar{T} = \frac{Q}{R \ln \left(\frac{2\xi}{r_o^2 - r_i^2} \right)} \quad (43)$$

where

$$\xi = \int_{r_i}^{r_o} \exp(Q/RT) r dr \quad (44)$$

Substitution of the pin temperature profile into eq. (44) allows numerical evaluation of the diffusion-averaged temperature for each row of rods, and a DIFFUSE analysis was made at each of these temperatures for a single characteristic grain representative of an entire rod. The steady-state inventory for each grain was scaled up for the entire rod, then for each row, to give new values of total row-by-row blanket inventory. These results are also shown in Fig. 10, and gave a total blanket inventory of 12.6 g. These results agree very well with the previous methods, with less than 5% variation from the results of the non-DIFFUSE numerical approximation method. Thus, reasonable inventory results can be obtained for an entire rod using only one DIFFUSE-analyzed characteristic grain at a temperature calculated from eq. (43).

This result shows the single grain model to be ideal for transient analyses: by calculating the temperature profile for a rod under different

power levels, an average temperature can be determined for each power level at which to analyze the characteristic grain with DIFFUSE. By assuming step changes in the power levels, the transient behavior of a rod can be simply modeled by a series of consecutive DIFFUSE runs for the same grain at different temperatures and generation rates, with the resultant diffusive concentration profile within the grain from one run used as an initial concentration profile for the next run. DIFFUSE is set up to perform exactly this kind of sequential analysis, thus making transient analyses very convenient.

9.6. TRANSIENT TRITIUM INVENTORY

9.6.1. STARTUP

Reactor startup was modeled for two different situations. The first considered helium coolant flow at 100% of full power status. The second considered significantly reduced helium flow during the transients to achieve higher operating temperatures and lower transient tritium inventories.

The transients modeled were at 10%, 20%, 50%, and 80% of full power, each lasting 3.5 days, followed by 100% power. The percentage of full power was used to scale the volumetric heating rates for ceramic and cladding accordingly, and likewise for tritium generation rate. The rise in coolant temperature from module entrance (constant at 250°C) to each row of pins was taken as a corresponding percentage of the temperature rise at full power. The heat transfer coefficient from cladding to coolant was adjusted accordingly. These values were used to calculate the temperature profiles inside the pin for each row at each transient power level, and these results

used to calculate a characteristic temperature for each rod during each transient, to use as DIFFUSE input as mentioned previously.

With each characteristic grain evaluated by DIFFUSE over several transients, the diffusive inventory can be obtained from the DIFFUSE output at any point during the two weeks prior to full power. After two weeks the input parameters were set at full power values, and the inventory soon returned to its steady-state values. Thus, after eleven DIFFUSE runs, each with several transients and each analyzing a characteristic grain for each row of pins, the results are scaled up for total blanket mass per row. The total transient inventories are obtained as a function of time for each row, and summed for the total blanket inventory.

The transient results involving 100% helium flow gave total blanket inventories of 226 g after 3.5 days at 10% power, 621 g after 3.5 days at 20% power, 429 g at 50% power, 61 g at 80%, and a short time at 100% returned the total inventory to the steady-state value of 12.6 g. These results are plotted in Fig. 11, and the row by row results in Figs. 12 and 13. The inventories are obviously much higher than steady-state during 10%, 20%, and 50% power levels, reaching several hundred grams with the biggest contribution from the first row and its dominant tritium generation rate. The characteristic temperatures ranged from 550 to 590°K at 10% power, 590 to 650°K at 20%, 680 to 830°K at 50%, 780 to 1010°K at 80%, and 840 to 1130°K at full power. The temperatures fall within the regime of very slow tritium diffusion for 10 to 50% power, leading to rapid build-up of the bred tritium within the pins.

These significant inventories could lead to safety-related concerns of large blanket inventory, especially for protracted startup times at these temperatures. For this reason, the same startup regime was re-analyzed under

Fig. 11. Total tritium inventory during startup transients.

Fig. 12. Inventory at end of each transient (rows 1 - 5).

Fig. 13. Inventory at end of each transient (rows 6 - 11).

conditions of restricted helium flow, reducing the rate of cooling and allowing higher temperatures to be achieved within the ceramics. For the 10% and 20% power transients, the helium flow rate was set at 20% of that at full power. Thus, at 20% of full power the same coolant temperature should be obtained for each row as at 100% full power. Likewise, at 50% and 80% of full power the helium flow rate was set at 50% and 80% of full power, again giving the same coolant temperatures. For 10% of full power, with helium flow set at 20% the rise in temperature between blanket inlet and the row in question was taken to be one half that at full power. The volumetric heating rate and tritium generation rate were scaled down from full power according to the transient power level as mentioned previously. New temperature profiles across the pins were recalculated for each transient, characteristic temperatures evaluated for each transient at each row and the transient analyses rerun as before.

Due to considerably higher ceramic temperatures under these conditions of controlled coolant flow, the total blanket inventories were greatly reduced from the uncontrolled flow transients. The results give total inventories of 153 g at 10% after 3.5 days, 40.2 g at 20%, 25.6 g at 50%, and 16.5 g at 80%. These results are also shown in Fig. 11.

The results for these two cases clearly show the need for tailoring the helium flow rate to the transient power levels. An extended startup could lead to massive inventories for uncontrolled conditions. Even with controlled flow, a gradual inventory build-up might be a concern for low power levels over an extended period of testing. In this case, supplemental pre-heating of the coolant would alleviate such concerns. Further study of the minimum practical helium flow rates is also warranted: the minimum of 20% of full

power flow is a conservative estimate, to avoid problems with flow instabilities.

9.6.2. SHUTDOWN

For shutdown, two different cases were analyzed: (1) coolant flow at 20% that for full power and (2) a combination of coolant flow reduction and heating such that the coolant temperature ranged from 470°C to 500°C across the pins. This second scenario can be thought of as "baking" the blanket to speed its purging of tritium inventory. Both scenarios assume afterheat volumetric generation of 1% that of full power and zero tritium generation.

For the first case, that of simply reducing the helium flow, the temperature of the coolant entering the blanket was again 250°C, and the temperature rise at each row of pins was taken to be 1/20 that of full power. These new coolant temperatures, 1% volumetric heating, and a corrected heat transfer coefficient were used to calculate new temperature profiles across the pins as before, and diffusion-averaged temperatures were again calculated for each pin. These temperatures were used as input to DIFFUSE along with zero generation rate as an additional step following the analysis for the three year run at 100% power. Thus, the characteristic grains start with their steady-state inventories, then instantaneous shutdown occurs at which no more tritium is generated but the steady-state concentration profiles within each grain are now subjected to the much lower temperatures at shut-down. As can be expected, tritium diffusion from the grains slows down drastically. The diffusive inventories are modeled for a year after shutdown, with total blanket inventory calculated as before.

Figure 14 shows that, after two weeks, the total inventory is reduced from the steady-state full-power inventory of 12.6 g to 12.4 g, only a 1.6%

Fig. 14. Total tritium inventory after shutdown (no heating).

decrease in total inventory. At the end of six months at these shutdown conditions an inventory of 12.1 g still remains and after one year an inventory of 11.7 g, indicative of the almost negligible rate of diffusion at these cold shutdown temperatures. With all the characteristic temperatures near 540°K, only 7.1% of steady-state inventory is purged after one year.

In comparison, the characteristic grains are then modeled for hot shutdown, with the coolant temperatures mentioned before. Again, new row by row temperature profiles were calculated, average temperatures evaluated from these, and the previous analyses rerun at these higher temperatures. As could be expected, the result was a significantly accelerated tritium purge from the blanket. Figure 15 shows that, after only one week, the total inventory was reduced from 12.6 g to 0.45 g. After two weeks, inventory was reduced to an insignificant 0.02 g. Thus, with characteristic temperatures ranging from 750 to 780°K and no tritium generation after shutdown, tritium purging can be considered complete after only two weeks.

For less than 13 g steady-state inventory, a question could arise whether such baking with the extra expense of heating the coolant is actually needed. However, if the final blanket is not as "clean" as our idealized model would indicate, this analysis shows that baking could rapidly clean out the major portion of the blanket inventory after shutdown. Such auxiliary heating should alleviate inventory concerns in a post-shutdown environment.

9.7. ADDITIONAL DIFFUSE MODELING

9.7.1. COMPLETELY DENSE PINS

DIFFUSE was used to model a hypothetical case for steady-state inventory and release for this rod design, but using a ceramic pin of 100% theoretical

Fig. 15. Total tritium inventory after shutdown.

density rather than 85% as assumed in this study. The purge channel was retained but the tritium was considered to be generated in and diffuse through one giant hollow cylindrical crystal of LiAlO_2 rather than a minute spherical grain. The boundary conditions were taken to be a perfectly reflecting outer surface (i.e. an inner oxide layer on the cladding) and zero concentration on the inside purge channel surface.

These 100% dense pins were analyzed for rows one, five, and eleven. The values for inner and outer surface temperatures were taken from the original 85% dense pins (although detailed analysis would require recalculation of temperature based on the changed thermal conductivity). DIFFUSE assumes a linear temperature profile rather than the actual parabolic design, slightly overestimating inventory and underestimating release. The same tritium generation rates were used as in the original pin design.

For row one, the minimum temperature is 1017°K and the maximum is 1410°K. After three years of steady-state operation, DIFFUSE evaluated the inventory of each pin to be 21.0 g of tritium, or over 350 kg for the first row over the entire blanket, an extremely high inventory. For row five, with temperatures from 809 to 891°K, the inventory for each pin was found to be 1.76 g and over the whole blanket nearly 30 kg. For row eleven, with temperatures from 900 to 1155°K, the single pin inventory was 6.30 g and the total blanket inventory about 40 kg. Obviously, the first row inventory will dominate the total due to the high generation rate.

The complement of high inventory is low release. Over three years, the entire first row would release 6.7 kg into the purge stream, or 1.7% of the total tritium generation. With their colder operating temperatures, rows five and eleven would release only a few hundredth of a percent of their total generation: all of row five would release a mere 1.0 g tritium, and row eleven

only 6.9 g. Except for the relatively hot first row rods, tritium release over three years would be negligible.

The importance of these results is the lack of any significant release for most of the pins over the entire blanket lifetime, raising the possibility of using 100% dense LiAlO_2 pins and batch processing the retained tritium at the end of blanket life. Elimination of the purge channels as superfluous would provide more breeder mass in the blanket and a higher breeding ratio. More significant is an increase of over 17% in breeder mass in using 100% vs 85% density, with its major impact on tritium breeding ratio. With no purge channel, total release from the colder outer zones of the pins through the oxide barrier of the cladding would likely be insignificant. Structural integrity of the ceramic during burn-up would be enhanced, considering the results of the FUBR-1A experiment [19]. Actual permeation losses over lifetime would likely be less than with permeation through the purge system.

The main disadvantage to this concept is of course very high inventories, but this should not affect safety concerns as long as all pins remain intact over lifetime. Retention of tritium the entire three years would lead to enhanced decay to helium prior to extraction, but DIFFUSE indicates total decay to be 8% of all bred tritium, insignificant considering the 17.6% increase in breeder mass and breeding ratio with 100% vs 85% dense LiAlO_2 . A major concern is the presence of any grain boundaries in the 100% dense pin, leading to more rapid diffusion than our ideal single grain cylinder. This problem would be alleviated by retaining the purge channels to handle such diffusion.

Although the inventory levels with these dense pins appears frightening, the structural integrity of the pins over blanket lifetime mitigates the

safety concerns. The advantages obtainable from these dense pins might encourage future consideration.

9.7.2. OTHER INVENTORY PHENOMENA

All previous DIFFUSE analyses assumed a zero tritium concentration on the surface of a spherical grain, analogous to previous modeling efforts. However, TRIO [6] assumed a significant surface inventory under a pure helium purge stream to explain the experimental results. Solubility effects are known to be significant for Li_2O . Concern exists over irradiation effects and the possibility of production of traps for tritium which might significantly increase inventory.

As a very preliminary indication of the versatility of the DIFFUSE code, several analyses were made under different conditions. Using input parameters for LiAlO_2 felt to be reasonable (due to the lack of relevant experimental data), both Sievert's Law (solubility effects) and surface recombination boundary conditions were compared to zero surface concentration over a range of temperatures, all other parameters held constant. In addition, an analysis was made assuming a trap concentration of 1% atomic fraction and a trapping energy of 2 eV. Results are presented in Fig. 16.

For all temperatures below 1100°K, these other phenomena show no effect on total inventory, indicating diffusive inventory to be overwhelming. However, at higher temperatures, measurable additions to diffusive inventory are apparent. Radiation trapping under these conditions is insignificant, but surface recombination becomes significant and solubility very significant as temperature becomes very high. However, with an ambient tritium gas pressure of one torr assumed, these solubility results may overstate actual operating concentrations.

Fig. 16. Tritium inventories, different surface phenomena.

These limited results are not significant without further analysis, but the potential of DIFFUSE for additional analyses into these phenomena is obvious.

9.7.3. RADIATION TRAPPING

Some parametric analysis using DIFFUSE has been done [23] to estimate significant tritium trapping regimes under irradiation, i.e. combinations of temperature, trap concentration, and trapping energy with which significant contributions are made to total inventory. The results are not presented here, but will be briefly summarized.

No experimental determination of tritium trapping energies in lithium ceramic breeders due to irradiation has been made. Trapping energies for helium in metals is known to be 1.5 to 2.5 eV, but the ionic nature of the diffusing triton prevents direct comparison. DIFFUSE analysis of tritium trapping in LiAlO_2 was made at two temperatures over a range of trapping energies from 1.5 eV to 4.5 eV, all other parameters held constant, and an assumed trap concentration of 1 appm. At 773°K, trapped inventory was insignificant, reaching only a few percent of diffusive inventory at a 4.5 eV trap energy. At 1273°K, diffusive inventory is greatly reduced, and trapped inventory equals diffusive at about 4 eV and greatly exceeds diffusive at 4.5 eV.

For an assumed trap energy of 3.5 eV and a temperature of 1273°K, the relative inventories were compared as a function of trap concentration. At 1 appm traps, trapped inventory is insignificant. At a trap concentration of about 10^{-4} atom fraction, the two inventories are equal. At higher trap concentrations, the trapped inventory rapidly increases.

Finally, given a trap concentration of 1% atomic fraction, trapped and diffusive inventories were compared between 700 and 1475°K for trap energies of 2.5 eV and 3.5 eV. For 2.5 eV the trapped inventory is predominant at 750°K, equals diffusive at about 825°K, and rapidly becomes insignificant to that of diffusive at higher temperatures. For 3.5 eV, the trapped inventory is overwhelming at all but the highest temperature, where it is still an order of magnitude above the diffusive inventory.

Lack of experimental analysis of radiation damage effects in LiAlO_2 prevents direct application of DIFFUSE results to blanket design and engineering. These limited results suggest trapping energies approaching 3 eV are significant for high trap concentrations i.e. at high lithium burn-up fraction. For low trap concentrations (1 appm) the trapped inventory becomes significant at higher temperatures, while an inverse temperature effect occurs for 1% atomic fraction of traps. Much more analysis is required to determine the significance of radiation effects on tritium inventory. Hypotheses of tritium trapping mechanisms remain rudimentary at best [24,25].

9.8. SUMMARY AND RECOMMENDATIONS

This section of the pin-type solid breeder blanket study is the first detailed attempt at modeling tritium inventory and release within a fusion reactor blanket based on actual tritium generation and thermal hydraulic profiles, rather than a simple average unit cell extrapolation or some assumed exponential profiles.

The DIFFUSE 83 code was found to give inventory results consistent with previous modeling efforts and with the general spherical grain model.

The inventories for this blanket design calculated using DIFFUSE were found to be very satisfactory, less than 14 g at steady-state for a 129,000 kg LiAlO_2 blanket. This result is reasonable compared with a BCSS LiAlO_2 blanket inventory calculated by GA Technologies.

DIFFUSE was found to be very useful in approximating tritium inventories during transient startup/shutdown modes. The evaluations of transient inventories in this study appear to be the most detailed to date. The results suggest the need for controlling coolant flow during start-up to maintain high breeder temperatures and low tritium inventory, and the use of pre-heated coolant to "bake" out tritium inventory after shut-down.

DIFFUSE modeling of breeder pins of 100% theoretical density indicates very limited tritium release from the LiAlO_2 ceramic, suggesting batch processing of the pins for tritium extraction at the end of blanket lifetime.

Preliminary analysis of other surface and radiative trapping effects shows DIFFUSE to be potentially a very useful tool in approximating and evaluating experimental results. Additional DIFFUSE analysis of these effects given the available experimental data is warranted.

Another logical area for DIFFUSE analysis is tritium permeation of the pin cladding and structure within the blanket. This area will be pursued in the future.

A real need for modeling of tritium behavior in ceramic breeders is a broader experimental data base. Order of magnitude approximations and unknown diffusive and trapping mechanisms for tritium are inappropriate for detailed modeling of tritium behavior and accurate estimates of tritium inventory, permeation, and release within fusion reactor solid breeder blankets.

REFERENCES

1. C. E. Johnson and G. W. Hollenberg, "Recent Advances in the Development of Solid Breeder Blanket Materials," J. Nucl. Mater., 122/123 (1984) 871-881.
2. M. Abdou et al., "Blanket Comparison and Selection Study - Final Report," Argonne National Laboratory Report ANL/FPP-84-1 (1984).
3. M. Abdou et al., "FINESSE: A Study of the Issues, Experiments and Facilities for Fusion Nuclear Technology Research and Development," Interim Report, Vol. 3, PPG-821/UCLA-ENG-84-30 (1984).
4. D. Guggi et al., "Diffusion of Tritium in Single Crystal Li_2O ," J. Nucl. Mater., 118 (1983) 100.
5. D. Brünig, D. Guggi, and H. R. Ihle, "The Diffusivity of Tritium in the System $\text{Li}_2\text{O}-\text{Al}_2\text{O}_3$," in Proc. 12th Symp. on Fusion Technology, (Julich, FRG, 1982), EUR 7983 EN, pp. 543-548.
6. R. G. Clemmer et al., "The TRIO Experiment," Argonne National Laboratory Report ANL-84-55 (1984).
7. M. Tetenbaum and C. E. Johnson, "Partial Pressures of H_2O Above the Diphasic $\text{Li}_2\text{O}(\text{s})-\text{LiOH}(\text{s,l})$ System," J. Nucl. Mater., 126 (1984) 25-29.
8. D. Donne and Dorner, "Some Considerations on Tritium Control in a He-cooled Ceramic Blanket for the NET Reactor," KFK-3755 (1984) p. 39.
9. M. Tetenbaum, A. K. Fischer, and C. E. Johnson, "An Investigation of the Solubility of LiOH in Solid Li_2O ," Fusion Technology, 2 (1985) 53-56.
10. JANAF Thermochemical Tables, 2d ed., D. R. Stull and H. Prophet, project directors, NSRDS-NBS37 (June 1971).
11. C. C. Baker et al., "STARFIRE - A Commercial Tokamak Fusion Power Plant Study," Argonne National Laboratory Report ANL/FPP-80-1 (1980) p. 10-94.

12. M. Abdou et al., "FINESSE," ibid., p. 5-88.
13. M. Abdou et al., "Blanket Comparison and Selection Study," ibid., p. 6.3-103.
14. op. cit., p. 8-147.
15. op. cit., p. 6.3-108.
16. M. Abdou et al., "FINESSE," ibid., p. 5-106.
17. K. Noda et al., "Radiation Damage in Sintered Li_2O Pellets - ESR Study," J. Nucl. Mater., 91 (1980) 234.
18. G. E. Johnson and A. K. Fischer, "Blanket Materials for Fusion Reactors: Comparisons of Thermochemical Performance," J. Nucl. Mater., 130 (1985) 445-453.
19. G.W. Hollenberg, "Fast Neutron Irradiation Results on Li_2O , Li_4SiO_4 , Li_2ZrO_3 , and LiAlO_2 ," J. Nucl. Mater., 122/123, (1984) 896-900.
20. M. I. Baskes, "DIFFUSE 83," SAND-83-8231 (1983).
21. M. I. Baskes et al., "Tritium Permeation Considerations in the MARS Direct Converter," J. Nucl. Mater., 122/123, (1984) 1511-1517.
22. M. Abdou et al., "FINESSE," ibid., p. 5-87.
23. R. Martin, "Effects of Radiation Damage on Tritium Diffusion in Fusion Solid Breeders," unpublished manuscript.
24. J. A. Shearer et al., "Tritium Diffusion in Lithium-Oxide Solid Breeder Materials," Annual Meeting of the American Nuclear Society (Detroit, MI, June 12-27, 1983).
25. M. Abdou et al., "Blanket Comparison and Selection Study," ibid., p. 6.3-73.

CHAPTER 10

MATERIALS DATA BASE AND DESIGN EQUATIONS FOR THE UCLA SOLID BREEDER BLANKET

Shahram Sharafat, Robert Amodeo and Nasr M. Ghoniem

CONTENTS

10.1.	INTRODUCTION	10-1
10.2.	MATERIAL PROPERTIES	10-1
10.3.	PHENOMENOLOGICAL AND EMPIRICAL EQUATIONS	10-3
10.3.1.	Ferritic Steels: HT-9, 2 1/4 Cr - 1 Mo	10-4
10.3.2.	Breeder Materials: Li_2O , LiAlO_2	10-9
10.3.3.	Selected Plots for Material Properties	10-15
10.4.	PHENOMENOLOGICAL SWELLING EQUATIONS FOR SOLID BREEDER AND NEUTRON MULTIPLIER MATERIALS	10-26
10.4.1.	Theory	10-26
10.4.2.	Swelling of Solid Breeder Material	10-29
10.4.3.	Beryllium Swelling	10-34
10.5.	SINTERING PHENOMENA IN BREEDER CERAMICS	10-39
10.5.1.	Introduction	10-39
10.5.2.	The Effect of Porosity	10-40
10.5.3.	Sintering	10-43
	REFERENCES	10-47

10.1. INTRODUCTION

The need for a complete and coherent material data base for fusion reactor systems has been an important issue for some time now. Since the choices for materials used in fusion reactors are becoming more apparent, it is important to be able to quickly access this data to facilitate reactor design. The problem with the current choices of materials is that there is not enough data available to supply complete information on all the important material properties under anticipated fusion operating conditions. In places where there is a deficit in data, it is therefore necessary to extrapolate information to support analysis which covers ranges of undetermined values.

The philosophy of a data base is one of expansion and modification. This will lead to a constantly growing collection of most recently acquired information. Based on this philosophy special care has been given to the structure, the accessibility and ease of modification. The data base is developed primarily for use on Personal Computers (PC's). The PC enhances accessibility due to their widespread use. The final version of the data base will have the format of a tree-structure, which will allow the user to access the information in form of raw data, plots or equations. In many instances the equations are not simple fits to the data but we have developed phenomenological equations based on analytical interpretations.

In Section 10.2. we will list materials and properties investigated for this blanket study. Section 10.3. is a list of phenomenological equations and mathematical fits for all materials and properties considered. Section 10.4. describes our efforts to develop a swelling equation based on the few experimental data points available for breeder materials. In Section 10.5. we investigate the sintering phenomena for ceramics.

10.2. MATERIAL PROPERTIES

The present reactor study investigated the following materials as candidates for the Fusion Power Core (FPC): ferritic steels HT-9, and 2 1/4 Cr-1 Mo, blanket breeder ceramics LiAlO_2 and Li_2O ; and blanket neutron multiplier material beryllium. Li_2O was ruled out as a candidate for solid breeder material due to its high anticipated swelling behavior. Nevertheless we will discuss some of our findings about Li_2O in Section 10.4.

The data on all materials was compiled from various sources, mainly the Materials Handbook for Fusion Energy Systems [1], and recent experimental investigation by Hollenberg [2] and Rasneur [3]. The Materials Handbook includes a comprehensive work on the ferritic steel 2 1/4 Cr - 1 Mo, of which much information has been determined over the last decade. Mechanical properties data and thermal data most relevant reactor blanket designs are listed in Table 1. Some of the data on the ferritic steel HT-9 is also derived from the Materials handbook. Most of the data used for this report, however, comes directly from the Sandvik Steel Co. reports on HT-9 mechanical properties [4].

Other data on the ferritic steels include steady state thermal creep correlations [5]. The expression used for swelling of the ferritic steel 2 1/4 Cr - 1 Mo is from work by Gelles and Puigh [6] on irradiation induced swelling. Since observed swelling is a bilinear phenomenon, this fit to the data produces a smooth transition from one swelling rate to another at the critical value of dose. The expression used for the ferritic steel HT-9 is based on a fit from work by Ghoniem [7]. It does not produce a smooth transition, and therefore the value of swelling should be set to zero below the incubation dose. The correlation used for irradiation creep of HT-9 is also based on an expression previously developed for 2 1/4 Cr - 1 Mo [6].

Information on material properties of lithium aluminate and lithium oxide is derived chiefly from Hollenberg [2] and Rasneur's work [3]. Some of the data displayed in this paper is in raw format, and requires transforming into suitable units for proper use. In addition, much of the information was sparse and scattered, and extrapolation techniques were used to fill in the data. In particular, the phenomenon of swelling and process of sintering of these breeder ceramics have not been measured in much detail. However, correlations were developed from these data and are discussed in section 3.

In order to determine the tritium breeding behavior of LiAlO_2 , Rasneur [3] examined properties of very homogeneous microtextures of porous LiAlO_2 in an in-situ tritium extraction experiment. From a large field of homogeneous microstructures, empirical equations were deduced as a function of porosity, grain diameter and temperature. Mechanical and thermal properties examined include: ultra sound velocity, Young's modulus, ultimate compressive strength, creep and thermal shock. Physico-chemical properties such as

TABLE 1
Summary for Blanket Material* Properties [Current Status (12/85)]

Ferritics	
<u>HT-9</u> Young's Modulus Shear Modulus Poisson's Ratio Ultimate Tensile Strength Instantaneous Thermal Expansion Mean Thermal Expansion Linear Thermal Expansion Steady State Thermal Creep Irradiation Creep	<u>2 1/4 Cr - 1 Mo</u> Young's Modulus Shear Modulus Poisson's Ratio Ultimate Tensile Strength Thermal Expansion Thermal Conductivity Void Swelling Steady State Thermal Creep Irradiation Creep
<u>Lithium Oxide</u> Young's Modulus Shear Modulus Poisson's Ratio Sintered-Thermal Conductivity Sphere Packed-Thermal Conductivity Creep Rate Thermal Expansion Specific Heat Fission Gas Swelling Fracture Strength	<u>Breeder</u> <u>Lithium Aluminate</u> Young's Modulus Shear Modulus Poisson's Ratio Sintered Thermal Conductivity Sphere Packed-Thermal Conductivity Creep Rate Thermal Expansion Specific Heat Fission Gas Swelling Ultra Sound Velocity Ultimate Compressive Strength

* Only the fission gas (He) swelling behavior for beryllium was investigated.

electrical resistivity and sensitivity to air were also investigated. Raseneur found porous LiAlO_2 to be an insulator even up to 600°C . Exposed to air, LiAlO_2 will react with H_2O vapor to form LiOH molecules. LiOH will then react with CO_2 to form Li_2CO_3 compounds. Raseneur's experiments showed that porous lithium aluminate exposed to air will have about 10 monolayers corroded after one year, i.e. less than 1% for an average grain size of $0.4\mu\text{m}$ in diameter. In water however, the reaction on identical samples was total after about only three months.

10.3. PHENOMENOLOGICAL AND EMPIRICAL EQUATIONS

This section lists the data base equations which are the results of our phenomenological investigations and purely empirical fits performed by us or by mentioned authors.

We start with ferritic steels HT-9 and 2 1/4 Cr - 1 Mo and conclude this collection with breeder material properties listed in Table 1.

10.3.1. FERRITIC STEELS

1. HT-9

Young's Modulus YM (T °F) MPa

$$YM = (A_1 + A_2T + A_3T^2) \times 10^3$$

Poisson's Ratio PR (T °F)

$$PR = B_1 + B_2T + B_3T^2$$

Shear Modulus SM (T °F) MPa

$$SM = YM/2(1 + PR)$$

Ultimate Tensile Strength [8] UTS (T K) MPa

$$473 < T < 873$$

$$UTS = C_1 + C_2T + C_3T^2 + C_4T^3$$

Instantaneous Thermal Expansion Coefficient ITE (T °C) $10^{-6}/^{\circ}\text{C}$

$$21 < T < 650$$

$$ITE = D_1 + D_2T + D_3T^2$$

Mean Thermal Expansion Coefficient MTE (T °C) $10^{-6}/^{\circ}\text{C}$

$$37 < T < 650$$

$$MTE = (E_1 + E_2T + E_3T^2 + E_4T^3)/(T - 25)$$

Linear Thermal Expansion LTE (T °C) mm/m

$$21 < T < 650$$

$$LTE = F_1 + F_2T + F_3T^2 + F_4T^3$$

Steady State Thermal Creep Rate [9] TCR (σ ksi, T K) s^{-1}

$$TCR = (B'/kT)(\sigma - \sigma_{10})^3 e^{(-Q1/kT)}$$

$$B' = 7.385 \times 10^{-3}$$

$$\sigma_{10} = aT + C \text{ ksi}$$

$$a = -0.2185 \text{ ksi/K}$$

$$C = 198.1783$$

$$Q1 = 1.23 \text{ eV}$$

Irradiation Creep [9] IC (σ_{eff} ksi, Φ n/cm²-s, t s) %

$$IC = K' \sigma_{eff}^{1.5} \Phi t$$

$$K' = 0.4 \times 10^{-4} (\text{MPa})^{-1} (10^{22} \text{ n/cm}^2) (\%)$$

Void Swelling [9] VS (δ DPA, T °C) %

$$VS = C'(\delta - \delta_0) e^{-((T - 425)/59)^2}$$

$$C' = 0.025 \% / \text{DPA}$$

$$\delta_0 = 90 \text{ DPA}$$

2. 2 1/4 Cr - 1 Mo

Young's Modulus YM (T °F) MPa

$$YM = (G_1 + G_2 T + G_3 T^2 + G_4 T^3) \times G_5$$

Poisson's Ratio PR (T °F)

$$PR = H_1 + H_2 T + H_3 T^2$$

Shear Modulus SM (T °F) MPa

$$SM = YM/2(1 + PR)$$

Ultimate Tensile Strength [8] UTS (T K) MPa

$$473 < T < 873$$

$$UTS = I_1 + I_2 T + I_3 T^2 + I_4 T^3$$

Thermal Expansion Coefficient TE (T °C) $10^{-6}/^{\circ}\text{C}$

$$20 < T < 700$$

$$TE = J_1 + J_2 T + J_3 T^2$$

Thermal Conductivity TC (T °C) W/cm-K

$$25 < T < 700$$

$$TC = K_1 + K_2 T + K_3/T + K_4 T^2$$

Steady State Thermal Creep Rate * TCR (σ ksi, T K) s^{-1}

$$TCR = (A'/kT)(\sigma - \sigma_{20})^4 e^{(-Q2/kT)}$$

$$\ln A' = 17.2898$$

$$\sigma_{20} = aT + C$$

$$a = -0.0628$$

$$C = 51.7929$$

$$Q2 = 2.767 \text{ eV}$$

Irradiation Creep [9] IC (σ_{eff} ksi, Φ n/cm²-s, t s) %

$$IC = K' \sigma_{\text{eff}}^{1.5} \Phi t + D S \sigma_{\text{eff}}$$

$$K' = 0.4 \times 10^{-4} (\text{MPa})^{-1} (10^{22} \text{ n/cm}^2) (\%)$$

D = swelling enhanced creep coefficient

$$= 2.7 \times 10^{-5} (\text{MPa})^{-1} (10^{22} \text{ n/cm}^2)$$

S = fractional swelling rate ($\%/10^{22} \text{ n/cm}^2$)

Void Swelling [9] VS (Φ n/cm², t s) %

$$VS = VS_1 - D$$

$$VS_1 = R \left[\Phi t + \frac{1}{\alpha} \ln \left[\frac{1 - \exp[\alpha(\gamma - \Phi t)]}{1 + \exp(\alpha t)} \right] \right]$$

D = Densification Term

$$= D(1 - e^{(-\lambda \Phi t)})$$

D = steady state density (= -0.08%)

λ = transition parameter (= 3 (10²² n/cm²))

γ = swelling incubation parameter

$$= C_1 \exp(C_2(T - C_3)^2)$$

C_1 = transition parameter

$$C_2 = 2.0 \times 10^{23} \text{ n/cm}^2$$

$$C_3 = 5.0 \times 10^{-5} \text{ (K)}^{-2}$$

α = swelling rate (= 0.5 (10²² n.cm²)⁻¹)

$$R = 0.25\% / 10^{22} \text{ n/cm}^2$$

Table of Constants for Ferritic Steel Equations

1. HT-9

A1: 213.28	B1: 0.2762
A2: -4.799×10^{-2}	B2: 8.9309×10^{-5}
A3: -4.065×10^{-6}	B3: -6.262×10^{-8}
C1: 299.7298	D1: 9.2207
C2: -1.11815	D2: 1.5161×10^{-2}
C3: 1.98127×10^{-3}	D3: -1.0624×10^{-5}
C4: -1.179135×10^{-6}	
E1 = -235.2	F1: -0.23520
E2 = 9.2207	F2: 9.2207×10^{-3}
E3 = 7.5806×10^{-3}	F3: 7.5806×10^{-6}
E4 = -3.5412×10^{-6}	F4: -3.5412×10^{-9}

2. 2 1/4 Cr - 1 Mo

G1 = 31.1	H1 = 0.254
G2 = -1.359×10^{-2}	H2 = 1.54×10^{-4}
G3 = 2.505×10^{-5}	H3 = -1.26×10^{-7}
G4 = -2.007×10^{-8}	
G5 = 6.89475×10^3	
I1 = 152.7886	J1 = 11.281
I2 = -0.5938	J2 = 6.97817×10^{-3}
I3 = 1.43787×10^{-3}	J3 = -3.8036×10^{-6}
I4 = -1.060566×10^{-6}	
K1 = 0.64986	
K2 = -0.2784×10^{-3}	
K3 = -82.462	
K4 = 6407.375	

10.3.2. BREEDER CERAMICS

1. Lithium Oxide (Li₂O)

$$T_m = 1433 \text{ }^{\circ}\text{C}$$

$$\rho_{Li} = 0.93 \text{ g/cm}^3$$

$$k = 3.4 \text{ W/mK}$$

Notations are as follow:

T - temperature

T_m - melting point temperature

ρ - density

P - porosity

A. Young's Modulus (E)

$$E = 140 \exp(-4P) [(1-T)\exp(1-T_m/T)] \text{ GPa}$$

B. Poisson's Ratio (η)

$$\eta = 0.25$$

C. Shear Modulus (G)

$$G = \frac{E}{2(1+\eta)} \quad (\text{GPa})$$

D. Thermal Expansion Coefficient (α)

$$\alpha = a + 1.72 \times 10^{-2} T$$

$$a = 16.05 \text{ for } T \text{ in } ^{\circ}\text{K}$$

$$a = 20.75 \text{ for } T \text{ in } ^{\circ}\text{C}$$

E. Fracture Strength (σ_f)

$$\sigma_f = \sigma_o [1 - 44 \exp(-7000/T)] \quad (\text{MPa})$$

$$\sigma_o = 108 d^{-0.4} \exp(-10P) \quad (\text{MPa})$$

d - grain diameter

T - in $^{\circ}\text{K}$

F. Thermal Conductivity (k)

SPHERE PACKED ($\approx 37\%$ SPF)

$$400^{\circ} \leq T \leq 1100 \text{ }^{\circ}\text{K}$$

- (1) Unirradiated
 - (a) Helium fill-gas pressure = 6 atm.
 $k = 14.220 - 2.076 \ln(T) + 0.0506 (\ln T)^2$ W/mK
 - (b) Helium fill-gas pressure = 2 atm.
 $k = 21.683 - 4.669 \ln(T) + 0.259 (\ln T)^2$ W/mK
 - (c) Helium fill-gas pressure = 1 atm.
 $k = 15.594 - 3.295 \ln(T) + 0.182 (\ln T)^2$ W/mK
- (2) Irradiated ($\Phi t > 0.5 - 2 \times 10^{26}$ n/m²)
 - (a) Helium fill-gas pressure = 6 atm.
 $k = -0.612 + 1.679 \ln(T) - 0.1905 (\ln T)^2$ W/mK
 - (b) Helium fill-gas pressure = 2 atm.
 $k = 8.937 - 1.339 \ln(T) + 0.00387 (\ln T)^2$ W/mK
 - (c) Helium fill-gas pressure = 1 atm.
 $k = 16.035 - 3.681 \ln(T) + 0.223 (\ln T)^2$ W/mK

SINTERED

$$400^\circ \leq T \leq 1100^\circ \text{K}$$

- (1) Unirradiated
 - (a) TD = 100%
 $k = 42.011 - 5.407 \ln(T)$ W/mK
 - (b) TD = 84%
 $k = 72.404 - 16.146 \ln(T) + 0.899 (\ln T)^2$ W/mK
 - (c) TD = 70%
 $k = 49.701 - 11.724 \ln(T) + 0.708 (\ln T)^2$ W/mK
- (2) Irradiated ($\Phi t > 0.5 - 2 \times 10^{26}$ n/m²)
 - (a) TD = 85%
 $k = 45.602 - 11.532 \ln(T) + 0.76 (\ln T)^2$ W/mK
 - (b) TD = 70%
 $k = 29.353 - 7.751 \ln(T) + 0.545 (\ln T)$ W/mK

G. Creep Rate

$$\epsilon = A \exp(-E_c/kT) \text{ (hr}^{-1}\text{)}$$

It is assumed that Li₂O will follow approximately the creep rate of UO₂. Constants for the creep equations have been correlated using melting point temperatures.

$$\begin{aligned}
 A &= 140 \\
 \sigma &= 1.49 \times 10^4 \text{ (kN/m}^2\text{)} \\
 E_c &= 2.03 \text{ (eV)} \\
 &\text{for } 570^\circ \leq T \leq 900^\circ\text{C}
 \end{aligned}$$

H. Specific Heat (cp)

$$\begin{aligned}
 \alpha &= A_1 + A_2 T + A_3 / T^2 \text{ J/kgK} \\
 A_1 &= 2.5179 \times 10^3 \\
 A_2 &= 0.3328 \\
 A_3 &= 8.382 \times 10^{-7} \\
 &\text{for } 306 \leq T \leq 1073 \text{ }^\circ\text{K}
 \end{aligned}$$

I. Swelling ($\Delta V/V$)

$$\frac{\Delta V}{V} = 8.87 \times 10^{-30} \left[\frac{t f G t}{N^{1/3}} \right]^{3/2} \quad (*)$$

$$\begin{aligned}
 f &= \text{fraction of Helium retained} \\
 G &= \text{Helium production rate (cm}^{-3}\text{sec}^{-1}\text{)} \\
 t &= \text{time (sec)} \\
 N &= \text{Bubble number density (cm}^{-3}\text{)}
 \end{aligned}$$

2. Lithium Aluminate

$$\begin{aligned}
 T_m &= 1610 \text{ }^\circ\text{C} \\
 \rho_{Li} &= 0.28 \text{ g/cm}^3 \\
 k &= 1.6 \text{ W/mK}
 \end{aligned}$$

A. Ultra Sound (1MHz) Velocity (c)

$$c = c_0 \left(1 - \frac{P}{0.7} \right) (\text{km}^{-3})$$

$$\text{where: } c_0 = 8.4 \text{ kms}^{-1}$$

$$P = \text{Porosity}$$

Temperature dependence:

c is slowly decreasing by 3 to 4% from $t = 25^\circ\text{C}$ to 700°C .

B. Young's Modulus (E)

$$E = \rho c^2$$

where ρ = apparent density

c = ultra sound velocity

$$E = E_0 (1-P) \left(1 - \frac{P}{0.7}\right)^2 \quad (\text{GPa})$$

$$\text{where } E_0 = \rho_0 c_0^2 \quad (\text{GPa})$$

$$\text{for } \rho_0 = 2.615 \text{ g/cm}^3$$

$$E_0 = 184.5 \quad (\text{GPa})$$

$$E = 184.5(1-P)\left(1 - \frac{P}{0.7}\right)^2 \quad (\text{GPa})$$

C. Poisson's Ratio

$$\eta = \eta_0 / (1 + 0.1P)$$

$$\eta_0 = 0.25$$

D. Ultimate Compressive Strength (UCS)

$$\text{UCS} = \frac{2[\exp(-10P)]}{\sqrt{d}} \quad (\text{GPa})$$

where: d = diameter of LiAlO_2 grain (in microns)

$$\text{For: } d = 0.42 \mu$$

$$P = 0.25$$

the temperature ($^{\circ}\text{K}$) dependence was found to be

$$\text{UCS} = \frac{\exp(-10P)}{\sqrt{d}} \ln \left[\frac{2200}{T} \right] \quad (\text{GPa})$$

valid for $293^{\circ} < T < 1273^{\circ} \text{K}$

E. Thermal Creep

i

Creep experiments were carried out at 700°C under three loads 50, 80 and 100 MPa or samples with 0.25 porosity and 0.42μ grain diameter.

(1) Under 100 MPa

$$\frac{\Delta L}{L} = -0.0135 \ln \frac{t}{72} \quad 200 < t < 1000 \text{ h}$$

- (2) Under 80 MPa

$$\frac{\Delta L}{L} = -0.0045 \ln \frac{t}{29} \quad 300 < t < 2000 \text{ h}$$

- (3) Under 50 MPa

$$\frac{\Delta L}{L} = -0.00143 \ln \frac{t}{150} \quad 300 < t < 2000 \text{ h}$$

Extrapolation of (3) to 20 years leads to a creep of 1% at 700°C.

F. Thermal Conductivity (k)

SPHERE PACKED (= 87% SPF)

$$500^\circ\text{K} \leq T \leq 1200^\circ\text{K}$$

- (1) Unirradiated

- (a) Helium fill-gas pressure = 6 atm.

$$k = 25.670 - 5.518 \ln(T) + 0.297 (\ln T)^2 \quad (\text{W/mK})$$

- (b) Helium fill-gas pressure = 2 atm.

$$k = 8.806 - 1.075 \ln(T) \quad (\text{W/mK})$$

- (c) Helium fill-gas pressure = 1 atm.

$$k = 6.897 - 0.827 \ln(T) \quad (\text{W/mK})$$

- (2) Irradiated ($\Phi t > 0.5 - 2 \times 10^{26} \text{ n/m}^2$)

- (a) Helium fill-gas pressure = 6 atm.

$$k = 8.385 - 0.999 \ln(T) \quad (\text{W/mK})$$

- (b) Helium fill-gas pressure = 2 atm.

$$k = 6.823 - 0.803 \ln(T) \quad (\text{W/mK})$$

- (c) Helium fill-gas pressure = 1 atm.

$$k = 5.387 - 0.619 \ln(T) \quad (\text{W/mK})$$

SINTERED

$$500^\circ \leq T \leq 1200^\circ\text{K}$$

- (1) Unirradiated

- (a) TD = 100%

$$k = 21.01 - 2.65 \ln(T) \quad (\text{W/mK})$$

- (b) TD = 84%

$$k = 14.43 - 1.81 \ln(T) \quad (\text{W/mK})$$

- (c) TD = 70%

$$k = 9.83 - 1.22 \ln(T) \quad (\text{W/mK})$$

(2) Irradiated ($\Phi t > 0.5 - 2 \times 10^{26}$)

(a) TD = 84%

$$k = 11.097 - 1.373 \ln(T) \quad (W/mK)$$

(b) TD = 70%

$$k = 6.45 - 0.77 \ln(T) \quad (W/mK)$$

G. Specific Heat (c_p)

The following measurements are available for a porosity of 0.22 and $d = 0.43 \mu$ grain diameter

Temperature ($^{\circ}C$)	C_p (cal/g $^{\circ}C$)
400	0.330
500	0.348
600	0.365

H. Linear Expansion Factor ($\Delta L/L$)

(T_n $^{\circ}C$)

$$\frac{\Delta L}{L} = 7.43 - 2.95 \ln(T) + 0.495 (\ln T)^2 \quad (\%)$$

I. Swelling ($\Delta V/V$)

$$\frac{\Delta V}{V} = 8.87 \times 10^{-30} \left[\frac{T f G t}{N^{1/3}} \right]^{3/2}$$

where: T = temperature ($^{\circ}K$)

G = Helium production rate ($cm^{-3} sec^{-1}$)

N = Bubble number density (cm^{-3})

f = Fraction of helium retained

10.3.3. SELECTED PLOTS FOR THE FOLLOWING MATERIAL PROPERTIES
(see pages 10-16 through 10-25)

HT-9 and 2 1/4 Cr - 1 Mo:

Young's Modulus

Shear Modulus

Poisson's Ratio

Ultimate Tensile Strength

Li₂O and LiAlO₂:

Thermal Conductivity

Sintered Irradiated 84%, 70% TD

Sintered Unirradiated 100%, 84%, 70% TD

Thermal Expansion Coefficient

Specific Heat

Creep Rate

HT-9 YOUNG'S/SHEAR MODULI

HT-9 POISSON'S RATIO

Ultimate Tensile Strength -- HT-9

2 1/4 Cr - 1 Mo YOUNG'S/SHEAR MODULI

2 1/4 Cr - 1 Mo POISSON'S RATIO

Ultimate Tensile Strength -- 2 1/4 Cr - 1 Mo

THERMAL CONDUCTIVITY LI20 -SINT- -IRRAD-

THERMAL CONDUCTIVITY LI20 - SINT - UNIRR

Thermal Expansion Coefficient - Lithium Oxide

Specific Heat - Lithium Oxide

Lithium Oxide Creep Rate

Thermal Expansion Coefficient - Lithium Aluminate

Specific Heat - Lithium Aluminate

10.4. PHENOMENOLOGICAL SWELLING EQUATIONS FOR SOLID

BREEDER AND NEUTRON MULTIPLIER MATERIALS

10.4.1. THEORY

The behavior of Solid Breeder Materials under irradiation has been the subject of various experiments. Hollenberg [2] conducted experiments on Li_2O , LiAlO_2 and Li_4SiO_4 ceramics in the FUBR-1A experiment in EBR-II. He reported results on helium retention and swelling of these materials after $\approx 1\%$ Li burnup ($\approx 3 \times 10^{20}$ captures/cm³) as a function of irradiation temperature. Figure 1 shows the percent helium retentions in Li_2O and LiAlO_2 .

In Li_2O , helium retention is a strong function of temperature, in LiAlO_2 , however, almost no temperature dependence was detected. Furthermore compared to Li_2O , $\gamma\text{-LiAlO}_2$ has very low helium retention capabilities. The cause of swelling is believed to be helium bubbles forming during irradiation. Thus the different retention rates of helium in Li_2O and $\gamma\text{-LiAlO}_2$ manifest themselves into different swelling rates. Fig. 2 lends some qualitative support for the relationship between helium retentions and swelling. However, the low and athermal helium retention behavior in $\gamma\text{-LiAlO}_2$ are not known at present. Microstructure examination is needed to ascertain the helium bubble morphology in order to explain experimental findings. Since it is believed that swelling has its microstructural origin in helium behavior, we will develop a simple swelling equation based on the following gas behavior assumptions:

- * All retained gas is trapped in bubbles
- * All bubbles are of the same size
- * Van der Waal's Equation of State (EOS) is used
- * Bubbles are in mechanical equilibrium with the solid
- * Re-solution is neglected

The Van der Waal's EOS is most commonly used to describe the thermodynamic state of fission gases in bubbles. It can be written as

$$p \left(\frac{1}{\rho_g} - B \right) = kT \quad (1)$$

where p is the pressure of a gas of molecular density ρ_g at temperature T . The van der Waal's parameter B can be regarded as an expression for the

Fig. 1. Helium retention in the FUBR-1A experiment (after Hollenberg [2])

Fig. 2. Diametrical swelling of Li₂O, Li₃SiO₄, Li₂ZrO₃, and LiAlO₂ after achieving a burnup of 3 x 10²⁰ captures/cm (after Hollenberg [2])

10.4.2. SWELLING OF SOLID BREEDER MATERIAL

Thus we have an expression for swelling as a function of temperature, helium production rate and irradiation time. Note that swelling scales as $t^{1.5}$ which is commonly found experimentally.

To use eq. (8), knowledge about the helium number density is required. The experiment [2] provides us with three swelling data points. Using an approximate value of ≈ 1000 ergs/cm² for bubble surface tension, we find from eq. (8) and the experimental value for swelling and helium retention at 700°C a bubble number density of $N \approx 1.5 \times 10^{-14}$ cm⁻³. Using Eq. 6, we can now estimate the average bubble radius at specified temperatures. Table 2 shows the estimates for N and R using the experimental data points [2].

TABLE 2
Estimates for bubble number densities and radii using above model and experimental swelling data [2] for LiAlO₂

T (°C)	*ΔV/V (%)	R (Å)	N x 10 ¹⁴ (cm ³)
500	0.12	192	0.382
700	0.09	112	1.533
900	0.07	69	5.017

* Experimental data

Assuming a temperature insensitive bubble number density for $500 \leq T \leq 900$ °C we find a slight increase in swelling as a function of temperature, while experimental evidence points to a slight decrease. In Table 3 we compare experimental to analytical results.

TABLE 3
Experimental [2] and Analytical [Eq. 8]
Swelling Results for LiAlO₂ for $500 \leq T \leq 900$ °C

T(°C)	ΔV/V (%)	
	Exp.	Anal.
500	0.12	0.015
700	0.09	0.092
900	0.07	0.122

We can now estimate swelling rates of LiAlO_2 for our blanket conditions. As a demonstration of our model we can use a helium generation rate of about $5 \times 10^{13} \text{ He/cm}^3/\text{sec}$, typical for 5 MW/m^2 neutron wall loadings. Furthermore we assume a bubble number density of about $1.5 \times 10^{14} \text{ cm}^{-3}$ and an athermal helium retention fraction of 3%. Fig. 3 shows the results of our phenomenological equation. Under these conditions the swelling is estimated to $\approx 6.7\%$ at 900°C and 3.3% at 500°C after 3 years of irradiation.

Using the approach outlined above, we have also used Hollenberg's experimental swelling data for Li_2O [2]. Table 4 shows the experimental findings for swelling and helium retention at various temperatures.

TABLE 4
Experimental swelling and helium retention data [2] for Li_2O

T ($^\circ\text{C}$)	$\Delta V/V$ (%)	F_r (%)
500	0.6	60
700	1.8	43
900	1.6	20

The model combined with these experimental data points allows us to estimate the bubble number density (N) and bubble radii (R). Table 5 shows these estimates.

TABLE 5
Estimate for bubble number densities and radii using the above model and experimental data [2] for Li_2O

T ($^\circ\text{C}$)	* $\Delta V/V$ (%)	R ($\text{\AA}$)	$N \times 10^{15} \text{ (cm}^{-3}\text{)}$
500	0.6	61.5	6.19
700	1.8	195	0.58
900	1.6	323	0.12

* Experimental values

Fig. 3: Volumetric Swelling of LiAlO_2 Exposed to $\sim 5 \text{ MW/m}^2$ of Neutron Wall Loading as a Function of Irradiation Time.

We see an increase in bubble radius as a function of temperature for Li_2O , while for LiAlO_2 (Table 2) we note a decrease in bubble size. This is consistent with the swelling results measured by Hollenberg [2]. In LiAlO_2 thermal emission of helium atoms from bubbles is probably the major cause for bubble shrinkage thus leading to a decrease in swelling with increasing temperatures. Our model does not reflect this behavior because of the simplifying assumptions we had to make due to lack of knowledge of helium behavior in ceramics. We are assuming constant size and constant bubble densities in describing the swelling behavior. These assumptions are more justified for Li_2O ceramics since swelling more than doubles over the 500 to 900 °C temperature range. Thus our model's predictions are in better agreement for Li_2O than for LiAlO_2 . Table 6 compares our model result to experimental data.

TABLE 6
Experimental [2] and analytical [eq. 8] swelling
results for Li_2O for $500 < T < 900^\circ\text{C}$

T (°C)	$\Delta V/V$ (%)	
	Exp.	Anal.
500	0.6	0.58
700	1.8	1.65
900	1.6	1.55

Experimental data for Li_2O show a highly temperature sensitive swelling rate and helium retention fraction. Because of this we no longer can assume constant bubble number densities and retention fractions without introducing large errors in the model. Because of this, we have used the values of Table 4 to evaluate swelling rates for Li_2O at the various temperatures. The results of applying our swelling equation (eq. 8) to our reactor conditions are demonstrated in Fig. 4. We find our model to follow experimental trends predicting a slightly higher swelling rate for 700°C than for 500 or 900 °C. For our model to be used at intermediate temperatures between 500 and 700°C, one could use a simple linear extrapolation of the bubble number densities and fraction of helium retained from Tables 4 and 5.

Fig. 4: Volumetric Swelling of Li_2O Exposed to $\approx 5 \text{ MW/m}^2$ of Neutron Wall Loading as a Function of Irradiation Time.

It should be noted, however, that our approach is purely a phenomenological one, which does not necessarily take into account the actual physical behavior of helium atoms in ceramics under irradiation. Our model does not include important parameters such as porosity, grain size, dislocation network or irradiation produced damage. A model describing these phenomena would require a major effort combining experiment with basic analytical research over a period of many years.

10.4.3. BERYLLIUM SWELLING

The swelling equation (eq. 8) can also be applied to beryllium. Beryllium has a large (n,2n) reaction associated with helium production. Furthermore, the 14 MeV neutrons create high FKA energies. Because of these characteristics, swelling is of major concern in beryllium. Effects of helium production and irradiation-induced dimensional changes in Be have been the subject of experimental investigations using fission spectrums by Beeston and co-workers [8,9]. They irradiated Be-specimens in ETR and ATR up to 3.5×10^{22} n/cm² (E > 1 MeV). Beeston's et. al. [8,9] measurements represent low neutron energy, low fluence irradiation conditions. Because the effects of fast neutron, high fluence irradiation are not known at the present time, we have to base our model on Beeston's data. Thus our simple model, although applicable to fission conditions, has to be used cautiously when applied to fusion conditions. As was the case before, we need to know the bubble number densities, the fraction of helium atoms retained and the surface energy of beryllium.

Beeston gives an empirical equation for the number density of helium bubbles for an annealing temperature between 400° and 600°C.

$$N = 1.4 \times 10^{14} \exp(0.45/kT) \text{ cm}^{-3} \quad (9)$$

No measurements for the fraction of helium atoms retained were reported. Thus we will take the conservative approach of trapping all produced helium atoms inside the bubbles. Surface energy measurements quote values ranging from 1 to 2 J/m² [10-12]. Wolfer et. al. [13] have also investigated beryllium swelling behavior. They [13] use a similar simple swelling model and report best agreement between model and data for a surface energy value of 2 J/m². Thus we will use the 2 J/m² value for the surface energy of beryllium in our model. Table 7 compares Beeston's data [8,9] to our model.

TABLE 7

Experimental [10,11] and analytical swelling results for Be irradiated to 50,000 appm at $400^\circ < T < 600^\circ\text{C}$

T ($^\circ\text{C}$)	$\Delta V/V$ (%)	
	Exper.	Analyt.
400	5	4.54
450	9	6.61
500	12	9.24
550	17	12.5
600	20	16.4

We have applied the simple swelling model to the present blanket design. Fig. 5 shows the average pin temperature as a function of distance from the first wall. The maximum Be-pin temperature is $\approx 440^\circ\text{C}$ while the LiAlO_2 hottest pin reaches $\approx 1000^\circ\text{C}$. The six Be-pins are all of the same size. The five front LiAlO_4 pins are roughly half the size of the following six rows of breeder pins. Fig. 6 shows the helium atom production rate in the LiAlO_2 pins as a function of distance from the first wall. The helium atom production rate in the beryllium rods is depicted in Fig. 7. Note that the helium production rate is roughly an order of magnitude larger in Be than it is in the breeder pins.

In Fig. 8 we have shown the volumetric swelling of the LiAlO_2 breeder pin at the end of three years. We have assumed a constant helium atom production rate. Thus not allowing for burnup of lithium atoms. Nevertheless, the maximum swelling occurs in the front row of breeder pins with a top value of $\approx 1.3\%$ after 3 years. This level of swelling would be tolerable for blanket lifetimes of 3 - 4 years.

The high helium-generation rate in Be-rods turns out to be the life limiting phenomena for our blanket design. In Fig. 9 we have depicted the beryllium volumetric swelling at various irradiation times. After 1 year of irradiation the maximum swelling occurring in the first row of Be-rods

Fig. 5: Blanket Pin Average Steady State Temperature as a Function of Distance from the First Wall for a Neutron Wall Loading of $\approx 5 \text{ MW/m}^2$.

Fig. 6. Helium production in LiAlO_2 breeder pins as a function of distance from the first wall

Fig. 7. Average helium production in Be rods as a function of distance from the first wall

Fig. 8. Average volumetric swelling of LiAlO_2 breeder rods exposed to 5 MW/m^2 neutron wall loading after 3 years of irradiation

Fig. 9. Volumetric swelling of Be rods exposed to 5 MW/m^2 neutron wall loading after 1, 2, and 3 years of irradiation

reaches 7.7%. This value increases to unacceptable limits of 20 and 30% after 2 and 3 years respectively. Although these levels of swelling seem to limit the blanket lifetime to about one year, one should note that no allowance was made for Be-burnup during the irradiation time. Furthermore, at high levels of swelling, microstructural evolutions will play an important role. High swelling rates coupled with thermal cycling lead to the formation and propagation of cracks. If bubbles form predominately on grain boundaries, interlinkage of bubbles can lead to gas releases high enough to reach swelling peaks. Another important microstructural evolution is the role of dislocation network formation during irradiation. Dislocations can sweep bubbles from the matrix to grain boundaries, or act as gas tunnelling channels. Both phenomena lead to an increase of gas release from the matrix. Furthermore, dislocation climb is considered a major creep mechanism that can accommodate volumetric swelling. Due to lack of consideration of microstructural phenomena, the high swelling rates for Be-rods cannot be considered accurate. They only point to a potential problem which has to be investigated in great detail experimentally and analytically.

10.5. SINTERING PHENOMENA IN BREEDER CERMANICS

10.5.1. INTRODUCTION

Two configurations for solid breeders have been considered: pressed and sintered pellets and sphere-pac. Cold pressing and sintering techniques were developed for Li_2O , Li_4SiO_4 , and $\gamma\text{-LiAlO}_2$ samples irradiated in ORR [14] and EBR-II irradiation experiments [15].

For LWR and fast reactor fuel rods sphere-pac technology has been developed and is considered attractive for solid-breeders.

The sphere-pac solid breeder configuration could potentially allow for higher thermal conductance due to the elimination of pellet clad gaps. This configuration assumes that internal shifting of the sphere-pac will accommodate expansions or contractions of solid breeder material.

A sound understanding of the factors that govern the temperature distribution within the solid-breeder material is essential to successful prediction of breeder performance over long periods of time and in an intense radiation field.

The temperature distribution, which depends strongly on porosity, influences solid-breeder performance in two important ways:

1. High temperatures: Solid-state reactions that would be immeasurably slow at lower temperatures proceed at rates sufficient to produce significant changes in material properties during the life-time of the breeder in the reactor. Phenomena primarily affected by high operating temperatures include:
 - a. Grain growth
 - b. Densification (sintering)
 - c. Fission-product diffusion
2. Steep temperature gradients: High temperatures can close pores which migrate up a temperature gradient; important constituents of solid-breeder material, such as oxygen, Li, and fission products, are redistributed from their initial concentration profiles (which are usually uniform); thermal stresses resulting from the temperature gradients cause breeder material either to deform plastically in regions of high temperature or to crack in low-temperature zones.

Because porosity determines both temperature and temperature gradients in solid breeders, the next section will outline some effects of porosity.

The following section looks at the possibility of sintering of sphere-pac solid breeder material.

10.5.2. THE EFFECT OF POROSITY

Oxide fuels are generally fabricated by sintering pellets of pressed powdered UO_2 or mixed UO_2 - PuO_2 at high temperatures (typically 1700°C) for a predetermined length of time. By control of the sintering conditions, materials of any desired density between 80 and 90% of theoretical density can be produced.

In the sphere-pac route, high density ceramic spheres of controlled sizes are produced and then subsequently loaded with the assistance of low-energy vibration.

The high density ceramic spheres are produced in three steps:

1. Preparation of a special solution
2. Gelation of droplets to give semi-rigid spheres
3. Drying and sintering these spheres to a high density.

Inasmuch as porosity in a ceramic body invariably reduces its thermal conductivity, it would appear desirable to eliminate all internal pores or voids in the fuel fabrication process.

However, a certain amount of fabricated porosity is useful in accommodating the fission products that accumulate during irradiation; porosity is a means of minimizing fuel swelling. The density of both thermal and fast reactor fuels are well below theoretical value. The porosity of the fast reactor [(U, Pu) O_{1.96}, $\rho \approx 90\%$ TD] is purposefully made greater than that of the thermal reactor fuel [UO_{2+x} $\approx 92\%$ TD], because of the larger burnups required in the former. (Larger burnups are required to enhance breeding-ratios.

Since controlled solid breeder material porosity is a design variable, it is important to be able to predict the effect of porosity on solid-breeder properties, in particular on thermal conductivity and consequently fission gas (³T) diffusivity.

Theoretical analysis of the effect of porosity on thermal conductivity has been hampered by the number of variables that must be considered. One variable that appears in most theoretical models is the volume porosity (P), defined by:

$$P = \frac{\text{volume of pores}}{\text{volume of pores} + \text{volume of solid}}$$

In addition, the geometry and physical properties of the individual pores may also be important. Pore geometry is defined by its size, shape and orientation with respect to the direction of heat flow. Physical properties that may be significant are the emissivity of the solid and the thermal conductivity of the gas trapped within the pore (if any).

In 1954, Loeb [16] treated the thermal conductivity in a manner that permitted many of the secondary variables mentioned in the preceding paragraph to be properly accounted for. In 1966, Biancheria [17] was able to theoretically account for the effects of pore shape.

The well-known Loeb equation describing conductivity of a porous material is:

$$k = k_s(1-P) \quad (9)$$

where k = thermal conductivity of a porous material

k_s = thermal conductivity of a solid material

P = porosity given by

$$P = 1 - \left(\frac{\rho}{\rho_s}\right) \quad (10)$$

where ρ/ρ_s is percentage of theoretical density. Loeb's equation has been found to underestimate the porosity effect on the thermal conductivity of UO_2 . This deficiency has been remedied by inserting an adjustable parameter to yield:

$$k = k_s(1-\alpha P) \quad (11)$$

Equation (11) is called the modified Loeb equation. Values of the parameter α from 2 to 3 have been determined by fitting this equation to UO_2 thermal-conductivity measurements.

Blancheria's [17] analysis of the porosity effect yield the following formula:

$$\frac{k}{k_s} = \frac{1-P}{1+(\alpha-1)P} \quad (12)$$

However, contrary to the purely empirical nature of the parameter α in equation (11), α in equation (12) can be evaluated for equal-sized pores of a particular geometry randomly distributed in the solid. For spheres $\alpha = 1.5$; for axisymmetric shapes such as ellipsoids of revolution, the sphere factor α is greater than 1.5.

In theory, the thermal conductivity of porous ceramics can be determined by using equations 9, 11 or 12. However, extensive measurements of UO_2 thermal conductivity have shown that the theoretical formulas do not agree well enough with experiments to serve as design equations.

This trend will probably hold true for solid-breeder material and purely empirical formulas should be used when reliable numerical values are needed.

Yung Y. Liu, M.C. Billone and R. Clemmer [18] have performed thermal-conductivity measurements on irradiated and unirradiated Li_2O -sintered pellet and sphere-pac $\gamma\text{-LiAlO}_2$, which could be utilized to compare to theoretical values and to produce empirical equations for k_{eff} of Li_2O and $\gamma\text{-LiAlO}_2$.

10.5.3. SINTERING

Lithium oxide pellets in the form of cylinders with 1 in. in diameter by 1 in. in height have been prepared for the TFTR lithium blanket module (LBM) program [19].

To facilitate the release of bred tritium, the density of these pellets is purposely kept well below theoretical densities (TD). Takahashi and Kikuchi [20] have demonstrated that as the density of Li_2O pellets is increased above 86%, the fraction of closed porosity remains essentially constant at 3% to 4% of the pellet volume.

Sphere-pac requires three sizes of spheres to achieve about 88% smear density. If a lower density is acceptable, two sizes will yield about 85% smear density [21].

The contact area between adjacent microsphere plays an important role in the thermal conductivity. Although there is only a ~3% difference in density between ternary and binary mixtures of UO_2 , experiments [22] show that the effective conductivity of the ternary mixture of UO_2 (1.75 W/m-k) is much higher than that of a binary mixture (0.9 W/m-k).

Fuel-gap conductance has also been measured for sphere-pac fuel and pellets of identical fuel composition [23]). These measurements show a gap conductance of $7.3 \text{ kW/m}^2 \text{ }^\circ\text{C}$ for pellet fuel and $19.3 \text{ kW/m}^2 \text{ }^\circ\text{C}$ for sphere-pac fuel. This reduction of contact resistance between the breeder and its clad can significantly change temperature distributions of the breeder material.

From the point of view of thermal conductivity and lithium loading in the blanket, the density of the solid breeder material should be maximized, whereas from the point of view of tritium recoverability porosity, in particular open porosity, should be maximized.

One of the challenges encountered in solid-breeder material design lies in determining this optimum density for the breeder material. Once determined, what will the upper temperature limit be to insure no further densification?

To shed light on some of these issues let us examine some sintering mechanisms which could occur in sphere-pac solid breeder material.

During sintering, an originally porous compact is transformed into a dense ceramic. Most commonly pores become more spherical (i.e., closed) in shape and smaller in size as firing continues. The free energy that gives rise to densification is the decrease in surface area and lowering of the surface free energy by the elimination of solid-vapor interfaces. This usually takes place with the coincidental formation of new, but lower-energy, solid-solid interfaces. Thus restructuring of the contact area between the microsphere causes a change in shape and shrinkage of the pore.

The different mechanisms for material transfer during sintering processes are:

- a. Evaporation and Condensation
- b. Viscous flow
- c. Surface diffusion
- d. Grain-boundary or Lattice diffusion
- e. Plastic deformation.

In treating fission fuel material the two mechanisms shown to be important are: Evaporation-Condensation and Diffusion. In the absence of temperature gradients vapor phase material transfer can occur due to vapor pressure differences as a function of particle curvatures.

For micron-range particle size the vapor pressure has to be larger than 10^{-4} - 10^{-5} atm for evaporation-condensation to be significant.

Vapor pressures reach significant values for Li_2O above 1200°C and for LiAlO_2 above 1400°C , both well above proposed upper temperature limits.

However, if vapor pressures are low, material transfer may occur more rapidly by diffusion. Diffusion is equivalent to the migration of lattice vacancies in the opposite direction. Material can migrate from the grain

boundaries between particle to the neck area between two microspheres resulting in equivalent motion of particle centers toward one another. This results in simultaneous changes in pore size and pore shape and a corresponding decrease in overall porosity.

Kingery [24] has derived an expression for the rate of growth of bond formation (x). His finding, $x \propto t^{1/5}$, are verified experimentally:

$$\frac{x}{r} = \left[\frac{40\gamma D_x \Omega^3}{kT} \right]^{1/5} r^{-3/5} t^{1/5} \quad (13)$$

Using equation (13), Kingery [24] derived an expression for the shrinkage of a compact:

$$\frac{\Delta V}{V} = 3 \left[\frac{20\gamma \Omega^3 D_v}{\sqrt{2} kT} \right]^{2/5} r^{-6/5} t^{2/5} \quad (14)$$

From this expression he concluded that shrinkage is not strongly time dependent [$(\Delta V/V) \propto t^{2.5}$] but size dependent. The sintering rate is roughly proportional to r^{-1} [$(\Delta V/V) \propto r^{-6/5}$]; as the particle size is decreased, the rate of sintering is raised.

The other variable affecting sintering rates is the vacancy diffusion coefficient which is sensitive to temperature and impurities. Because the vacancy diffusion coefficients have not been well established for solid-breeder material use of Kingery's equations can be misleading in determining sintering behavior of γ -LiAlO₂.

Some experimental measurements [25] indicate the onset of sintering in sphere-pac γ -LiAlO₂ around 1200°C.

Great uncertainty remains as to the effect of irradiation on the sintering rate.

The kinetics of the sintering process depends on the location of the pores (which are the vacancy sources) relative to the grain boundaries (which are the vacancy sinks). Porosity close to the grain boundaries is more rapidly eliminated than porosity in the middle of the grain for in the latter case vacancies must diffuse through the bulk.

The rate of shrinkage of a porous solid is governed by the flux of vacancies reaching the grain boundary from the pores.

In Kingery's model, the only means by which vacancies left a pore and entered the adjacent solid was by diffusion.

In solid-breeder material exposed to radiation, however, vacancies can be ejected into the solid by the disruptive action of primary-knock on-atoms (PKA's) passing through or near a pore. This is a type of re-resolution process generally associated with fission-gas bubbles. Although a pore can be thought of as consisting of $(4/3)\pi R_p^3/\Omega$ vacancies, the vacancies are not ponderable particles that can acquire kinetic energy by collision with high-speed particles (e.g., a fission fragment or a lattice knock-on). Therefore, pore shrinkage by fission fragments or PKA's must be described by a form of macroscopic re-resolution, in which a passing fragment blasts solid from one side of a pore to the other, trapping some vacancies in the deposited side in the process. The buried vacancies are considered to be redissolved and to acquire the mobility of single vacancies in the lattice.

Because the pores in ceramic fuels and thus in solid-breeder material are considerably larger than fission-gas bubbles, it is unlikely that a pore can be completely converted to vacancies by a single re-resolution event. Hence re-resolution is considered to reduce the size of the pores but not to change the total pore concentration. In sphere-pac solid breeder material we have highly dense microspheres with the porosity concentrated in large volumes between the spheres. In this case it is even more unlikely that re-resolution will have any effect on the shape or size of the pores and consequently on the sintering rate.

The other irradiation effect that could be considered is the substantial vacancy production rate due to collision cascades and fission spikes. In densification models neither of the vacancy sinks in the solid (grain boundaries) exhibits a preference for interstitials or vacancies; so an excess of one type of point defect in the solid cannot be built up. Only dislocations are biased sinks, and they are not considered. If they were, swelling rather than shrinkage would probably occur, for this is the prime effect of the presence of dislocation sinks in void swelling in metals.

The above discussion shows that unrealistic assumption concerning the distribution of the pores, their initial size, the neglect of grain growth

concurrent with densification, and the effects of radiation on re-solution and pore shape have to be made in order to model sintering of solid-breeder material.

Therefore, we have to rely on experiments to set guide-lines in the development of solid-breeder material.

REFERENCES

1. Materials Handbook for Fusion Energy Systems, DOE/TIC-10122, John W. Davis, Coordinator (April 1980).
2. G. W. Hollenberg, "Fast Neutron Irradiation Results on Li_2O , Li_4SiO_4 , Li_2ZrO_3 and LiAlO_2 ," J. Nucl. Mater., 122&123 (1984) 896-900.
3. B. Rasneur, "Tritium Breeding Material: $\gamma\text{-LiAlO}_2$," Fusion Technol., 8 (July 1985).
4. R. J. Amodeo and N.M. Ghoniem, J. Nucl. Mater., 122, 1-3 (May 1984) 91.
5. R. J. Amodeo and N. M. Ghoniem, "Development of Design Equations for Ferritic Alloys in Fusion Reactors," NED/Fusion, 2 (1985) 97-110.
6. D. S. Gelles and R. J. Puigh, "Alloy Development for Radiation Performance," Semi-Annual Progress Report DOE/ER/0045/11 (Sept. 1983).
7. N. M. Ghoniem and R. W. Conn, "Assessment of Ferritic Steels for Steady State Fusion Reactors," International Atomic Energy Agency, in Proc. on Fusion Reactor Design and Technology, IAEA-TC-392/62, Vol. II (1983) p. 389.
8. J. M. Beeston, M. R. Martin, C. R. Brickman, G. E. Korth and W.C. Francis, in Symp. on Materials Performance in Operating Nuclear Systems, Ames Laboratory, Ames, Iowa, August 1978, Nucl. Metall., 19, CONF-730801, pp. 59-87.
9. J. M. Beeston et al., "Comparison of Compression Properties and Swelling of Beryllium Irradiated at Various Temperatures," Idaho National Engineering Laboratory Report EGG-FT-6608 (1984).
10. R. S. Barnes and G. B. Redding, Nucl. Energy, 10 (1959) 22.
11. J. B. Rich, G. B. Redding and R. S. Barnes, J. Nucl. Mater., 1 (1959) 73.
12. L. E. Muir, Interfacial Phenomena in Metals and Alloys, (Addison-Wesley Publ. Corp., Reading, MA, 1975) p. 124.

13. W. G. Wolfer and T. J. McGarville, "Swelling of Beryllium," DAFS, DOE/ER-0046/19 (Nov. 1984).
14. L. Yang, R. Medico, W. Baugh and K. Schultz "Irradiation Study of Lithium Compound Samples for Tritium Breeding Application," J. Nucl. Mater., 103 (1981) 585.
15. Program Review Documents of the Task Force on Material and Irradiation Effects for Solid Breeders During 1980-1983, C.E. Johnson, Coordinator, Argonne National Laboratory.
16. A. L. Loeb, J. Amer. Ceram. Soc., 37 (1954) 96.
17. A. Biancheria, Trans. Amer. Nucl. Soc., 9 (1966) 15.
18. Y. Y. Loeb, M. C. Billone and R. Clemmer, "Solid Breeder Tritium Recovery," Blanket Comparison and Selection Study, Argonne National Laboratory Report ANL-FPP-84-1 (1984).
19. "TFTR Lithium Blanket Module (LBM) Program, Preliminary Design Report," GA Technologies Report GA-G16616 (December 1981).
20. T. Takahashi and T. Kikuchi, JAERI-M 7518 (1978).
21. "Gel-Spere-Pac Fuel for Thermal Reactor -- Assessment of Fabrication Technology and Irradiation Performance," Oak Ridge National Laboratory Report ORNL-5469 (November 1979).
22. M. J. Ades, "Thermal Conductivity of Sphere-Pac Fuel," Exxon Nuclear Company Report DOE/ET/34215 (July 1981).
23. "Fast Breeder Reactor Oxide Fuels Development -- Final Report," A. L. Lotts, Comp., Oak Ridge National Laboratory Report ORNL-49-1 (November 1973).
24. W. D. Kingery, Introduction to Ceramics, (Wiley, New York, 1967).
25. G. W. Hollenberg, Private Conversation.

CHAPTER 11

THERMAL RESPONSE OF A PIN-TYPE FUSION REACTOR BLANKET DURING STEADY AND TRANSIENT REACTOR OPERATION

Steven P. Grotz and Nasr M. Ghoniem

CONTENTS

11.1. INTRODUCTION	11-1
11.2. STEADY-STATE THERMAL ANALYSIS	11-4
11.2.1. First Wall	11-4
11.2.2. Breeder and Neutron Multiplier	11-7
11.2.3. Fractional Power Operation	11-17
11.3. TRANSIENT THERMAL ANALYSIS	11-22
11.3.1. Power Ramps	11-22
11.3.1.1. First Wall Response	11-22
11.3.1.2. Breeder Rod Response	11-23
11.3.2. Primary Loop Preheat	11-23
11.4. SUMMARY	11-29
NOMENCLATURE	11-30
REFERENCES	11-31

11.1. INTRODUCTION

The thermal analysis of the blanket examines both the steady-state and transient reactor operations. The steady-state analysis covers full power and fractional power operation whereas the transient analysis examines the effects of power ramps and blanket preheat. The blanket configuration chosen for this study is a helium cooled solid breeder design. The blanket design is summarized below. In this paper we first discuss the full power, steady-state temperature fields in the first wall, beryllium rods, and breeder rods. Next we examine the effects of fractional power on coolant flow and temperature field distributions. This includes power plateaus of 10%, 20%, 50%, 80%, and 100% of full power. Also examined are the restrictions on the rates of power ramping between plateaus. Finally we discuss the power and time requirements for pre-heating the primary loop from cold iron conditions up to startup temperature (250°C).

The results of the thermal analysis have been used in the structural analysis of the first wall and the beryllium and LiAlO_2 rods [1,2] and the tritium inventory calculations [3].

REFERENCE BLANKET DESIGN

The reference blanket design chosen for this study is a helium cooled solid breeder concept. The breeder material is lithium aluminate (LiAlO_2) which is in the form of metal clad rods. The cladding material is 9-C, a low activation ferritic steel similar to HT-9. A neutron multiplier zone of bare beryllium rods is included to enhance the tritium breeding. The blanket is divided toroidally into 24 sectors each consisting of 45 modules. The sector configuration is shown in Fig. 1 and a single module cross section is shown in Fig. 2.

The coolant flow path starts in the module shell and first wall and then enters the blanket. It then cools the neutron multiplier zone followed by the tritium breeding zone. Notice that the breeding section is divided into two zones. The larger rods, toward the back of the blanket are sized to operate near $T_{\text{MAX}} = 1100^\circ\text{C}$. This reduces the diffusive tritium inventory. Selection of the pin sizes is discussed in Sect. 11.2.2. Table I summarizes the major

Fig. 1. One of 24 toroidal blanket sectors [5]

Fig. 2. Cross section of a single blanket module

TABLE I
REACTOR AND BLANKET MODULE OPERATING CONDITIONS

Reactor type	Continuous burn d-t tokamak with a helium cooled solid LiAlO_2 blanket
Major radius	5.7 m
Minor radius	1.56 m
Elongation	1.6
Number of toroidal sectors	24
Number of blanket modules/sector	45 (full poloidal coverage)
Blanket module type	Helium cooled, pressurized module of 9-C structure (low activation)
Module poloidal height	0.28 m
Module mean toroidal length	1.50 m
Fusion power	2850 MW
Thermal power	3463 MW
First wall 14 MeV neutron flux	5 MW/m^2
First wall surface heat flux	1 MW/m^2
Coolant pressure	5 MPa
Coolant inlet temperature	250°C
Coolant outlet temperature	550°C
Coolant mass flow rate	$2.21 \times 10^3 \text{ kg/s}$
Thermal efficiency, gross	35%
Electric output, gross	1034 MW

operating parameters of the blanket design. A more detailed description of this blanket design can be found in Ref. [4].

11.2. STEADY-STATE THERMAL ANALYSIS

11.2.1. FIRST WALL

The general configuration of the helium cooled first wall is shown in Fig. 3. This design is similar to the tokamak first wall design chosen for the Blanket Comparison and Selection Study (BCSS) [5]. Typical first wall dimensions are also shown on Fig. 3. The thermal analysis was an iterative process in which work was done in parallel with the first-wall stress analysis (Ref. [1]). Optimization of the first wall involved several design variables including peak and average temperature, coolant pressure and pressure drop, peak stress, and first wall lifetime.

First wall optimization includes the effects of two additional configuration changes. First we examined the effects of cooling fins inside the coolant channel. This concept is illustrated in Fig. 3. The other design change is the use of 3-D surface roughening. Both of these first wall alterations affect the temperature fields and the coolant pressure drop.

Numerical methods were used to calculate the 2-D temperature fields. The finite element heat conduction code, TACO2D [6], was used for the analysis.

The applied heat loads include a surface heat flux of 1 MW/m^2 and an average volumetric heat generation rate of 60 MW/m^2 . The latter value has been calculated from the neutronics analysis [4] while the former is based upon the BCSS guidelines for a neutron wall load of 5 MW/m^2 .

The equation for the turbulent heat transfer coefficient [Eq. (1)] is obtained from Ref. [7]. (NOTE: The nomenclature is summarized at the end of the paper.)

$$h = 0.022 \frac{k}{D_H} \text{Re}^{0.8} \text{Pr}^{0.4} \left(\frac{T_w}{T_f} \right)^{-0.18} \quad (1)$$

The temperature effects on Prandtl number are small over the film temperature drop, therefore:

Fig. 3. Helium-cooled first wall configuration (shown are the three cases studied: no fins and one or two internal fins)

$$\text{Pr}^{0.4} \text{ (for helium at } 250^\circ\text{C)} = 0.85 \quad . \quad (2)$$

A further simplification of Eq. (1) can be made in the temperature ratio term. Preliminary analysis has shown that the film temperature drop, $\Delta T_f = T_w - T_f$, on the plasma side of the coolant channel is such that $80^\circ\text{C} < \Delta T_f < 320^\circ\text{C}$. With the bulk coolant temperature, $T_f = 250^\circ\text{C}$ (513 K), we see the following range of the temperature ratio.

$$0.92 < \left(\frac{T_w}{T_f}\right)^{-0.18} < 0.97 \quad . \quad (3)$$

In this analysis an average temperature ratio of 0.95 is chosen, thus simplifying Eq. (1) to the following:

$$h = 0.018(k/D_H)\text{Re}^{0.8} \quad . \quad (4)$$

In addition to Eq. (4), which applies to smooth wall tubes, we examined the effectiveness of wall roughness. Specifically, we chose 3-D roughening which approximately triples the heat transfer coefficient [7]. The penalty for this enhanced heat transfer is a seven-fold increase in the friction factor. To minimize the coolant pressure drop due to roughness, we only roughen the side of the coolant channel nearest the plasma since this surface has the highest heat flux entering the coolant.

The other critical thermal-hydraulic parameter is the coolant pressure drop. In general, the pressure drop is given by Eq. (5).

$$\Delta P = (L/D_H) f_g (\rho v^2/2) \quad , \quad (5)$$

The smooth wall friction factor, f_g , is given by [8]:

$$f_g = 1/[2 \log_{10}(\text{Re}/\sqrt{f_g}) - 0.3]^2 \quad . \quad (6)$$

For the cases involving surface roughening, the average friction factor, $\bar{f}$, is estimated using

$$\bar{f} = 7f_s \left(\frac{A_R}{A_W} \right) + f_s \left(\frac{A_W - A_R}{A_W} \right) \quad (7)$$

Equation (7) states that the average friction factor is equal to the sum of the local friction factors weighted by their area fraction.

The following first wall variations have been examined:

1. No fins, smooth walls,
2. No fins, roughness on front surface,
3. One fin, smooth walls,
4. One fin, roughness on front surface,
5. Two fins, smooth walls,
6. Two fins, roughness on front surface.

The choice of Case 1 allows simple comparison to the BCSS studies. Cases 2 through 6 are included as heat transfer enhancement methods. The desire is to decrease the peak and average first wall temperature without sacrificing first wall thickness. We also investigated the effect of fin length on the maximum and average temperatures and the coolant pressure drop.

Table II summarizes the results from Cases 1 to 6. Temperature contours for Cases 2, 4, and 6 are shown in Figs. 4a, 4b, and 4c, respectively. The fin length variation and its effect on maximum and average temperature and pressure drop are shown in Fig. 5.

The maximum surface temperature of a simple first wall channel can be reduced from 827° to 605°C with slight modification of the geometry. On the high heat flux side of the coolant channel, 3-D surface roughening greatly reduces the film temperature drop thus leading to the lower structure temperature. The addition of internal fins has lesser impact on structure temperature. An optimum choice appears to be Case 4, one fin with roughening. The structure temperature is 621°C, and the required first wall pumping power is 6 MW(e). This is not unreasonable for a 1000 MW(e) gas-cooled power plant.

11.2.2. BREEDER AND NEUTRON MULTIPLIER

The flow geometry in the breeder and neutron multiplier zones is cross flow of helium around rod bundles. Because the pins are grouped into three zones of different outside diameters, an average heat transfer coefficient for

TABLE II
SUMMARY OF FIRST WALL PARAMETRIC ANALYSIS

Case No.	Description	$T_{MAX}(^{\circ}C)$	$V_{He}(m/s)$	$\Delta p(MPa)$	$P_{pump}[MW(e)]$
1	Simple Channel, Smooth	827	76	0.0023	1.7
2	Simple Channel, Rough	676	76	0.0049	3.6
3	One Internal Fin, Smooth	703	91	0.0037	2.7
4	One Internal Fin, Rough	621	91	0.0080	5.8
5	Two Internal Fins, Smooth	651	127	0.0255	18.9
6	Two Internal Fins, Rough	605	127	0.0548	40.6

Fig. 4. First wall temperature contours for the case with (a) no internal fin, (b) one internal fin, and (c) two internal fins. All three cases had surface roughening on the plasma side of the coolant channel

Fig. 5. The effect of internal fin length on maximum first wall temperature and helium pumping power (first wall only)

each zone is required. The average heat transfer coefficient for cross flow in rod bundles is given by [9]

$$\bar{h} = 0.33 CF(k/D)(G_m D/\mu)^{0.6} Pr^{0.3} \quad (8)$$

Scaling factor C is an empirical coefficient that does not vary significantly from unity. F is a factor that depends on the number of rods in each zone, as shown in Table III. In our design, F is always between 0.95 and 0.92 for each zone, individually. Since each zone is in close proximity to the others, we will assume that the number of rows of pins that affect the local flow paths is large and therefore F is approximately unity.

TABLE III
SCALING FACTOR, F,
FOR AVERAGE HEAT TRANSFER COEFFICIENT FOR FLOW ACROSS ROD BUNDLES

No. of rows	1	2	3	4	5	6	7	8	9	10
F	0.68	0.75	0.83	0.89	0.92	0.95	0.97	0.98	0.99	1

(Triangular lattice)

The rod lattice for each zone is a triangular pitch. The layout is shown in Fig. 6 with the various dimensions for each of the three zones tabulated in Table IV.

TABLE IV
SUMMARY OF PHYSICAL LAYOUT OF THE
BERYLLIUM AND BOTH $LiAlO_2$ ZONES

Composition	$d^{(1)}$	$s^{(1)}$	% Struc ⁽²⁾	Void	Re
Beryllium	1.7	0.3	66	34	26,900
$LiAlO_2/9-C$	1.22	0.2	67	33	16,100
$LiAlO_2/9-C$	3.00	0.3	75	25	67,700

(1) Centimeters

(2) Breeder + cladding

Fig. 6. Rod lattice used in the beryllium zone and both LiAlO_2 zones
(see Table IV for specific dimensions)

An effect that is only encountered in the breeder pins is multi-layer heat flow. The pins are similar to fission fuel rods with the breeder, LiAlO_2 , surrounded by a helium filled gap and a 9-C cladding. One large uncertainty of this configuration is the conductance of heat across the helium gap. Ideally, we would like no gap. However, assembly of the rods requires a small clearance gap to allow insertion of the breeder into the cladding. An initial gap of 0.1 mm was assumed for this analysis. Based on the ECSS analysis [5] we find that the gap heat flow in our design is governed by the gas phase conductance. This results from the gap size being much larger than the surface roughness. Typical surface roughness is 6×10^{-6} m while the gap is 10^{-4} m. Therefore, in the thermal analysis we model the breeder pin as three concentric cylinders of LiAlO_2 , He, and 9-C.

The first analysis performed was a 1-D heat flow in concentric, volumetrically heated cylinders. The breeder temperature as a function of radius is given by

$$T(r) = T_f + \frac{Q_{Li}''' r_o^2 + Q_c''' (r_{co}^2 - r_{cl}^2)}{2r_{co}} \left(\frac{t_c}{k_c} + \frac{1}{h} \right) + \frac{Q_{Li}''' r_o t_g}{2k_g} + \frac{Q_{Li}'''}{4k_{Li}} (r_o^2 - r^2) \quad (9)$$

Results of the 1-D analysis are illustrated in Figs. 7a and 7b. Figure 7a presents $T(r)$ for the inner and outer pins of breeder zone 1 (rows 1 and 5) and Fig. 7b presents $T(r)$ for the same pins of breeder zone 2 (rows 6 and 11). Equation (9) has also been used to calculate the temperature profiles within the beryllium rods. The maximum temperature and temperature gradients of the Be are found to be quite small. This is a result of using bare Be rods and the good thermal conductivity of the Be. Typically the maximum Be temperature is 430°C and the minimum is 410°C .

The diameter of the breeder pins in the final design is based on iteration between the neutronics calculations and the blanket heat transfer. The maximum allowable temperature for LiAlO_2 is set at 1100°C . For a given volumetric heat generation rate we can use Eq. (9) to calculate the pin diameter

Fig. 7. Radial temperature gradients in the two breeder zones.
 (a) Curves A and B are for the pins in rows 1 and 5, respectively;
 (b) Curves A and B are for the pins in rows 6 and 11, respectively.
 (Row numbers are defined in Fig. 2.)

that results in a 1100°C centerline temperature. It is conceivable that we could apply Eq. (9) to each row of pins, thereby designing a blanket with pins of many different sizes, and each one having $T_{MAX} = 1100^{\circ}\text{C}$. This would result in a large portion of the breeder material operating near T_{MAX} which results in a minimum blanket tritium inventory. For reasons of blanket design simplicity, we opted for a two-zone blanket model.

The 1-D thermal analysis is sufficient for calculating average temperature, radial temperature gradients, and maximum temperature of each pin. There are, however, 2-D effects that need to be taken into account. Specifically we need to calculate the azimuthal variation of the cladding temperature so that we can determine the pin deflection due to rod bowing. See Ref. [2] for the rod bowing analysis.

The 2-D effects are: azimuthal variation of heat transfer coefficient and coolant temperature, non-uniform helium gap size due to breeder/cladding eccentricity, and variation of the volumetric heat generation rate across the breeder and the cladding. The latter effect is most pronounced in the first row of breeder pins. In these pins, the peak-to-minimum heat generation rate is 2.9.

Of these 2-D asymmetries, the most difficult to quantify is the variation of the heat transfer coefficient. Early experiments on flow around a single cylinder (Lohrish, 1929 [10,11]) have shown that the local Nusselt numbers and hence the local heat transfer coefficients vary azimuthally from 220% ($Re = 50,000$) up to 475% ($Re = 4000$). Typical azimuthal variations of Nu and the heat transfer coefficient are shown in Fig. 8 (calculated from Ref. [10]). The situation becomes more complicated as we go from a single cylinder to banks of cylinders. Lohrish, also in 1929, performed some experiments with rod bundles that revealed that there was still an azimuthal dependence on h .

Based on Lohrish's result, we chose to let $h(\theta)$ vary in the following fashion:

$$h(\theta) = h_{AVG}[1.5 - 0.625(\theta/90)] \quad ; \quad 0 < \theta < 90^{\circ} \quad , \quad (10)$$

$$h(\theta) = 0.875 h_{AVG} \quad ; \quad 90^{\circ} < \theta < 180^{\circ} \quad , \quad (11)$$

Fig. 8. Azimuthal variation of Nusselt number for flow around a single cylinder

where h_{AVG} is calculated from Eq. (8). Equations (10) and (11) produce a linearly decreasing heat transfer coefficient from the front of the pin to its midplane and a constant h from the midplane to the back of the pin. The variation of h is illustrated in Fig. 9.

The remaining 2-D asymmetries are more easily applied. The neutronics calculations produce the local volumetric heat generation rates. In the first row of breeder pins, the heat generation within the $LiAlO_2$ ranges from 60w/cm^3 up to 179w/cm^3 . The coolant temperature was assumed to vary linearly with angle around the pin. An energy balance on each row of pins yields the total coolant temperature rise per pin. Finally, the eccentricity of the cladding and breeder was modeled as a line-of-contact at the front of the pin and an azimuthally varying gap around the pin. Figure 10 summarizes all the 2-D asymmetries.

The effects of the 2-D asymmetries were examined for three rows of pins. The first row of each breeding zone, since they have the highest volumetric heating in their respective zones, and the last row of pins in the blanket since the cladding temperature is highest. This latter case could cause problems because of the thermal creep of 9-C [2].

The cladding temperature distributions for the three cases are shown in Fig. 11. A typical temperature contour plot is shown in Fig. 12. These contours are for the first row of breeder pins (o.d. = 1.22 cm).

As stated earlier, the temperature profiles have been used to determine the rod bowing deflection due to thermal expansion [2]. The calculations reveal that thermally induced rod deflections are small ($\Delta X \approx 0.5\text{ mm}$) and can be considered to have no consequence on reactor operation.

11.2.3. FRACTIONAL POWER OPERATION

The objective of the blanket thermal analysis during fractional power operation is to determine what changes will occur in the temperature fields and what the resulting effect will be. Initially, we observe that the thermal stresses in the reactor are reduced. Both the surface and volumetric heating are reduced resulting in lower thermal gradients. This leads us to conclude that at low power the reactor should operate quite safely. However, from Eq. (9) we see that the breeder temperature will be lower. The result is a

Fig. 9. Azimuthal variation of Nusselt number for flow normal to rod bundles

Fig. 10. Summary of the 2-D asymmetries found on the breeder pins. Included are non-uniform heat generation, azimuthal variation of the heat transfer coefficient, helium gap eccentricity, and increased coolant temperature around the pin

Fig. 11. Azimuthal variation of cladding temperature for breeder pins in row 1 (curve A), row 6 (curve B), and row 11 (curve C)

Fig. 12. Temperature contours for breeder pins in row 1 (symmetric half shown). The jagged horizontal lines define zones of different rates of volumetric heat generation used in finite element analysis.

decrease in tritium diffusion and an increase of the diffusive tritium inventory in the blanket.

To model the thermal behavior of the breeder at fractional power we rewrite Eq. (9) as follows:

$$T(r, \eta) = T_F + \left[\frac{Q_{Li}''' r_o^2 + Q_c''' (r_{co}^2 - r_{ci}^2)}{2r_{co}} \left(\frac{r_c}{k_c} + \frac{1}{n^{0.6} h} \right) + \frac{Q_{Li}''' r_o t_g}{2k_g} + \frac{Q_{Li}'''}{4k_{Li}} (r_o^2 - r^2) \right] \eta \quad (12)$$

In Eq. (12), η is defined as the operating power divided by the reactors maximum power capacity. For example, if the reactor is operating at 50% power, $\eta = 0.5$. Optimum thermal efficiency is obtained if the primary coolant inlet and outlet temperatures are kept constant. Therefore, the coolant mass flow rate must decrease linearly with η . The heat transfer coefficient calculated from Eq. (8) scales as (velocity)^{0.6} which accounts for the $\eta^{0.6}$ scaling factor for h in Eq. (12). As the flow decreases for smaller η the flow regimes change from turbulent to transition to laminar. The transition occurs at $\eta = 0.05$ which implies that Eq. (12) is valid for $0.05 < \eta < 1.0$. This range is virtually the full range of power for the reactor.

Figure 13 illustrates the effect of steady fractional power operation on breeder pin temperature. Also shown on Fig. 13 is the total tritium inventory of the blanket (from Ref. [3]). We see a pronounced increase in inventory at low power levels because the tritium diffusion coefficient is a strong function of temperature.

One method of decreasing the tritium inventory in the event of prolonged operation at low power is to alter the coolant flow conditions. For example, at 50% power we can raise the coolant inlet temperature from 250° to 350°C while maintaining the outlet at 550°C. Now the coolant temperature and the breeder temperature for each row of pins will be higher. The result is a decrease in diffusive tritium inventory. For the specific example given, the tritium inventory decreases by about 20%. The penalty paid for this type of flow control is increased helium pumping power and lower net efficiency.

Fig. 13. Effect of steady, fractional power operation on the maximum and average breeder temperatures and the total blanket tritium inventory

Another method of decreasing tritium inventory at low power is to periodically allow the breeder rods to run hotter than normal. This could be achieved by decreasing the coolant mass flow rate at prescribed intervals. The tradeoff between plant efficiency and tritium inventory should be examined before adopting either of these methods of control.

11.3. TRANSIENT THERMAL ANALYSIS

11.3.1. POWER RAMPS

As stated in Sect. 11.2.3, steady power generation at fractions of full power does not place strict requirements on the operation of the reactor. Increased tritium inventory is the primary concern and judicious plant operation via coolant flow control will reduce that problem.

A more serious concern relating to fractional power operation is the time rate-of-change from one power plateau to the next. In a fusion plasma the rate-of-change of power production can be quite fast: typically on the time scale of energy confinement, about one second [12]. If the power loading on the first wall and blanket are increased on this time scale severe transient thermal fields could develop. These transients result from the slow fluid response time in the primary loop. The coolant flow time constant can range from 10s to 100s of seconds. The longest time constant is the time required to reach thermal equilibrium after a change in the level of power production. In the reference blanket design the time constant for thermal equilibrium is governed by thermal diffusion out of the multilayer breeder rods. We will now discuss the calculation and consequences of various time constants.

11.3.1.1. First Wall Response

The first wall thermal response to abrupt changes in power load followed by slow changes in coolant flow have been studied. Using the finite element model presented in Sect. 11.2.1 we are able to produce temperature histories for the first wall under various conditions. Two wall load conditions were examined: a severe rate-of-change of power (10%/sec) and a moderate rate-of-change (10%/min). In combination with these power ramps were the associated changes in coolant flow rate. We looked at three flow ramps; 10%/sec, 10%/min and 1%/min. The combinations of these rates yields 6 different cases that

cover a wide range of operating scenarios. Each case starts at 50% of full power and ramps up to full power. The entire ramping cycle lasts from 5 sec to 50 min depending on the specific case.

The results of these calculations are plotted in Fig. 14. The most severe conditions exist when the change in coolant flow rate is slower than the change in power production. The worst case is Case 3 in which the power increases at 10%/sec and the coolant flow increases at 1%/min. In this situation the structure reaches a maximum temperature of 702°C and then slowly cools to 621°C. We conclude from this analysis that the rate of power ramp-up should never be faster than the rate of coolant flow ramp-up. It is desirable to ramp power and coolant flow at the same rate. This dictates that the plasma power ramp rate will be determined by the available coolant flow ramp rate.

11.3.1.2. Breeder Rod Response

The thermal response of the multilayer breeder rods has been examined. The worst case is a step change in power followed by a slow ramp-up of coolant flow. However, based on the conclusion of the first wall response analysis we only consider moderate and equal ramps of power and coolant flow to be realistic.

The breeder rods studied were the first row of 3-cm-diam pins (row 6). These pins were chosen because they have a longer thermal time constant than the smaller pins in rows 1-5. The ramp rates for power and flow are 10%/min. The time response of the centerline temperature in the pins is shown in Fig. 15. We see that the centerline temperature follows closely the power ramp and reaches equilibrium at 600 sec. This is 300 sec after the end of the power ramp. It is interesting to note that even for a step change in power and flow the time to reach equilibrium is about 400 sec. This indicates that a safety margin exists within the breeder in the event of unforeseen rapid power excursion.

11.3.2. PRIMARY LOOP PREHEAT

The final topic of this paper is a discussion of the requirements for primary loop preheating. It is desirable from a reactor operations viewpoint to perform hot functional tests prior to reactor power generation. These tests will ensure that no problem will be produced from thermal expansion and

Fig. 14. Thermal response of the first wall surface temperature under various power and coolant flow ramps.
 A. Power increases at 10%/sec; Flow increases at 1%/min
 B. Power increases at 10%/sec; Flow increases at 10%/min
 C. Power increases at 10%/sec; Flow increases at 10%/sec
 D. Power increases at 10%/min; Flow increases at 10%/sec

Fig. 15. Response of the breeder centerline temperature to a 10%/min increase in fusion power. The dashed line shows the temperature response following a step change in power production

reactor pressurization. To perform the hot functional testing the primary loop must be heated with an auxiliary heat source. For the purposes of plant design and operation it is necessary to know the power and time required for preheat. In this analysis we assume that the primary coolant, helium, is used as the heat transfer medium.

The model of the primary loop and the method of solution are similar to those developed at UCLA by Taghavi and Ghoniem [13]. In their analysis the individual primary loop components are modeled as thermal capacitance "lumps." The analysis presented here is similar with slight modifications to account for the physical differences in the specific reactor designs.

The lumped capacitance model requires a thermal time constant for each component. This time constant τ_i is given by Eq. (13).

$$\tau_i = \left(\frac{\rho C_p V}{A h} \right)_i \quad (13)$$

The time constants for the primary loop components are tabulated in Table V.

All of the primary loop components are initially at ambient temperature, 20°C. During the hot functional testing we intend to operate all components at 250°C. Thus, our task is to determine the time required to reach this temperature and the total energy needed.

Since we assume that there are no heat loss mechanisms, the preheat energy is easily determined. The energy required is given by Eq. (14).

$$E = \sum_1^N (mC_p)_i \Delta T \quad (14)$$

where i represents an individual component and $\Delta T = 230^\circ\text{C}$. Evaluating the summation for the conditions specified gives the total energy required, 9.1×10^{10} J (25.3 MW-hr). We must next calculate the time required to preheat. From this information we can determine what the heater power must be.

We will assume that the heater can continually produce hot helium with an outlet temperature of 400°C which is 150°C hotter than the desired primary loop final temperature. Figure 16 illustrates the temperature history of the primary loop components during preheat. We see from Fig. 16 that the first

TABLE V
PREHEAT TIME CONSTANTS FOR THE PRIMARY LOOP COMPONENTS

Component, i	τ_i (sec) ⁽¹⁾	Total Number of Components with τ_i
1/2 Primary piping	750	2
First wall plus module shell	57	1,080
Beryllium rods	24	64,800
LiAlO ₂ rods (OD = 1.22 cm)	45	86,400
LiAlO ₂ rods (OD = 3.0 cm)	150	45,360

(1) Time constant τ is defined as $\tau = \rho c_p V / hA$.

Fig. 16. Temperature response of the primary loop during reheat

half of the primary piping heats quite rapidly, reaching the desired temperature of 250°C in 20 min. This is expected since there is always hot gas (400°C) in contact with this structure. The blanket modules reach 250°C after 180 min of preheat. The average thermal power required for this cycle is 8.4 MW.

It should be noted that during most of the preheat cycle the temperature of the first half of the primary piping is 400°C, which is 150°C hotter than during normal operation. If this is unacceptable, then it may be desirable to heat up the primary loop in reverse order. Thus we would heat up the hot leg of the primary piping first, then the blanket and finally the cold leg of the primary. Now the hot leg, which normally operates at 550°C, will be at 400°C for the majority of the preheat cycle.

11.4. SUMMARY

Detailed thermal analysis has been performed on a pin-type solid breeder blanket. A first wall with internal fins and 3-D surface roughening is used. The maximum temperature is 620°C and at the beginning-of-life there is 5 mm of structure between the coolant and plasma. The breeding blanket is divided into two zones of 9-C clad LiAlO_2 pins. The pins in each zone are sized to operate near T_{MAX} of the breeder (1100°C).

Fractional power operation of the blanket modules was examined. During steady fractional power operation, no adverse effects are seen except for increased tritium inventory due to lower average temperatures within the breeder. A more interesting condition is observed during power ramps between fractional power plateaus. Specifically, when the fusion power and coolant flow are not increased at the same rate, the first wall experiences atypical temperature behavior. This condition could lead to structure temperatures that are higher than the allowable.

Finally, we examined the requirements of primary loop preheat. The results indicate that three hours of preheat operation will heat the loop to 250°C with an average power consumption of 8.4 MW.

NOMENCLATURE

A	area, m^2
A_R	roughened area of first wall channel, m^2
A_W	total first wall channel area, m^2
C	empirical coefficient for flow across rod bundles
c_p	specific heat, J/Kg K
D_H	hydraulic diameter, m
F	scaling factor for flow across rod bundles
f_s	friction factor for smooth tubes
G_m	mass flow rate per unit area, $Kg/m^2 s$
h	heat transfer coefficient, $w/m^2 K$
k	thermal conductivity, $w/m K$
k_c	thermal conductivity of the cladding, $w/m K$
k_g	thermal conductivity of the helium filled gap, $w/m K$
k_{Li}	thermal conductivity of the $LiAlO_2$, $w/m K$
L	length, m
m	mass, Kg
Δp	pressure drop, Pa
Pr	Prandtl number
Q_c'''	volumetric heat generation rate in the cladding, w/m^3
Q_{Li}'''	volumetric heat generation rate in the $LiAlO_2$, w/m^3
Re	Reynolds number
r_{ci}	inner radius of the cladding, m
r_{co}	outer radius of the cladding, m
r_o	outer radius of the $LiAlO_2$, m
T_f	bulk coolant temperature, K
ΔT_f	coolant film temperature drop, K
T_w	wall temperature, K
t_c	cladding thickness, m
t_g	gap thickness, m
V	volume, m^3
ρ	density, Kg/m^3
η	fraction of full power ($0 \leq \eta < 1$)
θ	angle measured around pins, degrees
τ	thermal time constant, s
μ	viscosity, $Kg/m s$

REFERENCES

1. G. E. Orient and N. M. Ghoniem, "Elastic Structural Analysis of the Pin-Type Solid Breeder Blanket First Wall," Chapter 14 this report; also UCLA Report UCLA-ENG-8609/PPG-935 (February 1986).
2. J. P. Blanchard and N. M. Ghoniem, "The Bowing of Solid Breeder and Multiplier Rods in a Pin-Type Fusion Blanket," Chapter 13 this report; also UCLA Report UCLA-ENG-8605/PPG-931 (February 1986).
3. R. Martin and N. M. Ghoniem, "Modeling of Tritium Transport in a Pin-Type Solid Breeder Blanket," Chapter 9 this report; also UCLA Report UCLA-ENG-8608/PPG-934 (February 1986).
4. P. Levin and N. M. Ghoniem, "Neutronic Optimization of a LiAlO_2 Solid Breeder Blanket," Chapter 8 this report; also UCLA Report UCLA-ENG-8607/PPG-933 (February 1986).
5. D. L. Smith, G. D. Morgan, et al., "Blanket Comparison and Selection Study — Final Report," Argonne National Laboratory Report ANL/FPP-84-1 (September 1984).
6. P. J. Burns, "TACO2D-A Finite Element Heat Transfer Code," Lawrence Livermore National Laboratory Report UCID-17980 (January 1982).
7. R. D. Boyd, C. P. C. Wong and Y. S. Cha, "Technical Assessment of Thermal-Hydraulics for High Heat Flux Fusion Components," Sandia National Laboratories Report SAND84-0159 (January 1985).
8. H. Schlichting, Boundary-Layer Theory, 7th Ed. (McGraw Hill, New York, 1979) p. 611.
9. M. M. El-Wakil, Nuclear Heat Transport (International Textbook Co., New York, 1971) p. 247.
10. M. Jacob, Heat Transfer, Vol. 1. (Wiley & Sons, New York, 1949) pp. 607-610.
11. W. Lohrlich, Forschungsarb. a.d. Geb. d. Ingenieurwes., No. 322 (1929) 46.
12. R. W. Conn, N. M. Ghoniem and F. Najmabadi, "START/TM, A Study of Start-up and Fractional Power Operation of Tandem Mirror Fusion Reactors," UCLA Report PPG-764 (January 1984).
13. K. Taghavi and N. M. Ghoniem, "Primary Loop Conditioning and Design Constraints of Li-Pb Cooled Tandem Mirror Reactors During Start-up/Shutdown Operations," Nucl. Engr. Des./Fusion, 1, 4 (1984).

CHAPTER 12

THERMOMECHANICAL ANALYSIS OF SOLID BREEDERS IN SPHERE-PAC, PLATE, AND PELLET CONFIGURATIONS

James P. Blanchard and Nasr M. Ghoniem

CONTENTS

12.1.	INTRODUCTION	12-1
12.2.	SPHERE-PAC BREEDER CONFIGURATION	12-2
12.2.1.	Unconstrained Thermal Stress	12-2
12.2.1.1.	Uniform Temperature Change	12-3
12.2.1.2.	Uniform Volumetric Heating	12-3
12.2.2.	Constrained Thermal Stresses	12-5
12.3.	PLATE CONFIGURATION	12-6
12.3.1.	Unconstrained Plates	12-7
12.3.1.1.	Thermal Stresses	12-9
12.3.1.2.	End Effects	12-10
12.3.1.3.	Creep Relaxation of Swelling Deformations ..	12-10
12.3.2.	Constrained Plates	12-14
12.3.3.	Conclusions	12-16
12.4.	PELLET CONFIGURATION	12-16
12.4.1.	Problem Description	12-17
12.4.2.	Stress Analysis	12-17
12.4.2.1.	Axisymmetric Thermal Field	12-19
12.4.2.2.	Non-Axisymmetric Thermal Field	12-21
12.4.3.	Results	12-24
12.4.4.	Conclusions	12-26
REFERENCES		12-27

12.1. INTRODUCTION

Fusion blankets featuring solid breeders have been proposed in many different configurations. The "Blanket Comparison and Selection Study" (BCSS) [1], for example, includes breeder-in-tube, breeder-outside-tube, and layered designs. As discussed in the study, each concept has advantages and disadvantages associated with its various components. In this report we will assess the thermomechanical behavior of three common configurations and discuss the implications on blanket design.

The first configuration studied is called sphere-pac. It features small breeder spheres of three different diameters, thus allowing efficient packing and minimal void fraction. The concept originated as an attempt to minimize thermal stresses in the breeder and improve the predictability of the breeder-structure interface heat conduction. It remains a viable option for commercial fusion.

The second concept features plates of solid breeder wrapped with thin steel cladding. The thermal stresses in large plates can reach high levels, so small rectangular blocks will be used in most designs. This shifts much of the load to the cladding, which should be thin for good breeding and thick for good mechanical strength. In general the breeder is made as thin as possible, to maximize the breeding ratio, so the cladding's integrity will likely be the life-limiting issue of this concept.

The third breeder configuration is in the form of pellets clad by steel tubes. This design is similar to fission fuel rods, but the analysis for fusion differs in two ways:

1. The coolant in fission reactors flows parallel to the fuel rods' axes, whereas fusion designs can be either parallel or cross flow.
2. The flux source in fission reactors is the fuel itself, while in fusion it is the plasma. As a result, the damage gradient over the cross section of a solid breeder pin may be more severe than in fission rods.

The major thermomechanical issue of the pin-type designs is cracking, which would impair the thermal performance of the blanket. Fortunately, the pins can be sized to prevent cracking under normal operation.

In this report we have treated each blanket generically, dealing with basic issues rather than design specifics. Our basic philosophy is to avoid cracking of the breeder if at all possible. It can be argued that cracking could be allowed, but this would sacrifice predictability of the blanket thermal performance and tritium release characteristics. Proper design can and should minimize breeder cracking.

12.2. SPHERE-PAC BREEDER CONFIGURATION

Because the use of relatively large blocks of solid breeder can lead to severe interaction with the surrounding structure, the sphere-pac concept is an attractive configuration that achieves high net density while reducing the blanket stresses. Three different sphere diameters are used to minimize the voids in the breeder zone. It is assumed, by those who advocate this concept, that the spheres are relatively free to move around as they expand, thus reducing the thermal stresses due to constraint. In this sense, the spheres are believed to act much like a fluid.

Unfortunately, it may not be sufficient to assume that the spheres are unconstrained. One might expect that if the sphere diameter is more than about half of the diameter of the surrounding structure relative motion may be reduced, causing the spheres to act more like a solid. Additionally, sintering could inhibit the idealized behavior of the breeder, leading to cracking or breakage. The following analysis attempts to quantify the possible effects of solid breeder pellet constraint.

12.2.1. UNCONSTRAINED THERMAL STRESS

Initially we consider the stresses in a free sphere subject to a uniform temperature rise ΔT_b , and uniform volumetric heating q''' . If we assume linear, elastic material behavior and no angular dependence of the temperature, the displacements and stresses are [2]:

$$\begin{aligned}
U &= \frac{(1+\nu)}{1-\nu} \frac{\alpha}{r^2} \left[\int_0^r Tr^2 dr + \frac{2(1-2\nu)}{1+\nu} \frac{r^3}{b^3} \int_0^b Tr^2 dr \right] , \\
\sigma_r &= \frac{2\alpha E}{1-\nu} \left(\frac{1}{b^3} \int_0^b Tr^2 dr - \frac{1}{r^3} \int_0^r Tr^2 dr \right) , \\
\sigma_\phi = \sigma_\theta &= \frac{\alpha E}{1-\nu} \left(\frac{2}{b^3} \int_0^b Tr^2 dr + \frac{1}{r^3} \int_0^r Tr^2 dr - T \right) , \quad (1)
\end{aligned}$$

where U is the radial displacement, T is the temperature change from the stress-free state, b is the sphere radius, α is the thermal expansion coefficient, E the Young's modulus, and ν is Poisson's ratio. All material properties are assumed to be homogeneous and independent of temperature. σ_r , σ_θ , and σ_ϕ are the normal stresses in the directions of the spherical coordinates r , θ , and ϕ .

12.2.1.1. Uniform Temperature Change

For a uniform temperature change, the stresses are zero and the radial displacement at the surface is given by

$$U = b\alpha\Delta T_b . \quad (2)$$

12.2.1.2. Uniform Volumetric Heating

For a uniform volumetric heating rate q''' , the temperature distribution is parabolic [2]:

$$T = \Delta T_b + \frac{q'''}{6k} (b^2 - r^2) , \quad (3)$$

where k is the thermal conductivity.

Substituting Eq. (3) into the stress and displacement equations (1), we obtain

$$\begin{aligned}
\sigma_r &= \frac{\alpha E q'''}{15(1-\nu)k} (r^2 - b^2) , \\
\sigma_\phi = \sigma_\theta &= \frac{\alpha E q'''}{15(1-\nu)k} (2r^2 - b^2) , \quad (4) \\
U(b) &= b\alpha \left(T_b + \frac{q''' b^2}{15k} \right) .
\end{aligned}$$

The stresses are plotted in Fig. 1. The radial stress σ_r is always compressive, while σ_θ and σ_ϕ are compressive near the center ($r = 0$) and tensile

Fig. 1. Thermal stresses in an unconstrained sphere subjected to uniform volumetric heating.

near the surface ($r = b$). The stress state is equi-triaxial compression near the center and equi-biaxial tension near the surface, so cracking will likely begin at the surface. Surface stresses in Li_2O spheres for various heating rates and sphere radii are given in Table I. As seen, the thermal stresses in an unconstrained 1 mm pellet are quite small, but those in 5 mm pellets exceed the fracture strength for Li_2O . Complete or partial constraint will only aggravate the potential problems.

12.2.2. CONSTRAINED THERMAL STRESSES

In some cases, adjoining spheres will not allow free accommodation of thermal expansion. The worst case is one in which the centers of two spheres remain at a fixed distance. This problem is addressed by calculating the change in the distance between centers of two free spheres due to a temperature change and then calculating the force required to restore them to their original position.

Roark [3] gives the stresses and displacements in two identical elastic spheres due to a point load P as

$$\Delta = 1.55 \left(\frac{P^2}{E^2 b} \right)^{1/3}, \quad (5)$$

$$\sigma_{c, \max} = -0.616 \left(\frac{PE^2}{b} \right)^{1/3},$$

TABLE I
MAXIMUM STRESSES IN BREEDER SPHERES

b (mm)	q''' (MW/m ³)	ΔT_b (K)	σ (unconstrained) (MPa)	σ (constrained) (MPa)
1	50	600	3.3	7000
1	50	0	3.3	260
1	10	0	0.7	120
5	50	0	82.0	1300

where Δ is the change in distance between the sphere centers (after free expansion) and $\sigma_{c,max}$ is the maximum compressive stress, which occurs at the contact surface between the two spheres. The maximum tensile stress is about $0.133 \sigma_{c,max}$ and occurs at the edge of the contact surface.

Setting $\Delta + U(b) = 0$, where $U(b)$ is given by Eq. (4), we obtain

$$P = Eb^2 \left[\frac{\alpha}{1.55} \left(T_b + \frac{q''' b^2}{15k} \right) \right]^{3/2}, \quad (6)$$

and the maximum stress is

$$\sigma_{c,max} = -0.616 E \left[\frac{\alpha}{1.55} \left(T_b + \frac{q''' b^2}{15k} \right) \right]^{1/2}. \quad (7)$$

The stresses given by this formula, ignoring fracture, are in Table I. Obviously, the stresses are very high, but one must consider the model used in this analysis before giving up on the sphere-pac concept.

Sphere-pac was originated to allow accommodation of thermal expansion or swelling, on the premise that relative motion of adjoining spheres would be possible. In general, the constraint of the breeder will be minimized and the resulting stresses will also be small. It is only in certain situations that free expansion is restrained and large stresses are developed. Even then, the constraint will not likely be as severe as those analyzed here. These considerations indicate that a thorough analysis of sphere-pac is difficult and testing is required to verify its feasibility.

12.3. PLATE CONFIGURATION

One common fusion blanket concept features plates of solid breeder contained by some structural material [1]. The breeders are often quite brittle so thermal stresses can easily lead to cracking in even an unconstrained plate. The cladding, which offers some restraint of the breeder's thermal expansion, compounds the problem. In addition, one would like to minimize the amount of structure in the breeding zone in order to increase the tritium breeding ratio; hence the cladding is generally very thin. In some cases, the cladding may actually be the weakest member of the system. In either case, analysis is needed to assess the integrity of a solid breeder blanket.

To deal with the interaction between breeder plates and their cladding, this report first analyzes the thermal stresses in an unconstrained plate that is essentially infinite in two directions, as seen in Fig. 2. The neutron current is in the z-direction and the temperature varies in both the y- and z-directions, leading to stresses in the z-direction. The impact of the cladding on this stress field is then calculated assuming that it acts like a simple truss member at the top of the plate ($z = 0$). Finally, possible advancements in this analysis are suggested.

12.3.1. UNCONSTRAINED PLATES

A neutron current in the z-direction, such as that in Fig. 2, will cause a temperature distribution that is independent of x, symmetric about $y = 0$ and which decays exponentially in the z-direction. These conditions imply that the plate will not bend, thus simplifying our analysis. Assuming that there are no stresses in the y-direction because the plate is thin and that there are also none in the x-direction because the uniform heat generation does not vary in that direction, a 1-D analysis is sufficient for the bulk of the plate. We begin with a general equation for the strains in the plate:

$$e^T = e^{el} + e^{th} + e^C + e^S, \quad (8)$$

where e^T is the total strain at a point in the breeder, e^{el} is the elastic strain, e^{th} is the thermal strain, e^C is the creep strain, and e^S is the swelling strain. Assuming linear elastic behavior, e^{el} is given by σ/E , where E is the Young's Modulus. By integrating Eq. (8) over a cross section in the x-y plane, one obtains:

$$\int e^T dA = \int \frac{\sigma}{E} dA + \int (e^{th} + e^C + e^S) dA. \quad (9)$$

If E is independent of position, $\int (\sigma/E) dA = F_z/E$ where F_z is the net force on the cross section. This must be zero because there are no applied loads. Additionally, e^T can be taken to be independent of x and y as long as the plate is thin and doesn't bend [4]. Therefore, $\int e^T dA = e^T \int dA$, yielding

$$e^T = \frac{1}{2t} \int_{-t}^t (e^{th} + e^S + e^C) dy, \quad (10)$$

where the integral over x has canceled out.

PLASMA

Fig. 2. Model used for breeder plate analysis.

Substituting Eq. (10) into Eq. (8), the stress is found to be

$$\sigma_z = E(\bar{e}'' - e'') \quad , \quad (11)$$

where

$$e'' = e^{th} + e^s + e^c \quad ,$$

and

$$\bar{e}'' = \frac{1}{2l} \int_{-l}^l e'' dy \quad . \quad (12)$$

The stress is thus determined by the difference between the average and local strains.

12.3.1.1. Thermal Stresses

In this section, swelling and creep strains are ignored. Assuming that the thermal expansion coefficient α is independent of position and temperature, Eq. (11) becomes

$$\sigma = E\alpha(\bar{T} - T) \quad , \quad (13)$$

where

$$\bar{T} = \frac{1}{2l} \int_{-l}^l T dy \quad . \quad (14)$$

Ignoring conduction in the x-z plane, the temperature is given by [5]

$$T = T_c + \frac{q''' l^2}{2k} \left[1 - \left(\frac{y}{l} \right)^2 \right] \quad , \quad (15)$$

where T_c is the coolant temperature, q''' is the uniform volumetric heating rate and k is the breeder thermal conductivity. Integrating over the thickness, one finds

$$\bar{T} = T_c + \frac{q''' l^2}{3k} \quad , \quad (16)$$

and

$$\bar{T} - T = - \frac{q''' l^2}{2k} \left[\frac{1}{3} - \left(\frac{y}{l} \right)^2 \right] \quad . \quad (17)$$

This quantity, $(\bar{T} - T)$, depends on z only as q''' depends on z , hence it exhibits an exponential decay due to the decay in the neutron flux.

Substituting Eq. (17) into (13) one obtains

$$\sigma_z = - \frac{Eq''l^2}{2k} \left[\frac{1}{3} - \left(\frac{y}{l} \right)^2 \right] \quad (18)$$

For typical reactor conditions, the stresses at $z = 0$ are plotted in Fig. 3. The peak tensile stress is approximately 200 MPa, while the peak compressive stress is -100 MPa. The tensile peak exceeds the fracture stress of the Li_2O , so cracking will occur.

12.3.1.2. End Effects

Because there are no applied tractions on the breeder surfaces, σ_z must be zero at $z = 0$. Therefore Eq. (13) is only valid some distance away from the surface. Arguments similar to St. Venant's principle [6] imply that Eq. (13) is valid only for $z > 2l$ and σ_z decreases to zero as z approaches zero. Therefore, decreasing the length of the plate or cutting it into small blocks will decrease the stresses, thereby reducing the likelihood of fracture.

This phenomenon was verified using SAP IV, a finite element structural analysis code [7]. As shown in Fig. 4, the stress reaches the value given by Eq. (18) in a distance of about $1.6 l$. Therefore blocks of breeder that are less than 3.2 times as long as they are thick will likely have reduced stresses.

12.3.1.3. Creep Relaxation of Swelling Deformations

If creep and swelling are included in the analysis, the steady-state stresses can either be above or below the initial thermal stresses. In this analysis, we will assume a swelling equation of the form

$$e^S = \frac{1}{3} \frac{\Delta V}{V} = f(T) \delta \quad (19)$$

where δ is the accumulated dose and $f(T)$ gives the temperature dependence. In addition, we will assume

$$\dot{e}^C = A \sigma_z \quad (20)$$

Fig. 3. Axial stresses near top of plate subjected to neutron flux

Fig. 4. Effect of free surface on the axial stress distribution at the top of the plate.

where A is independent of temperature and $\dot{e}^c$ is the creep strain rate in terms of the dose δ . In other words,

$$\dot{e}^c = \int_0^\delta A \sigma_z d\delta' \quad . \quad (21)$$

Substituting Eq. (21) into Eq. (12), one obtains

$$\dot{e}^c = \frac{1}{2l} \int_{-l}^l \int_0^\delta A \sigma_z d\delta' dy = \frac{A}{2l} \int_0^\delta F_z d\delta' = 0 \quad , \quad (22)$$

where F_z , the applied force, is taken to be zero for any cross section. Substituting Eqs. (19), (21), and (22), in Eq. (11), one obtains

$$\sigma_z = \sigma_z^{th} + E\delta [\overline{f(T)} - f(T)] - E \int_0^\delta A \sigma_z d\delta' \quad (23)$$

where σ_z^{th} is given by Eq. (18) and

$$\overline{f(T)} = \frac{1}{2l} \int_{-l}^l f(T) dy \quad . \quad (24)$$

Equation (23) is a first order ODE which is easily solved, giving

$$\sigma_z = \sigma_z^{th} \exp(-EA\delta) + \frac{\overline{f(T)} - f(T)}{A} [1 - \exp(-EA\delta)] \quad . \quad (25)$$

The initial thermal stress is relaxed exponentially by the creep, while the swelling stresses, which would otherwise increase linearly with dose, are limited to a value determined by the ratio of the swelling and creep rates.

As seen in Eq. (25), stresses in the breeder can become large even if the thermal stresses are tolerable. Unfortunately, the data on the swelling and creep of solid breeder is quite limited, so further analysis will be set aside until these uncertainties are removed.

12.3.2. CONSTRAINED PLATES

The interaction of the solid breeder and its cladding is a complex problem, requiring a 3-D global analysis for a complete description. In order to get a rough estimate of the expected problem, we will assume that the major point of interaction is at the top, $z = 0$, and that the clad and breeder behave as truss members.

Consider two truss members of length $2l$ that are connected at the ends. The first is made of steel with thickness τ ; the second is solid breeder with thickness s . While the breeder is actually quite long, it can be modeled as a truss element with some effective thickness determined by the amount of material which is influenced by the cladding attached to the top.

For the two truss members, we find

$$P_C = K_C(\Delta_C - \Delta_{0C}) ,$$

and

$$P_B = K_B(\Delta_B - \Delta_{0B}) , \quad (26)$$

where P is the load per unit length in the x -direction, K is the stiffness ($K = E\tau/2l$), Δ is the total displacement of the end, and Δ_0 is the initial displacement due to thermal expansion.

Because the bars are connected, one requires

$$\Delta_B = \Delta_C ,$$

and

$$P_B + P_C = 0 , \quad (27)$$

where B denotes the breeder and C the cladding. Setting

$$\Delta_0 = 2\alpha T , \quad (28)$$

and solving for P_C yields

$$P_C = \frac{E_C \tau}{1 + (E_C \tau / E_B s)} (\alpha_B - \alpha_C) \bar{T} , \quad (29)$$

where s is the effective thickness of breeder influenced by the cladding. To approximate s , SAP IV was used to analyze a thin truss member attached to the top of a long plate. The stiffness of a 1-cm-thick Li_2O plate was calculated by calculating the load and deflection at the top, due to differential expansion of a 0.25-mm-thick cladding. The breeder stiffness was found to be nearly equal to the cladding stiffness, so

$$\sigma_C = P_C/\tau = \frac{1}{2} E_C(\alpha_B - \alpha_C)\bar{T} \quad (30)$$

This result will depend on the geometry and materials, but it is adequate for comparison of various configuration options. Stresses predicted by Eq. (30) can be very large because σ_C is proportional to the average temperature rise $\bar{T}$, rather than the gradient $\bar{T} - T$. Therefore, unless $\alpha_B - \alpha_C$ is quite small, the cladding is likely to fracture.

To relieve the stresses due to interaction, a designer may try curving the cladding to allow for expansion. For a semi-circular cladding of radius ℓ , Timoshenko and Goodier [6] determine the stiffness:

$$K_C = \frac{E_C t^3}{6\pi \ell^3} \quad (31)$$

giving

$$P = \frac{(E_C t/3\pi)(\frac{\tau}{\ell})^2}{1 + (\frac{\tau}{\ell})^2 \frac{E_C \tau}{3\pi E_B s}} (\alpha_B - \alpha_C)\bar{T} \quad (32)$$

Because the cladding stiffness is reduced by the factor $(1/3\pi)(\tau/\ell)^2$, we can assume $K_C/K_B \ll 1$, giving

$$\sigma_C = P_C/\tau = \frac{1}{3\pi} E_C \left(\frac{\tau}{\ell}\right)^2 (\alpha_B - \alpha_C)\bar{T} \quad (35)$$

This simple design change has reduced the stress by the factor $(2/3\pi)(\tau/\ell)^2$, which is approximately 0.002 for typical dimensions. Even if bending of this circular cladding is considered, the reduction in stress is significant.

12.3.3. CONCLUSIONS

The analysis of solid breeder stresses is difficult, complicated by uncertainties in reactor designs, material properties and modeling. The rather simple analyses presented here are nonetheless useful because they provide insight into scaling relationships and possible design changes. They also allow one to consider both swelling and creep, without reverting to complex codes, which can be costly.

It is apparent from this analysis that large solid breeder plates will most likely fracture because of thermal stresses. To avoid such a possibility, small blocks have to be packed into rectangular cladding canisters. However, the flat interaction surfaces of this configuration leads to further problems. It is advisable to contain solid breeders in tubular-type cladding configurations. For typical reactor designs, no analysis will be reliable enough to preclude thorough component testing, so the analysis presented here may suffice until blanket design progresses further.

12.4. PELLET CONFIGURATION

Because solid breeders are inherently brittle, they are prone to cracking due to thermal gradients, swelling gradients, and external loads or constraint. Since the breeders are not needed to function as structural materials, this cracking is not hazardous, yet it can impede reactor performance. As the breeder pellets crack, their net conductivity and tritium diffusion behaviors change. Additionally, the restructuring and fragmentation can impede purge flow, increasing the tritium inventory. Before cracking is allowed, the designer must understand the pellet behavior under expected operating conditions.

In this section, thermal stresses in solid pellets are calculated for a variety of temperature fields, including azimuthal variations. Because the rods are oriented perpendicular to both the neutron current from the plasma and the coolant flow, the temperature will vary radially and azimuthally. The thermal stresses, therefore, will also be functions of r and θ . Axial stresses will be treated by assuming that the pellets are in a state of plane stress.

12.4.1. PROBLEM DESCRIPTION

The model chosen for the analysis is seen in Fig. 5. It features a solid cylinder of radius b , with no body forces or surface tractions. The thermal field is assumed to be of the form:

$$T = T_0 - T_{\text{ref}} = (T_c - T_{\text{ref}}) + T_1 \left(\frac{r}{b}\right) f_1(\theta) + \Delta \left(\frac{r}{b}\right)^2 \left[1 + \frac{T_2}{\Delta} f_2(\theta)\right] \quad , \quad (34)$$

where T represents the components of the thermal field that cause stress, T_0 is the operating temperature, and T_{ref} is a zero-stress reference temperature. The thermal field consists of four major components:

1. $T_c - T_{\text{ref}}$ is the difference between the operating temperature at the center of the pellet and the reference temperature.
2. $T_1(r/b)f_1(\theta)$ represents an azimuthal variation that is linear in r .
3. $\Delta(r/b)^2$ is obtained when $f_1(\theta) = f_2(\theta) = 0$ and the volumetric heating q''' is constant. In this case,

$$\Delta = -q'''b^2/4k \quad , \quad (35)$$

where k is the thermal conductivity.

4. $T_2(r/b)^2 f_2(\theta)$ represents an azimuthal variation that is quadratic in r .

The functions f_1 and f_2 are general functions of θ which depend upon the flow parameters and neutronics. In this section they will be expanded in a Fourier series and only the first term will be analyzed.

12.4.2. STRESS ANALYSIS

To simplify the calculations, the four components of the thermal field will be treated separately and the resulting stresses will be superimposed.

Fig. 5. Model used for breeder pellet analysis.

12.4.2.1. Axisymmetric Thermal Field

12.4.2.1.1. Uniform Temperature. For an axisymmetric thermal field and plane stress conditions, Olander [8] gives the following equation for the radial displacement u :

$$\frac{d}{dr} \left[\frac{1}{r} \frac{d}{dr} (ru) \right] = (1 + \nu) \frac{d}{dr} (\alpha T) \quad , \quad (36)$$

where ν is Poisson's ratio and α is the coefficient of thermal expansion. Assuming α and ν are independent of r and T , Eq. (36) can be integrated once to yield

$$\frac{d}{dr} (ru) = (1 + \nu) \alpha T r + 2 C_2 r \quad , \quad (37)$$

where C_2 is an arbitrary constant.

For constant temperature, Eq. (37) can be integrated to yield

$$u = \left[\frac{1}{2} (1 + \nu) \alpha (T_c - T_{ref}) + C_2 \right] r + \frac{C_1}{r} \quad . \quad (38)$$

In order to have finite displacement at $r = 0$, one requires $C_1 = 0$.

To obtain the stress field, one requires stress-strain and strain-displacement equations. Timoshenko and Goodier give the axisymmetric strain-displacement equations [9] as

$$\begin{aligned} \epsilon_r &= \frac{\partial u}{\partial r} \quad , \\ \epsilon_\theta &= \frac{u}{r} + \frac{1}{r} \frac{\partial v}{\partial \theta} \quad , \\ \gamma_{r\theta} &= \frac{1}{r} \frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial r} - \frac{v}{r} \quad , \end{aligned} \quad (39)$$

and the plane stress constitutive equations [10] as

$$\begin{aligned}\sigma_r &= \frac{E}{1-\nu^2} [\epsilon_r + \nu\epsilon_\theta - (1+\nu)\alpha T] \quad , \\ \sigma_\theta &= \frac{E}{1-\nu^2} [\epsilon_\theta + \nu\epsilon_r - (1+\nu)\alpha T] \quad , \\ \tau_{r\theta} &= G\gamma_{r\theta} \quad .\end{aligned}\tag{40}$$

Substituting Eq. (38) into Eq. (39) and then into Eq. (40), one obtains

$$\sigma_r = \sigma_\theta = \frac{E}{1-\nu} \left[C_2 - \frac{1}{2} (1-\nu)\alpha(T_c - T_{ref}) \right] \quad .\tag{41}$$

Using the boundary condition $\sigma_r(r=b) = 0$, one finds that the pellet is stress free, i.e.,

$$\sigma_r = \sigma_\theta = 0 \quad .\tag{42}$$

Hence, the average pellet temperature has no influence on cracking and only gradients (radial or azimuthal) cause stresses.

The displacement, u , is given by

$$u = \frac{1}{2} \alpha (T_c - T_{ref}) r \quad .\tag{43}$$

12.4.2.1.2. Uniform Volumetric Heating. Starting from Eq. (37), one inserts $T = \Delta(r/b)^2$ and integrates to obtain

$$u = \frac{1}{4} (1+\nu)\alpha\Delta \frac{r^3}{b^2} + C_2 r + \frac{C_1}{r} \quad .\tag{44}$$

Again, finite displacement requires $C_1 = 0$.

Using Eqs. (39) and (40), the stresses are found to be

$$\begin{aligned}\sigma_r &= \frac{E}{1-\nu^2} \left[(1+\nu)C_2 - \frac{1-\nu^2}{4} \alpha\Delta \left(\frac{r}{b}\right)^2 \right] \quad , \\ \sigma_\theta &= \frac{E}{1-\nu^2} \left[(1+\nu)C_2 - \frac{3(1-\nu^2)}{4} \alpha\Delta \left(\frac{r}{b}\right)^2 \right] \quad .\end{aligned}\tag{45}$$

For $\sigma_r(r=b) = 0$, $C_2 = (1/4)(1-\nu)\alpha\Delta$, the stresses are

$$\begin{aligned}\sigma_r &= \frac{1}{4} E\alpha\Delta \left[1 - \left(\frac{r}{b} \right)^2 \right] , \\ \sigma_\theta &= \frac{1}{4} E\alpha\Delta \left(1 - 3 \left(\frac{r}{b} \right)^2 \right) ,\end{aligned}\quad (46)$$

and the displacement is

$$u = \frac{1}{4} \alpha\Delta r \left[(1 - \nu) + (1 + \nu) \left(\frac{r}{b} \right)^2 \right] . \quad (47)$$

12.4.2.2. Non-Axisymmetric Thermal Field

12.4.2.2.1. $T = T_1(r/b)\cos\theta$. Because of the azimuthal variation introduced here, the previous analysis must be modified. The procedures will involve both stress and displacement potentials, as suggested by Timoshenko and Goodier [11].

To find a particular solution, one introduces a displacement potential ψ , such that

$$u = \psi_{,r} , \quad v = (1/r)\psi_{,\theta} , \quad (48)$$

where v is the azimuthal displacement and $\psi_{,r}$ represents $\partial\psi/\partial r$. For plane stress, the governing equation for ψ is

$$\nabla^2 \psi = \psi_{,rr} + \frac{1}{r} \psi_{,r} + \frac{1}{r^2} \psi_{,\theta\theta} = (1 + \nu)\alpha T . \quad (49)$$

If ψ is expanded in a cosine series, i.e., $\psi = \sum_{n=0}^{\infty} \psi_n \cos n\theta$, then the particular solution will involve only ψ_1 because of $n=0$ the assumed thermal field. Substituting into Eq. (49), one obtains

$$\psi_{1,rr} + \frac{1}{r} \psi_{1,r} - \frac{1}{r^2} \psi_1 = (1 + \nu)\alpha T_1 \left(\frac{r}{b} \right) . \quad (50)$$

the particular solution to this equation is

$$\psi_1 = -\frac{1}{4} (1 + \nu)\alpha T_1 \frac{r}{b} \left(b^2 - \frac{1}{2} r^2 \right) , \quad (51)$$

and, from Eq. (48), one finds

$$u = -\frac{1}{4} (1 + \nu) \alpha T_1 \left(b - \frac{3}{2} \frac{r^2}{b} \right) \cos \theta \quad ,$$

$$v = \frac{1}{4} (1 + \nu) \alpha T_1 \left(b - \frac{1}{2} \frac{r^2}{b} \right) \sin \theta \quad ,$$

$$\sigma_r = -\frac{1}{4} E \alpha T_1 \left(\frac{r}{b} \right) \cos \theta \quad ,$$

$$\sigma_\theta = -\frac{3}{4} E \alpha T_1 \left(\frac{r}{b} \right) \cos \theta \quad ,$$

$$\tau_{r\theta} = -\frac{1}{4} E \alpha T_1 \left(\frac{r}{b} \right) \sin \theta \quad . \quad (52)$$

Since this is only a particular solution, there are no arbitrary constants that allow us to impose the proper boundary conditions. Therefore, a problem featuring applied surface tractions must be superimposed upon Eq. (52) to yield the proper solution. This problem is best solved using a stress potential ϕ .

The surface tractions required to produce a traction-free surface condition when added to Eq. (52) are

$$\begin{aligned} \sigma_r(r = b) &= \frac{1}{4} E \alpha T_1 \cos \theta \quad , \\ \tau_{r\theta}(r = b) &= -\frac{1}{4} E \alpha T_1 \sin \theta \quad . \end{aligned} \quad (53)$$

To solve this problem, introduce ϕ such that

$$\begin{aligned} \sigma_r &= \frac{1}{r} \phi_{,r} + \frac{1}{r^2} \phi_{,\theta\theta} \quad , \\ \sigma_\theta &= \phi_{,rr} \quad , \\ \tau_{r\theta} &= -\frac{\partial}{\partial r} \left(\frac{1}{r} \phi_{,\theta} \right) \quad . \end{aligned} \quad (54)$$

The governing equation is

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right) \left(\phi_{,rr} + \frac{1}{r} \phi_{,r} + \frac{1}{r^2} \phi_{,\theta\theta} \right) = 0 \quad , \quad (55)$$

and the solution is $\phi = Ar^3 \cos\theta$, where A is an arbitrary constant. This leads to

$$\begin{aligned}\sigma_r &= 2Ar \cos\theta, \\ \sigma_\theta &= 6Ar \cos\theta, \\ \tau_{r\theta} &= 2Ar \sin\theta.\end{aligned}\quad (56)$$

Using Eq. (53) one finds $A = (1/8b)E\alpha T_1$ and

$$\begin{aligned}\sigma_r &= \frac{1}{4} E\alpha T_1 \left(\frac{r}{b}\right) \cos\theta, \\ \sigma_\theta &= \frac{3}{4} E\alpha T_1 \left(\frac{r}{b}\right) \cos\theta, \\ \tau_{r\theta} &= \frac{1}{4} E\alpha T_1 \left(\frac{r}{b}\right) \sin\theta.\end{aligned}\quad (57)$$

Adding Eqs. (57) and (52) to obtain the total solution, one again finds that the pellet is stress-free. This is not surprising if one considers the nature of the assumed thermal field. When $T = \Delta(r/b)\cos\theta$, the contours of constant temperature are straight lines perpendicular to the coolant flow. In other words, T varies linearly with distance from the plasma. This, intuitively, is a zero-stress situation.

12.4.2.2.2. $T = T_2(r/b)^2 \cos\theta$. In this case, the particular solution to Eq. (49) is

$$\psi = -\frac{1}{6} (1 + \nu) \alpha T_2 r \left(b - \frac{2r^2}{5b}\right) \cos\theta, \quad (58)$$

and the stresses are

$$\begin{aligned}
\sigma_r &= -\frac{1}{5} E\alpha T_2 \left(\frac{r}{b}\right)^2 \cos\theta \quad , \\
\sigma_\theta &= -\frac{4}{5} E\alpha T_2 \left(\frac{r}{b}\right)^2 \cos\theta \quad , \\
\tau_{r\theta} &= -\frac{1}{5} E\alpha T_2 \left(\frac{r}{b}\right)^2 \sin\theta \quad .
\end{aligned} \tag{59}$$

To obtain a traction-free surface, one solves Eq. (55) for

$$\begin{aligned}
\sigma_r(r=b) &= \frac{1}{5} E\alpha T_1 \cos\theta \quad , \\
\tau_{r\theta}(r=b) &= \frac{1}{5} E\alpha T_1 \sin\theta \quad ,
\end{aligned} \tag{60}$$

and adds the result [from Eq. (56)] to Eq. (59) to obtain the final solution:

$$\begin{aligned}
\sigma_r &= \frac{1}{5} E\alpha T_2 \left(\frac{r}{b}\right) \left(1 - \frac{r}{b}\right) \cos\theta \quad , \\
\sigma_\theta &= \frac{3}{5} E\alpha T_2 \left(\frac{r}{b}\right) \left(1 - \frac{4}{3} \frac{r}{b}\right) \cos\theta \quad , \\
\tau_{r\theta} &= \frac{1}{5} E\alpha T_2 \left(\frac{r}{b}\right) \left(1 - \frac{r}{b}\right) \sin\theta \quad .
\end{aligned} \tag{61}$$

12.4.3. RESULTS

Using the material parameters in Table II, stresses are tabulated in Table III for various reactor parameters and pellet dimensions. In all cases the stress given is the hoop stress σ_θ at the surface, which is the maximum tensile stress in the pellet.

For typical parameters of a 1-cm-diam pellet and 50-MW/m³-volumetric heating, the peak tensile stress for $T_2 = 0$ is 215.6 MPa, well above the fracture strength of the pellet. To just avoid the cracking that would result from uniform heating, the pellet radius would have to be changed to 6.5, 3.75, and 2.9 mm for heating rates of 10, 30, and 50 MW/m³, respectively.

TABLE II
LiAlO₂ PROPERTIES

$E = 230 \times 10^3 \text{ MPa}$
$\alpha = 1.2 \times 10^{-5} \text{ } ^\circ\text{C}^{-1}$
$\nu = 0.23$
$k = 2 \text{ W/mk}$
$\sigma_{\text{fracture}} = 72.8 \text{ MPa (tensile)}$

TABLE III
MAXIMUM TENSILE STRESSES IN SOLID BREEDER PELLETS

b (mm)	q''' (MW/m ³)	T_2 (°C)	$\sigma_{\text{axisymmetric}}^{(a)}$ (MPa)	$\sigma_{\text{nonsymmetric}}^{(b)}$ (MPa)
5	50	-20	215.6	11.0
2.9	50	0	72.8	0
3.75	30	0	72.8	0
6.5	10	0	72.8	0
5	10	-20	43.1	11.0
5	10	-10	43.1	5.5

(a) $T_2 = 0$

(b) $q''' = 0$

The effect of azimuthal temperature variation is seen in the last column of Table III. Even for $T_2 = 20^\circ\text{C}$, which would represent a difference of 40°C between the front and back of the pellet, the stresses are only 11 MPa. For pellets with diameters on the order of 1 to 2 cm, T_2 should be much less than 20°C and the corresponding stresses should be quite small. This indicates that the nonsymmetric effects are relatively unimportant, so axisymmetric thermal fields could be used in a complex, nonlinear inelastic analysis without sacrificing the integrity of the solution.

12.4.4. CONCLUSIONS

According to the calculations presented here, cracking of solid breeder pellets can be avoided by the proper choice of their radii. This optimal pellet size is reasonable when thermal stresses are considered. Other factors not discussed are stresses due to the interaction of irradiation creep and swelling. The equally important problem of breeder-cladding interaction is discussed elsewhere in the report.

Another significant result of this work is the relatively small effect of azimuthal variations in the thermal field. This indicates that further analysis can be carried out assuming axisymmetric behavior.

REFERENCES

1. M. A. Abdou et al., "Blanket Comparison and Selection Study," Argonne National Laboratory Report ANL/FPP-84-1, 1984.
2. Franklin Institute Research Laboratories, Thermal Stress Techniques in the Nuclear Industry, (American Elsevier, New York, 1985).
3. R. J. Roark, Formulas for Stress and Strain, (McGraw-Hill, New York, 1975) p. 516.
4. T. H. Lin, Theory of Inelastic Stresses, (Wiley & Sons, New York, 1968) p. 128.
5. A. J. Chapman, Heat Transfer, (Macmillan, New York, 1967) p. 55.
6. S. P. Timoshenko and J. N. Goodier, Theory of Elasticity, (McGraw-Hill, New York (1934) p. 39.
7. K. Bathe et al., "SAP IV, a Structural Analysis Program for Static and Dynamic Response of Linear Systems," Earthquake Engineering Research Center Report EEC 73-11 (1974).
8. D. R. Olander, Fundamental Aspects of Nuclear Reactor Fuel Elements, U.S. Energy Research and Development Office, NTIS TID 26711-P1 (Nat. Tech. Info. Serv., Springfield, VA, 1976) p. 574.
9. S. P. Timoshenko and J. N. Goodier, op. cit., p. 76.
10. Ibid., p. 442.
11. Ibid., p. 481.

CHAPTER 13

BOWING OF SOLID BREEDER FUEL PINS AND MULTIPLIER RODS IN A PIN-TYPE FUSION BLANKET

James P. Blanchard and Nasr M. Ghoniem

CONTENTS

13.1.	INTRODUCTION	13-1
13.2.	ANALYSIS	13-2
13.2.1.	Beam Theory	13-2
13.2.2.	Stresses	13-4
13.2.3.	Constitutive Equations	13-5
13.3.	RESULTS	13-6
13.3.1.	Breeder Rod Bowing	13-6
13.3.2.	Initial Deflections	13-6
13.3.3.	Time-Dependent Behavior	13-7
13.3.4.	Relative Bowing in Different Breeding Zones	13-8
13.3.5.	Be Multiplier Rods	13-10
13.4.	CONCLUSIONS	13-11
	REFERENCES	13-12

13.1. INTRODUCTION

When a solid breeder is encased in a metal tube, its designer must use analyses of both the deflections and stresses in order to determine the number and type of supports to construct along the rod. For a thorough analysis, one must include such "loads" as thermal expansion, body weight of both the pellets and tube, and swelling. In addition, models for thermal and irradiation creep must be incorporated to model time-dependent response. For this report, analytical methods will be used to allow use of the most general, up-to-date material models.

The breeder rods, as shown in Fig. 1a, are modeled as Euler beams which are simply supported at each end and in the center. The thermal and radiation damage fields are assumed to be constant along the rod, varying only over the cross section. This leads to symmetry about the central support, allowing analysis of only half the length of the model as shown in Fig. 1b. The applied force per unit length q is equivalent to the weight loading, while arbitrary temperature and swelling variations over the cross section are also considered.

Fig. 1. Beam models used for bowing analysis.

13.2. ANALYSIS

13.2.1. BEAM THEORY

For a Bernoulli-Euler beam under axial and transverse loadings, the axial strain ϵ_z is given by

$$\epsilon_z = \epsilon_0 - x \frac{d^2 v}{d\xi^2} \quad , \quad (1)$$

where ϵ_0 is the strain at the central axis ($x = 0$) and v is the transverse beam deflection [1]. To relate ϵ_z to the stress σ_z , one requires a constitutive relation, which for a 1-D stress state is

$$\epsilon_z = \frac{\sigma_z}{E} + \alpha T + e^s + e^c \quad , \quad (2)$$

where αT , e^s , and e^c represent the thermal, swelling, and creep strains, respectively.

Combining Eqs. (1) and (2), multiplying by x , and integrating over the cross section, one finds the following second order equation for the displacement:

$$-EI \frac{d^2 v}{d\xi^2} = M + M_T + M_S + M_C \quad , \quad (3)$$

where I is the moment of inertia of the cladding cross section, E is its Young's Modulus and

$$\begin{aligned} M &= \int x \sigma_z dA \quad , \\ M_T &= E \int x \alpha T dA \quad , \\ M_S &= E \int x e^s dA \quad , \\ M_C &= E \int x e^c dA \quad . \end{aligned} \quad (4)$$

Using the standard sign conventions, the moment M can be related to q ; equilibrium requires

$$\frac{d^2 M}{d\xi^2} + q = 0 \quad . \quad (5)$$

Integrating Eq. (5) twice and using the boundary condition $M(L/2) = 0$, yields

$$M = C_1 \left(\xi - \frac{L}{2} \right) - \frac{1}{2} q \left(\xi^2 - \frac{L^2}{4} \right) \quad , \quad (6)$$

where C_1 is an arbitrary constant. Substituting Eq. (6) into Eq. (3) and integrating twice gives

$$\begin{aligned} -EIv = & C_2 \xi + C_3 + \frac{1}{2} \xi^2 C_1 \left(\frac{\xi}{3} - \frac{L}{2} \right) - \frac{1}{8} \xi^2 q \left(\frac{1}{3} \xi^2 - \frac{L^2}{2} \right) \\ & + (M_T + M_S) \frac{1}{2} \xi^2 + \int_0^\xi \int_0^\eta M_C d\psi d\eta \quad . \end{aligned} \quad (7)$$

Using the displacement boundary conditions $v(0) = v'(0) = v(L/2) = 0$, the constants are found to be

$$\begin{aligned} C_2 = C_3 = 0 \quad , \\ C_1 = \frac{5}{16} qL + \frac{3}{L} (M_T + M_S) + \frac{24}{L^3} \int_0^{L/2} \int_0^\eta M_C d\psi d\eta \quad , \end{aligned} \quad (8)$$

and Eq. (7) becomes

$$\begin{aligned} EIv = & \frac{1}{4} \xi^2 \left(1 - 2 \frac{\xi}{L} \right) (M_T + M_S) + \frac{1}{64} \xi^2 L^2 \left(1 - \frac{4}{3} \frac{\xi}{L} \right) \left(1 - 2 \frac{\xi}{L} \right) q \\ & + 6 \frac{\xi^2}{L^2} \left(1 - \frac{2}{3} \frac{\xi}{L} \right) \int_0^{L/2} \left(\frac{L}{2} - \eta \right) M_C d\eta - \int_0^\xi (\xi - \eta) M_C d\eta \quad , \end{aligned} \quad (9)$$

where Green's theorem has been used to reduce the double integral in Eq. (7) to a single integral, i.e.,

$$\int_0^\xi \int_0^\eta M_C d\psi d\eta = \int_0^\xi (\xi - \eta) M_C d\eta \quad . \quad (10)$$

13.2.2. STRESSES

To determine the axial stress we return to Eq. (2). Substituting Eq. (1) for ϵ_z and integrating over the cross section, ϵ_0 is found in terms of the axial force on a cross section F , and the inelastic strains:

$$\epsilon_0 = \frac{F + F_T + F_S + F_C}{EA} \quad , \quad (11)$$

where A is the cross-sectional area and

$$\begin{aligned} F &= \int \sigma dA \quad , \\ F_T &= E \int \alpha T dA \quad , \\ F_S &= E \int e^S dA \quad , \\ F_C &= E \int e^C dA \quad . \end{aligned} \quad (12)$$

Inserting Eq. (11) into Eq. (2) gives

$$\sigma = \frac{F + F_T + F_S + F_C}{A} - E(\alpha T + e^S + e^C) - xE \frac{d^2 v}{d\xi^2} \quad . \quad (13)$$

From Eq. (9), $d^2 v/d\xi^2$ is found to be

$$\begin{aligned} EI \frac{d^2 v}{d\xi^2} &= \frac{1}{2} (1 - 6 \xi/L)(M_T + M_S) \\ &\quad + L^2/32 (1 - 8 \xi/L)(1 - 2 \xi/L)q \\ &\quad + 12/L^2 (1 - 2 \xi/L) \int_0^{L/2} (L/2 - \eta) M_C d\eta - M_C \quad , \end{aligned} \quad (14)$$

and the stress equation [Eq. (13)] becomes

$$\begin{aligned} \sigma &= \frac{F + F_T + F_S + F_C}{A} - E(\alpha T + e^S + e^C) - \left(\frac{M_T + M_S}{2I} \right) x \left(1 - \frac{6\xi}{L} \right) \\ &\quad - qL^2 x / 32I (1 - 2 \xi/L)(1 - 8 \xi/L) \\ &\quad - \frac{12x}{IL^2} (1 - 2 \xi/L) \int_0^{L/2} (L/2 - \eta) M_C d\eta + M_C x / I \quad . \end{aligned} \quad (15)$$

13.2.3. CONSTITUTIVE EQUATIONS

To solve this rod bowing problem, a time-dependent or steady-state thermal field must be determined. To calculate the equivalent moments and forces of Eqs. (4) and (12) also requires equations for swelling and thermal and irradiation creep in terms of time (or dose) and temperature.

In this study, the swelling of the HT-9 cladding was assumed to conform to the following general relation [2]

$$\frac{\Delta V}{V} (\%) = S(\delta - \delta_I) \exp\left[-\left(\frac{T - 425^\circ\text{C}}{590^\circ\text{C}}\right)^2\right] \quad , \quad (16)$$

where S is the swelling rate (%/dpa), δ is the dose (dpa), δ_I is the incubation dose (dpa), and T is the cladding temperature ($^\circ\text{C}$). If δ is less than δ_I , $\Delta V/V$ is taken to be zero. Using data for 2-1/4 Cr-1 Mo gathered by Gelles and Puigh [3], S and δ_I were chosen to be 0.02%/dpa and 90 dpa, respectively. A 30-dpa incubation dose will also be studied.

Again using data from Gelles and Puigh, irradiation creep was modeled according to

$$\frac{de^c}{d\delta} (\text{dpa}^{-1}) = C\sigma^{1.5} \quad , \quad (17)$$

where C was found to be $8.511 \times 10^{-8} \text{ dpa}^{-1} \text{ MPa}^{-1.5}$.

The thermal creep correlation for HT-9 takes a different form [4]:

$$e_{th}^c = \frac{2289}{T} (|\sigma| - \sigma_0)^3 \exp\left(-\frac{1.427 \times 10^4}{T}\right) \quad . \quad (18)$$

Here e_{th}^c (yr^{-1}) is the thermal creep rate and σ_0 (MPa) represents a temperature dependent back stress given by

$$\sigma_0 = 1366.4 - 1.507 T \quad , \quad (19)$$

where T is the temperature (K). This formula predicts no thermal creep for temperatures below 500°C and stresses below 200 MPa, so irradiation creep dominates thermal creep in this analysis.

13.3. RESULTS

13.3.1. BREEDER ROD BOWING

Excessive deflections in a bank of solid breeder rods can severely impair reactor performance. Relative displacements of neighboring rods can choke off coolant flow paths, leading to hot spots and premature failure. In extreme cases, the rod cladding may buckle, causing changes in thermal gap conductances, and possibly tube rupture. Therefore, the peak rod deflections due to thermal, gravitational, and swelling effects must be kept small.

For the rod dimensions and reactor parameters analyzed here, the deflections due to swelling were by far the most severe. The bowing caused by the pellet weight was inconsequential because of the low density of the breeder and the thermal deflections were also relatively small, even for a 2-m-long rod with only one internal support. The final support spacing will depend on the peak bowing caused by swelling, as well as by the allowable cladding stresses.

13.3.2. INITIAL DEFLECTIONS

The rod bowing due to the pellet weight and thermal field is primarily driven by an azimuthal temperature variation over the cladding's cross section. In the first few breeder pins, the temperature gradient is due mostly to a gradient in the volumetric heating, so the front (plasma side) is hotter than the rear. On the other hand, pins near the back of the blanket are hotter at the rear of the tube because the heat transfer coefficient is highest there. This basic difference in the thermal fields causes the front pins to bow towards the plasma, while the rear pins deflect in the opposite direction.

The initial deflections of a 2-m-long rod with one internal support are shown in Fig. 2. The ends are assumed to be simply supported, while there is no axial restraint. The peak deflection is under 0.5 mm, which is negligible. Unfortunately, the swelling deformation is somewhat larger, so additional internal supports must be added to reduce the peak displacements to an acceptable level.

Fig. 2. Initial (thermal) deflections of a 2-m-long breeder rod

The model developed previously features only one internal support, so care must be taken if it is used to analyze multiple internal supports. To approximate the deflections of a rod with 50 cm between supports, the existing model can be used by setting L equal to 100 cm. The deficiency in this approach is seen in Fig. 2, which shows that the slope continuity condition at the center of the rod clearly is not met. Fortunately, the model is conservative because the calculated deflections will be greater than those in the actual blanket. The swelling behavior will determine the final choice for the number of internal supports.

13.3.3. TIME-DEPENDENT BEHAVIOR

Because the void swelling of HT-9 has a temperature variation that is Gaussian [Eq. (16)], the thermal gradient over the rod's cross section leads to an even harsher swelling gradient. Hence, one might expect swelling deformations to be quite severe. As shown in Fig. 3, the peak deflection after eight full power years overwhelmed the initial deflections, which are essentially zero on this scale. These deflections should be small enough to allow a four year life.

Fig. 3. Rod deflections over eight full-power years

The effect of the support spacing is shown in Fig. 4 which shows the peak deflections of rods with 100 and 50 cm between supports. In the case of the smaller spacing, the peak deflection is only about 1 cm after 6 years. Considering that the model over-estimates the rod deflections, this should be an adequate choice for the design.

13.3.4. RELATIVE BOWING IN DIFFERENT BREEDING ZONES

The deflections in the first breeder zone differ from those in the second zone for three reasons:

1. The second zone contains stiffer rods because of the larger diameter,
2. The thermal gradients are different, as discussed in Section 13.3.2,
3. The damage rate decreases with distance from the plasma.

All of these factors lead to lower deflections in the back zone.

In Fig. 5, the peak deflections are shown for the first row of each zone. Because of the 90 dpa incubation dose, there is no significant deformation for

Fig. 4. Bowing of rods with different support spacings

Fig. 5. Comparison of the peak deflections in two different breeder zones

the first three years. The first rod then begins to deflect towards the plasma at a peak rate of about 0.5 cm/yr, whereas the second rod shows no bowing through five years. The deflections here are quite small, leading to minimal tube contact during five full power years.

This rosy picture is altered by uncertainties in the swelling correlation for HT-9. If the incubation dose is smaller, the rods will begin to bow significantly at a lower dose and the peak deflections will be larger at a given time. In Fig. 6 we see the peak deflections for a 30 dpa incubation dose. The first zone begins bowing after about a year and the second zone begins after about two years. Because the two rods are bowing in opposite directions, the most important rod interaction will involve the multiplier rods and the first breeder zone.

13.3.5. Be MULTIPLIER RODS

In terms of bowing, the multiplier rods have two distinct advantages over the breeder rods. First, the multiplier rod is solid, so its moment of inertia is much larger than the breeder rods. Second, the thermal conductivity of Be is much higher than the effective conductivity of the breeder pellet and its cladding, so the temperature gradient over the cross section is much smaller. The resulting deflections are very small.

For a 2-m-long Be rod with one internal support and a linear temperature dependence, Eq. (9) predicts a peak deflection of

$$v_{\text{peak}} = 0.015 \Delta T \quad , \quad (20)$$

where ΔT ($^{\circ}\text{C}$) is the front-to-back temperature difference over the cross section. Thermal calculations similar to those for the breeder rods predict a difference of only 2°C , leading to a deflection of less than 1 mm.

Because the temperature is nearly constant in the Be rods, the swelling gradient will depend only on the damage gradient. The damage gradient is expected to be small in the individual multiplier rods because of their relatively small diameter, so the Be rod deflections should remain low. More detailed neutronics calculations are required before this assumption is validated.

Fig. 6. Peak deflections in two breeder zones with reduced incubation dose

13.4. CONCLUSIONS

The most important outcome of this bowing analysis is the determination of the number of internal breeder rod supports required for good thermal performance. Although many effects were considered, the swelling deformations were the most restrictive in terms of the peak deflections. It appears that three internal supports should be sufficient to keep the rod bowing below acceptable levels without significantly raising the structure-to-breeder ratio.

The most severe interaction problem involved the last multiplier row and the first breeder row. Because the Be rods deflect very little, this problem can be alleviated by leaving enough space between these two zones. The spacing can be tighter elsewhere. A more detailed analysis of the multiplier rod bowing caused by the damage gradient must be analyzed before this can be verified.

REFERENCES

1. T. H. Lin, Theory of Inelastic Stresses, (Wiley & Sons, New York, 1968) p. 128.
2. R. W. Conn and N. M. Ghoniem, "Assessment of Ferritic Steels for Steady-State Fusion Reactors," Fusion Reactor Design and Technology, IACA-TC-392/62, (International Atomic Energy Agency, Vienna, 1983) Vol. II, p. 389.
3. D. S. Gelles and R. J. Puigh, "Evaluation of Ferritic Alloy Fe-2-1/4 Cr - 1 Mo After Neutron Irradiation-Irradiation Creep and Swelling," Hanford Engineering Development Laboratory Report HEDL-7405 (1983).
4. R. J. Amodeo and N. M. Ghoniem, "Development of Design Equations for Ferritic Alloys in Fusion Reactors," Nucl. Engrg. and Design/Fusion, 2 (January 1985) pp. 97-110.

CHAPTER 14

ELASTIC STRUCTURAL ANALYSIS OF THE PIN-TYPE SOLID BREEDER BLANKET FIRST WALL

George E. Orient and Nasr M. Ghoniem

CONTENTS

14.1.	INTRODUCTION	14-1
14.2.	MODELING	14-2
14.2.1.	Stiffness Matrix and Consistent Load Vectors for a 2-D Frame Element	14-2
14.2.2.	Application of 2-D Frame Elements to Analyzing Thin Planar Structures; Boundary Conditions	14-6
14.3.	RESULTS OF CALCULATIONS	14-7
14.4.	ANALYSIS OF THE SIDE WALL	14-13
14.5.	DISCUSSION OF LIMITATIONS AND UNCERTAINTIES	14-15
	REFERENCES	14-17

14.1. INTRODUCTION

A key to the successful design of a fusion blanket is the distribution of the amount of structural material in the first wall and the blanket enclosing structure. The first wall must support dynamic and static mechanical loads and at the same time sustain the thermal gradients and erosion resulting from the plasma heat and particle fluxes. Dynamic and static loads require high flexural rigidity, which is influenced by flow channels designed to cool the high heat flux incident on the first wall. Since the first wall temperature is roughly proportional to its thickness, thermal stresses require a thin first wall. On the other hand, a thick wall is necessary to withstand surface erosion due to particle fluxes. A feasible design must therefore represent an optimal solution satisfying such conflicting requirements.

The structural design of the breeder blanket must not only satisfy the steady state operating conditions, but must also meet cyclic conditions due to start-ups, shut-downs and non-steady operation. Fatigue of the first wall, crack propagation under cyclic load and residual stress build-up are some of the most important issues related to repetitive loading conditions. Another problem arising from the time history of the loads is thermal shock due to power ramps. One also has to consider emergency situations such as loss of pressure.

Throughout its life, the blanket will be subjected to a radiation environment causing radiation induced creep, swelling, and embrittlement. Although creep has a tendency to relieve high stresses due to thermal gradients, it can cause large displacements. Under axially restrained conditions creep buckling and high stresses due to swelling may be important. Loss of the structural material due to erosion must also be considered so that the module can sustain the loads under end of life conditions. A satisfactory structural design requires that all of these operating conditions be met without loss of structural integrity of the system.

To address some of the issues described above and gain some insight into scaling, a displacement type finite element model, based on classical beam theory, has been developed.

14.2. MODELING

The first wall structure is modeled here as an assembly of curved I-beams. Then the resulting thermoelasticity problem is solved using a displacement type finite element formulation. The curved beam is discretized in 2-D frame elements. The following assumptions have been adopted:

1. A homogeneous, isotropic, linearly elastic material,
2. Linear thermal expansion,
3. Cross sectional dimensions are small compared to the radius of curvature.

14.2.1. STIFFNESS MATRIX AND CONSISTENT LOAD VECTORS FOR A 2-D FRAME ELEMENT

When deriving a displacement type finite element formulation, one has to choose a displacement field which satisfies the following conditions:

- It can describe the rigid body motions of a particular element,
- It has sufficient freedom to allow kinematic continuity between the elements,
- It can produce a displacement pattern corresponding to the constant strain state of the elements. This requirement is necessary for the element in order to be able to converge to the true strain state upon refining the partition of a body in a general strain state.

Consider a two-dimensional frame element given in Fig. 1. Its behavior for small displacements is a superposition of that of a bar and a beam. Let $u(x)$ and $v(x)$ denote the axial and transverse displacements in the local coordinate system. Later quantities will be expressed in a global coordinate

Fig. 1. Forces and displacements on a 2-D frame element

system. The global quantities will be denoted by a prime. The nodal displacement quantities associated with a frame element are the axial and transverse components of the motion, u , v and the rotation at the nodal points, β . The corresponding force and moment quantities are F_x , F_y and M . Subscripts 1 and 2 refer to the arbitrarily chosen "beginning" and the "end" of the element. Transverse and axial distributed line loads are denoted by p and q , respectively. Consider a linear interpolation for $u(x)$ and a cubic interpolation for $v(x)$. The rigid body motion of a bar is a translation which is clearly included in a linear displacement field. Since the nodal degrees of freedom are u_1 and u_2 in the axial direction, inter-element continuity is guaranteed. For a bar, the extensional strain is equal to the first derivative of the axial displacement field; consequently, a linear interpolation function permits a constant strain state. The rigid body motions of a beam are uniform transverse translation and a rotation which are included in a cubic field. There are four generalized nodal degrees of freedom, because classical beam theory requires that the displacement and the slope to be continuous between elements. This makes it necessary to choose at least a cubic interpolation function. The constant strain mode for thin beams is a constant curvature, which requires a displacement at least quadratic in the axial coordinate. Thus,

$$u(x) = N_a^T u_a = [1-\xi] \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (1)$$

$$v(x) = N_v^T u_a = [1-3\xi^2+2\xi^3 \quad d(\xi-2\xi^2+\xi^3) \quad 3\xi^2-2\xi^3 \quad d(-\xi^2+\xi^3)] \begin{bmatrix} v_1 \\ \beta_1 \\ v_2 \\ \beta_2 \end{bmatrix} \quad (2)$$

where ξ denotes x/d and d is the length of the element. The axial strain is expressed in terms of the displacement field as follows (Euler-Bernoulli beam theory):

$$\epsilon_x = \frac{du}{dx} - y \frac{d^2 v}{dx^2} \quad (3)$$

The axial stress is given by linear stress-strain relations:

$$\sigma_x = E(\epsilon_x - \alpha T) \quad (4)$$

and the strain energy is:

$$U = \frac{1}{2} (\epsilon_x - \alpha T) \sigma_x = \frac{E}{2} (\epsilon_x - \alpha T)^2 \quad (5)$$

The total potential energy of the element is defined as:

$$V = U + V_E \quad (6)$$

where the potential energy of the external forces is given by:

$$V_E = -F_{x1}u_1 - F_{x2}u_2 - \int_0^d p(x)u(x)dx - F_{y1}v_1 - F_{y2}v_2 - M_1\beta_1 - M_2\beta_2 - \int_0^d q(x)v(x)dx \quad (7)$$

Applying the theorem of minimum potential energy, $\delta V=0$ leads to the complete stiffness relation:

$$\begin{bmatrix} X_1 \\ Y_1 \\ M_1 \\ X_2 \\ Y_2 \\ M_2 \end{bmatrix} = \underline{K} \begin{bmatrix} u_1 \\ v_1 \\ \beta_1 \\ u_2 \\ v_2 \\ \beta_2 \end{bmatrix} - \underline{P} - \underline{P}_T \quad (8)$$

$$\underline{K} = E/d \begin{bmatrix} A & 0 & 0 & -A & 0 & 0 \\ & 12/d^2 & 6I/d & 0 & -12/d^2 & 6I/d \\ & & 4I & 0 & -6I/d & 2I \\ & \text{symmetric} & & A & 0 & 0 \\ & & & & 12I/d^2 & -6I/d \\ & & & & & 4I \end{bmatrix} \quad (9)$$

$$\underline{P} = \begin{bmatrix} pd/2 \\ qd/2 \\ qd^2/12 \\ -pd/2 \\ qd/2 \\ -pd^2/12 \end{bmatrix} \quad \underline{P}_T = \begin{bmatrix} -F_T \\ 0 \\ M_T \\ -F_T \\ 0 \\ -M_T \end{bmatrix}$$

$$F_T = \int_A \alpha T(y,z) dA \quad M_T = \int_A y \alpha T(y,z) dA \quad (10)$$

where A denotes the area of the cross section and I is the moment of inertia. The transverse and axial loads are taken constant over the length of an element to obtain the consistent load vector in (10). The temperature field $T(x,y,z)$ has been replaced by

$$T(y,z) = \frac{1}{d} \int_0^d T(x,y,z) dx \quad (11)$$

which does not affect convergence if one takes fine enough partitions to capture the change of the thermal field in the x direction. It can be observed that thermal effects appear as a consistent load vector. It can also be shown that the stiffness matrix, $\underline{K}$ exhibits all the necessary rigid body motions and constant strain modes as it can be anticipated from the orders of interpolations used to describe the displacement field.

For practical applications, it is convenient to express the nodal displacements and forces in terms of global coordinates. The transformation is given by the following relations:

$$\begin{bmatrix} u' \\ v' \\ \beta' \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ \beta \end{bmatrix} = \underline{T}_1 \begin{bmatrix} u \\ v \\ \beta \end{bmatrix} \quad (12)$$

$$\underline{u}' = \begin{bmatrix} u'_1 \\ v'_1 \\ \beta'_1 \\ u'_2 \\ v'_2 \\ \beta'_2 \end{bmatrix} = \begin{bmatrix} \underline{T}_1 \\ \\ \\ \underline{T}_1 \\ \\ \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ \beta_1 \\ u_2 \\ v_2 \\ \beta_2 \end{bmatrix} = \underline{T} \underline{u} \quad (13)$$

$$\underline{F}' = \underline{T} \underline{F}$$

$$\underline{F}'_T = \underline{T} \underline{F}_T$$

$$\underline{K}' = \underline{K}^T \underline{K} \underline{T} \quad (14)$$

14.2.2. APPLICATION OF 2-D FRAME ELEMENTS TO ANALYZING THIN PLANAR STRUCTURES; BOUNDARY CONDITIONS.

In order to apply the previous method to the analysis of the first wall, we first define a discretization method of its structure. Then, it is assumed that the elements are interconnected at nodal points, and therefore, system stiffness relation is established. This takes a form similar to that of an element stiffness relation,

$$\underline{K} \underline{u} = \underline{F} \quad (15)$$

where $\underline{K}$ is a banded matrix. The bandwidth is determined by the numbering of the nodes.

Next, we modify the stiffness relation by including boundary conditions. For example, built-in conditions are achieved by setting both displacements and slope to zero at the boundary, i.e.,

$$u = 0, v = 0, \beta = 0 \quad (16)$$

while symmetry with respect to a horizontal axis is modeled by setting the axial displacement and the slope zero,

$$v = 0, \beta = 0 \quad (17)$$

Finally, a simple support condition is such that only rotation is allowed:

$$u = 0, v = 0 \quad (18)$$

To delete the i^{th} degree of freedom, we simply zero out the i^{th} row and i^{th} column of the system and replace K_{ii} by one. This guarantees that upon solving the equation, the i^{th} displacement component will be zero. Using element stiffness relations, we then solve for nodal forces and hence nodal stresses.

The modeling process described above is applied as follows: Taking advantage of symmetry one half of the first wall in one blanket module is considered. This is realistic, since the blanket structure and the loads are symmetric. The corresponding boundary conditions are (16) at the location where the neighboring modules meet and (17) at the middle of the segment. To discretize the structure, we divided it into five segments. Calculations to check accuracy using ten segments are also performed. It is found that approximately five segments give an accurate representation of the stress distribution. Integrals appearing in the expressions for thermal forces and moments are calculated by Simpson's rule using 12 partitions of the cross sections.

14.3. RESULTS OF CALCULATIONS

Fig. 2 shows the pure pressure stresses at beginning of life. It can be observed that the stresses are positive due to the internal pressure and the support gives rise to local bending. As a result, the stresses at the plasma side tend to be less positive, while the inner side stresses are increased. The design limit for the primary stresses at the operating temperature is approximately 170 MPa. To allow for a sacrificial layer for erosion, the plasma side of the first wall is thickened.

Results for thermal and pressure stresses for this design, based upon a temperature profile without a central fin as discussed in reference [1], are given on Figs. 3-6. Based on the assumption that the support behind the manifold allows thermal expansion of the whole unit, the base temperature for the thermal strain is taken as the inlet coolant temperature. The plasma

Fig. 2. Pressure stresses in the first wall, beginning of life

Fig. 3. Pressure and thermal stresses in the first wall beginning of life (one fin, no roughness)

Fig. 4. Pressure and thermal stresses in the first wall beginning of life (one fin with roughness)

Fig. 5. Pressure and thermal stresses in the first wall beginning of life (two fins, no roughness)

Fig. 6. Pressure and thermal stresses in the first wall beginning of life (two fins with roughness)

side of the first wall experiences compression due to the higher temperatures. The relatively high level of the stresses results from the interaction between the high bulk temperature and the supports, as well as the gradients across the first wall. It is found that bulk temperature is the more important factor here, since azimuthal variation of the stresses was not large.

In order to reduce the bulk temperature, fins in the coolant channels and surface roughening have been employed to improve heat transfer between the coolant and the structure. Figs. 3 and 4 show total stresses with one fin in the coolant channel and with and without surface roughening. Improvements due to surface roughening are relatively because only a small fraction of the total surface is assumed to be roughened. On Figs. 5 and 6, the effect of surface roughening can be observed when there are two fins present in the coolant channel. The stresses are lower than the values for the single fin configuration, primarily because of the lower bulk temperature. The values are relatively high, but referring to the design guidelines for secondary stresses published in reference [3] they are allowable. At operating temperature the allowable primary stress for C-9 is about 170 MPa. The design limit for secondary stresses which include thermal stresses is three times the value for the primary stresses.

Fig. 7 illustrates what the pressure stresses might look like at the end of life assuming three full-power years of operation and 1 mm/year erosion rate. The point here is that the pressure stresses are below the allowable limit. Stresses shown here are to be taken only as estimates, since many processes neglected here may significantly alter the elastic state of the structure.

Fig. 7. Pressure stresses in the first wall, end of life

14.4. ANALYSIS OF THE SIDE WALL

In order to attain inherent structural safety, the design of the blanket must be failure tolerant. Loss of pressure or rupture of a module must not start a catastrophic chain of failures propagating through the whole blanket. As the worst case one can consider a single module loaded by the internal pressure of the coolant, while the neighboring module suffered total loss of pressure. In order to be safe, a module has to be able to support the pressure without the equalizing pressure on the side walls resulting from the presence of the adjacent pressurized module. In principle, a tapered side wall could result in an optimal load carrying structure and minimal impact on the blanket performance, since the added structural material would appear at the back of the blanket. However, the loads and the geometry of our design would require excessively thick side wall at the inlet manifold. Alternatively, periodic supports as shown on Fig. 8 along the axis of the module can add the necessary stiffness to the side wall. The necessary spacing can be estimated as follows:

Consider the rectangular segment of the side wall bounded by the manifold, the supports and the cylindrical part of the first wall. Relaxing the connectivity conditions at the top of the segment, the boundary conditions are those shown in Fig. 8(b). Further relaxing the symmetry conditions at the vertical edges of the segment, simple supported conditions can be used. This modeling process is rather conservative, but will yield a safe geometry. As it turns out, it is necessary to assume a mechanical coupling between the side walls of the adjacent modules in order to achieve a flexural rigidity that permits reasonable spacing between the supports. The flexural rigidity of a double-channelled cross section as shown on Fig. 9. is given by

$$D = \frac{E}{1-\nu^2} \int_A y^2 dy = \frac{E}{1-\nu^2} ((h_1 + h_2 + h_3)^3 - (h_2 + h_3)^3 + h_3^3) \quad (19)$$

after neglecting the walls of the channels. The maximum stress occurs in the middle of the free edge and at the outer fiber it is given by [2]

Fig. 8. Side wall modeling process

Fig. 9. Cross section geometry of the coupled side wall

$$\sigma = \frac{E(h_1 + h_2 + h_3)}{2(1-\nu^2)} \frac{b^2}{D(b/a)^2} p\beta \quad (20)$$

where p is the coolant pressure, a is the distance between supports, b/a is the aspect ratio of the rectangular plate and β is a numerical factor tabulated for different aspect ratios on page 210 of reference [2]. For the current design $h_1=0.025\text{m}$ $h_2=0.01\text{m}$ and $h_3=0.005\text{m}$ and $b=0.6\text{m}$ resulting $D=7.84 \text{ MPa m}^2$. An aspect ratio $b/a=1/3$ yields 251 MPa maximum stress. Noting that it occurs at the middle of the free end in the model, which is a connection line to the cylindrical surface in the real structure this value is acceptable. Safer still, if $b/a=1/2$ then the maximum stress is 110 MPa, therefore supports at every 1.2 m will provide enough rigidity.

14.5. DISCUSSION OF LIMITATIONS AND UNCERTAINTIES

While this model is based upon a finite element model to the first wall geometry, it has not accounted for several important aspects. Future efforts may be necessary to resolve the importance of these aspects, which are discussed below:

1. Further elastic-plastic analysis is necessary to better understand the scaling of geometry, since the stresses exceed the elastic

limit. Furthermore, residual stress build-up due to cyclic load should also be calculated.

2. The built-in end condition at the joints with the side walls is valid, since there is no rotation and translation in the radial direction due to symmetry. Translation in the tangential direction is negligible compared to displacements in other locations, because temperatures are the lowest at the joint. Application of symmetry conditions in a continuous curved beam at the top of the arch may not be realistic, depending upon the real geometry at this point. If a channel is present there, a hinge type boundary condition may be a better approximation, if a significant portion of the cross section is removed. In this case, a detailed analysis is necessary to quantify stress concentrations around coolant channels.
3. Swelling is a significant phenomenon for an axially restrained structure, such as a curved member. The equivalent thermal field arising from swelling is superposed on the real temperatures and can cause high stresses.
4. Since the structure is made of a ferritic alloy (9-C), significant body forces may be present due to the magnetic field, and proper analysis should address this question. If the distribution of the magnetic body force field is known then the tangential and normal components can be calculated and input as distributed transverse and axial loads in equation (10).
5. Since the temperatures throughout the first wall structure are high, both thermal and irradiation creep are expected to play significant role during the lifetime of the blanket. A treatment of the creep contribution is discussed in reference [4] and is directly applicable to this problem.
6. Swelling strains could be easily incorporated similarly to the consistent loads due to thermal strains given in equation (10).
7. Time-dependent erosion can be included when a total time analysis is performed by changing the cross section geometry as the solution continues.

REFERENCES

1. S. Grotz and N. M. Ghoniem, "Thermal Analysis of a Pin-Type Blanket for Tokamak Reactors," Chapter 11 this report; also UCLA Report UCLA-ENG-8612/PPG-938 (1986).
2. S. Timoshenko and S. Woinowsky-Krieger, Theory of Plates and Shells, (McGraw-Hill, New York, 1959).
3. "Impurity Control and First Wall Engineering," Chapter VII, FED-INTOR/ICFW/82-17 (1981).
4. G. E. Orient, P. Gierszewski and J. G. Garner, "Elastic-Plastic Analysis of Slender First Wall Structures," Fusion Technol., 8 (1985) 568-591.

CHAPTER 15

A MODEL FOR THE MECHANICAL PELLET-CLADDING INTERACTION DURING POWER CYCLES

George E. Orient and Nasr M. Ghoniem

CONTENTS

15.1.	INTRODUCTION	15-1
15.2.	THEORETICAL FORMULATION	15-4
15.2.1.	Assumptions and Model Restrictions	15-4
15.2.1.1.	General Solution for the Pellet	15-5
15.2.1.2.	General Solution for the Cladding	15-7
15.2.1.3.	Boundary Conditions	15-7
15.2.2.	Method of Numerical Solution	15-8
15.2.3.	Effect of the Power Cycles	15-9
15.3.	MATERIAL CHOICES AND PROPERTIES	15-9
15.4.	RESULTS	15-11
15.4.1.	Effect of Solid Breeder/Cladding Compatibility	15-12
15.4.2.	Effect of Initial Gap Thickness	15-12
15.4.3.	Effect of Preheating	15-12
15.4.4.	Effect of the Temperature Ramp	15-17
15.4.5.	Effect of Cyclic Operation	15-17
15.4.6.	Effect of Pellet Swelling - Integrated Analysis	15-17
	REFERENCES	15-25

15.1. INTRODUCTION

The use of solid breeders in blanket components of fusion reactors is necessitated by the need for tritium breeding, with avoidance of problems arising from liquid metal breeders; such as corrosion and MHD pressure drop. However, solid breeders have a relatively low fracture strength and poor mechanical properties. Their ability to withstand mechanical loads and thermal gradients is greatly reduced, leading to intolerable design limits. Another problem with solid breeder materials is the high retention of tritium and helium if the breeder is fully dense. To allow for tritium diffusion, and as a practical solution to their mechanical problems, solid breeders can be produced as porous square cylinders of dimensions on the order of 1-3 cm. Such cylinders, called pellets, are encapsulated in a structural cladding material which provide for a barrier to their interaction with the coolant. Moreover, the cladding material carries existing structural and thermal loads.

Figure (1) shows the blanket configuration, where the material 9-C (low activation equivalent of HT-9) is used for the outside structure of the module, as well as for solid breeder pin cladding. The first wall is cooled with high velocity helium through coolant channels, while the pins are cooled by helium in a cross-flow configuration. The first zone is made up entirely of Beryllium pins for neutron breeding, and then two solid breeder zones follow. The configuration and dimensions of pins in these zones are shown in figure (2).

In order to minimize maintenance time, a satisfactory configuration has to withstand transient phenomena associated with power-ramps and shutdowns while giving optimal thermomechanical performance in reasonable lifetime. These objectives can be reached by selecting suitable geometry and material specifications as well as prescribed temperature histories. Finding the minimal allowable ramp rate for startup is important from the economy of the operation, since overly conservative startup can amount to substantial percentage of the power cycle. In the stage of preliminary design the main concern is failure of the cladding due to high stresses or large total strains. The following analysis provides a simplified tool for design based on macroscopic mechanical interaction between the breeder and the cladding. The next most important effect to include in the analysis is

Fig. 1. Blanket configuration

PIN STACKING: BREEDER ZONE 1

PIN STACKING: BREEDER ZONE 2

Fig. 2. Pin stacking

phenomena associated with failure of materials at the micro-scale such as stress corrosion, creep fracture and fatigue. Some of these aspects of pellet-cladding interaction are covered in reference [4].

15.2. THEORETICAL FORMULATION

15.2.1. ASSUMPTIONS AND MODEL RESTRICTIONS

Since the thermal field (reference [1]) exhibits only a slight azimuthal asymmetry, the problem is formulated based on an axisymmetric analytical solution for the radial temperature distribution. Interaction between the changing geometry and the thermal field has been neglected during the course of the analysis. The size of the developed gap is in the order of hundredth of a millimeter which justifies this simplification.

The large length-radius ratio implies plane strain conditions for most of the pin. This is the result of the different cross-sections constraining each other from axial movement. Plane strain formulation gives a conservative estimate on the radial deformations and the stress field near free surfaces, therefore it can be accepted for design purposes.

Small deformation theory is used throughout the entire analysis. The thickness of the cladding is assumed to be small compared to its diameter, which leads to thin shell formulation in the cladding.

Material models for the pellet include linear elastic behavior, linear thermal expansion and temperature dependent swelling. The cladding material was assumed to exhibit linear elastic behavior, linear thermal expansion, temperature dependent creep. All linear material properties are taken as constants, although temperature dependence, especially in the pellet may have an influence on the results.

Quasi-static solution method is employed, for the frequency of the changes in the thermal field are small compared to the structure's eigenfrequencies. The presence of material and geometrical imperfections is also neglected.

The time dependent temperature field is also represented in a quasi-static manner as follows:

$$T(r,t) = T_{\text{preheat}} + (R(r) - T_{\text{preheat}}) T(t)$$

where $R(r)$ is the steady state temperature profile. Swelling is assumed to act only at full power, that is, when the temperature profile is fully developed.

For this analysis densification of the pellet material was neglected due to sparsity of available data and in order to simplify the analysis. This material nonlinearity might also be an important component in the interaction. Its effect would lower the contact stresses, therefore the current analysis is conservative as far as analysis of the cladding is concerned.

15.2.1.1. General Solution for the Pellet

Using the notation shown on Fig. 3., the field equations corresponding to the above specified assumptions are as follows:

Strain-displacement relations:

$$\epsilon_r = \frac{u}{r} \quad (1)$$

$$\epsilon_t = \frac{du}{dr} \quad (2)$$

Strain - stress relations:

$$\epsilon_r = \frac{1}{E} ((1+\nu)\sigma_r - \nu\sigma_{kk}) + \alpha T + \epsilon^{sw}/3 \quad (3)$$

$$\epsilon_t = \frac{1}{E} ((1+\nu)\sigma_t - \nu\sigma_{kk}) + \alpha T + \epsilon^{sw}/3 \quad (4)$$

$$\sigma_{kk} = \sigma_{rr} + \sigma_{tt} \quad (5)$$

where r and t subscripts denote radial and tangential quantities respectively, ϵ^{sw} is the swelling strain, and the temperature measured from some base value, T_0 . The $1/3$ factor gives dilatational strains from the volumetric strain when infinitesimal strains are assumed.

Equilibrium equations for an axisymmetric stress state reduce to one single equation:

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_t}{r} = 0 \quad (6)$$

Fig. 3 Geometry, notations

Substituting the strain-displacement relations (1) and (2) into the constitutive equations (3) and (4), then the stresses in the equilibrium equation we arrive at the displacement form of the equilibrium.

$$\frac{d}{dr} \left[\frac{1}{r} \frac{d(ru)}{dr} \right] = \frac{1+\nu}{1-\nu} \frac{d}{dr} (T\alpha + \epsilon^{sw}/3) \quad (7)$$

The solution of this equation for an axisymmetric solid cylinder is:

$$u_r = \frac{(1+\nu)}{r} \int_a^r (\alpha T(r', t) + \epsilon^{sw}(r', t)/3) r' dr' + Cr - \theta(t) + Cr \quad (8)$$

$$\sigma_r = - \frac{E}{r^2} \int_a^r (\alpha T(r', t) + \epsilon^{sw}(r', t)/3) r' dr' + C \frac{E}{1-\nu} - \Phi(t) + CA \quad (9)$$

$$\sigma_t = \Phi(t) - E(\alpha T(r, t) + \epsilon^{sw}/3) + CA \quad (10)$$

Equation 2.8 represents the radial displacement of the pellet at any given position r while equations 2.9 - 2.10 give the two dimensional stress field within the cross-section of the pellet.

Introduction of auxiliary functions Φ and Θ will make the rest of the equations more compact.

15.2.1.2. General Solution for the Cladding

Strain-displacement relation:

$$2\epsilon = \frac{2u}{b+d} \quad (11)$$

Stress-Strain relation:

$$2\sigma_c = 2E(2\epsilon - 2\alpha 2T(t) - \epsilon_c - \epsilon^R) \quad (12)$$

where the notation is as follows:

ϵ_c : total strain

ϵ_c : accumulated creep strain

ϵ^R : residual strain present, zero at initial state

and superscript 2 denotes quantities associated with the cladding in general.

15.2.1.3. Boundary Conditions

Continuity of displacements:

$$u_r(b,t) - d = 2u(t) \quad (13)$$

Continuity of tractions:

$$-(p_0 + \sigma_r(b,t)) \frac{b+d}{h} = 2\sigma_c \quad (14)$$

where p_0 denotes the coolant pressure which produces compressive stress in the cladding. Since the radial stress in the pellet is negative after contact 14 gives the right signs. Equations (13) and (14) lead to the following integral equation for the unknown function $C(t)$:

$$C(t) = F(t) + \epsilon^c \left(-\frac{b+d}{h} (\Phi(t) + AC(t)), T(c), t \right) \quad (16)$$

where

$$\delta = \frac{b+d}{h} A + 2E \frac{b}{b+d}$$

$$F(t) = \frac{1}{\delta} \left[-\frac{b+d}{h} (\sigma(t) + p_0) - 2E \frac{\theta(t) - d}{b+d} + 2E(2\alpha T(t) + \epsilon^x + \epsilon^{sw}/3) \right] \quad (17)$$

and $\epsilon^c(\sigma(t), T(t), t)$, denotes the functional that gives the total creep strain due to a load history $\sigma(t)$ and temperature history $T(t)$. Assuming time-hardening which is identical to strain-hardening for continuous loads the creep strain functional has the following form:

$$\epsilon^c = \int_0^t \left[\frac{\hat{\sigma}e(\kappa, \theta, t')}{\delta t'} + \frac{\hat{\sigma}e(\kappa, \theta, t')}{\delta \theta} \frac{\delta \theta}{\delta t'} \right]_{\substack{\kappa=\sigma(t') \\ \theta=T(t')}} dt' \quad (18)$$

where $\hat{\sigma}e(\sigma, T, t)$ is the uniaxial creep response of the material to constant load σ and temperature T . Equation (2.16) is in the form of the familiar nonlinear Volterra equation of the second kind whose solution procedures are well established.

15.2.2. METHOD OF NUMERICAL SOLUTION

To simplify the treatment here let us introduce a compact notation for the integral equation (17) here as follows:

$$G(t) = F(t) + \int_0^t K(t, \alpha, G(\alpha)) d\alpha \quad (19)$$

Now we discretise the time domain and evaluate (19) at points $t=nh$, $n=0, 1, \dots, N$.

$$G(nh) = F(nh) + \int_0^{nh} K(nh, \alpha, G(\alpha)) d\alpha \quad (20)$$

Equation (20) is approximated by using the trapezoidal rule:

$$G(nh) = F(nh) + \frac{h}{2} (K(nh, 0, G(0)) + K(nh, nh, G(nh))) + h \sum_{m=1}^{n-1} K(nh, mh, G(mh)) \quad (21)$$

In the last equation $C(nh)$, the unknown quantity appears on the right hand side. Replacing it by $C((n-1)h)$ gives an estimate for $C(nh)$. Subsequent refined values can be obtained by substituting the current estimate in the right hand side of (21) in place of $C(nh)$. This process is repeated until $C(nh)$ attains a predefined relative accuracy. It can be shown that the method given here is equivalent to the implicit modified trapezoidal rule discussed in some detail in reference [3]. The order of the method is two, which means that the global error behaves like $O(h^2)$. This scheme works well on stiff systems although several further refinements could be introduced. The present system shows stiffness in the solution, since after the transients due to power variations die out the solution exhibits a slowly varying quasi-steady state behavior. Tracking this part of the curve accurately is important, because it also contributes to the total accumulated strains present at the beginning of the subsequent cycle. This, in turn, defines the behavior during the next transient.

15.2.3. EFFECT OF THE POWER CYCLES

To simulate real operational conditions, a typical time history shown in Fig. 4 is used. One cycle consists of a power ramp, operation at full power, shutdown and dwell time at preheat temperature. If creep deformation occurs during the cycle then there will be a residual plastic strain in the clad after it separated from the pellet. This effect is equivalent to a permanent increase in the diameter of the clad and causes it to reach contact in the subsequent startup at a later time within the cycle. Since the formulation given above is valid only during contact, the residual strains accumulated during contact and, the total time spent under swelling conditions has to be followed carefully in the course of the numerical solution.

15.3. MATERIAL CHOICES AND PROPERTIES

Two sets of materials are considered in this analysis. We will first analyze the combination of a Li_2O solid breeder clad with 2.25CrMo steel. Then we present the same analysis for $LiAlO_2$ and the 9-C martensitic steel. The corresponding material properties are listed below [2].

Fig. 4. Time history $T(t)$ for assumed cyclic loading

Pellet : Li_2O

Young's modulus : 60 GPa

Poisson's ratio : 0.25

Coefficient of thermal expansion : $26.6 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$

Swelling equation:

$$\epsilon_{\text{sw}} = \left[0.3102 \frac{kT}{\gamma} \frac{fGt}{N^{1/3}} \right]^{3/2}$$

where the volumetric strain is given in (m/m), temperature is in absolute scale and time is measured in seconds.

$$N = 125479.5 \exp(-0.001000826 T) \times 10^{-14} \text{ cm}^{-3} \quad (22)$$

$$\gamma = 6.24 \times 10^{14} \text{ eV cm}^{-3}$$

$$G = 5.299 \times 10^{13} \text{ cm}^{-3} \text{ s}^{-1}$$

$$f = 0.035$$

$$k = 8.615 \times 10^{-5} \text{ eV}^\circ\text{K}^{-1}$$

Cladding : 2.25Cr - 1Mo

Young's modulus : 200 GPa

Poisson's ratio : 0.25

Coefficient of thermal expansion: $13 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$

Creep equation:

$$\epsilon^c = \frac{B}{kT} (\sigma - \sigma_0)^3 \exp(-Q/kT) H(\sigma - \sigma_0) \quad (23)$$

where the creep strain is given in (m/m) and the temperature dependent "back-stress" is defined as:

$$\begin{aligned}\sigma_0 &= aT + c \\ B &= 13209.0 \\ k &= 0.0000862 \\ Q &= 2.767 \\ a &= -0.2185 \\ c &= 198.1783\end{aligned}$$

Temperature is on the absolute scale and the stress is measured in Mpa.

Pellet : LiAlO₂

Young's modulus : 71 GPa
Poisson's ratio : 0.25
Coefficient of thermal expansion : $11.96 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$

The swelling equation is the same as (2.22) with the following parameters:

$$N = (0.0000292T^2 - 0.0292T + 7.71) \times 10^{-14} \text{ cm}^{-3} \quad (24)$$

and

$$\begin{aligned}\gamma &= 6.24 \times 10^{14} \text{ cm}^{-3} \\ G &= 5.299 \times 10^{13} \text{ cm}^{-3} \text{ s}^{-1} \\ f &= 0.035 \\ k &= 8.615 \times 10^{-5} \text{ eV}^\circ\text{K}^{-1}\end{aligned}$$

Cladding : 9-C

Young's modulus : 180.0 GPa
Poisson's ratio : 0.265
Coefficient of thermal expansion : $12 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$

15.4. RESULTS

Although superposition of the phenomena driving the pellet-cladd interaction is not possible due to their nonlinearity, their effects can be studied qualitatively on separate analyses. In order to gain quantitative data, integrated analysis has to be done. In the following, we present results of calculations illustrating the effects of various important variables on the pellet-cladding mechanical interaction problem.

15.4.1. EFFECT OF SOLID BREEDER/CLADDING COMPATIBILITY

On Fig. 5. and 6. the stresses for different designs under the same thermal operating conditions are shown. Solid lines represent the thermoelastic stresses had plastic strains not developed, while broken lines correspond to the modelled interaction. Swelling of the pellet has been neglected for these runs. Since the coefficients of thermal expansion for the $\text{LiAlO}_2/9\text{-C}$ design are closer, the difference in the displacement of the pellet surface and the cladding is smaller. Consequently, the same elastic stress levels occur at significantly smaller gap sizes, which is advantageous both for thermohydraulics and operation purposes. In fact, for the $\text{LiAlO}_2/9\text{-c}$ design zero initial gap is safe as it will be shown in the integrated analysis.

15.4.2. EFFECT OF INITIAL GAP THICKNESS

Fig. 7 shows the stress histories in the cladding due to a ramp variation of the center-temperature for $\text{Li}_2\text{O}/2,25\text{Cr-1Mo}$ materials. At larger initial gap thickness contact occurs later, consequently the resulting thermal stresses are much lower. A mere 0.3mm reduction in the gap size resulted in contact at the preheating temperature which is indicated by high tensile stresses initially. This effect seems to drive the whole interaction at a given set of materials, which implies that tolerances will play a crucial role in operation when the breeder and cladding materials are incompatible thermomechanically.

15.4.3. EFFECT OF PREHEATING

The third most important effect in the interaction process. Fig. 8. shows the results for the same material choice, steady state temperature profile and geometry as those for Fig. 6. This time the system is not preheated to 550 °C. Since the coefficient of the thermal expansion for the cladding is slightly higher than that for the pellet, initially there is no contact when the pin is preheated. Compressive stress due to the coolant pressure develop in the cladding. Its value is about one third of the critical buckling stress, which represents adequate margin of safety. The advantage of preheating from structural point of view is the interaction occurring at cladding temperature which permits creep. Therefore stress relaxation can act during startup relieving the peak stress

Fig. 5. Stresses in the cladding, second zone, first row, no swelling, gap = 0.0 mm, preheating @ 550°C, LiAlO_2 /9-C design

Fig. 6. Stresses in the cladding, second zone, first row, no swelling, gap = 0.15 mm, preheating @ 550°C, $\text{Li}_2\text{O}/2.25 \text{ Cr-Mo}$ design

Fig. 7. Stresses in the cladding, second zone, first row, no swelling, gap = 0.12 mm, preheating @ 550°C, $\text{Li}_2\text{O}/2.25 \text{ Cr-1Mo}$ design

Fig. 8. Stresses in the cladding, second zone, first row, no swelling, gap = 0.0 mm, no preheating, LiAlO_2 /9-C design

significantly. Fig. 9.-10. show the stress relaxation for the characteristic locations of the blanket. Similar stress histories prevail, the first row of the second Li zone showing the highest peak stress. Further cyclic calculations were performed for this particular location.

15.4.4. EFFECT OF THE TEMPERATURE RAMP

On figures 6-11, one can observe that the sensitivity of the results to the temperature ramp rate is not significant. When the slope of the temperature ramp is low the cladding can creep more before steady state profile is attained and the resulting displacement helps to accommodate the thermal expansion of the pellet.

15.4.5. EFFECT OF CYCLIC OPERATION

Fig. 11 shows the effect of the residual strain for $\text{LiAlO}_2/9\text{-C}$ design. In the second cycle contact occurs later, therefore the stress level is lower as compared to the first cycle. For the particular geometry shown here it barely exceeds the value of the "back-stress" defining a threshold for creep, consequently creep strain is very low. The stresses tend to the "back-stress" asymptotically.

15.4.6. EFFECT OF PELLET SWELLING - INTEGRATED ANALYSIS

Fig. 12 illustrates the interaction under cyclic operation when swelling of the pellet is included for the same geometry and material selection as the previous figure. Swelling gradually builds up and instead of the decreasing stresses we see a slight increase and eventually higher stress levels. Another quantity limiting the lifetime of the pin is the accumulated plastic strain. Fig. 13. depicts the plastic strain and the elastic strain over four cycles for the $\text{LiAlO}_2/9\text{-C}$ design. Solid lines indicate the plastic strain, while broken line represents the elastic strain history. The elastic strains are decreasing which reflects the stress relaxation process, while the plastic strains are increasing. The plastic strains stay below 0.5%, and the plastic strain accumulated after the first startup is less than 0.05%. Extrapolating this value for the expected life of three years, the design is safe based on the 1% accumulated plastic strain criterion. On Fig. 14. and 15. the same stress and strain components can be seen for the $\text{Li}_2\text{O}/2.25\text{Cr-1Mo}$ design over one period. In this case swelling of the pellet

Fig. 9. Stresses in the cladding, first zone, first row, no swelling, gap = 0.0 mm, preheating @ 550°C, LiAlO_2 /9-C design

Fig. 10. Stresses in the cladding, second zone, last row, no swelling, gap = 0.0 mm, preheating @ 550°C, LiAlO_2 /9-C design

Fig. 11. Stresses in the cladding, second zone, first row, no swelling, gap = 0.0 mm, preheating @ 550°C, LiAlO_2 /9-C design, cyclic operation

Fig. 12. Stresses in the cladding, second zone, first row, with swelling, gap = 0.0 mm, preheating at 550°C, LiAlO_2 /9-C design, cyclic operation

Fig. 13. Strains in the cladding, second zone, first row, with swelling, gap = 0.0 mm, preheating at 550°C, $\text{LiAlO}_2/9\text{-C}$ design, cyclic operation

Fig. 14. Stresses in the cladding, second zone, first row, with swelling, gap = 0.15 mm, preheating at 550°C, $\text{Li}_2\text{O}/2.25 \text{ Cr-1Mo}$ design, one cycle

Fig. 15. Strains in the cladding, second zone, first row, with swelling, gap = 0.15 mm, preheating at 550°C, $\text{Li}_2\text{O}/2.25$ Cr-1Mo design, one cycle

is stronger, but the creep rate is also higher, therefore stresses remain low. On the other hand, accumulated plastic strain exceeds the 1% value by the end of the first power-cycle, which indicates failure. Because of this last finding and the sensitivity of the peak stress to the gap size, the LiAlO₂ design is recommended based on these structural arguments.

REFERENCES

1. S. Grotz and N. M. Ghoniem, "Thermal Analysis of a Pin-Type Blanket for Tokamak Reactors," Chapter 11 this report; also UCLA Report UCLA-ENG-8612 / PPG-938 (1986).
2. S. Sharafat, R. Amodeo and N. M. Ghoniem, "Materials Data Base and Design Equations for the UCLA Solid Breeder Blanket," Chapter 10 this report; also UCLA-ENG-8611 / PPG-937 (1986).
3. C. W. Gear, Numerical Initial Value Problems in Ordinary Differential Equations, (Prentice-Hall, New York, 1971).
4. J. Yu-Chen Yang, "A Model of Pellet Cladding Interaction to Simulate Operational Ramp Failure of Water Reactor Fuel" Ph.D. Thesis UCLA.
5. J. T. Boyle and J. Spence, Stress Analysis for Creep, (Butterworths, Boston/London, 1983).