THE COULOMB-NUCLEAR INTERFERENCE CORRECTIONS
TO MEASUREMENTS OF THE $\rho\rho$ TOTAL CROSS-SECTION
DIFFERENCE IN PURE SPIN STATES

Y. Watanabe

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THE COULOMB-NUCLEAR INTERFERENCE CORRECTIONS
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DIFFERENCE IN PURE SPIN STATES*

Y. WATANABE

Argonne National Laboratory
Argonne, Illinois 60439

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Abstract

It is pointed out that corrections due to the Coulomb-Nuclear interference could amount to an order of 1-2 mb at P_{lab} around 1 GeV/c in recently performed measurements of the pp -total cross-section difference in pure spin states.

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It is well known that the corrections due to the Coulomb-Nuclear interference are important for the pp-total cross section measurements done in the transmission technique. Recently experiments were performed to measure the pp total cross-section difference for the initial beam and target spin aligned either transversely or longitudinally.¹ Measurements revealed unexpectedly large values and rapid energy dependence. At first sight, one might think that spin dependent Coulomb corrections should be small. But it turned out that they are even more important than for the case of spin averaged total cross-section measurements.

In the transmission experiments, a set of circular counters is used to measure the attenuation of the beam through the target. The cross-section is obtained by extrapolating the observed attenuation at each counter to $\tau = 0$, where $t = -\tau$ is the four momentum squared, and τ_i is the τ -value subtended by the i -th transmission counter. The observed $\sigma(\tau_i)$ contains several components:

$$\sigma(\tau_i) = \sigma^{\text{Tot}} - \sigma^{\text{N}}(\tau_i) + \sigma^{\text{C}}(\tau_i) + \sigma^{\text{I}}(\tau_i)$$

$$\sigma^{\text{N}}(\tau_i) = \int_0^{\tau_i} \frac{d\sigma^{\text{N}}}{d\tau} d\tau$$

$$\sigma^{\text{C}}(\tau_i) = \int_{\tau_i}^{\tau_{\text{max}}} \frac{d\sigma^{\text{C}}}{d\tau} d\tau$$

$$\sigma^{\text{I}}(\tau_i) = \int_{\tau_i}^{\tau_{\text{max}}} \frac{d\sigma^{\text{I}}}{d\tau} d\tau$$

where N, C, and I stand for the nuclear, pure Coulomb and Coulomb-Nuclear interference respectively, and $\tau_{\max}$ is the maximum τ kinematically allowed. The equation can be understood as follows. The apparent cross-section is reduced because of the charged particles deflected into the counter, due to the nuclear interaction, while it is increased because of the incident protons scattered away from the counter due to the interactions other than the nuclear interaction, most importantly due to the Coulomb force.

The second term is proportional to τ at small τ and vanishes when extrapolated to $\tau=0$.

The total cross-section differences for pure spin states are defined as follows:

$$\Delta\sigma_L^{\text{Tot}} = \sigma^{\text{Tot}}\left(\begin{smallmatrix} \uparrow \\ \uparrow \end{smallmatrix}\right) - \sigma^{\text{Tot}}\left(\begin{smallmatrix} \uparrow \\ \downarrow \end{smallmatrix}\right)$$

$$\Delta\sigma_T^{\text{Tot}} = \sigma^{\text{Tot}}\left(\begin{smallmatrix} \uparrow\downarrow \\ \uparrow\downarrow \end{smallmatrix}\right) - \sigma^{\text{Tot}}\left(\begin{smallmatrix} \uparrow\uparrow \\ \uparrow\uparrow \end{smallmatrix}\right)$$

where L (T) indicates that the spins of the beam and the target are aligned longitudinal (transverse) to the beam direction.

To find out the expression for the Coulomb corrections, one writes down the corresponding spin dependent elastic cross-sections in terms of scattering amplitudes.²

$$\begin{aligned} \left(\Delta\frac{d\sigma}{d\tau}\right)_L &= -2 C_{LL} \left(\frac{d\sigma}{d\tau}\right)_0 = \frac{16\pi}{s-4m^2} \text{Re} \left(N_0 U_0^* - N_2 U_2^* \right) \\ \left(\Delta\frac{d\sigma}{d\tau}\right)_T &= - \left(C_{SS} + C_{NN} \right) \left(\frac{d\sigma}{d\tau}\right)_0 = \frac{8\pi}{s-4m^2} \text{Re} \left[\left(N_0 + U_0 \right) \left(N_2 - U_2 \right)^* - |N_1|^2 \right] \\ \left(\text{c.f.} \left(\frac{d\sigma}{d\tau}\right)_0 \right) &= \frac{4\pi}{s-4m^2} \left(|N_0|^2 + 2|N_1|^2 + |N_2|^2 + |U_0|^2 + |U_2|^2 \right) \end{aligned}$$

where $\left(\frac{d\sigma}{d\tau}\right)_0$ is the cross-section for spin averaged elastic scattering, C_{SS} , C_{NN} and C_{LL} are spin-spin correlation parameters, s is the total center of mass energy squared, and N_0 , N_1 , N_2 , U_0 and U_2 are exchange helicity amplitudes² and are defined in terms of s -channel helicity amplitudes; $\phi_1 = \langle ++|\phi|++ \rangle$, $\phi_2 = \langle --|\phi|++ \rangle$, $\phi_3 = \langle +-|\phi|+- \rangle$, $\phi_4 = \langle +-|\phi|-+ \rangle$ and $\phi_5 = \langle ++|\phi|-+ \rangle$:

$$N_0 = \frac{1}{2} (\phi_1 + \phi_3)$$

$$N_1 = \phi_5$$

$$N_2 = \frac{1}{2} (\phi_4 - \phi_2)$$

$$U_0 = \frac{1}{2} (\phi_1 - \phi_3)$$

$$U_2 = \frac{1}{2} (\phi_4 + \phi_2)$$

These amplitudes are sum of the nuclear and the Coulomb amplitudes. (Small phase of ~ 30 m rad, is neglected)³

$$N_i = N_i^N + N_i^C \quad (i = 0, 1, 2)$$

$$U_i = U_i^N + U_i^C \quad (i = 0, 2)$$

Coulomb amplitudes are explicitly given in Reference 3. For proton-proton scattering,

$$N_0^C = - \frac{\alpha}{\sqrt{s}} \frac{s-2m^2}{\tau} F_1^2$$

$$N_1^C = \alpha \left(\frac{s(s-4m^2)}{\tau} \right)^{\frac{1}{2}} \left(\frac{m}{s-4m^2} F_1^2 - \frac{\kappa}{2m} F_1 F_2 \right)$$

$$N_2^C = \frac{\alpha}{\sqrt{s}} \left(\frac{2m^2}{s-4m^2} F_1^2 - \kappa F_1 F_2 + \kappa^2 \frac{(s-2m^2)}{4m^2} F_2^2 \right)$$

$$U_0^C = \frac{\alpha}{\sqrt{s}} \left(\frac{m^2(s+2m^2)}{s(s-4m^2)} - \kappa F_1 F_2 - \frac{\kappa^2}{2} F_2^2 \right)$$

$$U_2^C = 0$$

Where $\alpha \approx 1/137$, $\kappa = \mu_p - 1 \approx 1.79$ is the anomalous magnetic moment of proton, F_1 and F_2 are the proton form factor; $F_1(\tau) = F_2(\tau) = \left(1 + \frac{\tau}{0.71}\right)^{-2}$. Among these amplitudes, N_O^C dominates at the relevant τ range, and we neglect other terms in estimating the corrections. More explicitly,

$$\Delta\sigma_L^I(\tau_i) = \frac{16\pi}{s-4m^2} \int_{\tau_i}^{\tau_{\max}} N_O^C \operatorname{Re} U_O^N d\tau$$

$$\Delta\sigma_T^I(\tau_i) = -\frac{8\pi}{s-4m^2} \int_{\tau_i}^{\tau_{\max}} N_O^C \operatorname{Re} \phi_2^N d\tau$$

$$\left(\text{c.f. } \sigma^I(\tau_i) = \frac{8\pi}{s-4m^2} \int_{\tau_i}^{\tau_{\max}} N_O^C \operatorname{Re} N_O^N d\tau \right)$$

The pure Coulomb corrections are negligible for the total cross-section difference experiments.

To estimate the amount of the corrections, one has to know the real part of the Nuclear amplitude that is interfering. At $t=0$, these values have been calculated by Grein and Kroll from the data of $\Delta\sigma_L$ and $\Delta\sigma_T$, together with the phase shift results, using the forward dispersion relations.⁴ Their values are reproduced in Table I for the P_{lab} range of 1-3 GeV/c, in our normalization convention. In order to integrate over τ , one needs to know the τ -dependence of these amplitudes, but the information is much less certain. Here we assume no τ -dependence, which might not be too bad an approximation in this P_{lab} range. Then the corrections are found to be:

$$\Delta\sigma_L^I(\tau_i) = -\frac{16\pi\alpha}{\sqrt{s}} \frac{s-2m^2}{s-4m^2} \operatorname{Re} U_O^N(0) \int_{\tau_i}^{\tau_{\max}} \frac{d\tau}{\tau \left(1 + \frac{\tau}{0.71}\right)^4}$$

$$\Delta\sigma_T^I(\tau_i) = +\frac{8\pi\alpha}{\sqrt{s}} \frac{s-2m^2}{s-4m^2} \operatorname{Re} \phi_2^N(0) \int_{\tau_i}^{\tau_{\max}} \frac{d\tau}{\tau \left(1 + \frac{\tau}{0.71}\right)^4}$$

The integration can be done analytically, and the calculated corrections at typical τ -values are given in Table II. These numbers are to be added before extrapolating to $\tau=0$. The corrections are 1-2 mb at P_{lab} near 1 GeV/c, and smaller at higher momenta. If the τ dependence is introduced to the nuclear amplitudes in the form of $\exp(-b\tau)$, these numbers are reduced by 20-30% for b up to 4.

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TABLE I The pp Amplitudes at $\tau=0$

P_{lab} (GeV/c)	P c.m. (GeV/c)	ReU_0^N (GeV) ⁻¹	I_{mO}^N (GeV) ⁻¹	ReU_2^N (GeV) ⁻¹	I_{m2}^N (GeV) ⁻¹	ReU_0^N (GeV) ⁻¹	I_{mO}^N (GeV) ⁻¹
0.600	0.287	1.951	1.449	-2.324	-0.033	2.025	-0.796
0.700	0.330	1.641	1.618	-2.495	0.047	1.931	-0.967
0.800	0.372	1.394	1.013	-2.687	0.028	1.896	-0.976
0.900	0.412	1.273	2.106	-2.870	-0.073	1.847	-0.858
1.000	0.451	1.212	2.528	-2.993	-0.207	1.728	-0.669
1.100	0.488	1.193	3.118	-3.083	-0.333	1.466	-0.479
1.200	0.524	1.139	3.877	-3.134	-0.467	1.142	-0.465
1.300	0.559	0.955	4.735	-3.132	-0.589	0.861	-0.653
1.400	0.592	0.300	5.489	-3.112	-0.698	0.825	-0.930
1.500	0.624	-0.296	6.019	-3.064	-0.791	1.030	-1.055
1.600	0.656	-0.873	6.381	-3.168	-0.877	1.152	-0.977
1.700	0.686	-1.166	6.684	-2.914	-0.940	1.201	-0.894
1.800	0.716	-1.515	6.961	-2.811	-0.994	1.221	-0.811
1.900	0.744	-1.818	7.227	-2.672	1.034	1.222	-0.731
2.000	0.772	-2.096	7.478	-2.505	-1.027	1.212	-0.654
2.200	0.826	-2.530	7.927	-2.215	-0.853	1.169	-0.516
2.400	0.877	-2.985	8.289	-2.101	-0.599	1.089	-0.395
2.600	0.926	-3.233	8.618	-2.113	-0.413	0.984	-0.320
2.800	0.972	-3.562	8.944	-2.149	-0.319	0.898	-0.288
3.000	1.017	-3.712	9.245	-2.166	-0.261	0.829	-0.268

TABLE II_a Corrections to $\Delta\alpha_L$

τ P _{lab}	0.001	0.002	0.003	0.005	0.010	0.020	0.030	0.050
1.00	1.747	1.498	1.338	1.151	0.901	0.660	0.528	0.373
1.10	1.318	1.124	1.012	0.871	0.683	0.502	0.403	0.286
1.20	0.927	0.791	0.712	0.613	0.482	0.355	0.285	0.204
1.30	0.639	0.545	0.491	0.423	0.333	0.246	0.198	0.142
1.40	0.564	0.482	0.434	0.374	0.294	0.218	0.175	0.126
1.50	0.656	0.560	0.504	0.435	0.342	0.253	0.204	0.147
1.60	0.686	0.586	0.528	0.456	0.359	0.266	0.214	0.154
1.70	0.674	0.576	0.519	0.448	0.353	0.261	0.211	0.152
1.80	0.649	0.555	0.500	0.431	0.340	0.252	0.203	0.147
1.90	0.619	0.529	0.476	0.411	0.324	0.240	0.194	0.140
2.00	0.586	0.501	0.451	0.389	0.307	0.228	0.184	0.133
2.10	0.551	0.471	0.425	0.366	0.289	0.214	0.173	0.125
2.20	0.520	0.445	0.401	0.346	0.273	0.202	0.163	0.118
2.30	0.484	0.413	0.373	0.322	0.254	0.188	0.152	0.110
2.40	0.451	0.385	0.347	0.300	0.236	0.175	0.142	0.102
2.50	0.415	0.355	0.320	0.276	0.218	0.161	0.130	0.094
2.60	0.382	0.327	0.295	0.254	0.200	0.149	0.120	0.087
2.70	0.355	0.303	0.273	0.236	0.186	0.138	0.112	0.081
2.80	0.329	0.281	0.254	0.219	0.173	0.128	0.104	0.075
2.90	0.303	0.264	0.238	0.205	0.162	0.120	0.097	0.070
3.00	0.269	0.247	0.223	0.192	0.151	0.112	0.091	0.065

TABLE II_b Corrections to Δa_T

τ P_{lab}	0.001	0.002	0.003	0.005	0.010	0.020	0.030	0.050
1.00	1.512	1.289	1.159	0.996	0.780	0.572	0.457	0.323
1.10	1.386	1.182	1.064	0.915	0.718	0.528	0.423	0.301
1.20	1.272	1.086	0.977	0.842	0.661	0.488	0.392	0.280
1.30	1.161	0.991	0.893	0.769	0.605	0.447	0.359	0.258
1.40	1.064	0.909	0.819	0.706	0.555	0.411	0.331	0.238
1.50	0.975	0.833	0.750	0.647	0.509	0.377	0.304	0.219
1.60	0.895	0.765	0.689	0.594	0.468	0.347	0.280	0.201
1.70	0.818	0.699	0.630	0.543	0.428	0.317	0.256	0.185
1.80	0.748	0.639	0.576	0.497	0.391	0.290	0.234	0.169
1.90	0.676	0.578	0.521	0.449	0.354	0.263	0.212	0.153
2.00	0.605	0.517	0.466	0.402	0.317	0.235	0.190	0.137
2.10	0.546	0.467	0.420	0.363	0.286	0.212	0.171	0.124
2.20	0.493	0.421	0.380	0.327	0.258	0.192	0.155	0.112
2.30	0.462	0.395	0.356	0.307	0.242	0.180	0.145	0.105
2.40	0.435	0.372	0.335	0.289	0.228	0.169	0.137	0.099
2.50	0.423	0.361	0.326	0.281	0.221	0.164	0.133	0.096
2.60	0.411	0.352	0.317	0.273	0.216	0.160	0.129	0.094
2.70	0.402	0.344	0.310	0.268	0.211	0.157	0.127	0.092
2.80	0.394	0.337	0.304	0.262	0.207	0.153	0.124	0.090
2.90	0.386	0.330	0.297	0.256	0.202	0.150	0.121	0.088
3.00	0.377	0.323	0.291	0.251	0.198	0.147	0.119	0.086