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CONSTITUTIVE MODELING OF CLIMAX STOCK GRANODIORITE
UNDER COMPRESSIVE LOADING

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CONSTITUTIVE MODELING OF CLIMAX STOCK GRANODIORITE UNDER COMPRESSIVE LOADING

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ABSTRACT

Described here is a constitutive relation for the stress-volume strain states of Climax Stock granodiorite under nonhydrostatic loading conditions. The simple functional form

$$\epsilon_v^e = a_0 + a_1 \tau + b_0 \exp(b_1 \tau)$$

is found using standard curve-fitting techniques; ϵ_v^e — the "excess volume strain" — is the difference between the volume strains under hydrostatic and nonhydrostatic conditions at the same mean stress, τ is the shear stress, and a_1, b_1 are fitting parameters. By comparison with Nugget sandstone under similar loading conditions, for which this functional form fitted the data, we find no relationship for Climax Stock granodiorite between the fitting parameters and other system variables such as mean stress or confining pressure. Therefore this relation is not yet suitable for use in computer codes that attempt to predict the response of rocks to stress.

INTRODUCTION

At LLL we are attempting to construct constitutive relations for compressive loading states of various materials. The intent is to describe the volume strain of the material as a function of its stress state. The approach may be phenomenological^{1,2} or empirical³ or both. The goal is to use these relations in computer codes that predict the response of the material — for example, a region of the earth — to stresses that might result from underground explosions or events leading to earthquakes.

Through our seismic monitoring research program, we have accumulated a large body of experimental data on the mechanical behavior of various geologic media. We are now examining these data to determine whether a simple analytic relation can be written to satisfy the following objectives:

- (1) To describe the stress-strain path under compressive loading for a given rock sample.
- (2) To predict the stress-strain paths under compressive loading for rocks of similar mechanical properties.

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(3) To predict the stress-strain paths under compressive loading for rocks of dissimilar properties.

The purpose is to develop relations that produce acceptable predictions from the fewest possible parameters, thus requiring the least amount of experimental data.

Thus far we have examined data on Nugget sandstone from Utah and on Climax Stock granodiorite from Area 15 of the Nevada Test Site. Previous work has been reported.⁴⁻⁶ For Nugget sandstone a simple constitutive relation was derived for stress-volume strain states under nonhydrostatic loading conditions that established a relationship between the fitting parameters and other system variables such as mean stress and confining pressure. This paper reports an attempt to apply that relation to Climax Stock granodiorite. It successfully describes the medium but does not relate the fitting parameters with the other system variables. Objective (1) has been satisfied and perhaps objective (2); this is not certain. Objective (3) has not yet been addressed.

METHOD

The approach here is phenomenological-empirical. The data base is analyzed, using graphical and statistical techniques, to discern "natural" functions that describe the data. Then an attempt is made to infer from these functions the physics underlying the rock's response. All nonhydrostatic compressive loading data are for uniaxial compression ($\sigma_1 > \sigma_2 = \sigma_3$).

The function used to describe dilatant strain in Nugget sandstone is

$$\epsilon_v^e = a_0 + a_1 \tau + b_0 \exp(b_1 \tau). \quad (1)$$

As shown in Fig. 1, ϵ_v^e , the "excess volume strain," is the difference between the volume strain in the stress states $(\tau, \bar{\sigma})$ and $(0, \bar{\sigma})$ — that is, between the volume strain-mean stress curves for nonhydrostatic and hydrostatic compressive loading. The fitting parameters a_i, b_i are assumed to be functions of the confining stress and are determined by regression analysis of $\epsilon_v - \tau$ data for each experiment. For Nugget sandstone the coefficients a_1 and b_1 show a linear dependence on the confining pressure σ_3 , whereas no relation is found between stress and a_0 or b_0 .

ANALYSIS

Available for analysis were data from 13 uniaxial compression experiments on samples of Climax Stock granodiorite. Of these, the results from three had been discarded because of experimental difficulties, leaving data from 10 experiments for evaluation.

Volume strains (ϵ_v), mean stress ($\bar{\sigma}$), and shear stress (τ) were calculated from the measured radial (ϵ_r) and longitudinal (ϵ_ℓ) strains and axial (σ_1) and radial (σ_3) stresses as follows:

$$\epsilon_v = (\epsilon_\ell + 2\epsilon_r), \quad (2a)$$

$$\bar{\sigma} = 1/3(\sigma_1 + 2\sigma_3), \quad (2b)$$

$$\tau = 1/2(\sigma_1 - \sigma_3). \quad (2c)$$

The analysis, similar to that for Nugget sandstone,⁴ proceeded in three steps. First we found an analytic hydrostat by curve-fitting to hydrostatic volume strain-mean stress data. Next we calculated the excess volume strain for nonzero shear stress states (see Fig. 1). Finally we fitted the excess volume strain using Eq. (1) and attempted to find a relation between parameters a_i , b_i and the confining pressure because, in each experiment, the only variable purposely changed was σ_3 . If such a relation exists, then Eq. (1) can be used in conjunction with the hydrostat as a predictive constitutive relation for any uniaxial stress path.

Hydrostat

We obtained the hydrostat using data from the hydrostatic portion of each uniaxial stress experiment (at pressures up to the confining pressure for that experiment) and from one quasi-hydrostatic experiment to 4 GPa.⁷ For Climax Stock granodiorite, three functional forms were tried:

$$\epsilon_H = \sum_{n=0}^4 a_n (\bar{\sigma})^n \quad (\text{polynomial}), \quad (3a)$$

$$v_{0/v} = \left[\frac{1}{B_0} (n\bar{\sigma} + 1) \right]^{\frac{1}{n}} \quad (\text{Murnaghan}), \quad (3b)$$

$$\epsilon_H = a_0 + a_1 \bar{\sigma}^{a_2} \quad (\text{power law}), \quad (3c)$$

Fig. 1. Mean stress-volume strain curves for hydrostatic ($\sigma_1 = \sigma_2 = \sigma_3$) and nonhydrostatic ($\sigma_1 > \sigma_2 = \sigma_3$) loading paths. The excess volume strain ϵ_v^e at the mean stress $\bar{\sigma}_1$ is defined as $\epsilon_v^e(\bar{\sigma}_1) = \epsilon_H(\bar{\sigma}_1) - \epsilon_v(\bar{\sigma}_1)$. The inset shows the loading conditions on the right circular cylinder samples.

where ϵ_H is the negative of the volume strain in units of 10^{-6} and the mean stress, $\bar{\sigma}$, is in MPa. For $\bar{\sigma} < 0.6$ GPa, the polynomial and power law forms worked equally well as measured by the χ^2 test. However, on inclusion of the 4.0-GPa data, the power-law form proved superior with $\epsilon_H = 53.7 \bar{\sigma}^{0.8521} \pm 300$, where the error is one standard deviation. Attempts to fit the data over finite intervals and to join the intervals by requiring approximate equality of slopes (the method of manual splines) did not significantly decrease the standard deviation or χ^2 . Examination of the residuals of the fitted equation, however, showed the data for each hydrostat to deviate systematically from the fitted curve (see Fig. 2). The importance of this in relation to analysis of the uniaxial compression portions of the experiment will be discussed below.

Dilatant Strain

No widely accepted precise definition of dilatant strain exists, but the term generally refers to a region of volume strain-mean stress for which the volume strain at a given mean stress, in nonhydrostatic compression, is less than that for hydrostatic compression. Since Ref. 4 describes in detail the scheme used to characterize the dilatant strain region, only a brief sketch is presented here. The excess volume strain ϵ_v^e is defined as

$$\epsilon_v^e = \epsilon_v(\bar{\sigma}) - \epsilon_h(\bar{\sigma}) \quad (5)$$

where ϵ_v and ϵ_h are the nonhydrostatic and hydrostatic volume strains, respectively, at the mean stress $\bar{\sigma}$. The hydrostatic volume strain is calculated using Eq. (4); the ϵ_v are experimental data. The excess volume strain, when plotted against shear stress, normally shows two distinct regions (see Fig. 3a), suggesting that

$$\epsilon_v^e(\tau, \bar{\sigma}) = f(\tau, \bar{\sigma}) + g(\tau, \bar{\sigma}). \quad (6)$$

Using standard curve-fitting procedures, we find

$$f(\tau, \bar{\sigma}) = a_0 + a_1 \tau, \quad (7a)$$

$$g(\tau, \bar{\sigma}) = \epsilon_v^e - f(\tau, \bar{\sigma}) = b_0 \exp[b_1 \tau] \quad (7b)$$

where the coefficients a_1 , b_1 are those listed in Table 1.

Fig. 2. Deviation of the hydrostatic portions of each experiment from the curve fitted to the data from all experiments. The systematic divergence of each hydrostat from the fitted hydrostat results in scatter of the fitting parameters in the expression for the excess volume strain.

Fig. 3. (a) The usual behavior of the excess volume strain observed in this work and in Ref. 4 consists of a linear region with positive slope at low shear stress. Data are from experiment CS8. (b) Anomalous behavior of the excess volume strain, due probably to incorrect hydrostatic strain measurements. Data are from experiment CS18.

Table 1. Coefficients for $\epsilon_v^e = a_0 + a_1 \tau + b_0 \exp[b_1 \tau]$.

Experiment	Confining Pressure, MPa	a_0	a_1	b_0	b_1
CS-1	0.1	53	0.317	$3.00(10^{-2})$	$4.821(10^{-3})$
CS-8	0.1	94	0.243	$5.47(10^{-3})$	$6.038(10^{-3})$
CS-6	20	34	2.784	12.83	$1.674(10^{-3})$
CS-2	50	320	2.746	4.97	$1.664(10^{-3})$
CS-5	50	-0.679	0.077	$1.95(10^{-5})$	$5.717(10^{-3})$
CS-3	600	0.151	0.154	—	—

Figure 3b shows what appears to be irregular behavior of the excess volume strain, the linear region having a negative slope. Since the idea of "excess volume strain" works only if the hydrostat of each rock sample differs from the average (fitted) hydrostat by at most a constant, a systematic irregularity could be explained by examining this assumption. However, because the variation between rock samples may be expected to be around 10%, this condition may be relaxed to requiring that only the individual hydrostats have the same functional form, i.e.,

$$\epsilon_H = a^{pb} \quad , \quad \bar{\epsilon}_H = \bar{a}^{p\bar{b}} \quad , \quad (8)$$

and that systematic errors (differences between a and $\bar{a}$ or b and $\bar{b}$) be limited to about 10% in the calculated strain. Here, ϵ_H is the hydrostat for an individual experiment, $\bar{\epsilon}_H$ is the hydrostat fitted to all experiments, and $P = \bar{\sigma}$ is the pressure. The assumption — Eq. (8) — can be tested by setting

$$\frac{\epsilon_H - \bar{\epsilon}_H}{\bar{\epsilon}_H} = \frac{\Delta \epsilon_H}{\bar{\epsilon}_H} = \frac{a}{\bar{a}} P^{b-\bar{b}} - 1$$

or

$$\log \left(1 + \frac{\Delta \epsilon_H}{\bar{\epsilon}_H} \right) = \log \left(\frac{a}{\bar{a}} \right) + (b - \bar{b}) \log P \quad (9)$$

and requiring that $\log [1 + (\Delta \epsilon_H)/\bar{\epsilon}_H]$ vs $\log P$ be linear.

Figure 4 shows the results of this test. All those experiments exhibiting "normal" excess volume strain — in the sense of Ref. 4 and Fig. 3a — have measured hydrostats similar to Eq. (6), Fig. 4a, while all those showing anomalous behavior have measured hydrostats different from Eq. (6), Fig. 4b. An assumption of errors in the stress measurement of either a constant $P' = P + \delta P$, or some proportional to the pressure, $P' = P(1 + \alpha)$, does not account for the discrepancy in the measured hydrostats. The strain measurements are suspect, however, since at low pressure ($P < 200$ bar) the volume strain for the anomalous experiments is much larger than for the fitted hydrostat [see Eq. (9) and Fig. 4b].

The anomalies might be due to the strain gages used to measure the linear strains. These gages are mounted on a thin copper jacket surrounding the rock sample. Normally the jackets are "seasoned" by applying about 10-MPa pressure before mounting the gages; we have assumed that the jackets are in permanent, intimate contact with the rock surface. However, in these experiments it is not uncommon to observe up to 0.2% strain after applying pressures up to 5 MPa, an effect almost certainly due to motion of the sample jackets rather than the sample itself. This and the "anomalous" behavior seen at low pressure and peaks of a few tens of megapascals, after which the $\log(1 + \Delta \epsilon_H/\bar{\epsilon}_H)$ vs $\log P$ curves are similar to the "normal" curves of Fig. 4a, suggest that the large hydrostatic volume strains measured at low pressures are due to jacket rather than rock sample deformation. Thus we can attribute the "anomalous" excess strain behavior to incorrect volume strain measurements at low pressures.

In those experiments showing normal excess strain behavior, an examination of the coefficients a_i , b_i in Table 1 shows no clear systematic relation. Since one of the goals of constitutive modeling is to find a predictive relation for the parameters involved, it is reasonable to examine other forms and sets of independent variables so that the parameters vary smoothly. A number of forms and variable sets were tried (see Table 2). None were as satisfactory as Eq. (1) in fitting the $\epsilon_v^e - \tau$ data.

Fig. 4. Normalized residuals of the difference between experimental and fitted hydrostats. (a) The straight line implies that the measured hydrostat has the same functional form as the fitted hydrostat. Data are from experiments CS5 and CS6. (b) The curve implies that the measured hydrostat has a different functional form than the fitted hydrostat. Data are from experiment CS19.

Table 2. Forms used in attempting to find a relation between excess volume strain and stress ($\epsilon_v^e = f + g$; $f = a_0 + a_1\tau$; $g = \epsilon_v^e - f$).

Form	Form
$\epsilon_v^e = \epsilon_v^e(\tau/\bar{\sigma})$	$\epsilon_v^e = b_0 \exp[b_1(\tau/\bar{\sigma})^2]$
$g = b_0 \exp[b_1\tau/\bar{\sigma}]$	$\epsilon_v^e = b_0 \exp[b_1\sqrt{\tau/\bar{\sigma}}]$
$g = g(\tau/\bar{\sigma})$	$\epsilon_v^e = b_0 \exp[b_1\tau/\bar{\sigma}]$
$\epsilon_v^e = \epsilon_v^e(\sqrt{\tau/\bar{\sigma}})$	$\epsilon_v^e = \epsilon_v^e(\tau)$
$g = b_0 \exp[b_1\sqrt{\tau/\bar{\sigma}}]$	$\epsilon_v^e = b_0 \exp[b_1\tau]$
$g = g(\sqrt{\tau/\bar{\sigma}})$	$\epsilon_v^e = a_0 \sinh(a_1\tau)$

CONCLUSIONS

Climax Stock granodiorite is a brittle, nonporous rock which, under uniaxial compressive loading, shows strong dilatancy and little or no compaction (loading below the hydrostat). The observed behavior may be described satisfactorily by the same model for dilatancy as for Nugget sandstone⁴: that is, an initial region of differential crack closure with volume strain linear in shear stress. This is followed by a catastrophic crack growth region in which the volume strain increases as an exponential function of shear stress.

It is disappointing that, for the granodiorite, no predictive relations for the fitting parameters are evident, precluding the development of a true constitutive relation. However, it is important to realize that the same functional form used to describe a dense sandstone also works well for a granodiorite. This suggests similar physical mechanisms responsible

for dilatant behavior in both rocks. Moreover, the simplicity of Eq. (1), the precision with which it can be fitted to the experimental data, and the inability of other functions to work so well suggests that an understanding of dilatancy requires examination of elastic deformation of flat cracks and a catastrophic growth mechanism that depends strongly on the existing crack density. The failure to provide predictive relations for the fitting parameters means that we must give more consideration to initial crack density and aspect ratio distribution, pay more attention to individual sample histories, and gain a more thorough knowledge of the rock microstructure.

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