CORRELATION EXPERIMENTS AND THE NONVALIDITY OF
ORDINARY IDEAS ABOUT THE PHYSICAL WORLD[†]

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ABSTRACT

It is shown that the predictions of quantum theory in certain correlations experiments are incompatible with ordinary ideas about the physical world. In particular the following theorem is proved: Consider situations involving two experimenters, one working in each of two space-like separated regions. Suppose each is apparently free to choose to perform in his region one of two alternative experiments. Assume that the results that would be obtained in each of the alternative cases conform to the statistical predictions of quantum theory. Then the experimental results in one region must, in some cases, depend on which experiment is performed in the space-like separated region. This theorem is akin to a theorem of J. S. Bell. However, Bell's theorem refers to hidden-variables, which may not exist in nature, whereas the present theorem deals directly with connections between the (macroscopic) results of possible measurements and physical variables subject to the control of experimenters.

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The question keeps arising whether quantum phenomena can be described in classical terms. No classical description of quantum phenomena has yet been found. But it is sometimes argued that this is simply an inadequacy of present-day science: some thinkers hold present-day quantum theory to be logically or metaphysically unacceptable, and expect quantum phenomena to be ultimately comprehensible in classical terms. However, it can be rigorously proved that quantum phenomena is definitely incompatible with classical ideas about the world. The proof given below is quite simple and direct. It is based on correlation phenomena, rather than the interference effects often considered in this connection.

Classical ideas about the physical world entail the following two properties:

1. Causal effects are transmitted over large distances by matter: there can be no causal link unless there is a possible material link.
2. Results of alternative possible experiments are definite: "What would have happened" if an alternative possible experiment had been performed can be supposed to be something specific, even though what that specific something actually is may be necessarily unknown.

What will be shown is that not both of these properties can actually hold true in nature.

Stated in more specific terms the two properties are these:

1. Material Causes. Let R_1 and R_2 be two regions of space separated by a large distance. Events in R_1 cannot depend on a decision made in R_2 unless it is possible for matter to travel

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from the location of the decision to the location of the events. Here the crucial property of matter is that it travels at finite velocity, or can be jammed out by intervening barriers.

2. Definite Results of Alternative Choices. Let E' and E'' be two alternative possible experiments. That is, one can decide to do either E' or E'' , but not both. Suppose E' has a finite number of distinct possible results. That is, if E' is performed then the observed result is precisely one of a finite set of distinct possible results. Let the distinct possible results be numbered in some definite way. Similarly, suppose E'' has a finite number of distinct possible results, which are numbered in some definite way. Let n' be the number of the result that occurs if E' is performed, and let n'' be the number of the result that occurs if E'' is performed. Since the experiments E' and E'' are alternatives, the results specified by n' and n'' cannot both be physically real. But one ordinarily thinks that since the unperformed experiment would have had some definite result if one had chosen to perform it, the numbers n' and n'' are both definite numbers, even though the value of only one is determined by actual experience. There is some definite number that specifies the result that would have been obtained if the "other" experiment had been performed, even though one can never know its actual value; or at least one can suppose there is some such number.

Property (1) is generally believed to be entailed by modern science. It is the basis, for example, of the contention that long range telepathic communications is scientifically impossible: Signals must have material conveyance, or at least the possibility

of such a conveyance. Property (2) asserts the results of alternative choices can be supposed to be definite, even though they cannot all be physically real.

At least one of these two properties is not satisfied in nature: the assumption that both are true is incompatible with the experimental facts.

The experimental facts used in the argument are results of experiments of a type now common in experimental physics. The precise experiments considered have not all actually been performed. But they are only slight variations of experiments that have been performed. Quantum mechanics is universally valid for all experiments that have been performed, and we assume it holds also for the slight variations we consider. All that stands in the way of strict verification is the conviction of scientists that nothing could be learned from examining such slight variations: Quantum mechanics would hold.

The strict technical result that is to be proved is therefore this:

Theorem A. If the results prescribed by quantum mechanics hold true in certain (correlation-type) experiments, then properties (1) and (2) do not both hold true in nature.

Proof. The argument is an adaptation of one given recently by J. S. Bell¹. Consider an experiment in which two low-energy protons are made to collide. Suppose each of the two protons emerging from the collision passes through an experimental apparatus consisting essentially of a region with a strong inhomogeneous magnetic field.

The direction of this field can be rotated by rotating the apparatus.

The two protons emerging from the collision will be moving in opposite directions (away from the collision region) along a certain line L . Let A_1 be the apparatus through which one proton passes and let A_2 be the apparatus through which the other proton passes. (A_1 and A_2 thus lie on L .) Let D_1 and D_2 be the direction of the fields in A_1 and A_2 , respectively. These directions will be chosen perpendicular to L . The angle between D_1 and D_2 will be called $\theta(D_1, D_2)$. (See Fig. A1 and A2.)

Fig. A1. Two protons emerge from a collision. One passes through A_1 and the other passes through A_2 . Both A_1 and A_2 lie on L .

Fig. A2. Both D_1 and D_2 are perpendicular to the line of flight L (which runs back into the paper in this figure). The angle between D_1 and D_2 is $\theta(D_1, D_2)$.

The proton that passes through A_1 will be deflected by the field in A_1 . This deflection is either in the direction of D_1 , or in the opposite direction. These two distinct possible results are labelled by $n_1 = +1$ and $n_1 = -1$, respectively. Similarly, the particle that passes through A_2 will be deflected either in the direction of D_2 or in the opposite direction, and these two distinct possible results are labelled by $n_2 = +1$ and $n_2 = -1$, respectively.

It is an experimental fact that the directions of the deflections of the two protons in such experiments are correlated. (These experiments are called correlation-type experiments.) Quantum mechanics tells us that for low energy protons and $\theta = 0$ one finds (almost always) that $n_1 = -n_2$. That is, if the particle passing through A_1 is deflected in the direction of D_1 , then the particle passing through A_2 will be deflected in the direction opposite to D_2 .

If the angle θ is not zero (or 180°) then the correlation is not exact. But there is still a statistical correlation. In particular, if one performs a large number N of similar experiments, all with the same θ , and lets n_{1j} and n_{2j} be the values of n_1 and n_2 in experiment j , where j runs from 1 to N , then the following relationship holds:

$$\frac{1}{N} \sum_{j=1}^N n_{1j} n_{2j} = -\cos \theta(D_1, D_2). \quad (A.1)$$

The special case $\theta = 0$ gives $\sum n_{1j} n_{2j} = -N$, because in this case $n_{1j} = -n_{2j}$ for all j , as mentioned above. (Actually,

equation (A.1) is a statistical result that holds with increasing accuracy as N increases. It is sufficient for the argument to assume that (A.1) holds, for example, to within several percent, for some sufficiently large N .

The quantum mechanical relationship (A.1) is inconsistent with properties (1) and (2). To see this, consider two different possible settings of D_1 . Let these be called D_1' and D_1'' . And consider two different settings of D_2 , called D_2' and D_2'' . Let n_{1j}' be the value n_{1j} would have if D_1 is chosen to be D_1' , and let n_{1j}'' be the value n_{1j} would have if D_1 is chosen to be D_1'' . Let n_{2j}' and n_{2j}'' be, similarly, the values of n_{2j} in the cases $D_2 = D_2'$ and $D_2 = D_2''$, respectively. Property (1) says that n_{1j} does not depend on whether D_2 is chosen to be D_2' or D_2'' , and that n_{2j} is similarly independent of the choice of D_1 . This is because A_1 and A_2 can be separated by very large distances and the decisions between D_1' and D_1'' , and between D_2' and D_2'' , can be delayed until the last minute. Property (2) says that one can define both n_{1j}' and n_{1j}'' , even though only one of them can actually be measured, and similarly for n_{2j}' and n_{2j}'' . We are saying that what the proton would do in a definite specific situation is some definite specific thing, regardless of whether we choose to create that particular situation or not.

To obtain the contradiction take $D_1' \perp D_2'$, and take D_2'' perpendicular to $D_1' = D_2'$. Then $\theta(D_1', D_2'') = 0$. Hence $\cos \theta(D_1', D_2'') = \cos 0 = 1$. Thus (A.1) gives

$$n_{1j}' = -n_{2j}' \quad (A.2)$$

as already mentioned. Because D_2'' is perpendicular to D_1' , $\cos \theta(D_1', D_2'')$ is zero. Thus (A.1) gives

$$\frac{1}{N} \sum n_{1j}' n_{2j}'' = 0 \quad (A.3)$$

which, in view of (A.2), implies that

$$\frac{1}{N} \sum n_{2j}' n_{2j}'' = 0. \quad (A.4)$$

One may choose D_1'' so that

$$\cos \theta(D_1'', D_2') = -1/\sqrt{2} \quad \text{and} \quad \cos \theta(D_1'', D_2'') = 1/\sqrt{2}.$$

Then (A.1) gives

$$\frac{1}{N} \sum n_{1j}'' n_{2j}' = 1/\sqrt{2} \quad (A.5)$$

and

$$\frac{1}{N} \sum n_{1j}'' n_{2j}'' = -1/\sqrt{2} \quad (A.6)$$

Subtraction of (A.6) from (A.5) gives

$$\begin{aligned} \frac{2}{\sqrt{2}} = \sqrt{2} &= \frac{1}{N} \sum n_{1j}'' (n_{2j}' - n_{2j}'') \\ &= \frac{1}{N} \sum n_{1j}'' n_{2j}'' (n_{2j}' n_{2j}'' - 1) \end{aligned} \quad (A.8)$$

where the fact that $n_{2j}' \times n_{2j}'' = 1$ is used. Taking the absolute

values of both sides and using the fact that the absolute value of a sum is less than or equal to the sum of the absolute values, one gets

$$\begin{aligned}
 |\sqrt{2}| &\leq \frac{1}{N} \sum_j |n_{1j}'' n_{2j}'' (n_{2j}'' n_{2j}' - 1)| \\
 &= \frac{1}{N} \sum_j |(n_{2j}'' n_{2j}' - 1)| \\
 &= \frac{1}{N} \sum_j (1 - n_{2j}'' n_{2j}') \\
 &= 1 - \frac{1}{N} \sum_j n_{2j}'' n_{2j}' .
 \end{aligned} \tag{A.9}$$

The last term on the right is zero, because of (A.4). Thus we obtain

$$|\sqrt{2}| \leq 1. \tag{A.10}$$

Squaring both sides one obtains the result that 2 is less than or equal to 1. This is false. Thus one of the premises of the argument is false. The premises were properties (1) and (2) and the validity of the quantum mechanical relationship (A.1). Thus if the quantum mechanical relationship (A.1) holds true {even approximately, for sufficiently large N } then properties (1) and (2) cannot both be true. Q.E.D.

That is, one cannot suppose that the result of an experiment that one elects not to perform would have had a well-defined and definite result (as it would have if one had elected to perform the experiment) unless one admits that causal effects can be transmitted over large distances without the possibility of material conveyance.

REFERENCES

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 - † This article, apart from the abstract, was originally submitted to the conference "Quantum Theory and Beyond" held in 1968 at Cambridge, England, and was widely distributed at that time. It is now being reissued to fill continuing requests. The statement of the theorem proved in it has been compressed in recent years to the statement that if the statistical predictions of quantum theory are true then the principle of local causes is false, where the principle of local causes states that the result of an experiment in one space-time region cannot depend on a variable subject to the control of an experimenter in a far-away space-like separated region. The notion of dependence occurring in this principle is based on the idea that the alternative unperformed experiments would have had definite results if they had been performed, and that these results can be represented by sets of (unknown) numbers. This requirement, which is property (2) of the present article, has more recently been called "contrafactual definiteness."
1. J. S. Bell, *Physics* 1, 415 (1965).