Higher Order Effects of Pseudoparticles in QCD*

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Abstract

We study gauge invariant Green's functions of quark-antiquark bilinear densities in massless, two-color QCD. We take into account non-zero-energy fermion modes, pseudoparticle solutions with topological charge $|v| > 1$, and n -point functions with $n > 2$. Some general properties of the $O(\hbar)$ approximation are developed, enabling us to isolate and define the terms which contribute to a general n -point function. We find the higher effects preserve the symmetry breakdown found earlier in the 2-point function $(U(2) \times U(2) \rightarrow SU(2) \times SU(2) \times U(1))$. It is shown that a previous 2-point function analysis is exact to order $\hbar$.

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I. Introduction

In a recent letter¹ we considered the $O(\hbar)$ effects of one pseudoparticle² configurations on lowest order n -point functions of quark-antiquark densities in the massless two-color formulation of QCD, following the work of 't Hooft.³ In the two flavor case this yielded two-point functions of scalar and pseudoscalar quark field bilinears. In Ref.1 we were able to complete the computation of 't Hooft and obtain a Green's function which in momentum space behaved like $|p|^{-4}$. We conjectured then that this term was one in a series of terms contributing to the two-point function, the other arising from configuration with more than one pseudoparticle.

In this paper we show (Sec.II) that, in fact, the two-point function receives $O(\hbar)$ contributions only from the one pseudoparticle sector. Thus the result of Ref.1 is exact to $O(\hbar)$ and will be modified only by effects higher order in $\hbar$. In general we find that the N -pseudoparticle configurations yield $O(\hbar)$ contributions to $(N \cdot N_f + 2m)$ -point functions where N_f is the number of flavors and m is a non-negative integer.

Following this result, we make a detailed study of the two-point function in the two-flavor case (Sec.III), with an explicit display of the now well known³ breakdown of the flavor group $U(2) \times U(2)$ to $SU(2) \times SU(2) \times U(1)$. Appendix A contains some simple $U(2) \times U(2)$ and $SU(2) \times SU(2)$ properties useful in analyzing the breakdown.

We then study the four-point functions in the two-flavor case. Two new features appear, namely the contributions of non-zero-energy fermion modes (Sec.IV,A) and higher order pseudoparticle configurations (Sec.IV,B). The symmetry properties of the four-point functions are examined and we

find, as in the case of two-point functions, that the flavor group is broken to $SU(2) \times SU(2) \times U(1)$, but no further. A plausibility argument is given (Appendix B) that this is true for all n-point functions in the $O(\hbar)$ approximation.

In Sec.V we summarize and comment upon our results.

II. The Determinantal Problem

We follow the vacuum analysis of Jackiw and Rebbi⁴ and Callan, Dashen and Gross⁵ in calculating the vacuum to vacuum transition in the presence of a source J . The different vacua are labeled by θ , and we have

$$\begin{aligned} \langle \theta' | \theta \rangle_{\text{out}}^J &= \delta(\theta - \theta') W(\theta) \\ &= \delta(\theta - \theta') \sum_{\nu=-\infty}^{\infty} e^{-i\nu\theta} W[J, \nu], \end{aligned} \quad (2.1)$$

where

$$W[J, \nu] = \int [DA_\mu]_\nu D\psi D\bar{\psi} \exp \int d^4x \{ \mathcal{L} + \bar{\psi} m \psi + \bar{\psi}_s J_s \psi_s \}. \quad (2.2)$$

Here ν is the topological charge and $[DA_\mu]_\nu$ in the functional integral means that the integration is to be carried over all field configurations with topological charge ν . $\mathcal{L}$ is the $SU(2)$ gauge (2-color) Lagrangian³ involving the gauge fields and gauge fixing and ghost terms, and m is

$$m(\nu) = - \gamma_\mu \left(\partial_\mu - i g \frac{\tau^a}{2} A_\mu^a(\nu) \right), \quad (2.3)$$

where the field $A_\mu^a(v)$ has topological charge v . The $O(\hbar)$ approximation of $W[J, v]$ has been studied in detail by 't Hooft³ in the case $v = \pm 1$. The $[DA_\mu]_v$ functional integration involves integration over collective coordinates corresponding to zero energy modes of A_μ , as well as the gaussian part arising from the finite energy modes. The integration over $D\psi D\bar{\psi}$ yields a factor $\det(\mathcal{M}(v) + J)$, where $\mathcal{M}(v)$ is given by (2.3) with $A_\mu^a(v)$ now standing for a classical solution of the gauge field equations with topological charge v . In this paper we are mostly concerned with calculating the above mentioned determinant, which carries the full J dependence. The n -point functions of quark-antiquark bilinears are determined by expanding the determinant in powers of J . The various other factors in $W[J, v]$ coming from collective coordinate integration and regularization effects have no effect on the symmetry properties of the n -point functions.

To calculate the determinant we write

$$\det(\mathcal{M}(v) + J) = \prod_{(a,s,\alpha)} \mathcal{E}_a(J, v), \quad (2.4)$$

where $\mathcal{E}_a$ is the eigenvalue

$$(\mathcal{M}(v) + J)_{st} |\mathcal{E}_a J s \alpha v\rangle = \mathcal{E}_a(J, v) |\mathcal{E}_a J s \alpha v\rangle. \quad (2.5)$$

Here s is the flavor index of the N_f fermionic states and α stands for the remaining degeneracy. We will calculate $\mathcal{E}(J)$'s as a power series in J using Rayleigh-Schrödinger perturbation theory. As is evident from (2.1) this expansion has to be done for each of the different v 's; in the following we do not write the v -dependence explicitly.

Before we go on with the perturbation expansion let us note the following properties of the unperturbed states which we take to be eigenstates of $\mathcal{M}$:

$$\mathcal{M} |E_a s \alpha\rangle = E_a |E_a s \alpha\rangle. \quad (2.6)$$

Because $\mathcal{M}$ is odd in γ -matrices we have

$$\gamma^5 \mathcal{M} \gamma^5 = -\mathcal{M}, \quad (2.7)$$

and the non-zero-energy fermionic eigenstates can then be chosen so that

$$\begin{aligned} \gamma^5 |E_a s \alpha\rangle &= |-E_a s \alpha\rangle \\ &\equiv |E_{-a} s \alpha\rangle. \end{aligned} \quad (2.8)$$

For zero energy solutions ($E_0 = 0$) the γ_5 property implies either degeneracy or linear dependence. 't Hooft's³ analysis for $|v|=1$ and Grossman's⁶ for $|v| \geq 1$ reveal that the latter is the case so that we have

$$\gamma^5 |0 s \alpha\rangle = -\text{sgn}(v) |0 s \alpha\rangle. \quad (2.9)$$

Let P_a denote the projection operator to the space spanned by eigenstates with energy E_a ;

$$P_a = \sum_{s, \alpha} |E_a s \alpha\rangle \langle E_a s \alpha|, \quad (2.10)$$

then we have also

$$\gamma^5 P_a \gamma^5 = P_{-a}. \quad (2.11)$$

Although a completely general source J could include arbitrary γ -matrices, for us a sufficiently general source [see (3.7) below] will have the property

$$\gamma^5 J \gamma^5 = J. \quad (2.12)$$

The perturbation expansion is given as follows. Let

$$\det(M+J) = \prod_{a>0} \Delta(E_a, J) \Delta(0, J) \prod_{a<0} \Delta(E_a, J), \quad (2.13)$$

where

$$\Delta(E_a, J) = \det_{(s,\alpha)(\dagger,\beta)} \langle E_a s \alpha | H(a, J) | E_a \dagger \beta \rangle.$$

Then the expansion of H is given by⁷

$$\begin{aligned} H(a, J) &= E_a P_a + P_a J P_a + \sum_{n=1}^{\infty} P_a J U^{(n)}, \\ U^{(n)} &= \sum'_{(n)} S^{k_1} J \dots J S^{k_n} J P_a, \\ S^k &= \begin{cases} -P_a, & \text{if } k=0, \\ \frac{1-P_a}{(E_a-M)^k}, & \text{if } k>0, \end{cases} \end{aligned} \quad (2.14)$$

where the sum $\sum'_{(n)}$ extends over such non-negative integers k_i , that

$$k_1 + k_2 + \dots + k_p \geq p, \quad (p=1, 2, \dots, n-1)$$

$$k_1 + k_2 + \dots + k_n = n.$$

Up to third order $H(a, J)$ is given by

$$\begin{aligned} H(a, J) = & E_a P_a + P_a J P_a + P_a J \frac{1-P_a}{E_a-m} J P_a \\ & + P_a J \frac{1-P_a}{E_a-m} J \frac{1-P_a}{E_a-m} J P_a - P_a J \frac{1-P_a}{(E_a-m)^2} J P_a J P_a + \dots \end{aligned} \quad (2.15)$$

Using the γ_5 properties noted before and that

$$\gamma^5 \frac{1-P_a}{E_a-m} \gamma^5 = - \frac{1-P_a}{E_a-m}, \quad (2.16)$$

we find

$$\gamma^5 H(a, J) \gamma^5 = -H(-a, -J). \quad (2.17)$$

This tells us, for example, that in the $\Delta(0, J)$ part only the terms odd in J contribute

$$\Delta(0, J) = \det \langle 0s\alpha | J + J \frac{1-P_0}{m} J \frac{1-P_0}{m} J - J \frac{1-P_0}{m^2} J P_0 J + \dots | 0t\beta \rangle. \quad (2.18)$$

Next note that using (2.8) and (2.17) we have

$$\begin{aligned}
\Delta(E_{-a}, J) &= \det \langle E_{-a} s \alpha | H(-a, J) | E_{-a} \neq \beta \rangle \\
&= \det \langle E_a s \alpha | \gamma^5 H(-a, J) \gamma^5 | E_a \neq \beta \rangle \\
&= (-1)^{g_a} \Delta(E_a, -J),
\end{aligned} \tag{2.19}$$

where g_a gives the degeneracy of the state of energy E_a (labeled by s, α).

Thus we can combine the products over positive and negative energy states as

$$\det(M+J) = \prod_{a>0} (-1)^{g_a} \Delta(E_a, J) \Delta(E_a, -J) \cdot \Delta(0, J). \tag{2.20}$$

But since $\Delta(E_a, J)\Delta(E_a, -J)$ is obviously even under $J \rightarrow -J$ we see that this product contributes only terms even in J . We have

$$\Delta(E_a, J)\Delta(E_a, -J) = \det \langle E_a s \alpha | H(a, J) P_a H(a, -J) | E_a \neq \beta \rangle \tag{2.21}$$

and up to second order in J

$$\begin{aligned}
H(a, J) P_a H(a, -J) &= P_a E_a^2 + 2 E_a P_a J \frac{1 - P_a}{E_a - m} J P_a \\
&\quad - P_a J P_a J P_a + \dots
\end{aligned}$$

Dividing out the E_a^2 contribution we finally get

$$\begin{aligned}
\det(M+J) &= \prod_{a>0} (-E_a^2)^{g_a} \\
&\times \det_{(s,\alpha)(t,\beta)} \langle 0s\alpha | J + J \frac{1-P_0}{m} J \frac{1-P_0}{m} J - J \frac{1-P_0}{m^2} J P_0 J + \dots | 0t\beta \rangle \\
&\times \prod_{a>0} \det_{(s,\alpha)(t,\beta)} \langle E_a s\alpha | 1 + 2 \frac{1}{E_a} J \frac{1-P_a}{E_a - m} J - \frac{1}{E_a^2} J P_a J + \dots | E_a t\beta \rangle
\end{aligned}
\tag{2.23}$$

The infinite product $\prod_{a>0} (-E_a^2)^{g_a}$ was calculated (in the $|\nu|=1$ sector) by 't Hooft³ using several renormalization techniques.

From the general form (2.23) we see that this determinant can only give rise to $(N_f \cdot g'_0 + 2m)$ -point functions, where g'_0 is the extra degeneracy of the zero energy states (labeled by α) and m is a non-negative integer. In deriving this result we have used only the γ_5 properties of the unperturbed states and J. Grossman⁶ has studied the degeneracy of the zero energy fermion modes for arbitrary ν using the Jackiw - Nohl - Rebbi⁸ pseudoparticles, and found that $g'_0 = |\nu|$. Thus from the N -pseudoparticle sector in a theory with N_f flavors, we can only have $(N \cdot N_f + 2m)$ -point functions in quark bilinears.

Some observations can now be made on the possible n -point functions that we can have. For example, if N_f is even we can only have even-point functions, whereas with odd N_f both even and odd-point functions are possible. For $N_f = 2$ case we see that two-point functions can come only from the $|\nu|=1$ sector; therefore the computations of Ref.1 are exact in the one loop approximation. The symmetry properties of that result will be described in Sec.III. The next highest n -point function for a $N_f=2$ theory is a four-point function. From the above we now see that we get contributions both

from the $|v|=1$ sector through higher order effects in (2.23) and from the lowest order zero energy modes in the $|v|=2$ sector. These will be described in Sec.IV.

III. Two-point functions

For pedagogical reasons we begin our analysis with two-point functions. Some of the results of this section are contained in Refs. 1 and 3.

When $N_f = 2$ only zero energy modes in the $|v|=1$ sector are relevant for the two-point function and the fermionic contribution to $W(J)$ is

$$\det_{st} \langle 0s | T | 0t \rangle = \det_{st} \int d^4x \psi_{s'}^{*(s)}(x) J_{s't'}(x) \psi_{t'}^{(t)}(x) \quad (3.1)$$

Here $\psi^{(s)}$ are the N_f normalized solutions of

$$\delta_{st} \not{\partial}_\mu (\delta_{ij} \partial_\mu - \frac{1}{2} i g \tau_{ij}^a A_\mu^a) \psi_{tj} = 0 \quad (3.2)$$

with A_μ^a the pseudoparticle solution for $v=\pm 1$. These solutions are³

$$\begin{aligned} \psi_{ti}^{(s)\pm} &= 2^{\frac{1}{2}} \pi^{-1} \lambda [(x_\mu - z_\mu)^2 + \lambda^2]^{-3/2} \delta_{st} u_i^\pm \\ &\equiv f(x) \delta_{st} u_i^\pm \end{aligned} \quad (3.3)$$

with

$$\not{\partial}^5 u_i^\pm = \pm u_i^\pm \quad (3.4)$$

and

$$\sum_i u_i^\pm u_i^{*\pm} = \frac{1}{2} (1 \pm \gamma^5).$$

(3.5)

Here the $-(+)$ solution goes with a pseudoparticle background field with topological charge $v=+1(-1)$. For a source diagonal in color we then have

$$\det_{S+} \langle 0_S | J | 0_+ \rangle = \det_{S+} \int d^4x |f(x)|^2 \text{Tr} \left(J_{S+} \frac{1}{2} (1 \pm \gamma^5) \right),$$

(3.6)

where the trace is over Dirac indices. Hence only that part of a generalized source which is proportional to 1 and γ_5 contributes and we may write in general

$$J_{S+} = \frac{1}{2} (j_\sigma + i j_\eta \gamma^5) \delta_{S+} + \frac{1}{2} (\vec{j}_{\pi'} + i \vec{j}_{\pi} \gamma^5) \cdot \vec{\tau}_{S+}$$

(3.7)

yielding

$$\det_{S+} \langle 0_S | J | 0_+ \rangle = \det_{S+} \int d^4x |f(x)|^2 j_{S+}^\pm(x)$$

(3.8)

where

$$j_{S+}^\pm(x) = (j_\sigma^{(\pm)} \pm i j_\eta^{(\pm)}) \delta_{S+} + (\vec{j}_{\pi'}^{(\pm)} \pm i \vec{j}_{\pi}^{(\pm)}) \cdot \vec{\tau}_{S+}$$

(3.9)

The effect of functional differentiation is simply to produce Green's functions which have the symmetry of $\det j^\pm$ as we shall see in detail presently. As a result the $U(2) \times U(2)$ flavor group is broken to $SU(2) \times SU(2) \times U(1)$.

The integration over the collective coordinates z and λ has been completed in Ref.1. Combining the $v=\pm 1$ terms, and expressing the result in terms of the Fourier transform $j_{st}^{\pm}(p)$, we have the two-point contribution

$$W^{(2)}(\theta) = \exp\left(-\frac{8\pi^2}{g^2}\right) \bar{g}^8 \mu_0^6 C \int d^4 p_1 d^4 p_2 (2\pi)^4 \delta^{(4)}(p_1 + p_2) \frac{1}{p_1^2 p_2^2} \\ \times \left\{ e^{-i\theta} \det_{st} j_{st}^+(p_s) + e^{i\theta} \det_{st} j_{st}^-(p_s) \right\}, \quad (3.10)$$

where C is a finite constant given in Ref.1 and μ_0 is the renormalization point. The curly bracket transforms like

$$\cos\theta \left[(j_0^2 + \vec{j}_a^2) - (j_\eta^2 + \vec{j}_{a'}^2) \right] + \sin\theta \left[-j_0 j_\eta + \vec{j}_a \cdot \vec{j}_{a'} \right], \quad (3.11)$$

which is a linear combination of $SU(2) \times SU(2)$ invariants. Performing the function variation we have the non-vanishing two-point functions

$$\langle 0 | T(\delta(x) \delta(y)) | 0 \rangle = 2C \exp\left(-\frac{8\pi^2}{g^2}\right) \bar{g}^8 \mu_0^6 \int d^4 p \, p^{-4} e^{-ip(x-y)} \cos\theta \\ \equiv \Delta(x-y) \cos\theta,$$

$$\langle 0 | T(\eta(x) \eta(y)) | 0 \rangle = -\Delta(x-y) \cos\theta,$$

$$\langle 0 | T(\delta(x) \eta(y)) | 0 \rangle = -\Delta(x-y) \sin\theta,$$

$$\begin{aligned}
\langle 0 | T (\pi_i(x) \pi_j(y)) | 0 \rangle &= \Delta(x-y) \cos \theta \delta_{ij}, \\
\langle 0 | T (\pi'_i(x) \pi'_j(y)) | 0 \rangle &= -\Delta(x-y) \cos \theta \delta_{ij}, \\
\langle 0 | T (\pi_i(x) \pi'_j(y)) | 0 \rangle &= \Delta(x-y) \sin \theta \delta_{ij},
\end{aligned}
\tag{3.12}$$

where σ , etc. are the fermion-antifermion field bilinears defined in Appendix A. The two-point function matrix displays explicitly the breakdown of chiral $U(2)$ to chiral $SU(2)$ for any θ . For $\theta \neq 0$, there is also a parity breakdown.

A comment concerning the p^{-4} singularity in the four fermion two-point function seems in order: It is interesting to note that if one takes p to be a space-like momentum transfer, then in the corresponding "potential"

$$\begin{aligned}
U(\vec{x}) &= \int d^3p \, e^{-i\vec{p} \cdot \vec{x}} (\vec{p})^{-4} \\
&= \lim_{|\vec{p}| \rightarrow 0} 4\pi |\vec{p}|^{-1} + \pi^2 |\vec{x}|
\end{aligned}
\tag{3.13}$$

the only $\vec{x}$ -dependence is in the finite part, which is linear in $|\vec{x}|$, suggestive of a quark confinement mechanism.

IV. Four-point Functions

A. One pseudoparticle contribution

As was noted at the end of Sec. II, in the case $N_f = 2$ the four-point functions receive contributions from both one- and two-pseudoparticle sectors. We will now consider the one-pseudoparticle sector.

We start with the expansion (2.23) and consider first the nonzero energy modes. Since the zero energy modes will always give at least order J^2 terms, we need to keep from the nonzero part only terms up to J^2 . We expand

$$\begin{aligned}
 & \prod_{a>0} \det_{(s,\alpha)(t,\beta)} \left\langle E_a s \alpha \left| 1 + 2 \frac{1}{E_a} J \frac{1-P_a}{E_a-m} J - \frac{1}{E_a^2} J P_a J + \dots \right| E_a t \beta \right\rangle \\
 &= \prod_{a>0} \left(1 + \sum_{s,\alpha} \langle E_a s \alpha \left| \frac{2}{E_a} J \frac{1-P_a}{E_a-m} J - \frac{1}{E_a^2} J P_a J \right| E_a s \alpha \rangle + \dots \right) \\
 &= 1 + \sum_{a>0} \sum_{s,\alpha} \langle E_a s \alpha \left| \frac{2}{E_a} J \frac{1-P_a}{E_a-m} J - \frac{1}{E_a^2} J P_a J \right| E_a s \alpha \rangle + \dots \quad (4.1)
 \end{aligned}$$

In the one pseudoparticle sector the zero energy states do not have extra degeneracy (beyond flavor), so we drop the label α . We will now also use the fact that $\langle 0s | J | 0t \rangle = \langle 0 | J_{st} | 0 \rangle$, where the matrix J_{st} has been given in (3.7). We then expand the zero energy contribution to fourth order in J

$$\begin{aligned}
 & \det_{s,t} \langle 0s | J + J \frac{1-P_0}{m} J \frac{1-P_0}{m} J - J \frac{1-P_0}{m^2} J P_0 J + \dots | 0t \rangle \\
 &= \det_{s,t} \langle 0s | J | 0t \rangle \left[1 - \sum_s \langle 0s | J \frac{1-P_0}{m^2} J | 0s \rangle \right] \\
 &+ \sum_s \langle 0s | \tau^2 \tilde{J} \tau^2 P_0 J \frac{1-P_0}{m} J \frac{1-P_0}{m} J | 0s \rangle + \dots \quad (4.2)
 \end{aligned}$$

where the appearance of the term $\tau^2 \tilde{J} \tau^2$ in the expansion of the determinant is a special property of 2×2 matrices, and $\tilde{J}$ is the transpose of J .

Multiplying (4.1) and (4.2) yields

$$\begin{aligned}
 \det(m+J) &= \prod_{a>0} (-E_a^2)^{g_a} \left\{ \det \langle 0s | J | 0s \rangle \right. \\
 &\times \left[1 + \sum_{a>0} \sum_{\alpha, s} \langle E_a s \alpha | \frac{2}{E_a} J \frac{1-P_a}{E_a - m} J - \frac{1}{E_a^2} J P_a J | E_a s \alpha \rangle \right. \\
 &\quad \left. \left. - \sum_s \langle 0s | J \frac{1-P_0}{m^2} J | 0s \rangle \right] \right. \\
 &\quad \left. + \sum_s \langle 0s | \tau^2 \tilde{J} \tau^2 P_0 J \frac{1-P_0}{m} J \frac{1-P_0}{m} J | 0s \rangle \right\} + \dots
 \end{aligned} \tag{4.3}$$

The terms in the square bracket can be further simplified. Let us first write the sums over intermediate energy levels explicitly [using also (2.8)] (summation over s, t, α, β is understood)

$$\begin{aligned}
 [] &= 1 + \sum_{a>0} \sum_{\substack{b>0 \\ a \neq b}} \langle E_a s \alpha | J | E_b t \beta \rangle \langle E_b t \beta | J | E_a s \alpha \rangle 2 E_a^{-1} (E_a - E_b)^{-1} \\
 &\quad + \sum_{a>0} \langle E_a s \alpha | J | 0s \rangle \langle 0s | J | E_a s \alpha \rangle 2 E_a^{-2} \\
 &\quad + \sum_{a>0} \sum_{b>0} \langle E_a s \alpha | J \gamma^5 | E_b t \beta \rangle \langle E_b t \beta | \gamma^5 J | E_a s \alpha \rangle 2 E_a^{-1} (E_a + E_b)^{-1} \\
 &\quad - \sum_{a>0} \langle E_a s \alpha | J | E_a t \beta \rangle \langle E_a t \beta | J | E_a s \alpha \rangle E_a^{-2} \\
 &\quad - \sum_{a>0} \langle 0s | J | E_a t \beta \rangle \langle E_a t \beta | J | 0s \rangle E_a^{-2} \\
 &\quad - \sum_{a>0} \langle 0s | J \gamma^5 | E_a t \beta \rangle \langle E_a t \beta | \gamma^5 J | 0s \rangle E_a^{-2}.
 \end{aligned} \tag{4.4}$$

Using (2.9) we see that the second, fifth and sixth terms cancel each other. Furthermore, since the double summation over a and b is symmetric, we can replace $2E_a^{-1}(E_a - E_b)^{-1}$ by $-E_a^{-1}E_b^{-1}$, and $2E_a^{-1}(E_a + E_b)^{-1}$ by $E_a^{-1}E_b^{-1}$. Then the other three terms can be combined to give

$$[\] = 1 - \sum_{a>0} \sum_{b>0} \sum_{(s,\alpha)(\beta)} E_a^{-1} E_b^{-1} \left\{ \langle E_a s \alpha | J | E_b \beta \rangle \langle E_b \beta | J | E_a s \alpha \rangle \right. \\ \left. - \langle E_a s \alpha | J \gamma^5 | E_b \beta \rangle \langle E_b \beta | \gamma^5 J | E_a s \alpha \rangle \right\}. \quad (4.5)$$

We can also write explicitly the intermediate sums in the last term of (4.3) giving the following expression for the fourth-order contribution

$$\Delta^{(4)} = \sum_{a>0} \sum_{b>0} \sum_{\alpha,\beta} E_a^{-1} E_b^{-1} \left\{ - \det_{s,t} \langle 0s | J | 0t \rangle \right. \\ \times \sum_{s,t} \left[\langle E_a s \alpha | J | E_b \beta \rangle \langle E_b \beta | J | E_a s \alpha \rangle \right. \\ \left. - \langle E_a s \alpha | J \gamma^5 | E_b \beta \rangle \langle E_b \beta | \gamma^5 J | E_a s \alpha \rangle \right] \\ + \sum_{s,t,u,v} \langle 0s | \tau^2 \tilde{J} \tau^2 | 0t \rangle \langle 0t | J | E_a u \alpha \rangle \\ \left. \times \langle E_a u \alpha | J (1 + \text{sgn}(v) \gamma^5) | E_b v \beta \rangle \langle E_b v \beta | J | 0s \rangle \right\}. \quad (4.6)$$

Next we project out the right and lefthanded parts, denoting

$$\frac{1}{2} (1 + \gamma^5) | E_a s \alpha \rangle = | E_a s \alpha, + \rangle, \\ \frac{1}{2} (1 - \gamma^5) | E_a s \alpha \rangle = | E_a s \alpha, - \rangle. \quad (4.7)$$

Since the zero energy state is righthanded for $v=1$, we get

$$\begin{aligned} \Delta_{v=1}^{(4)} = & \sum_{a>0} \sum_{b>0} \sum_{\alpha,\beta} 4 E_a^{-1} E_b^{-1} \left\{ - \det_{s,t} \langle 0s | J | 0t \rangle \right. \\ & \times \sum_{s,t} \langle +, E_a \alpha s | J | E_b \beta t, + \rangle \langle -, E_b \beta t | J | E_a \alpha s, - \rangle \\ & + \sum_{s,t,v} \langle 0s | \tau^2 \tilde{J} \tau^2 | 0t \rangle \langle 0t | J | E_a u \alpha, - \rangle \\ & \left. \times \langle +, E_a u \alpha | J | E_b v \beta, + \rangle \langle -, E_b v \beta | J | 0s \rangle \right\}. \end{aligned} \quad (4.8)$$

From this form we can clearly see the transformation properties of $\Delta^{(4)}$. Referring to Appendix A we find that J [given in (3.7)] sandwiched between righthanded (-) states transforms like M^- , and if it is between lefthanded (+) states like M^+ . Then $\Delta^{(4)}$ transforms like

$$\Delta_{v=1}^{(4)} \sim - \det(M^-) \text{Tr} (M^+ M^-) + \text{Tr} (\tau^2 \tilde{M} \tau^2 M^- M^+ M^-), \quad (4.9)$$

so that both terms break $U(2) \times U(2)$, but not $SU(2) \times SU(2)$.

Another interesting property is that for $\Delta_{v=1}^{(4)}$ to be nonzero it is necessary that both the left- and righthanded projections of nonzero-energy states are nonvanishing, but this is indeed the case.⁹

For a more explicit result we next separate the sources and the functions multiplying them. Because $\mathcal{M}$ is diagonal in the flavor index we have, e.g.,

$$\langle \pm, E_a s_a | J | E_b \beta, \pm \rangle = \int d^4x j_{st}^{\pm}(x) \cdot \left(\psi_{E_a, a}^{*\pm}(x) \psi_{E_b, \beta}^{\pm}(x) \right), \quad (4.10)$$

and so on. Here the brackets stand for summation over Dirac and color indices. Substitution in (4.8) gives finally

$$\Delta_{\nu=1}^{(4)} = 4 \int d^4x d^4y d^4z d^4w \left\{ -\frac{1}{2} \text{Tr}(\bar{j}^-(x) j^-(y)) \text{Tr}(j^-(z) j^+(w)) f(xyzw) \right. \\ \left. + \text{Tr}(\bar{j}^-(x) j^-(y) j^+(z) j^-(w)) g(xyzw) \right\}, \quad (4.11)$$

where

$$f(xyzw) = \sum_{a>0} \sum_{b>0} \sum_{\alpha, \beta} E_a^{-1} E_b^{-1} (\psi_0^*(x) \psi_0(x)) (\psi_0^*(y) \psi_0(y)) \\ \times (\psi_{E_a, \alpha}^{*-}(z) \psi_{E_b, \beta}^-(z)) (\psi_{E_b, \beta}^{*+}(w) \psi_{E_a, \alpha}^+(w)), \\ g(xyzw) = \sum_{a>0} \sum_{b>0} \sum_{\alpha, \beta} E_a^{-1} E_b^{-1} (\psi_0^*(x) \psi_0(x)) (\psi_0^*(y) \psi_{E_a, \alpha}^-(y)) \\ \times (\psi_{E_a, \alpha}^{*+}(z) \psi_{E_b, \beta}^+(z)) (\psi_{E_b, \beta}^{*-}(w) \psi_0(w)), \quad (4.12)$$

and

$$j_{st}^{\pm}(x) = (j_6(x) \pm i j_7(x)) \delta_{st} + (\vec{j}_{\pi'}(x) \pm i \vec{j}_{\pi}(x)) \cdot \vec{\tau}, \\ \bar{j}_{st}^{\pm}(x) = (j_6(x) \pm i j_7(x)) \delta_{st} - (\vec{j}_{\pi'}(x) \pm i \vec{j}_{\pi}(x)) \cdot \vec{\tau}. \quad (4.13)$$

Note $\bar{j}^{\pm} = \tau^2 j^{\pm} \tau^2$ transform under $SU(2) \times SU(2)$ like $j^{\mp}$. (See Appendix A.)

B. The two pseudoparticle sector

In the two pseudoparticle sector we do not have contributions to the four-point function from the nonzero-energy states due to the extra degeneracy of the zero-energy mode. We have instead

$$\Delta_{|v|=2}^{(4)} = \sum_{(s,\alpha)(\bar{s},\beta)} \langle 0 s \alpha | T | 0 \bar{s} \beta \rangle, \quad (4.14)$$

where α, β label the extra two-fold degeneracy. The zero-energy modes have been calculated by Grossman⁶ for the Jackiw-Nohl-Rebbi⁸ pseudoparticle.

For $v=2$ his result is

$$\psi_{mi\bar{s}}^{(\alpha)(s)} = (\bar{\alpha}^{\mu} \tau^2)_{im} g^{1/2} \partial_{\mu} \frac{\phi_a}{g} \delta_{s\bar{s}}, \quad (4.15)$$

where m and i are righthanded Dirac and color isospin indices, respectively, and

$$\begin{aligned} \bar{\alpha}^0 &= 1 \\ \bar{\alpha}^k &= -i \tau^k \end{aligned} \quad (4.16)$$

For the two pseudoparticle case

$$g = \frac{\lambda_1^2}{(x-x_1)^2} + \frac{\lambda_2^2}{(x-x_2)^2} + \frac{\lambda_3^2}{(x-x_3)^2}, \quad (4.17)$$

with ϕ_1, ϕ_2 any other linear combinations of the three functions $\lambda_i^2/(x-x_i)^2$, such that ϕ_1, ϕ_2, g are linearly independent.

Since the source J is diagonal in color isospin the sum over these indices in (4.14) results in the term

$$\sum_i \psi_{im}^{*(\alpha)} \psi_{in}^{(\beta)} = g \partial_\mu \frac{\phi_\alpha}{f} \partial_\nu \frac{\phi_\beta}{f} [(\bar{\alpha}^\mu \tau^2)^\dagger (\bar{\alpha}^\nu \tau^2)]_{mn}, \quad (4.18)$$

so that

$$\begin{aligned} \langle 0s\alpha | J | 0t\beta \rangle &= C \int d^4x \, g \partial_\mu \frac{\phi_\alpha}{f} \partial_\nu \frac{\phi_\beta}{f} \text{Tr} \left[\frac{1}{2} (1 - \gamma_5) \gamma^\nu \gamma^\mu J_{st} \right] \\ &= C \int d^4x \left\{ g \partial_\mu \frac{\phi_\alpha}{f} \partial_\mu \frac{\phi_\beta}{f} \text{Tr} \left[\frac{1}{2} (1 - \gamma_5) J_{st} \right] \right. \\ &\quad \left. + i g \left(\partial_\mu \frac{\phi_\alpha}{f} \partial_\nu \frac{\phi_\beta}{f} - \partial_\nu \frac{\phi_\alpha}{f} \partial_\mu \frac{\phi_\beta}{f} \right) \text{Tr} \left[\frac{1}{2} (1 - \gamma_5) \Sigma_{\mu\nu} J_{st} \right] \right\}, \end{aligned} \quad (4.19)$$

where C is a normalization constant and $\Sigma_{\mu\nu} = \frac{1}{2i} [\gamma^\mu, \gamma^\nu]$. Comparing this with (3.5) and (3.6) shows that now even a $\Sigma_{\mu\nu}$ type source can give nonzero contribution to the off-diagonal elements in the $\alpha\beta$ matrix. We do not study the corresponding Green's functions here but restrict ourselves to the source (3.7). (It is easy to see that they have the same chiral symmetry properties.) Then we have

$$\langle 0s\alpha | J | 0t\beta \rangle = \int d^4x \, j_{st}^-(x) f_{\alpha\beta}(x), \quad (4.20)$$

where

$$f_{\alpha\beta}(x) = C g(x) \partial_\mu \frac{\phi_\alpha(x)}{g(x)} \partial_\mu \frac{\phi_\beta(x)}{g(x)}. \quad (4.21)$$

Let us write the matrix in (4.14) in a block form

$$\Delta_{\nu=2}^{(4)} = \det \begin{pmatrix} \int d^4x j^-(x) f_{11}(x) & \int d^4y j^-(y) f_{12}(y) \\ \int d^4z j^-(z) f_{21}(z) & \int d^4w j^-(w) f_{22}(w) \end{pmatrix}. \quad (4.22)$$

Now using the identity (A...D are 2×2 matrices)

$$\det \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \det A \det D + \det B \det C - \text{Tr}(\bar{A} B \bar{D} C), \quad (4.23)$$

where $\bar{A} = \tau^2 \tilde{A} \tau^2$ we can write $\Delta^{(4)}$ finally in the form

$$\begin{aligned} \Delta_{\nu=2}^{(4)} = & \int d^4x d^4y d^4z d^4w \left\{ \frac{1}{4} \text{Tr}(\bar{j}^-(x) j^-(y)) \text{Tr}(\bar{j}^-(z) j^-(w)) \right. \\ & \times (f_{11}(x) f_{11}(y) f_{22}(z) f_{22}(w) + f_{12}(x) f_{12}(y) f_{21}(z) f_{21}(w)) \\ & \left. - \text{Tr}(\bar{j}^-(x) j^-(y) \bar{j}^-(z) j^-(w)) f_{11}(x) f_{12}(y) f_{22}(z) f_{21}(w) \right\} \end{aligned} \quad (4.24)$$

From this we again note that all terms break $U(2) \times U(2)$ into $SU(2) \times SU(2) \times U(1)$, but there is no further symmetry breaking.

To get the generating functional for the four-point functions one must then do the collective coordinate integrations which we will not do here. We would just like to note that although (4.17) has 15 parameters there are arguments⁸ that gauge invariant quantities like $\psi^+\psi$ depend only on 13 parameters; also note that the normalization coefficient C in (4.22) may depend on the parameters $\lambda_i, (x_i)_\mu$.

The anti-instanton contribution calculation proceeds in an analogous manner. The result can be obtained from (4.24) by replacing $(\pm)$ by $(\mp)$ in the sources and supplying the appropriate coefficient functions $f_{\alpha\beta}$ from (4.21), using the anti-instanton fermionic solutions. No change of symmetry breakdown occurs.

V. Summary

We have undertaken a calculation to check that the residual symmetry in massless two-color QCD is $SU(2) \times SU(2) \times U(1)$, taking into account multiple pseudoparticle solutions, higher order n -point functions, and non-zero-energy fermion modes. The results are in the affirmative: there is no further symmetry breaking in the $O(\hbar)$ approximation.

We are grateful for conversations with S. Pinsky.

Appendix A

Some notation and simple properties will be developed here for the analysis of the n-point function transformation behavior

We define

$$\begin{aligned}
 M_{st}^{\pm} &= \frac{1}{2} \bar{\psi} (1 \mp \gamma^5) \psi \delta_{st} + \frac{1}{2} \bar{\psi} \vec{\tau} (1 \mp \gamma^5) \psi \cdot \vec{\tau}_{st} \\
 &= \frac{1}{2} (\sigma \pm i \eta) \delta_{st} + \frac{1}{2} (\vec{\pi}' \pm i \vec{\pi}) \cdot \vec{\tau}_{st}, \quad (A1)
 \end{aligned}$$

where

$$\begin{aligned}
 \sigma &= \bar{\psi} \psi, \\
 \eta &= i \bar{\psi} \gamma^5 \psi, \\
 \vec{\pi} &= i \bar{\psi} \gamma^5 \vec{\tau} \psi, \\
 \vec{\pi}' &= \bar{\psi} \vec{\tau} \psi.
 \end{aligned} \quad (A2)$$

Under $U(2) \times U(2)$ rotations

$$\begin{aligned}
 \psi_L &\equiv \frac{1}{2} (1 + \gamma^5) \psi \rightarrow \psi_L' = U_L \psi_L, \\
 \psi_R &\equiv \frac{1}{2} (1 - \gamma^5) \psi \rightarrow \psi_R' = U_R \psi_R,
 \end{aligned} \quad (A3)$$

the $M^{\pm}$ transform according to

$$\begin{aligned}
M^+ &\rightarrow U_R M^+ U_L^\dagger, \\
M^- &\rightarrow U_L M^- U_R^\dagger.
\end{aligned}
\tag{A4}$$

It is then evident that $\det M^\pm$ is invariant under $SU(2) \times SU(2)$,

$$4 \det(M^\pm) = (\sigma^2 + \vec{\pi}^2) - (\eta^2 + \vec{\pi}'^2) \pm 2i(\sigma\eta - \vec{\pi} \cdot \vec{\pi}'), \tag{A5}$$

but not under $U(2) \times U(2)$. However, $\det M^+ M^-$ and $\text{tr} M^+ M^-$ are $U(2) \times U(2)$ invariant, leading to the chiral $U(2)$ scalars:

$$\begin{aligned}
&(\sigma \vec{\pi}' + \eta \vec{\pi} + \vec{\pi}' \times \vec{\pi})^2, \\
&\sigma^2 + \vec{\pi}^2 + \eta^2 + \vec{\pi}'^2.
\end{aligned}
\tag{A6}$$

$(\sigma, \eta, \vec{\pi}, \vec{\pi}')$ span an irreducible $U(2) \times U(2)$ representation containing the $SU(2) \times SU(2)$ irreducible representations $(\sigma, \vec{\pi})$ and $(\eta, \vec{\pi}')$. This follows from a property of 2×2 special unitary matrices ($\det U = 1$)

$$\tau^2 \tilde{U} \tau^2 = U^\dagger \tag{A7}$$

implying $\tau_{2\psi_{L,R}}$ transform like $\bar{\psi}_{L,R}$ under $SU(2) \times SU(2)$ and hence $\tau_{M^\pm}^2$ transform like $M^\mp$ under $SU(2) \times SU(2)$. The linear combinations

$$\begin{aligned}
M^+ + \tau^2 \tilde{M}^- \tau^2 &= \sigma + i \vec{\pi} \cdot \vec{\tau} \sim M^+, \\
-i(M^- + \tau^2 \tilde{M}^+ \tau^2) &= \eta - i \vec{\pi}' \cdot \vec{\tau} \sim M^+,
\end{aligned}
\tag{A8}$$

and their adjoints

$$\begin{aligned} \sigma - i \vec{\pi} \cdot \vec{\tau} &\sim M^-, \\ \eta + i \vec{\pi}' \cdot \vec{\tau} &\sim M^-, \end{aligned} \quad (A9)$$

form $SU(2) \times SU(2)$ irreducible matrices with indicated transformation properties, under $SU(2) \times SU(2)$. We are thus lead to the $SU(2) \times SU(2)$ bilinear invariants

$$\sigma^2 + \vec{\pi}^2, \quad \eta^2 + \vec{\pi}'^2, \quad \text{and} \quad \sigma\eta - \vec{\pi} \cdot \vec{\pi}'. \quad (A10)$$

The n-point function analysis in the text yields matrix elements of the form

$$\langle \psi_\alpha | J_{st} | \psi_\beta \rangle = \langle \psi_\alpha^+ | J_{st} | \psi_\beta^+ \rangle + \langle \psi_\alpha^- | J_{st} | \psi_\beta^- \rangle, \quad (A11)$$

where

$$y^5 | \psi_\alpha^\pm \rangle = \pm | \psi_\alpha^\pm \rangle, \quad (A12)$$

where it has been assumed,

$$y^5 J_{st} y^5 = J_{st}. \quad (A13)$$

For sources, diagonal in color, with the form

$$J_{st} = \frac{1}{2} (j_\sigma + i j_\eta y^5) + \frac{1}{2} (\vec{j}_{\sigma'} + i \vec{j}_{\pi'} y^5) \cdot \vec{\tau}_{st}, \quad (A14)$$

we have

$$\begin{aligned}
\langle \psi_\alpha^\pm | \bar{J}_{S+} | \psi_\beta^\pm \rangle &= \frac{1}{2} \int d^4x \sum_i \psi_{\alpha i}^* \psi_{\beta i} \\
&\times \left\{ (j_\sigma \pm i j_\eta) \delta_{S+} + (\vec{j}_{\pi'} \pm i \vec{j}_\pi) \cdot \vec{\tau}_{S+} \right\} \\
&\equiv \int d^4x f_{\alpha\beta}(x) j_{S+}^\pm(x),
\end{aligned} \tag{A15}$$

where i labels the color index, and

$$j_{S+}^\pm = (j_\sigma \pm i j_\eta) \delta_{S+} + (\vec{j}_{\pi'} \pm i \vec{j}_\pi) \cdot \vec{\tau}_{S+}. \tag{A16}$$

In the text we also use the property

$$\det(A + \vec{B} \cdot \vec{\tau}) = \frac{1}{2} \text{Tr} [(A + \vec{B} \cdot \vec{\tau})(A - \vec{B} \cdot \vec{\tau})]. \tag{A17}$$

Thus if

$$\begin{aligned}
\bar{j}^\pm &\equiv \tau^2 \tilde{j}^\pm \tau^2 \\
&= (j_\sigma \pm i j_\eta) - (\vec{j}_{\pi'} \pm i \vec{j}_\pi) \cdot \vec{\tau},
\end{aligned} \tag{A18}$$

then

$$\det j^\pm = \frac{1}{2} \text{Tr} (\bar{j}^\pm j^\pm). \tag{A19}$$

Appendix B

To obtain some gross features of the functional determinant it is instructive to consider the case when the source is constant. If we use the source (3.7) the perturbation mixes states of non-zero energy E_a and $-E_a$ but not states with $|E_a| \neq |E_b|$. The determinantal contribution from a pseudoparticle solution with topological charge v is then

$$\det(m + J) = \prod_{a > 0} \Delta'(a, J) \Delta'(0, J)^{|v|} \quad (B1)$$

where for $N_f = 2$,

$$\Delta'(a, J) = \det \begin{pmatrix} (E_a + \frac{1}{2}j_\sigma)1 + \frac{1}{2}\vec{j}_\pi \cdot \vec{\tau} & i\frac{1}{2}(j_\eta 1 + \vec{j}_\pi \cdot \vec{\tau}) \\ i\frac{1}{2}(j_\eta 1 + \vec{j}_\pi \cdot \vec{\tau}) & (-E_a + \frac{1}{2}j_\sigma)1 + \frac{1}{2}\vec{j}_\pi \cdot \vec{\tau} \end{pmatrix}, \quad (B2)$$

$$\Delta'(0, J) = \det \left(\frac{1}{2}(j_\sigma \pm i j_\eta)1 + \frac{1}{2}(\vec{j}_\pi \cdot \pm i \vec{j}_\pi) \cdot \vec{\tau} \right). \quad (B3)$$

Here the $+(-)$ sign goes with pseudoparticle of topological charge $-1(+1)$.

One can now use the identity (4.24) on (B2) and get

$$\begin{aligned} \Delta'(a, J) &= \left[(E_a + \frac{1}{2}j_\sigma)^2 - (\frac{1}{2}\vec{j}_\pi \cdot)^2 \right] \left[(-E_a + \frac{1}{2}j_\sigma)^2 - (\frac{1}{2}\vec{j}_\pi \cdot)^2 \right] \\ &\quad + \frac{1}{16} [j_\eta^2 - \vec{j}_\pi^2] \end{aligned}$$

$$\begin{aligned}
& + \text{Tr} \left\{ \left[(E_a + \frac{1}{2} j_\sigma) \mathbb{1} - \frac{1}{2} \vec{j}_\sigma \cdot \vec{\tau} \right] \left[\frac{1}{2} j_\eta + \frac{1}{2} \vec{j}_\eta \cdot \vec{\tau} \right] \right. \\
& \quad \left. \times \left[(-E_a + \frac{1}{2} j_\sigma) \mathbb{1} - \frac{1}{2} \vec{j}_\sigma \cdot \vec{\tau} \right] \left[\frac{1}{2} j_\eta + \frac{1}{2} \vec{j}_\eta \cdot \vec{\tau} \right] \right\} \\
& = \left[E_a^2 - \frac{1}{4} (j_\sigma^2 + \vec{j}_\sigma^2 + j_\eta^2 + \vec{j}_\eta^2) \right]^2 - \frac{1}{4} [j_\sigma \vec{j}_\eta + j_\eta \vec{j}_\sigma + \vec{j}_\sigma \times \vec{j}_\eta]^2, \tag{B4}
\end{aligned}$$

and

$$\Delta'(0, J) = \frac{1}{4} \left\{ (j_\sigma^2 + \vec{j}_\sigma^2) - (j_\eta^2 + \vec{j}_\eta^2) \pm i 2 (j_\sigma j_\eta - \vec{j}_\sigma \cdot \vec{j}_\eta) \right\}. \tag{B5}$$

Here we see clearly that the Δ 's are composed of $SU(2) \times SU(2)$ invariants only, so that no further symmetry breaking is visible. Note that in this constant source case, the non-zero-energy contribution, $\Delta'(a, J)$, is a $U(2) \times U(2)$ invariant; the term with lower symmetry, $SU(2) \times SU(2) \times U(1)$, only arises in $\Delta(0, J)$, the zero-energy contribution. The above computations are valid for all pseudoparticle configurations. The effect of different external pseudoparticle fields is to change the energy levels and possibly their degeneracy. The collective coordinate integrations will now give divergent coefficients to these generating functions. This is to be expected because with constant source we are developing a generating functional for n-point functions integrated over all n-points, i.e., expectedly divergent moments. Nonetheless, the lack of any further breaking of symmetry in the moments leads us to suspect that the residual symmetry is $SU(2) \times SU(2) \times U(1)$, taking into account all pseudoparticle solutions and all n-point functions.

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