ISOCRONOUS LIMITS ON UNEQUAL ENERGIES
FOR BUNCHED BEAM COLLISIONS

R. Diebold and M. Month

Prepared for
FNAL Summer Study
on the Uses of Energy Doubler
Aspen
June 28-July 9, 1976

NOTICE
This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

MASTER

OT W-31-109-ENG-38

ARGONNE NATIONAL LABORATORY, ARGONNE, ILLINOIS

operated under contract W-31-109-Eng-38 for the
U. S. ENERGY RESEARCH AND DEVELOPMENT ADMINISTRATION

ISOCRONOUS LIMITS ON UNEQUAL ENERGIES FOR BUNCHED BEAM COLLISIONS*

R. Diebold

Argonne National Laboratory

and

M. Month

Brookhaven National Laboratory

If it were not for the collider aspects of the ESD, it would be built to have the same central-orbit circumference as the main ring (MR). For proper collisions of bunched beams with unequal energies, the orbits in the two machines must have slightly different circumferences to compensate for the difference in velocities. Since the ESD will have the higher energy, it should have the larger circumference and should be constructed with an average radius (from the ring center) 1 or 2 cm greater than the MR. This difference in radius should be as large as possible without endangering the MR to ESD transfer which requires useable orbits of the same circumference in the two machines. For given aperture windows in the machines, this then sets a limit on the velocity difference in two beams colliding in a bunched mode. For the ESD set at several hundred GeV, the energy in the MR for useful bunched collisions will have to be greater than a limit probably in the region 100 to 150 GeV, depending on the useful apertures of the two machines.

For useful bunched beam operation, the revolution time $T = C/\beta c$

* Contribution to the 1976 Fermilab Summer Study of the Doubler as Collider. Work supported by the U. S. Energy Research and Development Administration.

(C = circumference, βc = velocity) must be the same for the two rings if the meeting point of the bunches is not to slip. This will require a master oscillator to lock the frequency and phase of the two rings. The required difference in circumference between protons with $\beta = 1$ and protons of finite energy, E , is

$$\frac{\Delta C}{C} = \frac{\Delta \beta}{\beta} \approx \frac{1}{2} \left(\frac{M_p}{E} \right)^2 ,$$

where M_p is the proton rest mass. To get a feeling for the size of the effect, we take $E = 100$ GeV and $C = 6.3$ km;

$$\Delta C = 28 \text{ cm.}$$

This change in circumference is much larger than the ~ 1 cm change generated by the "dog legs" used to bring the rings into collision. If the MR were set to 100 GeV and the ESD to a much higher energy, the 28 cm would imply an average difference in radius of the two rings of:

$$\langle \Delta R \rangle = \Delta C / 2\pi = 4.4 \text{ cm.}$$

As an example, we assume an average useful aperture of about 1 inch in each machine. The ESD would then be built with an average radius 2 cm larger than the MR. For beam transfer, when the orbit circumference in the two machines must be the same, the beam in the MR could be on average 1 cm displaced outward from the MR central orbit and then transferred to the ESD on an orbit 1 cm on average to the inside of the ESD central orbit as shown in Fig. 1. When colliding

100 GeV protons in the MR with high energy protons in the ESD, the MR beam would be 1.2 cm on average inside of the MR central orbit and the ESD beam 1.2 cm to the outside of the ESD central orbit.

Such orbits correspond to orbits of off-momentum particles and the actual distance off the central orbit is given by the local dispersion function η :

$$\Delta x = \eta \Delta p/p.$$

To calculate the $\Delta p/p$ needed for the beam with respect to the central orbit, we use the fact that the relative change in circumference, $\Delta C/C$, is proportional to $\Delta p/p$. The proportionality constant, the "momentum compaction", is easily obtained by noting that at transition energy the revolution frequency is independent of momentum, i. e. ,

$$\frac{\Delta C}{C} = \left(\frac{\Delta \beta}{\beta} \right)_{tr} .$$

As can be easily shown, for any particle with finite mass

$$\frac{\Delta \beta}{\beta} = \frac{1}{\gamma^2} \frac{\Delta p}{p} .$$

Thus we have

$$\frac{\Delta C}{C} = \frac{1}{\gamma_{tr}^2} \frac{\Delta p}{p}$$

where γ_{tr} is the transition energy of the machine in units of M_p .

Since the revolution times in the two rings must be the same for

useful bunched-beam collisions.

$$\frac{C_1}{\beta_1} = \frac{C_2}{\beta_2}$$

where subscripts 1 and 2 refer to the MR and ESD, respectively. Then

$$\Delta C = C_2 - C_1 = \frac{\beta_2 - \beta_1}{\beta_2} C_2 ,$$

or to a good approximation

$$\frac{\Delta C}{C} = \beta_2 - \beta_1 = \frac{M_p^2}{2} \left(\frac{1}{E_1^2} - \frac{1}{E_2^2} \right) .$$

The corresponding amount of momentum shift to achieve this difference in circumference is thus

$$\frac{\Delta p}{p} = \gamma_{tr}^2 \frac{\Delta C}{C} = \frac{E_{tr}^2}{2} \left(\frac{1}{E_1^2} - \frac{1}{E_2^2} \right) .$$

For $E_1 = 100$ GeV and $E_2 \gg E_1$, and taking $E_{tr} = 17.6$ GeV,

$$\frac{\Delta p}{p} = 1.55\% .$$

As previously discussed, this would likely be split between the two rings. For example, a full range of 0.8% in each would cover both this asymmetric case as well as the equal energy case (required for MR-to-ESD beam transfer).

We can now calculate the maximum excursion of these "off-momentum" orbits:

$$\begin{aligned}\Delta x_{\max} &= \eta_{\max} \Delta p/p \\ &= \frac{\eta_{\max} E_{tr}^2}{2} \left(\frac{1}{E_1^2} - \frac{1}{E_2^2} \right) .\end{aligned}$$

For the ESD lattice $\eta_{\max} = 3.9$ m and

$$\Delta x_{\max} = 6.1 \times 10^4 \left(\frac{1}{E_1^2} - \frac{1}{E_2^2} \right)$$

(units of cm, GeV). For $E_1 = 100$ GeV, $E_2 \gg E_1$, $\Delta x_{\max} = 6.1$ cm, to be compared with the $\langle \Delta R \rangle = 4.4$ cm calculated previously; i. e., the worst point in the lattice is a factor of 1.4 times worse than the average displacement. The special focusing near the crossing points may lead to even larger displacements. If the $\Delta x_{\max}$ of 6.1 cm were split evenly between the machines, a 3.0 cm aperture would be required in each for $E_1 = 100$ GeV. This may be optimistic, but since the aperture required goes as $1/E^2$, the space required drops rapidly as the energy increases. At 150 GeV, an aperture of 1.4 cm in each machine would suffice.

Fig 1. Schematic of average orbit offsets required to allow main ring operation at 100 GeV.