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M. Month
7/14/76

1976 Fermilab Summer Study, Aspen, Colorado

BNL-21682

Stacking in the Fermilab Doubler *

I. Introduction

The feasibility of stacking beam in a storage ring by the phase displacement technique, i.e. by the accumulation of momentum strips, is determined by a complicated interplay of many factors. It is the purpose of this note to comment on some of these factors, especially as they relate to stacking beam in the Fermilab doubler ring. No attempt is made to present a consistent solution.

An arbitrary division is made into five subject categories connected with the stacking process. They are listed as follows:

1. Momentum dilution, that is, the dilution of the longitudinal phase space area.
2. Rebunching the stack for acceleration.
3. The physical aperture used to create a stack of given current.
4. Beam loss during stacking in a superconducting environment.
5. Field errors due to random errors in the placement and support of the superconducting coils, including the amplification of the field errors for orbits displaced from the magnet center.

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The basic theory is given in the next few sections, and then is applied using doubler parameters.

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CONTRACT NO. E(30-1)-16

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II. Momentum Dilution

Stacking Efficiency

The formation of a stack is essentially the transformation of many small bunches of longitudinal phase space area into a coasting beam. Since the process is governed by the conservation of this area, it may be characterized by a single parameter, the stacking efficiency. It is defined to be the ratio of the stack momentum spread, with and without dilution--that is,

$$\eta_{\text{eff}} = \frac{\delta p[\text{no dilution}]}{\delta p[\text{with dilution}]} \quad (2.1)$$

where δp is the relative momentum spread, $(\Delta p/p)$.

Intrinsic Dilution

The phase displacement mechanism of stacking contains an intrinsic diluting process which has been interpreted as a form of statistical behavior. A computer model has been analyzed in statistical terms and has led to the simple expression,

$$\eta_{\text{eff}} = \left[1 + \frac{2\Gamma}{3\sqrt{n_p} \alpha(\Gamma)} \right]^{-1}, \quad (2.2)$$

which is a function only of the stable rf phase angle, ϕ_s . The quantity Γ is related to ϕ_s by

$$\Gamma = \sin \phi_s, \quad \text{and}$$

$\alpha(\Gamma)$ is the moving bucket factor. The dependence of the stacking efficiency on the number of stacked pulses, n_p , is in terms of $\sqrt{n_p}$, characteristic of statistical processes.

Systematic Errors

A few potential sources of error which will lead to momentum dilution arise in the transfer process and during the acceleration of the injected pulse into the stack. For example, there are timing and matching errors in the transfer procedure. Also, effects of noise and the closeness of the bucket unstable fixed point will inevitably lead to a momentum dilution of the stack, i.e. a decreased efficiency. There is however some measure of control over these types of errors.

Longitudinal Instability

A measure of the impact of longitudinal instabilities is the impedance, which relates the longitudinal electric field induced by the beam interacting with the surrounding medium to the beam current itself. The stacked beam is susceptible to impedances over a wide range of frequencies. Methods have been developed to cope with the large impedances resulting from resonant structures in the ring. However it appears that the accumulation of effects from many small discontinuities of the vacuum chamber can lead to large impedances at very high frequencies, typically, for high energy rings, in the microwave region. There are as yet no adequate methods to deal with this type of impedance. Thus, the beam must be designed so as to be capable of stabilizing itself through Landau damping. In particular the frequency spread in the beam must be sufficiently high so that incipient coherence (bunching) will be prevented from developing.

As the first few pulses are debunched to form the beginnings of a stack, the beam is highly susceptible to instability. A large momentum spread, i.e. high current stack, on the other hand, tends to be stable. An understanding of this fact has led to the development of procedures to circumvent the formation of unstable oscillations near the early stages of stack creation. Simply stated, what is done is to dilute the stack for the first few pulses. Then, dense beam is pumped into the stack while dilute tails are scraped away. In this way, density can be traded for time. The result is a stacking efficiency which can be made to approach the ideal (statistical) efficiency.

However, the effect of the high frequency impedance on the bunches about to be deposited in the stack cannot be overcome in this manner. There is a mild shielding phenomenon from a stack, but, by and large, each injected pulse is subject to instability at the same level. Therefore, the stack dilution due to the instability of the injected bunches is real. The only known means of avoiding it is sufficient Landau damping.

The instability of the bunches is not completely understood. However, it is known to manifest itself during the stacking process. As the voltage is reduced, the synchrotron frequency is reduced. Thus, an instability developing fast enough can cause parts of the bunch to blow up. The normal containment of such a tendency by the synchrotron motion, which can mix incipient coherence is absent for fast growth, meaning "fast" with respect to the period of synchrotron motion.

Although a complete theory is lacking, there is a phenomenological threshold criterion, which was obtained from experiments at the ISR. Computer simulation was also performed and tends to support the use of this criterion. Stated simply, it is an upper limit on the longitudinal impedance function present in the ring:

$$\left| \frac{Z}{n} \right| < \frac{0.4 |\eta| A_B}{e N_B} \left(\frac{\Delta p}{P} \right)_T, \quad (2.3)$$

where Z is the impedance,

n is the number of unstable oscillations in one revolution,

A_B is the invariant bunch area,

N_B is the number of particles per bunch,

$|\eta| = \left| \frac{1}{\gamma_{tr}^2} - \frac{1}{\gamma^2} \right|$, relates the beam frequency

spread, $(\Delta f/f)$, to the momentum spread, $(\Delta p/p)$,

$$\left(\frac{\Delta f}{f} \right)_T = \eta \left(\frac{\Delta p}{P} \right)_T,$$

T means that the spread contains 95% of the particles in the distribution, and

γ_{tr} , γ are the transition energy and beam energy respectively, in units of proton mass.

As implied above, the instability occurs as the voltage is reduced. Near the minimum voltage, bucket and bunch area become the same, and the full momentum spread of the bunch becomes

$$\frac{\Delta p}{p} = \frac{\pi}{2} \frac{A_B h f_{\text{rev}} F(\Gamma)}{p \alpha(\Gamma)} \quad (2.4)$$

where f_{rev} is the revolution frequency,
 h is the harmonic number of the rf system, assumed to
 be equal to the number of bunches stacked per pulse,
 and $\alpha(\Gamma)$, $F(\Gamma)$ are moving bucket factors.

III. Rebunching the Stack

Rebunching the stacked beam in preparation for acceleration is strongly related to the stacking efficiency and the general dilution that results during the stacking process. The connection can be tied to the minimum voltage required to capture the stack. In general the accelerating voltage should be significantly higher in order that the bucket be sufficiently removed from the stable bunch, preventing bucket leaks.

When a coasting stack is bunched, all buckets of the capture rf system are filled. Thus, in passing, we point out that if bunched beam collisions of a stacked doubler beam are to take place with the main ring beam, the harmonic number of the doubler rf system should be the same as that for the main ring rf system.

The minimum voltage for accelerating a stack of h bunches each of longitudinal phase space area $\bar{A}$ can be expressed as the inequality,

$$V > \frac{\pi^3 f_{\text{rev}}^2 |\eta| \bar{A}^2 h^3}{32 p \alpha^2(\Gamma)} \quad (3.1)$$

The bunch area, on the other hand, can be written in terms of A , the invariant area of each injected bunch from the main ring:

$$\bar{A} \geq \frac{n_p A}{\eta_{\text{eff}}} \quad (3.2)$$

where we have assumed the harmonic numbers of the main ring and doubler (stack-accelerating) rf systems to be the same. The equality in (3.2) holds for "ideal" stacking and perfect adiabatic capture.

Actually, some dilution is inevitable and an even higher capture voltage is required. Also, we have neglected the undoubted presence of an inductive impedance at rf frequencies. (A substantial inductive wall impedance has been observed and measured at the ISR). This means an even higher peak voltage to trap the stacked beam. Since high capture voltages translate into greater capture dilution, the conditions are not at all congenial for near-ideal capture.

IV. Physical Aperture Needed for Stacking

The physical aperture used in phase displacement stacking is made up of 4 components:

- Stack
- Septum
- Injected Beam
- Closed Orbit Errors plus Magnet Sagitta

In discussing the widths of stack and injected beam, it is assumed that momentum width and betatron width can be added arithmetically. An addition theorem is of course strongly distribution dependent,

but since the momentum distribution in either case does not have long tails, arithmetic addition does not appear to be unduly pessimistic.

The total stack width is

$$A_{st} = 2 a_{\beta} + \eta_{max} \delta_s , \quad (4.1)$$

where $a_{\beta} = \sqrt{\epsilon \beta_{max}}$ is the betatron 1/2 size,
 ϵ is horizontal emittance,
 β_{max} is value of the beta function at the kicker,
that is at a horizontal maximum,
 η_{max} is the value of the dispersion function at
the kicker,
and δ_s is the full momentum width ($\Delta p/p$) of the stack
[nominally, the 95% width].

The energy invariant emittance E is related to ϵ by

$$E = \beta \gamma \epsilon.$$

The stack momentum spread is related to the longitudinal invariant area, $\bar{A}$, of the stack by

$$\delta_s = \frac{f_{rev} h \bar{A}}{p} . \quad (4.2)$$

The injected bunch width is

$$A_{In} = 2 a_{\beta} + \eta_{max} \delta_p , \quad (4.3)$$

where δ_p is the full momentum width of the injected bunch, which can be written

$$\delta_p = \frac{4c}{\pi} \frac{A_B}{p l_B} \quad , \quad (4.4)$$

A_B is the invariant bunch area,

l_B is the total bunch length, which can be expressed by

$$l_B = c \left[\frac{8 A_B^2 |\eta|}{\pi^3 p f_{rev}^2 V h \cos \phi_s} \right]^{1/4} \quad . \quad (4.5)$$

If s is the septum width and Δ is the combined sagitta and closed orbit allowance, then the total aperture required is

$$A_p = A_{st} + A_{in} + s + \Delta \quad , \quad (4.6)$$

or,
$$A_p = 4a_b + \eta_{max} (\delta_s + \delta_p) + s + \Delta. \quad (4.7)$$

V. Beam Loss at Injection When Stacking

Because the doubler beam is in a superconducting environment, one must be concerned with the possibility of magnet quenches due to radiation heating of the superconducting coils, the source of this heating being particle loss. Since when the beam is stacked the magnets are operating at a low field, the required temperature rise to reach the critical, quench temperature, is relatively large. On the other hand, the phase displacement stacking method is a lossy process, beam loss being on the order of 50% of the injected pulse during acceleration into the stack at the ISR.

The use of scrapers and absorbers to prevent unstable particles from reaching the coils is therefore an important consideration in the design of a "stacking" doubler.

The scrapers can also be used to maximize the vertical beam density so as to maximize the luminosity. The principles behind the procedures for accomplishing this have been well demonstrated at the ISR.

VI. Random Field Errors due to Magnet Coils

The presence of current carrying coils just outside the beam aperture in superconducting magnets has the effect of making the magnetic field within the aperture sensitive to coil positioning and support errors. The immediate consequence is the creation of field multipole components at the magnet center which are strong functions of the radial distance to the coils. Furthermore, for orbits off the magnet center, the multipoles become amplified, with the amplification increasing rapidly as the coils are approached.

Since the stacking process requires an injected orbit off the center, the amplification effect could be significant. Simple estimates of resonance strengths induced by these random field errors can be made using a $\cos \theta$ current distribution in the coils. Although not strictly applicable to the doubler, such a model should be adequate for an order-of-magnitude estimate using doubler parameters. In this spirit, only one-dimensional resonances are considered. The parameters required are as follows:

- n_{mag} - the number of magnets containing errors,
 N_D - the number of statistically independent coil units
in error around each magnet aperture,
 β - the mean value of the amplitude function at the
azimuthal positions of the errors,
 $\langle \delta \rangle$ - the rms error in the location of each independent
coil unit,
and R_c - the radius of the coil block, taken to be the coil
inner radius.

The amplification factor for off center orbits is the ratio of multipole components off and on center and is a function only of the ratio of the displacement of the off center orbit, x_o , to the coil radius, R_c . Introducing

$$t = x_o/R_c, \quad (6.1)$$

the amplification factor for the multipole of order m ($m = 1$ corresponds to the dipole case) can be expressed as

$$r_{m-1}(t) = 0.7/(1-t)^{m+1/2}. \quad (6.2)$$

The resonance width for a linear resonance ($m=1$) is defined such that when the unperturbed tune reaches its edges, the betatron function reaches infinite magnitude. The non-linear resonance width for a resonance of order m ($m>2$) is defined as the tune shift from the resonance at which the unstable fixed points enter the beam betatron boundary. Their rms values, δv_m , are

for $m=2$,

$$\delta v_2 = r_1(t) \left(\frac{2}{N_b n_{\text{mag}}} \right)^{1/2} \frac{2\beta \langle \delta \rangle}{R_C^2}, \quad (6.3)$$

and for $m>2$

$$\delta v_m = r_{m-1}(t) \left(\frac{a_\beta}{2R_C} \right)^{m-2} \left(\frac{2}{N_b n_{\text{mag}}} \right)^{1/2} \frac{m\beta \langle \delta \rangle}{2R_C^2}, \quad (6.4)$$

where recall that a_β is the 95% betatron 1/2-size, $a_\beta = (\beta\epsilon)^{1/2}$.

Two interesting beam parameters are the rms closed orbit error, A_{co} , and the rms maximum β -variation $(\Delta\beta/\beta)_{\text{max}}$, near resonance. With Δ the tune distance from resonance, these are expressed by

$$A_{\text{co}} = r_0(t) \left(\frac{1}{N_b n_{\text{mag}}} \right)^{1/2} \frac{\beta \langle \delta \rangle}{|\Delta| R_C}, \quad (6.5)$$

and

$$(\Delta\beta/\beta)_{\text{max}} = \frac{\delta v_2}{2|\Delta|}. \quad (6.6)$$

VII. Estimates for the Doubler

The doubler parameters that we need and some results are listed in Table I. The resonance widths due to random errors in coil positioning are listed in Table II for various resonances.

TABLE I
 Doubler Parameters and Some Results

A. Longitudinal Stacking Parameters

Symbol	Units	Value	Comments
ϕ_s	degrees	50	
Γ		0.7660	
$\alpha(\Gamma)$		0.1213	
$F(\Gamma)$		0.3286	
n_p		10	number of pulses stacked
η_{eff}		0.43	
p	GeV/c	100	
γ		106.6	
γ_{tr}		18.8	
$ n $		2.74×10^{-3}	
h		1113	
f_{rev}	kHz	47.75	
N_B		1.8×10^{10}	protons per bunch
N		2×10^{13}	protons per pulse
A_B	eV-sec	0.1	Bunch Area, no dilution
$(\Delta p/p)_T$		2.26×10^{-4}	With Voltage Reduction
$ Z/n _{threshold}$	ohms	8.59	Maximum Allowable Impedance for Microwave Instability

B. Rebunching

Symbol	Units	Value	Comments
ϕ_s	degrees	10	
Γ		0.1736	
$\alpha(\Gamma)$		0.6963	
p	GeV/c	100	
h		1113	Doubler rf harmonic
$\bar{A}$	eV-sec	2.33	Ideal, no error dilution
$V_{\min}$	MV	0.935	No error dilution
$V_{\max}$	MV	4.0	$V_{\max} > V_{\min}$

C. Physical Aperture for Stacking

Symbol	Units	Value	Comments
$\beta_{\max}$	m	99.7	
$\eta_{\max}$	m	3.83	
E	rad-m	2×10^{-5}	particles per pulse = 2×10^{13}
ϵ	rad-m	0.19×10^{-6}	$p = 100$ GeV/c
a_β	mm	4.35	
δ_s		1.24×10^{-3}	Ideal Stacking
A_{st}	cm	1.35	
ϕ_s		0	
V	MV	2.0	
l_B	cm	58.0	
δ_p		6.59×10^{-4}	
A_{In}	cm	1.12	
s	cm	1.50	Similar to ISR
A	cm	1.00	Optimistic

D. Random Field Errors

Symbol	Units	Value	Comments
n_{mag}		400	Magnets closer to n_{max}
N_b		10	
β	m	81.6	Near n_{max} -consistent with value of n_{mag}
$\langle \delta \rangle$	mm	0.05	2 mil tolerance on coil positioning
R_c	cm	3.8	Coil ID = 7.6 cm
a_β	mm	3.94	$\epsilon = 0.19 \times 10^{-6}$ rad-m, $\beta = 81.6$ m
x_o	cm	2.28	Injection Orbit
t		0.6	
$r_o(0.6)$		2.8	CO Amplification
Δ		0.15	Distance of tune, ν , from resonance
A_{co}	cm	3.2	Injection Orbit, No Correction
$r_1(0.6)$		6.9	β -amplification
$(\Delta\beta/\beta)_{\text{max}}$		2.9	Injection Orbit, No Correction

Table II

Linear and Non-Linear Resonances, Stopband Widths

Resonance Order m	Central δv_m	r_{m-1} (0.6)	δv_m t = 0.6
2	0.126	6.9	0.872
3	4.91×10^{-3}	17.3	0.0850
4	3.39×10^{-4}	43.2	0.0147
5	2.20×10^{-5}	108.1	2.37×10^{-3}
6	1.37×10^{-6}	270.2	3.69×10^{-4}
7	8.25×10^{-8}	675.5	5.57×10^{-5}
8	4.88×10^{-9}	1688.8	8.25×10^{-6}

DRAFT
M. Month
7/14/76

-17-

VIII. Comments on Stacking

Stacking 10 pulses in the doubler, assuming no dilution except that which is intrinsic to the stacking process itself, leads to a longitudinal impedance limit $|Z/n| < 9\Omega$ with a physical aperture requirement, $A_p = 5$ cm. Because the injection orbit must be "close" to the coils, the field errors are amplified and have a significant effect on the injected beam. In fact since so much of the aperture is used, part of the stack will also be off-center and will also be influenced by amplified field errors. Uncorrected, the field errors seem unreasonable. On the other hand, field correction across the entire aperture is of such a complicated nature as to appear at first glance to be unmanageable.