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by

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PROFILE STUDY OF SHEAR STABILIZATION OF THE LOWER HYBRID DRIFT INSTABILITY IN THE REVERSE-FIELD SCREW PINCH

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ABSTRACT

A criterion due to N. Krall for magnetic shear stabilization of the Lower-Hybrid Drift Instability is applied to model profiles of the Reverse-Field Screw Pinch configuration. Conditions that can virtually eliminate this instability are found numerically, for gentle density profiles. However, shear-stabilization proves to be ineffective for sharper (but still reasonable-looking) profiles. If such profiles have to be lived with, it becomes necessary to rely either on finite-beta stabilization or on the fact that this instability possesses a threshold related to ion gyro-resonance.

I. INTRODUCTION

The Lower-Hybrid Drift Instability (LHDI) has recently become an object of concern with respect to its influence on anomalous resistivity and diffusion of plasma pinches. This is because of its large growth rate ($\propto \sqrt{\omega_{ce} \omega_{ci}}$), its minimal requirement on the drift velocity associated with the cross-field current ($v_{\text{drift}} \lesssim v_{\text{thi}}$), and its insensitivity to the electron-to-ion temperature ratio.¹

The role played by the LHDI in screw pinches can only be fully understood by including the effect of magnetic shear. Such a calculation has recently been performed by Krall,² who finds the following condition for stabilization of the LHDI.

$$K_n L_s < K_n r_{ci} + 1/K_n r_{ci} \quad .$$

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Here the shear length L_s is the length to be traversed in the radial direction such that the magnetic field vector rotates by 45° , K_n is the inverse density gradient scale length, $K_n = n^{-1} dn/dr$, and r_{ci} is the thermal-ion gyro-radius, $r_{ci} = v_{thi}/\omega_{ci}$, where $v_{thi} = \sqrt{2T_i/M_i}$ is the ion thermal speed, and $\omega_{ci} = e|\vec{B}|/m_i c$ is the ion gyro-frequency.

The term $K_n r_{ci}$ in Eq. (1) comes from a large drift approximation ($v_{drift} \gg v_{thi}$) and the term $(K_n r_{ci})^{-1}$ comes from a small drift approximation ($v_{drift} \ll v_{thi}$). The sum of these terms reduces properly to each limiting case, and constitutes a reasonable "patch" for the intermediate regime.

Although Krall's theory is formally nonlocal in that the radial dependence of the LHDI modes is explicitly treated, it is local in the sense that the magnetic fields and the density are assumed to be representable by locally uniform shear lengths and gradient lengths. Consequently, Eq. (1) must be applied point by point to a real pinch profile in order to obtain meaningful results.

By invoking reasonable model (analytical) profiles, we have found that Eq. (1) is generally satisfied in the core of a Reverse-Field Pinch (RFP) configuration, but usually breaks down near the edge of the pinch. Furthermore, it proved to require careful parameter adjustment to ensure that the breakdown radius occurred where the plasma density was negligible (i.e., small against its peak value). This was especially a problem for sharply defined pinch profiles. Several examples of these properties will be displayed in this report. Also, a short code will be presented into which the measured experimental profiles can easily be inserted. The present results indicate the parameter regime to be sought in the postimplosion phase of the RFP, such that the LHDI is rendered virtually harmless. One clear direction is to try to make the ion gyro-radius as small as possible against the pinch radius.

II. RESTATEMENT OF THE STABILITY CONDITION IN TERMS OF PROFILES OF DENSITY AND MAGNETIC FIELD

A. The Shear Length

We consider the left side of Eq. (1) and define the shear length in terms of the magnetic field directions at point r and point $(r + L_s)$. We have

$$\sin(45^\circ) = \frac{1}{\sqrt{2}} = \left| \frac{\vec{B}(r)}{B(r)} \times \frac{\vec{B}(r + L_s)}{B(r + L_s)} \right| \quad (2)$$

where

$$B \equiv |\vec{B}| = \sqrt{B_0^2 + B_z^2} \quad (3)$$

After expanding to first order in L_s , and performing some simple manipulations, Eq. (2) yields

$$\frac{\kappa_s}{\sqrt{2}} = \frac{B_z^2}{B^2} \left| \frac{d}{dr} \left(\frac{B_\theta}{B_z} \right) \right| \quad (4)$$

where

$$\kappa_s \equiv L_s^{-1} \quad (5)$$

In terms of the pitch wave number K_p , defined by

$$K_p = B_0 / r B_z \quad (6)$$

Eq. (4) reads

$$\frac{\kappa_s}{\sqrt{2}} = \frac{1}{1 + (K_p r)^2} \left| \frac{d}{dr} (K_p r) \right| \quad (7)$$

Thus, the shear length is directly related to the radial rate of change of pitch.

The magnitude of the inverse density-gradient scale is

$$\kappa_n = \frac{1}{n} \left| \frac{dn}{dr} \right| \quad (8)$$

Dividing Eq. (8) by Eq. (7) gives

$$\kappa_n L_s = \frac{1}{\sqrt{2}} \left[1 + (K_p r)^2 \right] \left| \frac{1}{n} \frac{dn}{dr} \right| \left/ \left| \frac{d}{dr} (K_p r) \right| \right. \quad (9)$$

It proves convenient to define

$$\kappa = (\kappa r)^2 \quad (10)$$

where κ^{-1} is a characteristic length for the radial density profile. Then Eq. (9) can be written

$$K_n L_s = \frac{1}{\sqrt{2}} \left[1 + (K_p r)^2 \right] \left| \frac{1}{\rho} \frac{d\rho}{d\chi} \right| \left/ \left| \frac{d}{dx} (K_p r) \right| \right. , \quad (11)$$

where $\rho(\chi)$ now signifies the density profile normalized to its value at $\chi = 0$, on axis. Finally, for numerical purposes, it proves convenient to rewrite Eq. (11) as follows.

$$K_n L_s = \sqrt{2} \left| \left(K_p r \right) \frac{d(\ln \rho)}{d\chi} \right/ \frac{d}{d\chi} \left(\ln \left[1 + (K_p r)^2 \right] \right) \right| . \quad (12)$$

B. The Ion Gyro-Radius

For $K_{n\text{ci}}$, we have

$$\begin{aligned} K_{n\text{ci}} &= \frac{1}{n} \left| \frac{dn}{dr} \right| \frac{v_{\text{thi}}}{\omega_{\text{ci}}} = \frac{1}{\rho} \left| \frac{d\rho}{dr} \right| \frac{v_{\text{thi}}}{eB/m_i c} \\ &= 2 \kappa^2 r \frac{1}{\rho} \left| \frac{d\rho}{dx} \right| \frac{r_{\text{ci}}^0}{B/|B_{z0}|} \\ &= 2 (\kappa r) \frac{1}{\rho} \left| \frac{d\rho}{dx} \right| \frac{(\kappa r_{\text{ci}}^0)}{|B_z/B_{z0}| \sqrt{1 + B_\theta^2/B_z^2}} , \end{aligned}$$

or

$$K_{n\text{ci}} = 2 x^{\frac{1}{2}} \left| \frac{d \ln \rho}{dx} \right| \frac{(\kappa r_{\text{ci}}^0)}{|B_z/B_{z0}| \sqrt{1 + (K_p r)^2}} , \quad (13)$$

where r_{ci}^0 is the ion gyro-radius on axis.

To summarize, Eq. (12) constitutes the left side of condition (1), and Eq. (13) plus its reciprocal constitutes the right side. Thus the stability condition has now been reconstructed in terms of the radial profiles of density, axial (toroidal) magnetic field, and pitch. As seen in the next section, the pitch profile is itself determined by the density and axial field profiles through pressure balance.

III. THE EXPRESSION FOR THE PITCH PROFILE

The MHD equation for the equilibrium is

$$\nabla P = \frac{1}{c} \vec{J} \times \vec{B} \quad , \quad (14)$$

where P is the total pressure, $P = P_i + P_e$. In straight screw-pinch geometry, $\vec{J}$ and $\vec{B}$ lie in the flux surfaces, which are concentric cylinders. Hence, the only nontrivial component of Eq. (14) is the radial component. Therefore, after application of Ampere's law, $\vec{J} = (c/4\pi) \nabla \times \vec{B}$, Eq. (14) can be written as

$$\frac{d}{dr} \left(P + \frac{B_z^2}{8\pi} + \frac{B_\theta^2}{8\pi} \right) + \frac{B_\theta^2}{4\pi r} = 0 \quad . \quad (15)$$

If one notes that

$$r^{-2} (d/dr) (r^2 B_\theta^2) = d B_\theta^2 / dr + (2/r) B_\theta^2 \quad ,$$

then one can write Eq. (15) as

$$d(r^2 B_\theta^2) / dr = -r^2 d(8\pi P + B_z^2) / dr \quad \text{or}$$

$$d(\chi B_\theta^2) / d\chi = -\chi d(8\pi P + B_z^2) / d\chi \quad , \quad (16)$$

where χ is defined by Eq. (10). After integration from $\chi' = 0$ to χ , with an integration by parts, Eq. (16) can be written as

$$B_\theta^2 = \frac{1}{\chi} \int_0^\chi (8\pi P + B_z^2)' d\chi' - (8\pi P + B_z^2) \quad (17)$$

where the prime on the integrand emphasizes that it is a function of the integration variable χ' . Thus, for given pressure and toroidal field profiles, the poloidal field profile or, equivalently, the pitch profile is determined. In terms of the pitch defined by Eq. (6), Eq. (17) can be written as follows.

$$1 + (K_p r)^2 = \left\{ \frac{1}{\chi} \int_0^\chi [\beta_o \rho(\chi') + B_z^2(\chi')] d\chi' - \beta_o \rho(\chi) \right\} / B_z^2(\chi) \quad , \quad (18)$$

where $\beta_0 = 8\pi n_0 T / B_{20}^2$ is the local beta on axis, and $B_z(\chi) = B_z(\chi) / B_{20}$ is the toroidal field normalized to its value on axis. A flat T-profile is assumed.

IV. MODEL DENSITY AND FIELD PROFILES

To make further progress, we next define analytic profiles to simulate RFP conditions, plug these profiles into Eqs. (12), (13), and (18), and then examine the stability condition (1) numerically, as a function of radial position. An MHD stable RFP configuration is characterized by a hollow pressure profile and a reversal in the toroidal magnetic field direction.³ We shall choose a density profile of the form

$$\rho(\chi) = \left[1 + \chi / (1 - \chi_{\max}) \right] e^{-\chi} \quad (19)$$

For $\chi_{\max} = -\infty$, this is a Gaussian profile. For $0 < \chi_{\max} < 1$, this is a hollow profile, with $\rho_{\max}$ occurring at $\chi = \chi_{\max}$, followed by essentially a Gaussian decline for $\chi > \chi_{\max}$. Later in this report, we shall consider the effects of pinch profiles that drop off more steeply. TABLE I gives values of $\rho_{\max}$ for various $\chi_{\max}$. Recall that $\rho(0) = 1$.

For the toroidal field profile, we choose

$$B_z = \frac{1 - A e^{-\alpha\chi}}{1 - A} \quad (20)$$

In general, the toroidal field is much stronger on axis than at the wall. This implies $A \gg 1$. Then B_z runs from 1 on axis to $-(A - 1)^{-1} \approx -A^{-1}$ at large χ .

The exponent factor, α , is related to the position of the reversal point χ_0 by $\alpha \chi_0 = \ln A$ or

$$\alpha = (\ln A) / (\kappa r_0)^2 \quad (21)$$

Thus, r_0 denotes the radius at which B_z changes sign.

These two profiles, Eqs. (19) and (20), involve three dimensionless parameters, $\chi_{\max}$, A , and (κr_0) . The pitch, Eq. (18), involves a fourth parameter, β_0 .

TABLE I

$\chi_{\max} = 0.1$	0.3	0.5	0.7	0.9
$\rho_{\max} = 1.005$	1.06	1.21	1.66	4.07

Finally, the right side of Eq. (1) involves $(k r_{ci})$ as given by Eq. (13), which introduces a fifth parameter, namely (kr_{ci}^0) , the dimensionless ion gyro-radius on axis. Thus, even for these simple model profiles, the LHDI shear stabilization condition has a radial dependence that must be fixed by five dimensionless parameters, $\chi_{\max}$, β_0 , A , (kr_0) , (kr_{ci}^0) . We shall now investigate the trends induced by the separate variation of each parameter.

V. THE CODE

The author has written a short code to examine Eq. (1) at each radial position. This code is presented here. Since the wall position must be placed inside of the point where the toroidal flux ϕ_z changes sign,³ the code also keeps track of ϕ_z vs position (kr) .

```

PROGRAM SHEAR
C CALCULATES LOWER HYBRID DRIFT SHEAR STABILIZATION FOR REP.
C 1/K IS RADIAL DENSITY SCALE LENGTH.
C X=(KR)*.02 REPRESENTS RADIAL POSITION
C DENSITY IS NORM TO ON AXIS VALUE
C BZ IS NORM TO AXIS VALUE
C PR MEANS PITCHER
C XMAX=LENS HOLLOWNESS PARAMETER
C XMAX=-1000 IS ALMOST A GAUSSIAN
C XMAX BETWEEN ZERO AND ONE IS HOLLOW
C BETA IS THE LOCAL BETA ON AXIS
C A=1-DES(BZ(AXIS)/BZ(INFINITY))
C R0 MEANS KRC MEANS BZ REVERSAL POINT.
C RCI MEANS KRCI MEANS ION GYRO RADIUS ON AXIS.
C ASSUME FLAT TEMPERATURE PROFILES
C RADIAL CUTOFF WHEN TOROIDAL FLUX=0
C
C STATEMENT FUNCTIONS
DENS(U)=(1+U/(1-XMAX))*EXP(-U)
BZ(U)=(1-A*EXP(-ALFA*U))/(1-A)
BZ2(U)=BZ(U)**2
C
C ENTER FIVE PARAMETER VALUES
XMAX=-1000
BETA=.3
C REQUIRED FOR IDEAL MHD STABILITY REASONS
A=10.
C CORRESPONDS ROUGHLY TO EXPERIMENT
R0=1.
C THIS GETS REVERSAL POINT INTO PINCH
R02=R0**2
ALFA=ALOG(A)/R02
RCI=.1
C THIS CORRESPONDS TO TION=100EV,BZ=5KG,I/K=4CM.
RCI=.03
RCI=.07
PRINT 5, XMAX, BETA, A, R0, RCI
5 FORMAT(3X,5F10.4//)
C
C STORY.
C NICK KRALL SHEAR STAB CONDITION IS
C KN*LS<.LT, (KN*RCI+RECIPROCAL)
C WHERE KN=GRAD(N)/N AND

```

```

C      WHERE LS=45DEGREE SHEAR LENGTH.
C      DEFINITIONS
C      LHS=AKNLS, RHS=AKNRC1, BOTH F(X).
C      PR2P1=PR**2+1
C
C      INITIALIZE
C      X=0
C      DX=0.0001
C      SUM=0.
C      SUMBZ=0.
C      KOUNT=0
C
C      START LOOP, 10 TO 100.
10  KOUNT=KOUNT+1
C      IF(KOUNT.GT. 50000) GO TO 200
C      200 IS PROGRAM EXIT
C      X=X+DX
C      RX=SQRT(X)
C      XM=X-DX
C      CALC TOROIDAL FLUX
C      PARTBZ=DX*BZ(XM)
C      SUMBZ=SUMBZ+PARTBZ
C
C      TOR FLUX=SUMBZ GOES POS FOR NORM BZ, THEN BACK TO ZERO=END.
C      THEREFORE
C      IF(SUMBZ .LT. 0.) GO TO 200
C
C      NOW DO SUM FOR SHEAR FORMULA.
C      PART=DX*(BETA*DENS(XM)+BZ2(XM))
C      SUM=SUM+PART
C      IF(KOUNT.EQ. 1) GO TO 10
C      THIS IS SO THAT SUM/X IS OK AT XM.
C
C      NOW AVOID REVERSAL NGBND
C      IF(ABS(R02-X) .LT. 0.1) GO TO 10
C
C      NOW CALC THE REST OF THE SHEAR FORMULA.
C      PR2P1=(SUM/X-BETA*DENS(X))/BZ2(X)
C      THIS QUANTITY AT XM SHALL BE CALLED PR2P1M.
C      PR2P1M=((SUM-PART)/XM-BETA*DENS(XM))/BZ2(XM)
C      PR=SQRT(PR2P1-1.)
C
C      DLP=(ALOG(PR2P1)-ALOG(PR2P1M))/DX
C      DLDENS=(ALOG(DENS(X))-ALOG(DENS(XM)))/DX
C
C      AKNLS=ABS(SQRT(2.)*PR*DLDENS/DLP)
C      LHS DONE
C
C      RHS1=ABS(SQRT(4.*X/PR2P1)*DLDENS*RC1/BZ(X))
C      AKNRC1=RHS1+1./RHS1
C      RHS DONE
C      STAB=AKNLS/AKNRC1
C      STAB .LT. 1 MEANS LHD1 IS SHEAR STABILIZED.
C
C      PRINT PACKAGE
C      RHO=DENS(X)
C      FIELD=BZ(X)
C      IF(MOD(KOUNT,1000) .EQ. 0) 50,100
50  PRINT 60, RX, STAB, RHO, FIELD, SUMBZ
60  FORMAT(5X,5F20.5/)
100 GO TO 10
C      END OF LOOP
200 CONTINUE
300 CONTINUE
END

```

VI. RESULTS

The main quantity calculated by the code is $STAB(\kappa r)$, or $S(\kappa r)$ for short here, which represents the left side of Eq. (1) divided by the right side. When $S(\kappa r) < 1$, Frell's shear stabilization condition is satisfied. Most runs were performed on the PDP-10 which carries seven figures in single precision. Occasionally, the CDC-7600 was used as a check (thirteen figures in single precision), and the results differed only in the fourth figure.

To study trends, we choose the following reference values for the five parameters.

$$\chi_{\max} = -1000, \beta = 0.3, A = 10, \kappa r_o = 1.0, \kappa r_{ci}^o = 0.1 \quad (22)$$

Unless specifically indicated otherwise, the parameters will have these values, the last four of which were chosen to be in the neighborhood of those occurring in the Los Alamos RFP experiments.

The results are presented in terms of the following quantities.

κr_S : radial position at which $S(\kappa r) = 1$.

κr_ϕ : radial position at which the toroidal flux changes sign.

ρ_S : the density ratio $\rho(\kappa r_S)/\rho_{\max}$.

The object is to make ρ_S as small as possible. Then, presumably the LHDI will not constitute a great danger. The following tables indicated the various trends.

TABLE II
DENSITY HOLLOWNESS

$\chi_{\max} = -1000$	0.1	0.4	0.7
$\kappa r_S = 1.15$	1.28	1.29	1.28
$\rho_S = 0.26$	0.55	0.63	0.75
$\kappa r_\phi = 2.07$	2.07	2.07	2.07

TABLE III
LOCAL BETA ON AXIS

$\beta = 0.1$	0.2	0.3	0.4
$\kappa r_S = 1.18$	1.16	1.15	1.14
$\rho_S = 0.25$	0.25	0.26	0.27
$\kappa r_\phi = 2.07$	2.07	2.07	2.07

TABLE IV
TOROIDAL FIELD RATIO, FIELD ON AXIS VS FIELD AT LARGE r

A	=	5	10	20	50
κr_s	=	1.34	1.15	1.04	0.92
ρ_s	=	0.16	0.26	0.33	0.43
κr_ϕ	=	1.73	2.07	>2.23	>2.23

TABLE V
FIELD REVERSAL POINT

κr_o	=	0.5	1.0	1.5	2.0
κr_s	=	0.73	1.15	1.38	1.43
ρ_s	=	0.58	0.26	0.15	0.13
κr_ϕ	=	1.00	2.07	>2.23	>2.23

TABLE VI
ION GYRO-RADIUS ON AXIS

κr_{ci}^o	=	0.01	0.03	0.07	0.1	0.3	0.5	1.0
κr_s	=	1.57	1.34	1.20	1.15	1.11	1.18	1.37
ρ_s	=	0.08	0.17	0.22	0.26	0.29	0.25	0.15
κr_ϕ	=	2.07	2.07	2.07	2.07	2.07	2.07	2.07

These results can generally be summarized as follows.

- G-i) Shear stabilization is always lost near the edge of the pinch. This implies that the finite-beta stabilization mechanism⁴ cannot necessarily be relied upon as an alternate form of protection against this instability.
- G-ii) The critical radius r_s and density ratio ρ_s appear to be very insensitive to the value of beta.
- G-iii) The critical radius generally occurs where the pinch density is still a very substantial fraction of its peak value.
- G-iv) The maximum allowable wall radius r_ϕ is always comfortably above both the pinch radius $1/\kappa$, and r_s .

More specific properties of the observed trends are listed below.

- T-i) Very hollow profiles are bad (large ρ_s).
- T-ii) Very large ratios of central to external toroidal field are bad.

T-iii) Internal reversal points are bad.

T-iv) Intermediate ion gyro-radius ($\sim \kappa^{-1}/3$) is bad. Improvement is found by going to small ion gyro-radius.

In an attempt to push the unstable region out to low relative densities ρ_s , we ran the code with trends (T-ii, iii, iv) all set simultaneously at favorable values. For trend (T-i), we settled for a moderately hollow density profile ($\rho_{\max} = 1.21$), since that is needed for MHD stability and moreover is naturally achieved by imploding onto a bias field. The parameters and the results of this attempt are given below.

We observe that ρ_s is now pushed down to the 5% range, and that this favorable result remains insensitive to the value of beta, as before. One more case follows.

Comparing TABLE VIII to TABLE VII, we see that a factor 2.5 increase in the ion gyro-radius still allows ρ_s to remain at a fairly low value.

TABLE VII

PARAMETER VALUES

$$\chi_{\max} = 0.5, \quad A = 4.0, \quad \kappa r_o = 2.0, \quad \kappa r_{ci}^o = 0.04$$

$$\frac{\rho(\kappa r_o)}{\rho_{\max}} = 0.15$$

Results:

For	$\beta =$	0.05:	$\kappa r_s = 2.36$	$\rho_s = 0.04$	$\kappa r_\phi > 3.00$
		0.10:	2.35	0.04	"
		0.50:	2.29	0.05	"

TABLE VIII

PARAMETER VALUES

$$\chi_{\max} = 0.5, \quad \beta = 0.1, \quad A = 4.0, \quad \kappa r_o = 2.0, \quad \kappa r_{ci}^o = 0.1$$

$$\text{Results:} \quad \kappa r_s = 1.93, \quad \rho_s = 0.16, \quad \kappa r_\phi > 3.0$$

VII. OTHER PINCH PROFILES

In the Los Alamos RFP experiments, magnetic probes are used to obtain time-resolved poloidal and toroidal magnetic field profiles. The slightly shaky assumption of equilibrium pressure balance in the postimplosion phase then permits the deduction of time-resolved plasma pressure profiles.⁵ Depending upon the time of measurement, some pressure profiles seem "gentle" like those in Fig. 1 and Fig. 2, and some are more sharply peaked with a steeper decline such as those in Fig. 3 and Fig. 4. Therefore, we have repeated the previous parameter study for a more sharply peaked density profile of the form

$$\rho(\chi) = \left[1 + \frac{\chi^2}{1 - \chi_{\max}} \right] e^{-\chi^2} \quad ; \quad \chi = (\kappa r)^2 \quad (23)$$

which implies a rapid radial decrease like $\exp[-(\kappa r)^4]$. For a given value of $\chi_{\max}$, this profile has the same peak density as that in Eq. (19). The old (gentle) and new (sharp) profiles can be contrasted by comparing Fig. (2) and Fig. (3), both with $\chi_{\max} = 0.5$.

FIG. 1.

A gentle pinch profile with internal reversal point ($\kappa r_0 = 0.8$) and unfavorable loss of shear stabilization ($\kappa r_s = 1.25$).

FIG. 2.

A gentle pinch profile with external reversal point ($\kappa r_0 = 2.0$) and concomitant improvement of shear stabilization ($\kappa r_s = 1.93$). Compare to Fig. 1.

FIG. 3.

A sharper pinch profile with external reversal point ($Kr_0 = 2.0$) showing worsened shear stabilization due to increased ($v_{\text{drift}}/v_{\text{thi}}$) near the edge of the pinch (compare to FIG. 2).

$$X_{\text{max}} = 0.8 \quad \beta = 0.1 \quad A = 10 \quad Kr_0 = 2 \quad Kr_{ci}^0 = 0.05$$

$$(Kr_0 = 1.5 \text{ ---})$$

$$\rho(x) = \left[1 + \frac{x^2}{1 - X_{\text{max}}} \right] e^{-x^2} ; \quad x = (Kr)^2$$

FIG. 4.

A sharp, very hollow pinch profile showing the insensitivity of the critical radius (Kr_s) to the position of the reversal point.

In this section, we present the results of parameter studies for a profile of the form Eq. (23), with $X_{\text{max}} = 0.8$, see Fig. 4. Unless otherwise indicated, the reference parameters have the following values.

$$X_{\text{max}} = 0.8, \quad \beta = 0.1, \quad A = 10, \quad Kr_0 = 1.5, \quad Kr_{ci}^0 = 0.05.$$

TABLE IX

	$\beta = 0.03$	$A = 3$	$Kr_0 = 0.5$	$Kr_{ci}^0 = 0.01$
Kr_s	1.22	1.26	1.24	1.39
ρ_s	0.57	0.48	0.53	0.22
Kr_ϕ	>3.00	1.66	>3.00	>3.00

TABLE X
(Reference Case)

$$\beta = 0.1 \quad A = 10 \quad \kappa r_o = 1.5 \quad \kappa r_{ci}^o = 0.05$$

$$\begin{aligned} \kappa r_s &= 1.24 \\ \rho_s &= 0.53 \\ \kappa r_\phi &> 3.00 \end{aligned}$$

TABLE XI

	$\beta = 0.3$	$A = 30$	$\kappa r_o = 3.0$	$\kappa r_{ci}^o = 0.1$
κr_s	1.25	1.23	1.24	1.19
ρ_s	0.52	0.56	0.53	0.66
κr_ϕ	>3.00	>2.80	>2.12	>2.20

From those tables, we conclude that for sharply defined pinch profiles such as the one shown in Fig. 4, the value of the critical radius at which the shear stabilization is lost is quite immune to large changes in all of the available parameters. For such profiles, the LHDI should be present at densities comparable to the peak density in the outer "wing" of the pinch.

VIII. DISCUSSION

To summarize, LHDI shear stabilization in the RFP configuration may be enhanced by simultaneous employment of the following three methods for sufficiently gentle density profiles. Firstly, the magnitude of the reversed toroidal field at the wall should be made as large as possible consistent with positive toroidal flux. Secondly, the reversal point should be kept out of the core of the pinch. Thirdly, the ion gyro-radius on axis should be as small as possible against the pinch radius. This last parameter can be made small either by scaling up the minor radius, or by increasing the magnetic field strength everywhere by some given factor. (If the latter alternative is chosen, then equilibrium demands that the total toroidal current also be increased by the same factor). If these three adjustments can be made, then it should be possible to shear stabilize out to radial positions involving quite low relative densities, provided the density profile is not too sharply decreasing.

For the sharp profiles considered in Section VII, it appears that there is no reasonable parameter regime that can produce satisfactory shear stabilization. However, Freidberg and Gerwin⁶ have found that, due to ion-gyro-resonance effects, the LHDI has a finite drift-velocity threshold amounting to $kr_{ci} \approx 0.07$ for $T_e \approx T_i$. Thus, by running the RFP in a small Larmor-radius mode, one should be able to prevent the occurrence of this microinstability throughout most of the density profile.

Finally, we mention that a code is provided herein, which can easily be modified to accept experimental profiles for study of the actual experimental situation with regard to shear stabilization of the LHDI.

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