POSITRON IMAGING WITH MULTIWIRE PROPORTIONAL
CHAMBER-GAMMA CONVERTER HYBRID DETECTORS

by

Dennis Ying-Hong Chu

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ABSTRACT

A large area positron camera was developed using multiwire proportional chambers as detectors and electromagnetic delay lines for coordinate readout. Honeycomb structured gamma converters made of lead are coupled to the chambers for efficient gamma detection and good spatial resolution. Two opposing detectors, each having a sensitive area of 48 cm x 48 cm, are operated in coincidence for the detection of annihilation gammas (511 keV) from positron emitters. Detection efficiency of 4.2 % per detector and spatial resolution of 6-7 mm FWHM at the mid-plane were achieved. The present camera operates at a maximum count rate of 24 K counts/min., limited by accidental coincidence.

The theory for the gamma converter is presented along with a review of the operation of the multiwire proportional chamber and delay line readout. Calculated gamma converter efficiencies are compared with the measured results using a prototype test chamber. The characteristics of the positron camera system is evaluated, and the performance is shown to be consistent with calculation.

The value of a positron camera lies in its high sensitivity, its ability to produce focused images showing the distribution of positron emitting radioisotopes within an object without the use of

collimators. Computer analyzed images of test patterns and clinical objects have been obtained to demonstrate this tomographic, or focusing, capability. Two schemes of image reconstruction to remove image background and produce true 3-D images are described. Further camera system modifications aimed to increase the count rate and improve the spatial resolution are also discussed.

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I. INTRODUCTION

During the past 10-15 years, radioisotope imaging has become an increasingly useful medical diagnostic method in the field of nuclear medicine. Without a doubt, this rapid progress is primarily due to the availability of many new and better radiopharmaceuticals and the development of improved radioisotope imaging devices. In radioisotope imaging, a gamma-emitting radioisotope with appropriate half life is introduced into a patient. The movement or localization of this radioisotope is monitored by detecting those gamma radiation which escape the body. In-vivo imaging can be divided into three general categories. (1) Localization studies: percentage localization of a radioisotope in an organ or region as compared to surrounding regions. (2) Dynamic function: passage of an administered radioisotope through an organ or region is observed. (3) Distribution studies: distribution of a radioisotope in an organ system is visualized.

An imaging system basically consists of three major components:

- (1) Detector unit which detects and locates the interaction sites of incoming radiation. (2) Image producing unit such as a collimator or pinhole, which projects images of radioactive subjects onto the detector. (3) Process and Display unit which translates data acquired by the detector into a "picture" of the distribution of radioisotope in an organ system. The earliest devices employed in radioisotope imaging were the scanners, fig.(1). With such a device, a detector equipped with a collimator scans over an object containing a gamma emitting radioisotope. A recording unit moves in synchronization with the

scanner. Upon detection of an event, the recording is activated. Consequently, both the intensity and distribution of gamma emitting radioisotopes can be mapped out in a two dimensional display. The first type of scanner introduced (Cassen et al., 1951;¹ Mayncord and Newberry, 1952)² employed a single cylindrical bore collimator and a calcium tungstate crystal as the gamma detector. The single-hole collimator was soon replaced by the multihole focused collimator (Newell et al; 1952)³ in which many tapered holes are used to increase the number of photons passing through the collimator. For large area scanning, multiple detectors equipped with sodium iodide (TI) crystals (Beck et al; 1967;⁴ Hindel and Gilson, 1967)⁵ form a high speed scanning system. Radioisotope scanning was first used to delineate the thyroid gland and to determine hot or cold nodules in the gland. As I-131 (364 Kev) was the isotope used, most scanners were designed for use with that radioisotope. However, radioisotope scanning soon found other fields of use such as localization of brain tumor, metastases in liver, and delineation of the kidneys. Though the scanner is a simple, effective device in localization studies, it is limited by the travelling speed of the probe in dynamic function studies.

The gamma camera, fig. (2), a stationary imaging device, views all parts of an object simultaneously. Data can be acquired more rapidly resulting in a higher sensitivity than with the scanner which can only view part of an object at a time. The shorter exposure time permits sequential pictures to be taken to illustrate movement of tracer compounds through an organ. Count rate for specific area

Fig. 1. Schematic configuration of a Scanner.

Fig. 2. Schematic configuration of a Gamma camera.

of interest can also be obtained. The stationary gamma camera is a recent development; but its basic principle is much older. Roentgen (1895) reported using a pin-hole aperture and photographic film to make images of the anode of his X-ray tube. The first camera introduced (Anger et al; 1956⁶) employed a single solid sodium iodide scintillation crystal viewed by an array of multiplier phototubes. In 1962, Bender and Blau⁷ described a gamma camera using a mosaic of sodium iodide crystal viewed by a position sensing multiplier phototube array. Shortly thereafter, a low energy gamma imaging device which employed an X-ray image amplifier with internal gamma detection phosphor was introduced (Ter-Pogossian, et al; 1963).⁸ In the following years, other types of position sensitive gamma detectors that did not require scintillation crystals were developed -- Spark chamber (Kellershohn et al; 1964),⁹ Multiwire proportional chamber (V. Perez-Mendez; et al; 1971),¹⁰ and solid state detector (Mc Cready et al; 1971,¹¹ Kaufman et al; 1973).¹² An inherent limitation to the rate of data acquisition by a gamma camera is the necessity of a collimator. A large fraction of gamma radiation originating from an object will not reach the detector, being stopped by the septa of the collimator. In addition, a collimator can only produce a two dimensional projection of a three dimensional object. Attempts have been made using rotating focused collimators¹³ to produce tomographic images, in which objects are seen best at the plane of focus while off-focus objects are seen as blurred background.

Another approach to viewing radioisotope distribution within a three dimensional object is to utilize the property of positron

emitting radioisotopes (Wreen et al; 1951;¹⁴ Brownell, Sweet et al; 1953).¹⁵

In the annihilation process of a positron with an electron, two 0.511 Mev gammas are emitted almost at about 180° . Two gamma detectors, placed on opposite sides of an object containing a positron emitter, detect the annihilation gammas in coincidence. The time-of-flight information, in addition to the inherent directional properties of two opposed gammas, give the location of the source, fig. (3). With the absence of collimators for positron imaging, high sensitivity can be achieved, resulting in lower dose to the patient. The inherent spatial resolution is only limited by the detection system and not by a collimator. Medically, there are many useful positron emitters, notable examples are C-11, N-13, O-15, F-18, Ga-68, Cu-64 and Ru-82, Table (1). These biologically important radionuclides have short half lives and high specific activity , and are suitable as radiopharmaceuticals. Since a positron imaging system operates in a coincident mode, low background count rate will improve counting statistics. A positron imaging scanner, utilizing the time-of-flight principle, was introduced in 1967 by Burnham, Aronow and Brownell.¹⁶ A stationary positron camera was first proposed by Anger and Rosentahl in 1959 as an extension to the gamma cameras.¹⁷ The imaging system developed consisted of an image detector and a focal detector (Anger et al; 1967).¹⁸ Instead of a truly three dimensional images, tomographic images would be obtained. In 1973, Brownell et al.¹⁹ built a positron camera consisting of two arrays of small NaI crystals. A positron camera consisting of two large NaI crystals is currently being developed by Muellehner et al.²⁰ The above

Fig. 3. Geometric configuration of a positron imaging system. The location of a radioactive source is given by the direction and the difference in time of flight of the annihilation gammas.

Table 1.

Positron emitters useful for nuclear medicine imaging

Isotope	$T_{1/2}$	Max β^+ E (MeV)	Positron Abundance (%)	Other Major Radiations (keV)	Production
C-11	20.3 min	.980	99.8		$^{10}\text{B} (p,\gamma)^{11}\text{C}$
N-13	10.0 min	1.200	100		$^{10}\text{B} (\alpha,n)^{13}\text{N}$
O-15	2.0 min	1.700	100		$^{14}\text{N} (d,n)^{15}\text{O}$
Rb-82	75 sec	3.150	94.6	777 (9%)	$^{85}\text{Rb} (p,4n)^{82}\text{Sr} \xrightarrow{\text{EC}} ^{82}\text{Rb}$
Ga-68	68 min	1.900	87.0	1,080 (4%)	$^{69}\text{Ga} (p,2n)^{68}\text{Ge} \xrightarrow{\text{EC}} ^{68}\text{Ga}$
F-18	110 min	.650	97.0		$^{20}\text{Ne} (d,\alpha)^{18}\text{F}$
Cu-64	12.8 hrs	β^+ .660 β^- .573	19.0	1,340 (.6%) β^- (40%)	$^{63}\text{Cu} (n,\gamma)^{64}\text{Cu}$

positron imaging systems employ gamma detectors consisting of scintillation crystals and multiplier phototubes.

Another type of position sensitive gamma detector is the multi-wire proportional chamber (MWPC) which has been successfully used for imaging radioisotopes with low energy gammas such as I-125, ²¹Xe-133. These multiwire proportional chambers have good spatial resolution (<1mm), large field of view, simplicity of design, low system cost, and good uniformity of response. However, due to the low interaction probability of energetic gammas with the gas in the chamber, MWPCs must be coupled to special gamma converters for efficient detection of 511 Kev annihilation gammas. It is the purpose of this thesis to describe and analyze such a MWPC-gamma converter hybrid system for positron imaging. A theoretical calculation of gamma converter detection efficiency is explained. The positron camera performance is described. An on-line computer system was used for data storage, data analysis, and image display. Results obtained for test and clinical objects demonstrate the capability of the camera. Further improvements and limitations of the present camera system are also discussed.

II. THEORY

2.1 Principle of positron imaging

2.1.1 Image formation

The principle of radioisotope imaging is, in a large part, analogous to optical imaging. A simplified optical camera system consists of a lens where light reflected from an object is directed onto a light sensitive film on which the image is formed. The earliest and the simplest type of lens is the pin-hole. Reflected light from an object passes through the pin-hole and forms an inverted image on the film. One significant difference between optical imaging and radioisotope imaging is the fact that gamma radiation cannot be refracted by any lens. In order to project the image of an object containing gamma emitters, collimators are required. Fig. (4) shows the projection of an object with two types of collimators, pin-hole and multichannel. A large portion of gammas do not reach the detector, being stopped by the septa of the collimator. This results in a low overall system sensitivity. The effective designs of collimators have been extensively investigated.

Another approach to this problem is to utilize the inherent property of positron annihilation. For proton-rich nuclei, transition to a stable state is achieved by converting a nucleon from a proton to a neutron with the emission of a positron and a neutrino from the nucleus. Excess energy in the transmutation process is shared by the positron and the neutrino in the form of kinetic energy. The electrically charged positron travels a short distance in the surrounding medium

Fig. 4. The projection of gamma emitting objects with pinhole (a.) and multihole (b) collimators.

(a few mm in tissue), transferring most of its energy in excitation and ionization of orbital electrons and finally combines with an electron forming an unstable pair, with subsequent annihilation of the electron-positron pair. This occurs mainly in the ground state of the system. In order to conserve momentum, two gammas of 511 keV each, emerge in opposite directions. This inherent directional property of back-to-back annihilation gammas forms the basis of collimatorless positron imaging. Two position sensitive radiation detectors are placed on opposite sides of an object containing a positron emitter. For a pair of annihilation gammas detected in coincidence, the line vector on which the source lies is specified. The exact position of a source on the line vector can, in principle, be determined by measuring the differences in the time of arrivals of the two annihilation gammas. This time-of-flight technique requires very high speed electronics for good spatial resolution. An alternative method of positron imaging which produces a tomographic image is the technique of seeking the "plane of best focus". A set of line vectors from coincidence detection of annihilation gammas is obtained and stored in the memory of a computer. A focal plane is selected, and the intersections of each line vector with the plane is calculated. The detailed tomographic reconstruction method will be explained in Ch. 4. Fig. (5) shows a sharply focused point source at the focal plane and the diffused effect of an off-focal plane source. This tomographic effect is used in positron imaging to reveal features at various depth within an object.

2.1.2 Gamma attenuation

One of the criteria in the choice of a radioisotope used for imaging is the energy of the emitted gammas. Gammas below 1 Mev interact with the matter of an object in two ways. In a photoelectric interaction, the energy of the gamma is totally absorbed, and an inner shell electron is released. In a Compton interaction, a gamma is scattered from an electron with partial loss of its initial energy, and at the same time the electron is scattered. The total interaction of gamma radiation (Photoelectric & Compton) behaves as an exponential function with the thickness of material traversed. At low energy, the absorption probability of gammas is high in tissue, and most of the gamma radiation emitted by a radioisotope cannot escape and be detected. One of the most useful radioisotopes used in nuclear medicine is Tc-99m. This radioisotope gives 140 Kev gamma radiation which is sufficiently energetic to escape a thick body organ without suffering appreciable attenuation, and it also has a short physical half life of 6.04 hrs. The mean free path for 140 Kev gammas in tissue with an average density of 1gm/cc is 6.7cm.²² One technique to discriminate against scattered gammas, which give incorrect position information of the source, is to use an energy window in the detector. Scattered gammas, which retain only part of the initial energy are rejected by the energy selection window and the result is a better image. However, the sensitivity (measured in counts/min/uCi) is different for sources that lie near the surface of an object close to the detector and for sources that are at a greater depth. This problem is less severe in the case of

positron imaging than that of single gamma imaging because the total thickness traversed by the two annihilation gammas is approximately the same from any depth of an object of uniform thickness.

The probability that one of the two annihilation gammas emerges from a medium without encountering any scattering is $\exp(-d/L)$ where L is the mean free path for 511 Kev gammas in the medium, and d is the distance between the point of annihilation and the surface of the medium.

The probability that both gammas emerge without scattering is given by

$$\exp(-d_1/L) \times \exp(-d_2/L) = \exp(-d_1-d_2/L) = \exp(-D/L)$$

The mean free path for 511 Kev gammas in tissue is 10.5 cm. D is the total attenuation length. As an example, we can calculate the average escape probability of annihilation gammas from a spherical object of tissue-equivalent density. For a spherical object of radius R and a positron emitter at distance $a \times R$ from the center, the average chord length $\bar{D}$ of a sphere and the average escape probability are given by

$$\begin{aligned} D(\theta) &= 2R (1 - a^2 \sin^2 \theta)^{1/2} \\ \bar{D}(\theta) &= \int D(\theta) \sin \theta \, d\theta \\ &= 2R \int_0^1 (1 - a^2 + a^2 \mu)^{1/2} \, d\mu \quad ; \mu = \cos \theta \\ \bar{P}_{\text{esc}} &= \int_0^1 \bar{D}(\mu) \exp(-\bar{D}(\mu)/L) \, d\mu \quad \bigg/ \quad \int_0^1 \bar{D}(\mu) \, d\mu \end{aligned}$$

The average escape probability, $\bar{P}_{\text{esc}}$, is calculated for a point source at some distance from the center of a sphere with diameter of 20 cm, fig. (6).

Fig. 5 A point source at the focal plane is sharply focused while an off focal plane source is blurred.

Fig. 6 The average escape probability, $\bar{P}_{esc}$, of annihilation gammas from a point source at various distances from the center of a spherical object of 10 cm. radius.

2.1.3 Gamma detection and localization

In a conventional gamma camera, the event rate is dependent on the following factors - the source activity, the attenuation within an object, the fraction of gammas that passes through the collimator, and the detection efficiency of the detector for the appropriate energy window setting. The attenuation and collimation problems have been discussed previously. Gamma detection is effected by converting the incident gammas into either light quanta or charged particles. In a NaI-phototube detector system, the gamma interacts with the scintillation crystal, and the light emitted is in turn detected by an array of multiplier phototubes. The amount of light shared by the cluster of phototubes gives the position information of the site of a gamma interaction.²³ In a solid state detector, the gamma radiation interacts with the materials, yielding charged electron-hole pairs. These pairs are collected by two electrodes on both sides of the detector. Localization of gamma interactions can be accomplished by the construction of orthogonal strips. A single wafer of detector material is etched in the form of parallel strips with electrical contacts on the surface. The strips on one face are at right angles to those on the other side. The position of interaction of an incident gamma with the detector is defined by the pair of strips (one from each face) on which the signal occurs. A third category of gamma position sensitive detector is the multiwire proportional chamber. Gamma radiation interacts with the gas atoms of the chamber releasing photoelectrons which then ionize the surrounding gas atoms. The ionization electrons are then drifted

into a multiplication region, and the resulting avalanche electrons produce detectable signals. The position of interactions are sensed by a set of orthogonal wires with delay line readouts.²⁴ When MWPCs are used for positron imaging, the interaction probability of 511 Kev annihilation gammas with the chamber gas is low, and they must be coupled to gamma converters to improve the detection efficiency. The detection efficiency is defined as the probability of detecting an incident gamma. Converter designs will be discussed later in this chapter.

2.2 Event rate considerations

2.2.1 True coincident rate

For a gamma emitting source which subtends a solid angle, G , with a radiation detector of intrinsic efficiency ϵ ; the singles count rate, R_S , is

$$R_S = G_S \epsilon N_0 \quad (2.1)$$

where N_0 is the rate at which gamma radiation is emitted. If a positron emitter is placed at the mid-plane between two identical detectors, the coincident detection rate for the annihilation gammas is

$$R_C = G_C \epsilon^2 N \quad (2.2a)$$

where N , in this case, is the rate of the annihilation of positrons.

The sensitivity, S , given in counts/min/ μ Ci, is therefore

$$S = 2.22 \times 10^6 G_C \epsilon^2 \quad (2.2b)$$

The singles rate at each detector, however, is doubled because of the paired gamma emission.

$$R_S = 2 G_S \epsilon N \quad (2.3)$$

For the particular case where the point source is at the center between the two detectors, and the solid angle, G , is identical for both expression (2.2a) and (2.2b), then $R_c/R_s = \epsilon/2$. At low detector efficiency, the true coincidence rate is only a fraction of the singles rate.

In general, the geometric factor is different for the single count rate and the coincident count rate calculation. The subtended solid angle in coincident detection diminishes rapidly if the point source is moved towards the detector edge and away from the mid-plane. The analytic expressions for solid angle calculation is presented in appendix(1A). Fig. (7) shows the variation of the solid angle with source position for coincident detection and the comparison of G_s and G_c at the mid-plane. If a detector has a thin sensitive volume compared to the mean free path of the incident gammas, then we also have to consider the detection efficiency variation due to the angle of gamma incidence. The probability of a gamma interaction with the detector is, in general, an exponential function of the length of gamma-conversion material traversed. Consequently, a detector is more efficient for gammas that enter the detector at an inclined angle than those at normal. The combined effect of solid angle variation and efficiency angular dependence with the source position for coincident detection rate is shown in fig.(8). The derivation is presented in Appendix (1B).

2.2.2 Random accidental coincidence rate

In any coincidence detection system, there is a finite coincident resolving time, T_r . If one on the annihilation gammas pair is detected by detector (1), the complement gammas from detector (2) must arrive

Fig. 7 Variation of the solid angle with position for coincident detection and the comparison with solid angle for single gamma detection at mid-plane.

FIG. 8. Detection efficiency variation with the source position for a thin detector.

within the resolving time to be counted as a valid coincident event. In the case where the complement gamma is not detected, and one of the gammas from a second annihilation process is detected by detector (2) within the resolving time, a random accidental coincidence is registered. This situation is depicted in fig. (9). The rate of singles which are not associated with the true coincidence is $(R_S - R_C)$. The random accidental rate due to singles in both detectors is

$2(R_{S1} - R_C)(R_{S2} - R_C)^2 T_R$. For a detector with low detection efficiency, i.e. $R_S \gg R_C$, the accidental rate, R_a , is simplified as

$$R_a = (2 G \epsilon N)^2 \times 2 T_R = 8 G_S^2 \epsilon^2 N^2 T_R \quad (2.4)$$

The accidental coincident events give displaced line vectors, and, consequently, incorrect position localization in the tomographic image reconstruction. It is, therefore, important to calculate the ratio of the accidental coincident rate to the true coincident rate, $p, (G_S = G_C = G \text{ is assumed})$

$$p = R_a / R_C = 8 G N T_R = R_C (8 T_R / \epsilon^2) \dots\dots\dots(2.5)$$

For example, if the time resolution $T_R = 300 \text{ nsec}$, the maximum true coincident rate is calculated for various detector efficiencies as functions of accidental to true coincidence rate, fig. (10). It should be emphasized that, in general, the ratio of accidental to real coincidence counts is larger for a source located near the edge of the detector than for one at the center for a given source activity and time

Fig. 9 Accidental coincidence in positron imaging.

Fig. 10 Calculations of coincident count rate for various detector efficiencies as functions of accidental to true coincidence rate.

resolution. This is due to the rapid fall-off of the solid angle, G_c .

2.2.3 Non-random accidental coincidences

Coincidence detection systems are subjected to other sources of non-random accidental coincidences. One of these is the cascade gammas, gamma radiation that is emitted along with the annihilation gammas of non-pure positron emitter. This type of accidental coincidence is dependent on the source-detector solid angle and the abundance of the cascade gammas relative to the annihilation gammas. Other sources of significant background coincidences come from the scattering at the source and scattering at the detector, fig. (11). The annihilation gamma scattering at the source has been considered earlier, and the accidental rate is dependent on the location of the source in the object and the size of an object. Another type of non-random accidental coincidence is due to the internal scattering in the detector itself. Incident gammas undergo multiple interactions with the gamma converter. Such scattering depends on the geometry and the type of conversion materials in the detector. The total coincidence rate is, therefore, a sum of the true coincidence (which includes source and detector scattering) and accidental coincidence (which includes cascade gammas).

2.2.4 Dead time correction

At high counting rate, the response time of each detector has to be taken into consideration. For a paralyzable system, the observed counting rate R_{so} , is simply

$$R_{so} = R_s \exp(-R_s \tau_d) \quad (2.6)$$

where τ_d is the dead time of the detection system, R_s is the theoretical singles count rate. Typically a dead time of 1-2 μ sec is associated

Fig. 11 Gamma scattering at the source (a) and at the detector (b).

with the read out system of a multiwire proportional chamber. As the true counting rate is increased, the observed counting rate, R_{so} , passes through a maximum when $R_s T_d = 1$

$$(R_{so})_{\max} = R_s / e = 1 / e \tau_d$$

In the case of coincidence counting, the observed coincident count rate is

$$R_{co} = R_c \exp(-R \tau_d) \quad (2.7a)$$

where R = total count rate in both detectors - true coincident rate,

$$\approx 2R_s - R_c$$

$$\begin{aligned} \text{and } \tau_d &= 1 / (2 R_s - R_c) = 1 / (2 e R_{so} - R_c) \\ &= \epsilon / (4 e R_{co} - R_c \epsilon) \approx \epsilon / 4 e R_{co} \end{aligned} \quad (2.7b)$$

From eq. (2.7a), it is evident that the coincidence rate increases as the square of the detector efficiency. It also should be emphasized that for a given ratio of accidental coincident rate to true coincident rate, the true rate is also proportional to the square of the detection efficiency as indicated in eq. (2.5) and fig. (10). To minimize both accidental coincidences and dead time losses, it is important that a detector system should have short coincident resolving time and small dead time.

2.3 Gamma Converter efficiency

The use of multiwire proportional chambers (MWPC) for detection and spatial localization of gamma radiation is limited by the low conversion efficiency of the chamber gas. For low energy gamma radiation, the combination of a noble gas of high atomic number with pressurization

produces a satisfactory detection efficiency. Xenon at 4 atmospheres has been used for 60 Kev ²⁵gamma^s to obtain a detection efficiency of 50%. But as the energy of the gamma radiation increases, the detection efficiency decreases rapidly, and the spatial resolution degrades because the energetic conversion electrons travel further in the gas. If a solid plane converter is coupled to the active region of a MWPC, gamma radiation interacts with the solid converter yielding conversion electrons. Some of the conversion electrons escape the solid medium and enter into the gas regions, producing ionization electrons which are detected as in the case of a conventional MWPC. The useful thickness of such a plane converter is limited by the maximum range of the conversion electrons. This thickness is usually very small, and the problem can be visualized as a surface phenomena. A useful geometric structure which provides a large surface area per unit cross sectional area is a honeycomb. A gamma converter which has a honeycomb structure provides an enhanced area of surfaces as well as restricting the range of conversion electrons. The efficiency of such a converter depends on the following:

- (1) The probability of the gamma radiation interacting in the converter material and thereby transferring some of its energy to the electrons in the converter. This is a function of the total gm/cm^2 of converter.
- (2) The escape probability of these electrons[†] into the detector gas. This is a function of the mean distance to an escape surface and the range of the conversion electrons.

[†] For convenience, these electrons will be referred to as "conversion electrons"—although this is not the conventional meaning of the expression.

2.3.1 Conversion yield

Consider an array of square cross section cells with dimension as shown in fig.(12). Conversion electrons, resulting from either photo-electric or Compton interactions of incident gammas with the material, have to traverse the solid and escape into the gas region where ionization can occur. In general, a photon will traverse several cells depending on the orientation of the converter with respect to source position, fig.(13). A simple model is first described to expose the important parameters in the consideration of high conversion yield for gamma converter design. This will be followed by a more precise analysis and expressions for calculating the conversion yield.

For a cell, fig.(12), with total surface area A_s and cross section A_c , the probability of producing secondary ionization electrons per gamma

ϵ = interaction probability per unit cross section x probability of escape of conversion electrons.

$$\begin{aligned} \epsilon &= V_{\text{eff}} \Sigma \bar{P}_{\text{esc}} / A_c = (A_s t) (N\sigma) \bar{P}_{\text{esc}} / A_c \\ &= 4Ht (L - t) (N\sigma) (\bar{P}_{\text{esc}}) / (L + 2t)^2 \end{aligned} \quad (2.8)$$

Where V_{eff} is the effective conversion volume, Σ is the macroscopic cross section, N is the number of atoms per unit volume, σ is the conversion cross section per atom, and $\bar{P}_{\text{esc}}$ is average escape probability for conversion electrons produced within the wall thickness, t . H is the height of the converter and L is the length of a side of a square cell. The energy of conversion electron is under 500 kev for annihila-

Fig. 12 Schematic of a square cross section cell array with unit cell dimension specified.

Fig. 13 Detector geometry for a point source with wide angular spread.

tion gamma interactions, and the range of the most energetic conversion electrons in a solid material is rather small (300 μ m in lead). For cell dimension $L \gg t$, eq.(2.8) is simplified to

$$\epsilon = \frac{4 H t}{L^2} (N \sigma) (\bar{P}_{\text{esc}})$$

$$\epsilon = \frac{4 H}{L} t (N \sigma) (\bar{P}_{\text{esc}}) \quad (2.9)$$

The most direct way to increase the conversion yield is to increase the ratio of H/L , the height of the converter versus the cell size. As we shall see in the next sections, this ratio is limited by the collection efficiency of the ionization electrons, and the minimum threshold for detection. The above simple model does not include the effect of angular dependence nor the detailed profile of conversion electron escape. A more precise analysis which was used in the actual calculation of gamma converter efficiency, is described in the following. The probability of producing an escaped conversion electron per gamma is

$$\epsilon = \sum_j n_j = \sum_{j=1}^n \sum_{i=1}^2 \int_0^T Q_{ij} \frac{(L+2x)}{(L+T)} \frac{1}{T} dx P_{\text{esc}}(x, E_i) \quad (2.10)$$

where i indicates the type of interaction (photoelectric and Compton), n_j is the probability of emitting an electron into the j -th cell space, P_{esc} is the escape probability of electron with energy E_i at x , Q_{ij} is the probability that an i -th type electron is produced in cell j .

In the case of a point source with a wide angular spread, fig. (13), the average number of electrons produced per gamma over the solid angle subtended by the source and converter is calculated. For a given geometry, the geometric factor, G , is calculated, Appendix (2), and substituted into eq.(2.10) The simplified expression for conversion yield, ϵ , for a point source is

$$\epsilon = 2 G \left\{ \sum_{i=1}^2 \sigma_i \rho T \int_0^T \frac{(L + 2x)}{(L + T)^T} dx P_{\text{esc}}(x, E_i) \right\} \quad (2.11)$$

In order to calculate the conversion yield, an analytic form for the escape probability is required.

Spencer²⁶ used a numerical method of spatial moments to evaluate

P_{esc} .

$$P_{\text{esc}}(x, E_i) = \exp^A \left[1 - \frac{x}{R(E_i)} \right]^P \exp \left\{ - \frac{A}{1 - \frac{x}{R(E_i)}} \right\} \quad (2.12)$$

A is a function of source energy and scattering material, P is a constant, $R(E_i)$ is the residual range defined and tabulated by Nelms.²⁷

The values of coefficient A can be obtained from experimental data from Seliger's work.²⁸ The electron escape probability $P_{\text{esc}}(x, E_i)$ for a number of material has been measured by Selinger with a 2π beta counter with high detection efficiency down to electron energies of a few hundred electron volts, fig. (14). Eq.(2.12) is relatively insensitive to the value of P .²⁹ Jeavons has calculated the values of A for the best fit value of $P = -3$.

The residual range $R(E_i)$ is determined from the kinetic energy of the two types of conversion electrons, photoelectric and Compton. In

Fig. 14 The electron escape probability for electrons of different energies from Lead, Silver, and Aluminum as functions of true range in the materials, ref. (28).

photoelectric interactions, the energy of conversion electrons is given by the difference between the incident gamma energy and the binding energies of the electrons.

$$E_p = E_\gamma - E_k$$

Contribution from L,M shells should be included for high Z materials. Since the Compton interaction yields a continuous spectrum, an average Compton electron energy is used to calculate the escape probability. The classical theory of Compton scattering has been described by Evans.³⁰ The energy of the scattered electron as a function of scattering angle $E_c(\theta)$, and the differential scattering cross section for unpolarized radiation, $d\sigma(\theta)$, can be expressed as

$$E_c(\theta) = h\nu_0 \alpha (1 - \cos \theta) / 1 + \alpha (1 - \cos \theta), \quad \alpha = \frac{h\nu_0}{m_0 c^2}$$

$$d\sigma(\theta) = r_0^2 d\Omega \left[\frac{1}{1 + \alpha(1 - \cos \theta)} \right]^3 \frac{(1 + \cos^2 \theta)}{2} \times \left\{ 1 + \frac{\alpha^2 (1 - \cos \theta)^2}{(1 - \cos^2 \theta) [1 + \alpha(1 - \cos \theta)]} \right\}$$

where $h\nu_0$ is the energy of incident photon, r_0 is the classical electron radius, θ is the angle between the direction of incident photon and the scattered photon. The average energy of the scattered electron can be written as

$$\bar{E}_c(\theta) = \frac{\int E_c(\theta) d\sigma(\theta)}{\int d\sigma(\theta)}$$

2.3.2 Intrinsic conversion yield

In the simple case where the converter is a plane surface of thickness greater than the optimum thickness and the detection of conversion electrons is restricted to one side of the converter as in the case of Geiger counters, the conversion yield, ϵ , is given by

$$\epsilon = \frac{\sum_{i=1}^2 \iint \sigma_{i,p} \exp(-\sigma_{i,p} \times \sec\theta) \sec\theta P_{esc}(x, Ei) dx d\Omega}{\int d\Omega} \quad (2.13)$$

The intrinsic conversion yield of various materials for different energies can now be calculated with the analytical form for the electron escape probability. The intrinsic conversion yields of various types of cathode materials have also been measured.³¹ Both measured and calculated results of ϵ as functions of gamma energy are compared in fig. (15).

2.3.3 Extraction efficiency

In the previous sections, we have considered the probability of conversion electron emissions into the gas region within the cells. The number of ionization electrons produced is a function of the energy of a conversion electron and the length of the ionization track. The number of ionization electrons collected is dependent on the extraction electric field profile. For a given chamber, the number of ionization electrons collected multiplied by the gain of the chamber (typically 10^4 - 10^6) must exceed the input noise level of the readout amplifier in order for an event to be detected.

In order to extract ionization electrons from the gas regions within the cells, an electric field is applied across the converter.

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Fig. 15 Measured (ref. 31) and calculated results of intrinsic conversion yield in various materials as functions of gamma energy.

Multiple lead strips are used and electrically biased to produce a drift field. To estimate the influence of field profiles on the extraction efficiency, we assume a cylindrical cell geometry Fig. (16). A solution to Laplace's equation $\nabla^2 \phi = 0$ gives the electric field both in the axial and radial direction, Appendix (3).

$$E_z(r,z) = -V_0/H - 2V_0/\pi S \sum_{n=1}^{\infty} \left\{ \frac{\sin(n\pi S/H)}{nI_0(n\pi R/H)} I_0(n\pi r/H) \cos(n\pi z/H) \right\}$$

$$E_r(r,z) = -2V_0/\pi S \sum_{n=1}^{\infty} \left\{ \frac{\sin(n\pi S/H) I_0'(n\pi r/H) \sin(n\pi z/H)}{nI_0(n\pi R/H)} \right\}$$

A profile of the electric field is shown in fig. (17). From the electric field profile, it can be seen that some ionization electrons are lost in the drifting process due to the termination of the field lines on the walls. The space within a cell can then be approximated by two concentric cylindrical regions, an inner cylinder where electric field lines are continuous along the length of the cell and an outside cylindrical shell where field lines terminate on the wall.

In this model, the probability of detecting a gamma ray which has produced a conversion electron is equal to the probability that the ionization track intersects the inner cylinder (intersection probability), and produces within it a sufficient number of secondary electrons (detection probability). Let $\bar{r}$ be the average radius of an inner cylinder where field lines are continuous, fig. (16). The ionization electrons released as a result of the interaction of the conversion electrons with the gas in the chamber will tend to spread out by diffusion in the course of their motion towards the anode wires. However, in our case, the rate of diffusion is assumed to be small

Fig. 16 A cylindrical cell geometry used for the calculation of electric extraction field.

Fig. 17 Schematic configuration of lead converters; showing the electric drift field lines.

and ionization electrons are considered to drift along the field lines. The electrons in the central region are extracted while electrons in the outer cylindrical volume are lost. The extraction volume fraction, $\bar{r}^2 / R^2$ as a function of R/l is shown in fig. (18). Next, we shall consider the probability that an ionization track will intersect the inner cylinder. The intersection probability, $P(\bar{r}/R)$, is a function of the ratio of the inner to outer cylinder radius. The derivation of the of the intersection probability is presented in Appendix (4), and it can be expressed as

$$P(\bar{r}/R) = 1 - \frac{4}{\pi} \int_0^{\mu_0} \mu \cos^{-1} \left[\mu \bar{r} / R (1 - \mu^2)^{1/2} \right] d\mu$$

$$\text{where } \mu_0 = \left[1 + (\bar{r}/R)^2 \right]^{-1/2}$$

The intersection probability as a function of R/l is also shown in fig. (18).

The detection probability, $D(E)$, is calculated from the track-length-distribution in a cylinder.³² Conversion electrons emitted from the wall produce ionization tracks of various length, depending on the orientation of these tracks, Fig.(19). The conversion electron range is assumed to be long compared to the tube diameter (this is the case for a tube radius on the order of 1-2mm and electron energy greater than 10 Kev). The effective track length distribution $T_e(r)$ through the inner tube is estimated by multiplying the track-length-distribution in the outer cylinder by the volume ratio $\bar{r}^2/R^2$

Fig. 18 Intersection probability $P(\bar{r}/R)$ and extraction volume $\bar{r}^2/R^2$ are shown as functions of R/H .

Fig. 19 The track-length-distribution in an infinite cylinder of radius R .

$$D(E_t) = \int_{E_t}^{E_{\max}} f(E) dE = \int_{r_t}^{r_{\max}} T_e(r) dr$$

E_t is the energy threshold for detection and r_t is the corresponding minimum track length. Total converter efficiency is then expressed as a product of conversion yield, intersection probability and detection probability,

$$\begin{aligned} \epsilon_t &= \epsilon \times \epsilon_{\text{extraction}} \\ &= \epsilon \times P(\bar{r}/R) \times D(E_t) \end{aligned}$$

Results of the calculated and measured converter efficiencies will be presented in Ch. 5.

2. 4. Electron drift velocity

From eq.(2.4), the accidental coincident rate was shown to be proportional to the coincidence resolving time. In our case, the resolving time corresponds to the time spread for drifting ionization electrons originating at various depths inside the cells to the top of the converter. The electron drift velocity is a function of the gas mixture used in the chamber, electric drift field, $\bar{E}$, and pressure of the gas mixture, p . For electrons, the application of an electric field cause an increase in the agitation energy. The agitation energy is frequently expressed in terms of the ratio η of its value in the presence of the field to its value in the absence of the field, $(3/2 kT)$. Superimposed on the agitation velocity, u , of the electrons in the presence of the field, where $mu^2/2 = \eta(3/2)kT$, is a drift velocity w in the direction of the field. A relationship between u and w can be derived from the following argument. Let λ be the mean free path of

an electron at unit pressure, each electron undergoes $u \times p/\lambda$ collisions per second. The average momentum lost is $(u \times p/\lambda) \times m\vec{w}$ where w is the drift velocity. On the other hand, the momentum gained through the electric field is $e\vec{E}$. Hence at equilibrium

$$\frac{u p m \vec{w}}{\lambda} = e \vec{E}$$

$$\vec{w} = (e/m) (\lambda/u) (\vec{E}/p)$$

High electron drift velocity can be achieved if the average agitation energy of electrons is lowered. Inelastic collision between electrons and gas molecules occur only when the electrons have an energy larger than the energy of the first excitation level of the gas molecules. In pure argon, where the first excitation level is 11.5 eV and electron agitation energy is, therefore, correspondingly quite high. This agitation energy is reduced considerably if methane gas (CH_4) is added to the argon because there are now more available energy levels. The mean free path increases rapidly with decreasing energy in the region between 1 eV and 10 eV. Both the increase of the mean free path and the decrease of the agitation energy help to increase the electron drift velocity. Electron drift velocity in various gas mixture has been investigated extensively.³³ Ar-CH_4 gas mixtures were chosen for our MWPC detector because they provide high electron drift velocities. Fig. (20) shows the electron drift velocity in Ar-CH_4 gas mixtures of various compositions as functions of E/p .

2.5. MWPC operation

(a). Electron amplification:- Incident gammas interact with the converter producing conversion electrons. A portion of the conversion electrons escape into the cell spaces where ionization electrons are produced.

Fig. 20 Electron drift velocity in Ar-CH₄ gas mixtures of various compositions as functions of E/p , ref.(33).

These ionization electrons are drifted into the sensitive area of the chamber. The electric potential and field distribution in a MWPC³⁴ has been calculated by Erskine. Consider a chamber with geometric configuration as shown in fig.(21), the electric potential and the electric field are given as follows:

$$V(x,y) = q \left\{ \frac{2\pi L}{S} - \ln \left[4\sin^2\left(\frac{\pi x}{S}\right) + 4\sinh^2\left(\frac{\pi y}{S}\right) \right] \right\}$$

$$E_x(x,y) = -\frac{\partial V}{\partial x} = \frac{\pi q \sin\left(\frac{2\pi x}{S}\right)}{S \left[\sin^2\left(\frac{\pi x}{S}\right) + \sinh^2\left(\frac{\pi y}{S}\right) \right]}$$

$$E_y(x,y) = -\frac{\partial V}{\partial y} = \frac{\pi q \sinh\left(\frac{2\pi y}{S}\right)}{S \left[\sin^2\left(\frac{\pi x}{S}\right) + \sinh^2\left(\frac{\pi y}{S}\right) \right]}$$

Fig.(22) shows the general feature of the field lines and the equipotentials for the geometry under consideration. At regions close to the anode wires ($x \ll s$, $y \ll s$), the electric field varies as $1/r$. When ionization electrons enter this active region, they are accelerated rapidly and gain sufficient energy to cause secondary ionization. In turn, these secondary ionization electrons acquire kinetic energy and cause further ionization. Thus an electron avalanche is initiated. The electron amplification depends on the size and the spacing of the anode wires and the type of gas mixture used. A detailed description of the operation of multiwire proportional chambers as well as useful discussion on the properties of various chamber gas mixtures has been given by Charpak.³⁵ In our case, an Ar-CH₄-Xe gas mixture was used in the chambers mainly because it provides a high electron drift

Fig. 21 Graphic definition of MWPC geometric parameters.

Fig. 22 Equipotentials and electric field lines in MWPC , ref. (41).

velocity. A small percentage (6%) of xenon is added to provide extra chamber gain. The effect of xenon on chamber gain has been reported by Kaufman et al.³⁶ A detailed analysis of the problem of electron gain in a proportional multiplication process is given by Rose and Korff.³⁷

The electron gain, A , is expressed as follows:

$$A = \exp \left\{ 2 \sqrt{\frac{a N_m r_o V_o}{\pi L/S - \ln(2\pi r_o/S)}} \left[(V_o/V_t)^{1/2} - 1 \right] \right\}$$

where a is the rate of increase of the ionization cross section with energy (cm^2/volt) and N_m is the molecular density (cm^{-3}), r_o is the radius of the anode wires, V_o is the potential at the anode and V_t is the threshold voltage for electron multiplication.

(b). Formation of induced pulses

If a charge N_e moves towards the anode wire and experiences a potential drop ΔV , the work expended is $N_e \Delta V$. This work is compensated by additional induced charge supplied to the anode.

$$N_e \Delta V = V_o \Delta Q$$

$$\begin{aligned} i &= \frac{\Delta Q}{\Delta t} = \frac{N_e}{V_o} \frac{\Delta V}{\Delta t} = \frac{N_e}{V_o} \frac{\Delta V}{\Delta r} \frac{\Delta r}{\Delta t} \\ &= \frac{N_e}{V_o} E(r) w \end{aligned}$$

where V_o is the voltage at the anode, N is the number of charged particles, w is the mobility of the ions, r is the radial distance from the center of a wire. Due to the fact that the avalanche is formed in the proximity of the anode wire, the work required to collect the electrons is small as compared to the amount of work expended in drifting the positively charged ions all the way to the

cathodes. Thus positive ion drifting is responsible for most of the induced positive signal formation at the two cathodes. The two cathode plane of wires are perpendicular to each other, and the positive signal induced on the wires of each cathode plane is capacitively coupled to a delay line. The time difference between the prompt anode signal and the arrival of the x, y delayed signals gives the location of an event. This coordinate readout will be described in more detail in this chapter.

2.6. Delay line operation

One of the methods of accomplishing position-to-time conversion is the use of distributed LC electromagnetic delay line. Perez-Mendez has developed a delay line design with rectangular cross section, especially suitable for MWPCs.³⁸ A delay line is characterized by an inductance per unit length, L_0 , due to the coil; and a capacitance per unit length, C_0 , due to the capacitive coupling between the coil and the grounded conductive strips. The important parameters of a delay line to be considered are: the delay per unit length, the characteristic impedance and the bandwidth of the delay line. All the above parameters are functions of distributed inductance and capacitance per unit length. Consider the delay line as a transmission line with uniformly distributed L and C. For an ideal transmission line

$$\frac{\partial I}{\partial z} = -C \frac{\partial V}{\partial t}$$

$$\frac{\partial V}{\partial z} = -L \frac{\partial I}{\partial t}$$

from which follows the wave equation

$$\frac{\partial^2 V}{\partial z^2} = LC \frac{\partial^2 V}{\partial t^2}$$

having the general solution $V(z,t) = V(t \pm z/v_0)$, where $v_0 = 1/\sqrt{LC}$ is the phase velocity. The time delay per unit length is, therefore, given by $\sqrt{LC}$. Typical values of position-to-time conversion ratio lie in the range of 5-20 nsec/mm. Similarly, the solution for the current is also of the form $I(z,t) = I(t \pm z/v_0)$. Therefore, the line has a characteristic impedance given by

$$Z_0 = \frac{dV}{dI} = v_0 L = \sqrt{L/C}$$

Amplifiers used for delay line readouts are designed such that the input impedance of the amplifier matches the characteristic impedance of the delay line. This impedance matching minimizes signal reflections and allow maximum signal transfer from the delay lines to the amplifier.

The performance of the delay line, as can be expected, deviates from that of an ideal transmission line. Due to the finite resistance of the wire conductors and lossy dielectrics at high frequencies, the amplitude of the waveform of a pulse is distorted to some degree. These frequency dependent loss mechanisms cannot be eliminated completely, but can be minimized by proper choice of construction materials. Perez-Mendez reports values for attenuation on the order of 1.5 db/ μ sec at 3.6 MHz, with increasing attenuation beyond 5 MHz³⁹. In addition to amplitude distortion, phase distortion

may occur in the delay line. This phenomenon is caused by the diminishing value of distributed inductance at higher frequencies, resulting in unequal phase velocities for different frequency components of a pulse. This phase distortion in the waveform of a pulse can be compensated by the addition of, frequency dependent, shunt capacity to ground.⁴⁰

A distinct advantage of the delay line readout method is the inherent ability to interpolate between signal coupling points for the maximum height. The centroid of a single pulse indicates closely the center of an ionization event even though the signal includes contribution from the adjacent wires. The time difference between the prompt signal from the anode plane and the delayed signals coming down on the X and Y delay lines corresponds to the position where an event occurs. This method of position-to-time transformation has successfully been used by Perez-Mendez et al, and spatial resolution as small as 0.15 mm have been obtained.

III. POSITRON CAMERA SYSTEM

3.1. General description

A schematic representation of the positron camera system is shown in fig.(23). The annihilation gammas from positron emitters are detected with two opposing detectors, The signals for each coincident event are electronically processed and the positions of the gamma interaction sites are digitized and then transferred via the computer memory to a mass data storage disc unit. The MWPC-gamma converter positron imaging system consists of three major groups of hardware. The first group includes MWPC chambers and gamma converters for 511 Kev annihilation gammas detection. Delay lines are used for the localization of gamma interaction sites. The second group of hardware includes electronic components such as low noise amplifiers for signal amplification and timing discriminators. A data selector unit was designed to perform decision logic functions such as valid coincident event selection and invalid event inhibit/reset. The interface unit consists of analog to digital conversion scalars, a 300 MHz clock and a buffering unit for data transfer to the computer. The third group of hardware consists of a Digital Equipment Co. (DEC) PDP 11/20 computer with an extended memory of 28 K. Peripherals include a display storage scope, two data storage disc units, a fast paper tape reader/punch and a Decwriter keyboard terminal. The software includes system programs for data acquisition, data processing, image display and image processing. The complete camera system is shown in fig.(24).

P = Prompt anode pulses

d = Delay line pulses

A = Amplifiers

Fig.23 Schematic configuration of the MWPC positron camera

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Fig. 24. The camera system, (left) detector system, (center) electronic processing unit, (right) computer and peripherals.

The sequence of data acquisition occurs as follows: If a pair of annihilation gammas is detected, 3 amplified signals from each detector (1 zero-time anode signal and 2 delay line signals) are applied to a multi-channel timing discriminator. The 6 discriminator outputs go into a data selector logic unit. At the same time 4 digitizing scalers are started by the anode signals, and the delay line signals are used to stop the scalers. If a valid event is detected, (the logic of valid event selection is given in sect.(3.6.3.), an interrupt signal is issued to the computer whereupon data is transferred via the buffer unit to the memory of the computer. If a pair of coincident events do not satisfy the data selection requirements, an inhibit/reset signal is issued by the data selector to all units. The operation and construction of hardware components are described in detail in the following sections, and data manipulation will be discussed in Ch.4.

3.2. MWPC construction

Four MWPC's with 48 cm x 48 cm sensitive area were used to form a pair of detectors. Each detector consisted of an assembly of two MWPCS and two gamma converters housed in an air-tight Aluminum container. Each chamber consists of two cathode planes of stainless-steel wires 50 microns in diameter and 2 mm spacing between wires. The anode plane consists of stainless steel wires 20 micron in diameter with a 3 mm spacing between wires, fig. (25). The support frames were made of Nema G-10 fiberglass; the two central frames and the outer frames are each 4 mm thick. All Nema G-10 surfaces were ground to

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Fig.25 Multiwire proportional chamber construction, the location of gamma converters is indicated.

insure uniform thickness for proper interplaner spacings. Reliefs were milled into these frames to accept the coupling strip PC boards which were epoxied into place. The cathode wires were terminated through 220 K Ω resistors to a common bus. The delay lines were mounted on the coupling strips with 0.1 mm thick mylar between them and the strips for insulation. Foam rubber strips were compressed into the space between the delay lines and the support frames to insure firm contact between the delay lines and the coupling strips, producing good capacitive coupling for the cathode signals. Detailed chamber construction techniques have been described by Valentine.⁴¹

3.3. Delay line construction

The design and properties of delay lines have been described in detail by Grove, Perez-Mendez et al.³⁸ Fig. (26) shows the basic construction of the delay line. The internal conductor which is grounded consists of a mylar foil with longitudinal copper bands on one side. They are connected at both ends to the ground pins on the plastic core. The other conductor consists of a closely wound helix of # 32 gauge copper Formvar wire. In the case where phase compensation is desired to reduce phase distortion, a mylar strip with etched metallic bands of copper or aluminum is cemented onto one of the flat side of the delay line. A set of 8 delay lines, 55 cm long, (2 for each MWPC) with delay to rise time ratio of 28:1 and a total delay of 1.1 μ sec were made for the positron camera.

3.4. Gamma converter construction

Because Pb is relatively inexpensive and readily available,

it was used for our converters. The optimum thickness of cell walls made of lead was calculated to be 75 microns. Photoetching and electroplating methods were employed to construct these thin walled converters. Bands of copper were photoetched onto sheets made of 50 microns of mylar clad on both sides with 50 microns of copper. Subsequently, a layer of lead 75 microns thick was electroplated onto the copper. Strips of selected width were cut from these sheets and corrugated with two meshing gears. The strips were then soldered together to form a honeycomb structure. Metal bands at the same height were connected by a common buswire. A drift field is provided to each cell by applying graded voltages to the buswires, fig. (27). We have made 4 converters of 48 cm x 48 cm sensitive area with 3.5 mm cell size. Each lead converter has 4 bands (3 mm per band) and the total height, including insulting gaps, is 15 mm. One converter is coupled to each of the two MWPCs in each detector. Extra space is provided for the addition of one more converters per MWPC making a total of 8 converters for the whole camera.

3.5. Detector assembly

A total of two MWPCs chambers, two gamma converters, and four delay lines were housed in one of two detector box. The Aluminum container box was made of 1 inch thick walls with 1/16 inch reinforced Aluminum window. The detector container was originally designed to withstand 2-3 atmospheres if pressurization should be required. A rectangular trough was milled into the Aluminum walls and a rubber gasket was compressed between the window and the side walls forming

- ① Plastic Core
- ② Floating Metal Strips On Mylar Base: Strips = 1.8mm Wide;
Gaps = 0.3mm Wide; Mylar = 25 Microns Thick
- ③ Winding = #30 Formvar Wire
- ④ 8 Copper Strips On Mylar Base: Strips = 1.8mm Wide;
Gaps = 0.3mm Wide; Mylar = 25 Microns Thick

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Fig. 26 MWPC delay line construction, ref. (38)

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Fig. 27 A section of the layered, honeycomb shaped gamma converter. Graded voltages are applied through bus-wires to individual cells.

an air-tight container. Each converter was spaced at 4 mm from one of the cathode planes of the chamber. A copper sheet was placed between the two MWPCs providing a physical signal decoupling screen between them. The two chamber-converter-delay-line assemblies were secured on studs anchored to the bottom of the container. Fig.(28) shows the detector assembly with the Aluminum window removed. The two detectors were then placed in position on a support rack. The detectors are separated by 56 cm , realizing a geometric acceptance of 28 % for a point source at the center of the field of view. Three external high voltage supplies, which provide voltage to the anodes, cathodes , and gamma converters, in both detectors, complete the detector assembly.

3.6. Electronics

A modified version of a charge-sensitive amplifier was specially designed to be suitable for signal detection and amplification in either the anode plane or the delay lines. Other electronic components, including timing discriminators and data selection unit are also described in the following sections.

3.6.1. Modified charge-sensitive amplifier (MCSA)

The design criteria of an amplifier for anode readout are good signal to noise ratio and bandwidth large enough to preserve the rise time of the signal for timing purpose. The additional design criteria for the cathode signal readout requires that the input impedance of the amplifier matches that of the delay line to eliminate signal reflections. The effect of the noise in a radiation detection system is twofold. First of all, if we have a distribution of signal

Fig. 28. Detector assembly showing the MWPC chamber and the gamma converter.

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amplitudes from a MWPC detector with a given gain, the lower detection threshold window which is set above the noise level determines the overall detection efficiency (ch.2). Secondly, the timing accuracy of the zero-cross discriminator which determines the spatial resolution is affected by the signal to noise ratio (Sect. 3.6.2). The analysis of noise has been presented in detail by Radeka⁴² and Goulding.⁴³ In the positron camera system, an important source of noise is the noise generated by the terminating resistor in the delay line. This dissipative element in the input circuit generates thermal **noise**. The level of noise generated is inversely proportional to the value of the terminating resistor which is in parallel with the input. We shall consider a method of replacing this dissipative terminating resistor by a non-dissipative impedance. The principle of "electronic cooling" of a delay line in order to reduce the noise was first introduced by Radeka.⁴⁴ A basic circuit for cooled damping with capacitance and the derivation of the input impedance is presented in Appendix (5). The introduction of a new time constant $R_o C_o$, in the forward gain path provide, in essence, a reflective impedance at the input. The input impedance is then given by

$$Z_{in} = \frac{(C_o + C_f)}{g_m C_f}$$

where g_m is the trans-conductance of the FET, C_f is the feedback capacitance. The input impedance of the MCSA has become purely resistive without any dissipative element. A schematic diagram of the MCSA is shown in fig. (29). A low-noise junction FET (2N4861) was

Fig. A schematic diagram of the Modified Charge-sensitive Amplifier.

chosen at the input stage of the amplifier to optimize signal to noise performance. Pulse - shaping is performed by a RC differentiator at the input to the operational amplifier ($\mu\text{L} 733$). At the output of the operational amplifier is a current driver followed by a diode clipping stage used to minimize pulse undershoot , which would cause double pulsing in the timing discriminator.

The MCSA has an effective bandwidth of (10MHz) and an adjustable gain from 50-2000. An equivalent noise level of $16\ \mu\text{V}$ is realized at the input of an operational MCSA. The effective input impedance can be varied from 500 ohms to 5 k ohms by adjusting C_0 . The MCSA can be used to amplify input signals of both positive and negative polarities by simply inverting leads in the intermediate operational amplifier stage. Additional differentiation is possible by adding an RC network at the output of the amplifiers. A typical anode signal from the MCSA has a rise time of 100 nsec and a fall time of 400 nsec with an output differentiator of time constant equal to 250 nsec. The delay line signals, however, are already differentiated through the capacitive coupling to the wire planes before input to the amplifiers, and have a rise time of 100 nsec with 250 nsec fall time. No further differentiation is necessary at the output of a delay line amplifier.

3.6.2 Timing discriminator

The position of a gamma interaction is determined by the time difference between the occurrence of a prompt anode signal and the delayed cathode signals, hence spatial resolution is critically dependent on the timing accuracy of the timing discriminator, making it one of

the most important components of the electronic unit. The timing discriminator is of the differentiating, zero-cross type⁴⁵, designed to locate the time at which the pulse reaches its peak amplitude. The time occurrence of peak (or zero-slope) should be identical for a family of pulses regardless of the pulse amplitudes.

The timing discriminator consists of a differentiation stage, an amplification stage, and a tunnel diode triggering stage. A schematic diagram of the timing discriminator is shown in Appendix (6). The input pulse is differentiated and amplified. The positive portion of the amplified bipolar signal sets the diode, and it is reset at the transition point when the polarity of the bipolar pulse goes from positive to negative. The differentiated input pulse was amplified to give a well defined transition point.

The principal factors affecting the timing accuracy are noise and the finite current differential for resetting of the tunnel diode. The multi-channel timing discriminator which was used has a time slew of 6 nsec over a dynamic range of 20 in signal amplitude. The time jitter associated with 100 mv signal and 20 mv noise is 8 nsec; but improves to 1 nsec as the input signal is increased to 1 volt, fig. (30). The output from the timing discriminator is fast NIM level (-700 mv). The anode signals from each of the four chambers pass through a single-channel analyzer where a band of input signal amplitudes can be selected by adjusting lower and upper-threshold levels.

3.6.3 Data Selector, Scalar, Clock

Outputs from all of the 12 channels of the timing discriminators

Figure 30 The timing discriminator jitter as a function of signal to noise ratio.

(4 from anode readouts and 8 from cathode readouts) are put into a data selector logical unit. This unit consists of a network of flip-flops and gates. A logic diagram is shown in fig.(31). A complete schematic diagram is presented in Appendix (6). A valid output signal from the data selector is initiated if all of the following conditions are met

Prompt anode signals

1. Only one of the two MWPC s is triggered in each of the upper and lower detector boxes. (2 chambers were housed in one detector box).
2. When one of the two chambers in a detector box is triggered, the coincident signal from one of the chambers in the opposite detector box must arrive within the coincidence resolving time.

Delayed cathode signals

3. If conditions (1) and (2) in the prompt signals are met, there must exist 4 delayed cathode signals associated with the chambers which were triggered.

An event is rejected by the initiation of an inhibit / reset signal if,

1. Either one of the conditions (1), (2), (3) is not satisfied.
2. A second prompt signal should occur on either one of the four chambers before a valid coincident signal is initiated.
3. More than one signal should occur on any one of the delay lines within delay line time (1.5 μ sec) before a valid signal is initiated.

Whenever any one of the four MWPCs is triggered, further signals from that particular chamber would be gated off for 1.5 μ sec while the

other detectors are still operative. This would prevent accidental coincidence and minimize the detector system dead time.

A set of four scalers, two for the upper detector and two for the lower detector are coupled in parallel to a 300 MHz clock. A prompt signal starts the scalers, while the associated x,y delay signals stop the scalers. The time difference between each prompt starting signal and the two delayed stopping signals is translated into digital counts in the scalers. If a valid coincidence event occurs, data is transferred via a buffering unit to a temporary buffer in the computer. For permanent storage, data can be transferred from the computer memory to a disc.

Derandomized data registers are used in the buffer interface electronics to speed up data transfer. A reset signal is initiated by the computer when data transfer is completed. All of the electronic components are housed in a Camac bin. Data Acquisition and processing by the computer will be explained further in Ch.(4).

IV. DATA PROCESSING

The software used in the data processing includes system programs (on paper tapes) supplied by DEC, and a positron camera program. The positron camera program (PCP) was written in assembly language and the program structure can be generally divided into 3 parts. The first part of the program transfers to memory the coded input parameters and the data. Input parameters such as total number of events required, focal plane position, detector separations etc. are coded in alphanumeric and are used throughout the program. The second portion of the program includes data unpacking, the focal plane intersection point calculation and other conversion. Image processing such as background subtraction, smoothing, detector uniformity correction is carried out in the last part of the program. The sequence of data acquisition, data processing, and image display is carried out as follows: - The total number of events required is specified via the keyboard input terminal. After the PCP program is started, continuous on-line data acquisition is monitored by the program. The PCP program is temporarily halted when the number of events required is collected. The data acquired on disc can be processed immediately, or at a later time at the user's convenience. Before the data is processed, image reconstruction parameters should be specified. Data is transferred to the computer and processed when the PCP program is continued. At the same time, the image can be displayed on the storage scope event by event. If image processing is desired, coded instructions are input to the program. Data is again recalled from the disc and processed

accordingly. The detailed data manipulation is described in the following sections.

4.1 Data collection

When a valid coincident event is detected, an interrupt signal is initiated by the data selector to the computer for data transfer. Each event consists of three 16-bit words. The bit arrangement is shown in fig. (32). The clock is a 10-bit buffer used as a real time marker in the data collection process. This clock is driven by a 60 HZ line frequency. After each data transfer to the computer memory core is completed, a reset signal is sent to the data selector, and data acquisition is continued. To minimize system dead time, a double buffering technique is used. Two buffers, 3072 words each, are set up in the memory core for temporary data storage. After the first buffer is filled, the data storage function is shifted to the second buffer and vice versa. While the second buffer is being filled, data in the first buffer is transferred to the disc during computer idling time between events. Each disc has a capacity of 1.2 million words. A theoretical maximum of 400k events can be stored on a single disc, but the actual data storage size is limited to 333k events with the remaining area occupied by system programs.

4.2 Image reconstruction and display

The number of coincident events required for an image reconstruction in our positron camera varies according to the complexity of an object. For a point source, a total of 1k coincident event is sufficient while in the case of a head phantom, it may require as many as 300k events to

produce a discernable image. Image reconstruction proceeds as follows:

- Input parameters, such as the position of the focal plane, number of events to be processed, etc., are specified on-line via the keyboard by the user. After the user starts the PCP program, each event consisting of 3 words is transferred serially from the disc to the CPU unit where data is unpacked into its proper chamber number and four coordinate digital readings, two for the upper detector and two for the lower detector. The intersection of a line connecting the two points with the selected focal plane is calculated. (The detailed tomographic image reconstruction and two other methods of 3-D reconstruction are described in the next sections.) The intersection point is displayed on the scope in one of two modes of image display which will be described later in this chapter. All events are processed in this manner, and hardcopies of the resulting image can be obtained with a Tektronix camera.

4.2.1 Tomographic reconstruction

A line joining the sites of interaction of a pair of annihilation gammas determines the line vector along which a point emitter lies. The intersection of two or more of these lines determines uniquely the locus of a point source. In the case of distributed sources, it is not possible to determine the position of the sources from the intersections of such a set of lines. The "plane of best locus" method is used to determine the distribution of positron emitters. In this method, the position of a focal plane parallel to the detectors is selected. The stored data is retrieved from the computer memory, and the intersection of all the line vectors with the selected focal plane is

calculated. In fig. (33), a line vector, $\bar{L}$, intersects the two detectors at (x_1, y_1, z_1) and (x_2, y_2, z_2) . The equation of a line passing through (x_1, y_2, z_1) and parallel to $\bar{L}$ is

$$\frac{(x - x_1)}{A} = \frac{(y - y_1)}{B} = \frac{(z - z_1)}{C} = K \quad 4.1$$

where $A = (x_2 - x_1)$, $B = (y_2 - y_1)$, $C = (z_2 - z_1)$ are the line directional components of $\bar{L}$. The equation of a plane passing through a point (x_0, y_0, z_0) and normal to a line vector $\bar{n} = \alpha\bar{i} + \beta\bar{j} + \gamma\bar{k}$ is

$$\alpha(x - x_0) + \beta(y - y_0) + \gamma(z - z_0) = 0 \quad 4.2$$

In the general case where a line parallel to $\bar{L}$ passing through (x_1, y_1, z_1) and intersects the above plane at a point (x', y', z') , then (x', y', z') must satisfy both eq.(4.1) and (4.2)

$$\left. \begin{aligned} x' &= AK + x_1 \\ y' &= BK + y_1 \\ z' &= CK + z_1 \end{aligned} \right\} \quad 4.3$$

$$\text{and} \quad \alpha(x' - x_0) + \beta(y' - y_0) + \gamma(z' - z_0) = 0 \quad 4.4$$

Substituting $(x'y'z')$ from eq. (4.3) into (4.4), we obtain

$$K = \frac{\alpha(x_0 - x_1) + \beta(y_0 - y_1) + \gamma(z_0 - z_1)}{\alpha(x_2 - x_1) + \beta(y_2 - y_1) + \gamma(z_2 - z_1)} \quad 4.5$$

To obtain the coordinate $(x'y'z')$, we substitute the constant K into eq.(4.3). In the particular case where the plane chosen is parallel to

Fig.32 The bit map of data for each coincident event. A_x and A_y are the coordinates for X and Y delay lines respectively in the upper detector. Coordinates for the lower detector are designated by B_x and B_y . The pair of chambers activated is coded in the 1st word.

Fig.33 Graphic representation showing a line L intersecting the upper and the lower detector planes.

the detectors and intercept the z-axis at a point z_0 , then

$$\alpha = \beta = 0$$

The equation for the plane is simplified as

$$\gamma(z - z_0) = 0$$

The intersection points at this plane are

$$x = x_1 + (x_2 - x_1) \cdot (z_0 - z_1) / (z_2 - z_1) \quad 4.6$$

$$y = y_1 + (y_2 - y_1) \cdot (z_0 - z_1) / (z_2 - z_1) \quad 4.7$$

Similarly, intersections can be obtained for planes normal to the x-axis or y-axis with an additional requirement that the two points are on opposite side of a focal plane. This condition is automatically satisfied for a plane normal to the z-axis. All of the stored data is processed according to the reconstruction algorithm, and the points of intersections are displayed as intensified dots on a scope screen. The image formed consists of the object image and the blurred superimposed images of objects at planes above or below the focal plane.

4.2.2 3-D reconstruction

Two schemes for overcoming this problem of superimposed images are being tested. One approach is to de-couple the image superposition for a number of planes using the tomographic images reconstructed on those planes (Method I). Deconvolution is performed in the frequency domain, and inverse Fourier Transform gives the true images at each plane. Another method of 3-D reconstruction uses the scalar field approach (Method II). Both of these methods will be reviewed briefly.

Method I

Fig. (34) shows the tomographic images of a point source at various

Fig. 34 Intersections of annihilation line vectors with focal planes from two point sources, one located at plane i and the other at plane j .

planes. Four coincident events are indicated. The tomographic images obtained at plane i has the superimposed image contributed by a point source at plane j . Similarly, the reconstructed image at plane j is blurred by a source at plane i . The extension from a point source to an object sources involves the convolution of an object with a point response function. Hence

$$\begin{aligned} t_1 &= o_1 * h_{11} + o_2 * h_{12} + o_3 * h_{13} + \dots o_n * h_{1n} \\ t_2 &= o_1 * h_{21} + o_2 * h_{22} + o_3 * h_{23} + \dots o_n * h_{2n} \\ t_3 &= o_1 * h_{31} + o_2 * h_{32} + o_3 * h_{33} + \dots o_n * h_{3n} \\ &\vdots \\ t_n &= o_1 * h_{n1} + o_2 * h_{n2} + o_3 * h_{n3} + \dots o_n * h_{nn} \end{aligned}$$

where t_n is the tomograms obtained at n^{th} plane

o_n is the object distribution at n^{th} plane

h_{ij} is the point response function at i -th plane from a point source at the j -th plane.

If we operate in the frequency domain by taking the Fourier Transforms,

the equation becomes

$$\begin{aligned} T_1 &= O_1 \cdot H_{11} + O_2 H_{12} + O_3 H_{13} + \dots O_n H_{1n} \\ T_2 &= O_1 H_{21} + O_2 H_{22} + O_3 H_{23} + \dots O_n H_{2n} \\ T_3 &= O_1 H_{31} + O_2 H_{32} + O_3 H_{33} + \dots O_n H_{3n} \\ &\vdots \\ T_n &= O_1 H_{n1} + O_2 H_{n2} + O_3 H_{n3} + \dots O_n H_{nn} \end{aligned}$$

where T_n , O_n , H_{ij} are the Fourier Transforms of the tomograms, object sources and the point response functions respectively.

This can be represented in a matrix form

$$\begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ \vdots \\ T_n \end{pmatrix} = \begin{pmatrix} O_1 \\ O_2 \\ O_3 \\ \vdots \\ O_n \end{pmatrix} \begin{pmatrix} H_{11} & H_{12} & H_{13} & \dots & H_{1n} \\ H_{21} & H_{22} & H_{23} & \dots & H_{2n} \\ H_{31} & H_{32} & H_{33} & \dots & H_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ H_{n1} & H_{n2} & H_{n3} & \dots & H_{nn} \end{pmatrix}$$

Finally, the object at each plane can be obtained by taking the inverse Fourier Transform of the matrix elements $O_1 \dots O_n$ obtained from dividing the tomogram matrix by the point response function matrix. In this reconstruction method, it is required that the point response function remain constant in shape, size and intensity as a point source moves over the camera field of view on a given plane, although it may be different for different planes, Fig. (35) shows the solid angle variation subtended by a point source on the boundary of an object and one in the interior along the same horizontal plane. In order to have a uniform point response function, only those events which satisfy the angular limit $0 \leq \theta_1$ are accepted, i.e. $|x_2 - x_1| \leq d$ and $|y_2 - y_1| \leq d$ where d is smaller than W , the width of the detector.

Method II

Another algorithm for three dimensional reconstruction using Fourier techniques was developed recently. The principle of the method is summarized here. Consider a point emitter which generates a set of straight lines in three spaces. For a set of these randomly oriented straight lines, it is possible to define a scalar field which is a function only of position relative to the point emitter. Thus a point emitter located at $\vec{r}_0$ generates a scalar field $\phi_0(\vec{r} - \vec{r}_0)$. For example, take an area element δA normal to the z -axis at a distance

$|\vec{r} - \vec{r}_0|$ from the emitter, fig. (36a). The probability that one of the lines will pass through the area element is

$$\frac{\delta A \cos \theta}{2\pi |\vec{r} - \vec{r}_0|^2}$$

and

$$\phi_0(\vec{r} - \vec{r}_0) = \frac{\cos \theta}{2\pi |\vec{r} - \vec{r}_0|^2} = \frac{(\vec{r} - \vec{r}_0) \cdot \hat{z}}{2\pi |\vec{r} - \vec{r}_0|^3}$$

where $\phi(\vec{r} - \vec{r}_0)$ is the scalar field defined as the average number of lines per unit area per event for lines emitted from $\vec{r}_0$ which pass through the oriented area element located at $\vec{r}$.

In general, for an arbitrary distribution of radioisotopes distribution ρ , a scalar field ϕ is generated and is given by the convolution

$$\phi(\vec{r}) = \int \rho(\vec{r}') \phi_0(\vec{r} - \vec{r}') d^3r' \quad 4.8.$$

where ϕ_0 is the scalar field associated with a point emitter described previously. The data from the positron camera measures the field ϕ . If enough coincident events are collected to provide good statistics, a density distribution ρ , can be calculated where both ϕ and ϕ_0 are known.

The Fourier transform of both sides of eq. (4.8) gives

$$\phi(\vec{p}) = S(\vec{p}) \phi_0(\vec{p})$$

where ϕ, ϕ_0, S are the Fourier transform of ϕ, ϕ_0, ρ respectively.

The solution of S , in momentum space is simply

$$S(\vec{p}) = \phi(\vec{p}) / \phi_0(\vec{p})$$

The density distribution can then be obtained by taking the inverse

Fig. 35 Angular limit for 3-D reconstruction.

Fig. 36a Geometric configuration for the calculation of scalar field generated by a positron source located at r_0 .

transform

$$\rho(\vec{r}) = \int S(\vec{p}) \exp(-2\pi i \vec{p} \cdot \vec{r}) d^3p$$

In practice, a numerical reconstruction of the density distribution of radioisotope is obtained by dividing the volume of interest into a lattice of discrete points and by using discrete Fourier transforms. This method of 3-D reconstruction also has to invoke the angular limit as in the case of multiplane reconstruction. Detailed discussions of noise instabilities, other scalar fields, the reconstruction algorithm, and methods of Compton scattering delineation can be found in ref.(47).

The two methods of 3-D reconstruction from the data obtained by a positron camera should provide better images than the simple tomographic reconstruction. Presently, data obtained from the positron camera is being used for testing and refinement of the above 3-D reconstruction programs.

4.2.3. Image display

Two modes of image display are available. In 'Dot Display' mode, a picture frame consisting of 1400 x 1400 addressable dots on the storage scope is used to represent the chamber active area, 500 mm x 500 mm. After the intersection point on the focal plane is calculated for an event, a dot corresponding to this position is activated. An image is formed when all of the stored events are processed similarly. The dot display provides only a quick overview of a reconstructed image. Another mode of display which allows much more flexibility in image processing is the 'Matrix Display' where each "point" in the image is represented not by a single dot but by a matrix element consisting of

a number of dots. The size of the matrix is chosen according to the resolution desired. For example, a 64×64 matrix used to represent an area of $500 \text{ mm} \times 500 \text{ mm}$ limits the resolution to about 7 mm per matrix element. If a 128×128 matrix is used to represent the same area, a finer resolution (4 mm per element) is realized. In our case, a 64×64 matrix was used for the image reconstruction and processing. A section of the memory core is assigned to represent the matrix. The content of the memory location corresponding to a matrix element is increased by 1 if an intersection point should fall within the limit of that matrix element. A number of image processing techniques can be used to improve the image quality.

4.3. Image processing

A number of image processing subroutines are available as options in the program. After all the events are processed and stored in matrix display mode, one or more of the following processing techniques can be applied.

Sensitivity correction

A point at the center of the mid-plane between two detectors subtended the largest solid angle. The solid angle diminishes rapidly as we move away from the center towards the edges of the detectors. It is also shown in Ch.2 that the converter efficiency is angular dependent. Therefore, in order to correct for both the solid angle effect and angular effect on sensitivity, a sensitivity correction matrix is determined experimentally as follows: A sheet source of uniform activity made of a lucite container filled with Ga-68 is placed at

mid-plane of the detectors. The total activity used should be low enough to minimize accidental coincidence countings. Coincident events are detected and are processed in matrix mode as before for object plane at a position corresponding to the sheet source. The matrix thus obtained constitutes the correction matrix where the differences of accumulated counts in the matrix elements reflect the sensitivity of the detectors with respect to source position. Fig. (36b) shows the relative counts for a row of elements in a 64 x 64 correction matrix taken along the center at the mid-plane with the experimental result lying, as would be expected, in the intermediate range between the two extreme cases of a thick detector and a thin detector described in sect. 2.2.1.

When the subroutine for sensitivity correction is called, the count in each element of the object matrix is multiplied by the ratio of maximum count and the count of the corresponding element in the correction matrix. This first order sensitivity correction does not take into account the non-uniform sensitivity for planes away from the mid-plane.

Smoothing

Smoothing is a statistical averaging process by which the accumulated counts in each element of the matrix are weighted together with counts in the immediate surrounding elements. In our program, we have available, as an option, a "five point smoothing" subroutine, in which an element is weighted with its four adjacent elements. For example, a smoothed elements, $\bar{a}_{ij}$, of a matrix with i columns and j rows

Fig. 56b. The relative counts for a row of matrix elements taken along the center at the mid-plane.

is calculated as follows:

$$(\overline{a}_{ij}) = \frac{(a_{i-1,j}) + (a_{i,j-1}) + (a_{i+1,j+1}) + (a_{i,j+1}) + (w_{ij}a_{ij})}{w_{ij} + 4}$$

The weighting factor, w_{ij} , is a variable integer having a range from 1-10. For large statistical fluctuation, a heavy smoothing, and therefore, a small value of the weighting factor is used.

Background subtraction and Contrast enhancement

Lower thresholding and upper thresholding are options available in background subtraction. Lower thresholding is specified as percentage of the maximum count in the matrix. For example, in a 10% background subtraction, matrix elements with 1/10 the maximum accumulative count would not be displayed. Similarly, in a 10% upper thresholding, matrix elements within 1/10 of the maximum count would be suppressed.

Normally, a range of 32 grey levels is divided between the lower count limit and the upper count limit of the matrix. In contrast enhancement, the range defined by the upper and lower thresholding is divided into 32 grey levels. As a result, for the same difference of counts between two matrix elements, the color contrast in the scope display is effectively enhanced with the expended grey scale.

V. A. PROTOTYPE MWPC-CONVERTER DETECTOR MEASUREMENTS

Prior to the construction of the MWPC positron camera, a prototype detector was constructed and tested. The construction of the scaled-down chamber and the converter is identical to those in the positron camera. The following performance characteristics were measured:

- (1) Converter detection efficiency
- (2) Spatial resolution
- (3) Coincidence resolving time

5.A1. Converter detection efficiency

As shown in Ch.2, sensitivity is proportional to the square of the system's detection efficiency. In order to test the detection efficiency for 511 Kev gammas with converters of various cell dimensions, we used a scaled-down MWPC. The test chamber has 25 cm x 25 cm sensitive area consisting of two cathode planes of stainless steel wires, 100 microns in diameter. The anode plane consists of stainless steel wires 25 micron in diameter. All planer spacings are similar to the true size MWPC chamber.

A Na-22 positron source ($\sim 100 \mu\text{Ci}$) was placed mid-way between the test chamber and a NaI crystal/photomultiplier detector. For a pure positron emitter, the detection efficiency is given by the ratio of the net coincidence count rate C_0 (coincidence-background-accidental) to the single count rate, S

$$\frac{C_0}{S} = \frac{N_0}{N_0} \frac{G \epsilon_1 \epsilon_2}{G \epsilon_1} = \epsilon_2 \quad 5.1.$$

where N_0 is the source strength, G is the geometric factor, ϵ_1 is the NaI detector efficiency, ϵ_2 is the detection efficiency of the converter

A correction has to be made for the Na-22 coincidence counts since a 1.27 Mev gamma is also emitted with every pair of annihilation gammas. For every 511 Kev gamma detected by the NaI detector which has an energy window set to accept unscattered primary gammas, the probability that a 1.27 Mev gamma would be detected by the chamber is

$$\frac{\Omega}{4\pi} \times \epsilon'_2 \times (1 - \epsilon_2)$$

where Ω is the solid angle subtended by the source and the converter.

Fig. (15) shows that the detection efficiency for 1.27 Mev gammas is approximately equal to that of 511 Kev gammas, so we assume $\epsilon'_1 \approx \epsilon'_2$. Then the modified net coincidence count rate versus singles is

$$\frac{C_m}{S} \approx \frac{N_O G \epsilon_1 \epsilon_2}{N_O G \epsilon_1} \left(1 + \frac{\Omega}{4\pi}\right) \approx \epsilon_2 \left(1 + \frac{\Omega}{4\pi}\right) \text{ for } \epsilon_2 \ll 1$$

The chamber detection efficiency, however, is dependent on the chamber gain. The chamber gain was calibrated with an Fe-55 source which gives monoenergetic 5.9 Kev gammas. The amplitude obtained from the anode with maximum chamber gain was 1 volt. The amplifier gain was set at 600 with 20 mv noise, equivalent to minimum detection threshold of 5 ion pairs. An energy spectrum was also obtained for the Na-22 source, fig.(37). It was found that over 70% of the signals are between 50 mv- 350 mv. The lower threshold of the timing discriminator was set at 30 mv, 10 mv above the noise level. It is evident that the detection efficiency is very sensitive to the lower threshold setting and the chamber gain. Table (2) compares the calculated values of detection efficiencies with the measured detection efficiencies of lead honeycomb gamma converters of various cell dimensions. In the

Fig. 37 The amplitude distribution obtained by a MWPC-gamma converter hybrid detector for a Na-22 source.

Table 2

Converter cell	<u>Surface area</u> <u>Cross sect. area</u>	ϵ_t (cal.) (%)	ϵ (exp.) (%)	ϵ (exp.) <u>plane</u>	Measured Extraction Efficiency
Pb(511 keV gamma)	4LH/L ²				
Plane converter	1	0.26	0.26	1.0	1.00
3mm x 3mm x 12mm	16	3.12	3.20	12.3	0.76
2.2mm x 2.2mm x 12 mm	22	4.29	4.20	16.2	0.73
2mm x 2mm x 4mm	8	1.56	1.55	5.4	0.74

numerical calculations, a uniform energy loss in the gas of 3.2 KeV per cm was assumed. The minimum track length was taken to be 0.4 mm, equivalent to ionization of 5 ion pairs. As can be seen from the table, the measured extraction efficiencies which vary between 0.73 - 0.83 are in good agreement with the calculated values.

5.A2. Spatial Resolution

The standard method of comparing spatial resolution capabilities among imaging systems is through the system's spatial frequency response, or modulation transfer function (MTF). This frequency response can be measured in several ways. Bar patterns and pie patterns are routinely used.⁴⁸ A direct way to measure the MTF of a system is to obtain the systems response to a line source, the so called line spread function (LSF). The MTF is simply the Fourier transform of the LSF with correction for finite line width.⁴¹

To obtain the LSF of the positron camera, the test chamber was again used because of its ease of experimental controls. For positioning an Fe-55 point source was put on top of a collimated slit 1 mm wide made of 1/4 inch Aluminum to ensure proper alignment between slit and one set of the chamber cathode wires. Starting signals from the anode wires and the stopping signals from the cathode wires were amplified and applied to two channels of a timing discriminator. The timing discriminator outputs were then applied to a time-to-amplitude converter (TAC), and the amplitude distribution which corresponds to the LSF of Fe-55 source was applied to a multichannel pulse height analyzer (PHA). The gain of the PHA was adjusted and calibrated to

give a maximum sensitivity of 10 channels to 7 mm. From sect.(3.6.2), we note that the timing accuracy of the discriminator is critically dependent on the signal to noise ratio. For the Fe-55 source with maximum chamber gain, we had a S/N ratio of 50. The range of 5.9 Kev electrons from an Fe-55 source in Ar-CH₄ is 1 mm. The short range, in combination with the electronic timing jitter gives an experimental resolution of 1.8 mm FWHM.

For the 511 Kev gamma which has a mean free path of 5.6 mm in lead, a 1 mm slit was formed by two 5 cm thick lead bricks. A Na-22 source was put on top of the slit, and a line spread function was obtained similar to the procedure described above. The FWHM for a Na-22 source was found to be 9 mm. The LSF obtained using Na-22 source is shown in fig. (38). For the Na-22 source, conversion electrons can emerge from either side of a converter wall and travel at most a distance equivalent to the cell size depending on its kinetic energy. However, some of the conversion electrons that arise near the top of the converter walls can have long ionization tracks if they escape into the gas region without hitting the surrounding walls. For a converter with cell dimension, d, the average spatial resolution measured in FWHM was estimated as

$$S_{FWHM} = \sqrt{(2x\bar{d})^2 + (\sigma \times v_{dl})^2 + (\delta \times v_{dl})^2} \quad 5.3$$

where $\bar{d}$ is the average track-length, equal to $0.73 d$, calculated from the track-length-distribution discussed in Ch.2. $(\int_0^{k_{max}} k f(k) dk)$; σ is the time jitter of the zero cross discriminator; δ is the time slew of the discriminator; v_{dl} is the speed of the delay line in

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Fig. 38. Line Spread Function (LSF) of a Na-22 source.

mm/nsec. The values of σ , δ are functions of the signal to noise ratio, sect.(3.6.2). From the energy spectrum of the chamber pulses, an average signal to noise ratio of 5 was used. ($\sigma = 8$ nsec, $\delta = 6$ nsec over a dynamic amplitude range of 20:1). This in combination with the delay line speed of 0.42 mm/nsec and converter cell size of 3.5 mm yields a calculated FWHM of 7-8 mm. The experimental spatial resolution obtained was a little higher than the calculated value.

5.A3. Coincident resolving time

The accidental coincidence rate is proportional to the coincidence resolving time of the detector system, eq.(2.4). This resolving time was measured with a Na-I detector and a test chamber operated in coincidence detection mode. A Na-22 source was placed between the two detectors. Signals from the NaI detector was used as the starting signals to a TAC and the corresponding coincidence signals from the chambers were used as the stopping signals. The TAC output was displayed on a PHA. Fig.(39) shows the drift time distribution of ionization electrons in Ar(70%)-CH₄(30%) gas mixture for a 4-layer converter, 15 mm high. The addition of 6% Xenon has little effect on the time spread. A coincidence gate was set in the data selector logic unit corresponding to the FWHM of the resolving time of 300 nsec.

Fig. 39. The drift time distribution of ionization electrons for a i-layer converter.

V.B. CHARACTERISTICS AND PERFORMANCE OF POSITRON CAMERA

Although the objective of the positron imaging is to produce informative pictures of spatial distribution of positron emitting radioisotope within a particular section of the body, it is also important that the sensitivity of the imaging system be sufficiently high to minimize the dose given to a patient. In addition to patient's comfort, a high data rate would also reduce image distortions as a result of patient's movement. The camera performance characteristics which would dictate the qualities of pictures and thus meaningful clinical results are as follows:-

1. Sensitivity of the imaging system (counts/min/ μCi)
2. Spatial resolution of the imaging system
3. Ratio of true coincidence to accidental/scattering coincidence

It should be noted that conditions (1), (2), and (3) are linearly independent. However, the limitation of the maximum data rate for the positron camera is imposed by condition (3). This will be clarified in the following sections.

5.B1. Camera Sensitivity

The chambers used in the camera system have a sensitive area of 50 cm x 50 cm. The signal amplitude, which is inversely proportional to the chamber capacity is smaller than that of the test chamber. To realize the full converter detection efficiency, the chamber gain was increased by adding xenon gas to the $\text{Ar}(70\%)\text{-CH}_4(30\%)$ gas mixture. With 6% of xenon added, the chamber gain was increased 2.5 times. In addition, the maximum voltage capacity of the camera chamber was

higher than that of the test chamber, which was probably due to better chamber construction. The chamber gain was increased sufficiently to give a minimum detection threshold equivalent of 5 ion pairs. To measure the sensitivity of the positron camera system, we used a $\text{Cu } 64$ positron source which decays 19 % of the time by positron emission. Approximately 0.5% of these decays are accompanied by the emissions of 1.34 Mev gammas. Small sections of different lengths were cut from the wire, and the activities were measured in a well counter. These were placed at the center of a plane equidistance from the upper and lower detector, and the coincidence count rate versus activity was measured. The energy windows of the timing discriminators for all anode prompt signals were open. The chamber gain for all of the chambers was calibrated and matched by using a ^{137}Cs source which gives monoenergetic gammas of 662 Kev, sufficient to penetrate the Aluminum windows of the detectors. The measured coincident count rates as functions of activities are shown in fig.(44). From eq.(2.2a), the coincident detection rate, R_c , is shown to be proportional to the source activity. Based on the total coincidence rate, the sensitivity of the camera system is measured to be 900 counts/min/ μCi . This gives a detection efficiency of 4.2% per detector for $G = 0.28$. Due to the dead time of the detector system, the observed coincidence count rate reaches a maximum of 2.5K counts/sec at 322 μCi activity. Using eq.(2.7b), we obtain a system dead time of 1.6 μsec .

5.B2. Camera Spatial Resolution

To determine the spatial resolution of the camera system, we used a Na-22 point source. The spatial resolution measured at the mid-plane of the camera system, however, is smaller than the value obtained for each detector. This can be seen clearly from fig.(41). A total of 10K events were collected. A reconstructed image in the plane of the point source was displayed in a matrix format on the scope. A row of matrix elements that cut across the center of the point source was selected, and the contents of the elements were displayed on a relative scale, fig.(42). Each element represents a spatial distance of 2 mm, and the spatial resolution in FWHM was measured to be 6-7 mm at the mid-plane.

5.B3. Ratio of true coincidence to accidental scattering coincidence

To measure the ratio of true coincident events to accidental/scattering coincident events as functions of activities, we again used the Cu-64 wire sources. To ensure that the positrons be stopped in the source ($T_e < 600$ Kev), each copper wire was immersed in a 10 mm diameter test tube filled with water. Each set of data was processed and the reconstructed image at the plane of the point source was displayed on a storage scope, fig.(43). Analysis was performed as follows: A square, 3 cm x 3cm was drawn around the image of the point source, and the sum of the events within the square (on-target events) was recorded. The events outside of the square (off-target events) consists of two components, accidental coincidenceas and scattering coincidences. Fig.(44) shows the on-target coincident events and

$$X_0 = \frac{X_1 + X_2}{2}$$

$$\xi = \sqrt{\Delta X_0^2} = \frac{1}{2} \sqrt{\Delta X_1^2 + \Delta X_2^2}$$

If $\Delta X_1 = \Delta X_2 = \delta$; Then

$$\xi = \frac{1}{\sqrt{2}} \delta$$

fig. 41 Comparison of spatial resolution at the mid-plane and spatial resolution at the detector plane of the positron camera.

Fig. 42. Spatial resolution of a point source at the mid-plane. Each matrix element represents a spatial distance of 2 mm.

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Fig. 43. Image of a point source. Events inside the square are designated as on 0 target events while events outside the square are off-target events.

off-target events as functions of source activities.

The accidental coincident counting rate increases with the square of source activity, eq.(2.4). In our data selector logic design, this accidental coincidence counting rate was reduced with a pile up rejection circuit. When two chambers within a detector were triggered, and at the same time one chamber of the other detector was triggered, the event was rejected, Ch.(3). The accidental counting rate was then modified as

$$R_a = \tau_r \left[(R_1 + R_2)(R_3 + R_4) - 2 \tau_r (R_1 R_2)(R_3 + R_4) - 2 \tau_r (R_3 R_4)(R_1 + R_2) \right]$$

where τ_r is the coincident resolving time, and $R_{1,2,3,4}$ are the singles rate of each chamber. The calculated and measured accidental rate are compared in fig.(44). At high source activity, the accidental coincidence become a dominant portion of the off-target events.

The scattering coincidence, however, increases linearly with source activity. This portion of off-target events was contributed from various sources. The major source of scattering occurs in the gamma converter. For every Compton interaction of an incident gamma with the lead walls, there is a probability that the scattered gamma will interact elsewhere in the converter and produce a conversion electron. There are two cases to be considered. In the first case, the primary conversion electron is not registered while the secondary conversion electron from the scattered gamma is detected. This would result in misplaced interaction. In the second case, both the primary and secondary conversions are detected. If the two events are separated far enough spatially, two distinct pulses would be detected

1. Total counts
2. Off-target count rate (1-3)
3. On-target count rate
4. Measured accidental count rate
5. Calculated accidental count rate

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Fig. 44 The measured coincident count rate, on-target count rate, off-target count rate and both measured and calculated accidental count rate as functions of source activity.

in the delay lines, and the coincident event would be rejected by the data selector. Let us estimate the total fraction of scattered events versus the unscattered events. Consider an incident gamma interacting with the lead wall, the probability that a conversion electron would be detected is

$$\epsilon_t = G_t \left\{ \sum_{i=1}^2 2 \sigma_i \rho T \int_0^T \frac{(L+2x)}{(L+T)T} dx P_{esc}(x, E_i) \right\} = G_t \left\{ \sum_{i=1}^2 D_i \right\}$$

G_t is the geometric factor and the quantities in the bracket { } have already been defined and are identical to eq.(2.11).

The probability that a conversion electron for a scattered gamma would be detected is

$$\epsilon_s = \left[G_t \times 2 \sigma_c \rho T \int_0^T \frac{(L+2x)}{(L+T)T} dx \right] \times G_s \left\{ \sum_{i=1}^2 D_i \right\}$$

The detection efficiency is taken to be unity for both the true events and the scattered events, i.e. every conversion electron is detected when emitted into the gas space. G_s is now the geometric factor for a gamma radiation source originating from the lead walls, (an average position for the primary interaction is taken at the middle of the converter, half the converter height). The ratio of scattered events versus true events is calculated for our gamma converter configuration and the calculated ratio as a function of lead wall thickness is shown in fig.(45). The fraction of scattered events increases rapidly with the wall thickness as can be expected. Other sources of scattering medium include detector window, converter backing material and chamber construction materials. At low activity, the fraction of off-target events was measured to be 0.36. This

fig. 45 The ratio of scattered events to true events as a function of converter wall thickness .

corresponds to 20% scattering in each detector. For our gamma converter which has a wall thickness of 150 μm , the scattering fraction is calculated to be 15%. This agrees with the measurement to first order.

5.B4. Effects of external scattering medium

The scattering of gamma radiation at the source has been considered in ch. 2 and fig. (6). For 511 Kev photons, the major interaction mechanism in tissue is Compton scattering. If one of the annihilation gammas is scattered at the source and is detected by the positron camera, a displaced line vectors constituted. At 511 Kev, scattering is predominantly in the forward direction. A 15% energy loss (which ordinary would be accepted by a scintillation camera pulse-height window) corresponds to scattering through 35 degrees, fig. (46). Thus most scattered events which are accepted by MWPC which does not have pulse height window would also be accepted by the scintillation positron cameras. The effects of scattering material in the path of the annihilation photons have been investigated. While the total count rate drops exponentially, as expected, the rate of on-target to total count rate drops more gradually, fig. (47). Using Cu-64 line source, spatial resolution of 6 mm and 7 mm FWHM are found for sources imbedded in 1.25 cm and 10 cm of lucite, respectively, Fig. (48). Thus, external scattering has relatively small effect on both signal-to - noise and spatial resolution, and the lack of energy resolution in the MWPC's positron camera does not significantly alter the performance of the

Fig. 46 Percentage energy change of incident gamma as a function of scattering angle.

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Fig. 47 Total count rate and on-target/total count-rate as functions of absorber thickness in the path of the annihilation gammas.

50 cm Full Scale
(a)

mm/bi:
(b)

Fig. 48 A.

50 cm Full Scale
(a)

mm/bi:
(b)

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Fig. 48 B

Fig. 48 A. Cu-64 line source in 1.25 cm of lucite, showing a spatial resolution of 6 mm FWHM.

B. Same source in 10 cm lucite. The resolution is 7 mm FWHM.

system.

5.B5. Imaging results

Test object imaging with Cu-64

Cu-64 ($T_{1/2} = 12.8$ hrs.) was readily obtained by irradiation a copper wire with thermal neutrons. A piece of #20 gauge copper wire, 25 cm long, was irradiated in The Berkeley Research Reactor (Triga Mark III) at 300 kw power for 30 min. It was then cut to form a ring source and two parallel line source. The total activity per object was approximately 20-50 μ Ci. Fig.(49) shows the reconstructed image of the 1.5 cm diameter Cu-64 ring source. The image of two line sources separated by 1 cm is shown in fig.(49).

Point source imaging with Cu-64

In order to show both the axial and tomographic reconstructions where focal planes are perpendicular and parallel to the detector planes respectively, two Cu-64 point sources separated by 10 cm were imaged. Fig.(50A) shows the axial reconstructed images on the source plane and planes on both side of the object separated by 1cm. Fig.(50B) shows the tomographic reconstruction images on planes of the sources and on a plane half-way between them.

Head phantom imaging with Ga-68

Ga-68 was used for the following head phantom studies. Ga-68 was obtained by 0.05 m EDTA elution from Ge-68 complexed to a cation exchange resin in a column. Ga-68 has an ideal half life ($T_{1/2} = 1.13$ hr) for brain imaging, and the long $T_{1/2}$ of parent Ge-68 (275 days) makes it a very useful imaging radioisotope. The eluted Ga-68 is

10 cm Full Scale

Fig. 49a. 1.5 cm diameter Cu-64 ring source.

50 cm Full Scale

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Fig. 49b. Two Cu-64 line sources separated by 1 cm.

Fig. 50A.
AXIAL RECONSTRUCTION, TWO POINT SOURCES

TOMOGRAPHIC RECONSTRUCTION

Fig. 50B

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Fig. 50A. Axial reconstruction of point sources
B. Tomographic reconstruction of point sources.

complexed to 0.1 M DTPA (Diethylene triamine penta-acetic acid) in 0.9% saline by incubating at 60^o C for 10 min. Ga-68 DTPA was chosen on the basis of the excellent track record of Tc-99m DTPA for brain imaging. A simple hollow head-phantom with a simulated tumor was filled with 30 μ Ci of Ga-68. The tumor region was labelled with an activity concentration five times greater than that in the remainder of the phantom. (5:1 activity concentration). The lesion is clearly visible in fig.(51). A corroborative test was also performed with a cylinder head-phantom with two simulated lesions. One lesion had activity concentration of 5:1 and the other, 10:1 . The head-phantom was immersed in a container filled with Ga-68 (concentration 5:1), simulation the peripheral activity around the skull. Fig. (52) shows the reconstructed images, one on focal plane of lesions (left), and 3 cm off focal plane of lesions (right).

Animal studies with F-18 and Ga-68

Bone imaging of a dog was performed using F-18 (e^+ (97%), EC(3%), $E_C^+ = 630$ Kev) which is a bone seeking pharmaceutical and has a half of 110 min. We injected 800 μ Ci of F-18 to the above animal in the form of fluoride ion in saline solution. The data was taken 2 hrs. after injection. Fig.(53) shows the processed image of a dog's head and shoulder area of a series of focal planes 2 cm apart. The tomographic images show distinctly the mandible in focus at 2 cm, elbows at 6 cm, calverreal cavity and humeral heads at 8 cm, and cervicle spine at 10 cm. A total of 300K events were used. Fig.(54) shows pictures converging the pelves region of a dog

XBB 756-4148

Fig. 51. Head phantom with 30 uCi of Ga-68 (left) Tumor to background tracer ratio of 10:1 (right) Same phantom with 5:1 tumor ratio.

Fig. 52. Ga-68 cylinder type phantom with two lesions.

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XBB 748-5311

Fig. 53. F-18 bone scan of a dog hours after injection of 800 uCi.
Images were reconstructed on planes separated by 2 cm.

XBB 767-6032

Fig. 54. Dog's Pelvis Region Labelled With Ga-68 EDTMP

labelled with Ga-68 EDTMP(Ethylene-Diamine-Tetramonophosphonic Acid).

The tomographic images were displayed at 1 cm apart. A total of 150K coincident events were collected.

Brain imaging with Ga-68 DTPA

The patient was injected with 1mCi of Ga-68 DTPA 2 hrs. prior to imaging. 100-200K coincidence events were accumulated over approximately 30 min. The coronal images were reconstructed in a 64 x64 matrix array as shown in fig.(55). Distances are given from the posterior aspect of the head. Note the tumor on the patient's right side. At the 5cm plane, the doughnut shape of lesion becomes apparent. On the lower left, conventional brain scintiphoto imaged with a Pho/Gamma IV camera 1 to 2 hours after the administration of 10-15 mCi of Tc-99m DTPA. The central necrosis which is evident in 5.0 cm plane, is not apparent in the conventional scan. On the right, an EMI scan following intravenous injection of 300 ml of Conray 60 shows the doughnut shape of the lesion. Ten more patients with known brain lesions were studied after obtaining informed consent. In eight of the ten positron brain studies, the lesions were clearly identified. The lesions in the remaining two were vaguely visible. From the above results, the quality of the tomography is evident by the discreetly focused image planes.

XBB 756-4145

Fig. 55. Brain imaging with Ga-68 DTPA: Coronal reconstruction of the head of a patient injected with 1mCi of Ga-68 DTPA 2 hours prior to imaging. Distances are given from the posterior aspect of the head.

VI. DISCUSSION AND CONCLUSION

6.1. Limitations and Improvements

In this chapter, we shall consider the limitations imposed on the present camera systems and future improvements. The detection efficiency of the present positron camera system can be doubled with the addition of two more gamma converters in each detector. The converters under construction have 2.5 mm cell size and lead wall height of 10 mm. The projected system detection efficiency per detector would be 9-10%. The sensitivity of the camera would be increased four-fold to 4-5K counts/min/ μ Ci, estimated from eq. (2.2b). The improvement in sensitivity would reduce the study time per patient from approximately 60 min. to 15 min. for a brain scan. The main disadvantage of having two converters per chamber is the degradation of spatial resolution. A photon can interact with either one of the two converters; the uncertainty of gamma localization introduced would be proportional to the separation between the converters and inversely proportional to the tangent of the inclined angle. An average spatial resolution degradation of 1 mm is estimated using computer simulation for a line source at mid-plane.

The gamma converter efficiency was shown to be directly proportional to the converter cell size, eq. (2.9). For our gamma converters, the cell size is limited by the diminishing extraction efficiency with small cell dimension. This can be seen from the rapid fall off of the intersection probability, fig. (18), and the lower extraction efficiency, Table (2), with decreasing cell size. The construction of small cell

size converter is also a problem using the present corrugation technique. The height of the converter is, however, limited when the drift time spread is taken into consideration. The increase of time spread with height limits the practical height of a gamma converter to 1-2 cm. To overcome both problems of construction and extraction efficiency, other converter design schemes utilizing high surface resistivity lead glass tubing as the conversion medium are under investigation. A more uniform drift field and higher electron collection efficiency can be obtainable with a converter made of lead glass tubing having a suitable surface conductivity ($10^8 - 10^{12}$ ohms/square). Calculations and some preliminary measurements have been made for lead glass of various percentage compositions of PbO and different wall thickness.⁵⁰ The detection efficiency increases with higher concentration of PbO and compares favorably with pure lead converter of comparable cell size. The cell size limitation, in the case of lead glass tubing, is dependent on the minimum energy threshold of the detection probability. The level of this threshold is determined by the noise of the detection system. As the size of a cell is decreased, the average track length of a conversion electron becomes shorter, and a smaller number of ionization electrons can be produced. As an example, the detection probability of various cell size is calculated from the track-length-distribution, fig. (19), for a minimum detection threshold equivalent of 5 ion pairs, Table (3). The detection probability decreases rapidly as we go below cell size of 1 mm. The cell size can be reduced further if we utilize microchannel plates which can be made of

lead-bismuth oxide glass with hole diameter between 20-200 microns.⁵¹ Signal detection, in this case, is effected by the detection of secondary electron emissions from the wall surface initiated by conversion electrons. In vacuum ($< 10^{-5}$ torr), electron multiplication of 10^6 are obtainable. The cascade electrons, in this case, are collected by a set of cross-grid wires placed over the converter, and the charge from each event is shared by a pair of orthogonal wires. Coordinate readout can be accomplished by the conventional delay line method. The advantages of microchannel plates are twofold. First, the large increase of surface area per unit area improves gamma converter efficiency significantly. Secondly, the spread in electron collection time is less than 1 nanosecond. The combined effects could make it quite possible to operate a positron camera, equipped with these microchannel plates converters, at a considerably higher event rate than the present camera. The technology of making large size microchannel plates economically is not available presently. However, further developments towards better and bigger microchannel plates is certain to continue in both industrial and academic research laboratories.

Table (3)

Glass tube dia. (2a)	$k = R(\min)/2a$ ⁺	Detection Prob. [*]
1 mm	0.52	73%
2 mm	0.26	87%
3 mm	0.17	91%

* Calculated from the track-length-distribution

+ k is the ratio of the minimum track length to glass tube diameter, calculated for a uniform energy loss of 130 ev per mm.

Next, we shall consider the improvements in spatial resolution. From eq.(5.3), it can be seen that the spatial resolution is predominantly dependent on the cell size. With diminishing cell size, the timing electronics become important for good spatial accuracy. A new version of timing discriminator has been designed. The time skew was reduced to < 200 psec for a dynamic range of 20:1. The time jitter has been improved to a few nsec. (2-3 nsec) for a signal to noise ratio as low as 3:1. These electronic improvements along with small cell size gamma converter, should give a significant improvement in the intrinsic spatial resolution of the positron camera system. The delay lines used presently have a delay of 2.4 nsec/mm. Slower delay lines which give a delay of 4nsec/mm have been constructed, and will be installed. From eq.(5.3), it can be seen that the position uncertainty contributed by the timing discriminator is reduced using slower delay lines, and the overall spatial resolution would be improved. These slower delay lines have delay to rise time ratio of 20:1 corresponds to a rise time of 110 nsec for a delay line 55 cm long. Consequently, there is no degradation to the rise time of the chamber pulses. With the improved electronics and slower delay lines, a spatial resolution of 4-5 mm FWHM at mid-plane is expected. The disadvantage of a slower delay line is the introduction of a longer dead time. The system's dead time would be extended to approximately 3.2 usec, and the maximum coincident event rate is estimated to be 1.25 K counts/sec.

In order to improve the ratio of on-target events to off-target events, we should focus our attention on the following: (1) accidental coincidence (2) internal detector scattering (3) external source scattering. The accidental coincidence can be minimized if the coincident resolving time is made small and a pure positron emitter is used. The advantage of using microchannel plates operated in vacuum has been mentioned above. It should also be noted that for a given event rate the accidental coincidence rate is lower for a camera with higher detection efficiency. The internal detector scattering comes primarily from the gamma converter itself. The only viable solution to this problem is to restrict its wall thickness to the optimum value and the use of multiple events rejection circuitry. External source scattering, however, can be minimized by shielding and it is also possible to remove external Compton scattering with deconvolution filtering in the digital processing as proposed by G. Chu.

6.2. Conclusion

A large area, positron camera using MMPC-gamma converter hybrid detectors has been constructed and tested. The system performance has been measured and some clinical results have been obtained. The tomographic capability of the positron camera was demonstrated. The low injected dose (1-2 mCi) required and the short half life of the positron emitters result in a 50% reduction in radiation exposure compared to a convention scan using 10-15 mCi Tc-99m. A data set is accumulated from a single positioning of the patient, saving repositioning time. The present camera operates at a maximum count rate of 24K counts/min with approximately 10% accidental coincidence and 35% scattering coincidence. The additions of two more converters per detector (a total of 8 gamma converters in the system) will approximately quadruple sensitivity. The replacement of improved timing electronics and slower delay lines is expected to lower the spatial resolution from 6-7 mm FWHM at mid-plane to 4-5 mm FWHM. Further improvements in detection efficiency and spatial resolution can be realized with lead glass converters and microchannel plates. A true 3-D reconstruction technique will improve the quality of the images by removing superimposed "backgrounds". Further refinement of images is possible with deconvolution of source scattering in digital processing.

Appendix (1)

A. Variation of solid angle with source position

The solid angle subtended by a point source and the detector varies with the position of the source. Consider a point source at the origin $(0, 0, 0)$, and the detector plane at $z = z_0$, as shown in the figure.

$$d\Omega = dA_{\perp} / r^2 = \frac{dx dy}{(x^2 + y^2 + z_0^2)^{3/2}} \times \frac{z_0}{(x^2 + y^2 + z_0^2)^{1/2}}$$

$$\Omega = \int_{y_1}^{y_2} \int_{x_1}^{x_2} \frac{dx dy}{(x^2 + y^2 + z_0^2)^{3/2}} \quad (1)$$

Integrating over y

$$y = (z_0^2 + x^2)^{1/2} \tan \theta = a \tan \theta$$

$$dy = (z_0^2 + x^2)^{1/2} \sec^2 \theta d\theta = a \sec^2 \theta d\theta$$

$$\Omega = z_0 \int_{x_1}^{x_2} dx \int_{y_1}^{y_2} a \sec^2 \theta d\theta / (a^2 + a^2 \tan^2 \theta)^{3/2}$$

$$= z_0 \left\{ \int_{x_1}^{x_2} \frac{dx}{(z_0^2 + x^2)^{3/2}} \int_{y_1}^{y_2} \frac{dy}{(z_0^2 + y^2 + x^2)^{1/2}} - y_1 \int_{x_1}^{x_2} \frac{dx}{(z_0^2 + x^2)^{3/2}} \int_{y_1}^{y_2} \frac{dy}{(z_0^2 + y^2 + x^2)^{1/2}} \right\}$$

Integrating over x

$$x = (z_0^2 + y^2)^{1/2} \tan \theta$$

$$\Omega = \left[\tan^{-1} \frac{y_2 x_2}{z_0 (x_2^2 + y_2^2 + z_0^2)^{1/2}} - \tan^{-1} \frac{y_2 x_1}{z_0 (x_1^2 + y_2^2 + z_0^2)^{1/2}} \right] \\ - \left[\tan^{-1} \frac{y_1 x_2}{z_0 (x_2^2 + y_1^2 + z_0^2)^{1/2}} - \tan^{-1} \frac{y_1 x_1}{z_0 (x_1^2 + y_1^2 + z_0^2)^{1/2}} \right] \quad (2)$$

For $y_2 = -y_1 = y_0$; $x_2 = -x_1 = x_0$

$$= 4 \tan^{-1} \left[\frac{x_0 y_0}{z_0 (x_0^2 + y_0^2 + z_0^2)^{1/2}} \right] \quad (3)$$

In the general case where a source is not situated at the center line of the detector, i.e. $y_2 \neq -y_1$ and $x_1 \neq x_2$, then eq. (2) has to be evaluated.

B. Variation of detector efficiency with source position

The probability of a gamma interaction with the detector is, in general, an exponential function of the length of the detector material traversed by the gamma ray. This track length is a function of the inclined angle of the incident gamma. The variation of average track length, $\bar{t}$, is calculated as a function of the source position, and the expression for detector efficiency is derived.

The length of a track traversed by a gamma ray is $t = h \sec \theta$, where h is the height of the detector material and θ is the polar angle as shown in the figure.

The average track length is given by

$$\begin{aligned}
 \bar{t} &= \int_{\Omega} t \, d\Omega / 2\pi \\
 &= \iint \frac{h \sec \theta}{2\pi} \times \frac{dx dy}{r^2} \times \cos \theta \\
 &= \frac{h}{2\pi} \iint \frac{dx dy}{(x^2 + y^2 + H^2)} \quad (4)
 \end{aligned}$$

This integral can be evaluated using the method presented in part (A), and will not be repeated here. The interaction probability of the incident gammas with the detector material, p_{int} , is a function of the track length, the cross-section and the density of the material.

$$p_{int} = \left[1 - \exp(-\bar{t} / L) \right] \quad (5)$$

L = mean free path of the gamma ray in the material

The detector efficiency which is defined as the probability of detecting an incident gamma is equal to the interaction probability, and the angular dependence can be evaluated for a given L .

Appendix (2)

To calculate the geometric factor G

The converter can be considered as two sets of infinitely long parallel walls running in the $\hat{x}$ and $\hat{y}$ directions. The height of the walls is h . A point source is located at a distance H_1 above the top of the converter and emits γ -rays in all directions. For the γ -ray photon shown in the diagram, let the coordinates of the points of intercept with the $z=0$ and $z=h$ planes be $(x_0, y_0, 0)$ and (X'_0, Y'_0, h) respectively.

$$X'_0 = x_0 H_1 / H, \quad Y'_0 = y_0 H_1 / H$$

The number of walls parallel to $\hat{x}$ -direction intercepted by the γ -ray path is equal to $m_x(\theta) =$ number of $\hat{x}$ parallel walls between y_0 and

y_0 - number of $\hat{x}$ parallel walls between Y'_0

$$\text{and } Y_0 \\ = \text{INT} \left(\frac{Y_\theta - Y_0}{W} \right) - \text{INT} \left(\frac{\frac{H_1}{H} Y_\theta - Y_0}{W} \right)$$

Similarly, the number of walls parallel to $\hat{Y}$ -direction intercepted by the γ ray path is

$$m_y(\theta) = \text{INT} \left(\frac{X_\theta - X_0}{W} \right) - \text{INT} \left(\frac{\frac{H_1}{H} X_\theta - X_0}{W} \right)$$

Then the total thickness t_θ traversed by the photon is

$$t_\theta = m_x(\theta) t(\sec \theta_1) + m_y(\theta) t(\sec \theta_2)$$

(we neglect the double counting in the regions common to both sets of walls)

From the geometry it can be seen

$$\sec \theta_1 = \frac{\sqrt{X_\theta^2 + Y_\theta^2 + H^2}}{|Y_\theta|}, \quad \sec \theta_2 = \frac{\sqrt{X_\theta^2 + Y_\theta^2 + H^2}}{|X_\theta|}$$

$$\text{Hence } t_\theta = t \sqrt{X_\theta^2 + Y_\theta^2 + H^2} \left\{ \frac{\text{INT} \left(\frac{Y_\theta - Y_0}{W} \right) - \text{INT} \left(\frac{\frac{H_1}{H} Y_\theta - Y_0}{W} \right)}{|Y_\theta|} \right. \\ \left. + \frac{\text{INT} \left(\frac{X_\theta - X_0}{W} \right) - \text{INT} \left(\frac{\frac{H_1}{H} X_\theta - X_0}{W} \right)}{|X_\theta|} \right\}$$

The number of i^{th} type electrons produced in the converter is

$$n_o = \int_0^{t_\theta} \sigma_i \rho e^{-\sigma_T \rho s} ds$$

$$= \frac{1 - e^{-\sigma_T \rho t_\theta}}{\sigma_T \rho} \sigma_i \rho$$

Hence the average number of i^{th} type electrons, Q_i produced by a γ ray photon emitted towards the converter within a solid angle Ω is given by

$$Q_i = \frac{\int_{\Omega} n_o d\Omega}{\int_{\Omega} d\Omega}$$

$$= \frac{\int_{\Omega} \left(\frac{1 - e^{-\sigma_T \rho t_\theta}}{\sigma_T \rho t} \right) d\Omega}{\int_{\Omega} d\Omega} \sigma_i \rho t$$

Comparing the expression for Q_i to the definition

$$Q_i = G \sigma_i \rho t$$

we can identify the quantity

$$\frac{\int_{\Omega} \left(\frac{1 - e^{-\sigma_T \rho t_\theta}}{\sigma_T \rho t} \right) d\Omega}{\int_{\Omega} d\Omega}$$

as the geometry factor G , which is independent of the type of electrons.

Appendix (3)

Solution for Extraction Electric Field

In order to extract ionization electrons from inside the cells of the gamma converter, an electric field is applied across the converter. Consider a cylinder with geometric parameters as specified in the figure. The conductive wall is separated by an insulating gap; the potential and the electric field inside the cylinder are calculated in the following.

In the regions of space where there is no charge density, the scalar potential satisfies Laplace's equation

$$\nabla^2 \phi = 0$$

In cylindrical coordinate

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad \dots\dots\dots (1)$$

Using separation of Variables

Let $\phi(r, z) = R(r) Z(z)$

$$\text{Then } \frac{1}{Z} \frac{\partial^2 Z(z)}{\partial z^2} = -\nu^2$$

Let $x = \sqrt{r}$

$$\frac{\partial^2 R(x)}{\partial x^2} + \frac{1}{x} \frac{\partial R(x)}{\partial x} - R(x) = 0 \dots\dots\dots(2)$$

Solution for eq. (2) is given by

$$\phi(r, z) = A_0 z + \sum_v (A_v \sin v z + B_v \cos v z) I_0(x)$$

Since $\phi(r, z)$ is an odd function in the z -direction,

$$\text{Therefore } \phi(r, z) = A_0 z + \sum_{v=1}^{\infty} A_v \sin(n\pi z/H) I_0(\sqrt{r})$$

$$v = n\pi/H$$

$$\phi(r, z) = V_0 z/H + \sum_{n=1}^{\infty} A_n \sin(n\pi z/H) I_0(n\pi r/H)$$

Now we have to evaluate A_n

$$f(z) = \sum_{n=1}^{\infty} A_n \sin(n\pi z/H) I_0(n\pi r/H)$$

$$\begin{aligned} \int_{-H}^H f(z) \sin(n\pi z/H) dz &= A_n I_0(n\pi R/H) \int_{-H}^H \sin^2(n\pi z/H) dz \\ &= \frac{1}{2} A_n H I_0(n\pi R/H) \end{aligned}$$

$$\text{Therefore } A_n = \frac{2}{H I_0(n\pi R/H)} \int_0^H f(z) \sin(n\pi z/H) dz$$

$$\begin{aligned}
&= \frac{2}{H I_0 (n\pi R/H)} \left[\int_0^S dz V_0 \left(1 - \frac{S}{H} \right) z/s \sin \left(\frac{n\pi z}{H} \right) \right. \\
&\quad \left. + \int_S^H dz V_0 \left(1 - \frac{z}{H} \right) \sin (n\pi z / H) \right] \\
&= \frac{2 V_0}{H I_0 (n\pi R/H)} \left[\left(\frac{1}{S} - \frac{1}{H} \right) \int_0^S z \sin (n\pi z / H) dz \right. \\
&\quad \left. + \int_S^H \sin (n\pi z/H) dz - (1/H) \int_S^H z \sin (n\pi z/H) dz \right] \\
&\int_0^S z \sin (n\pi z/H) dz = (H/n\pi)^2 \int_{\omega=0}^{n\pi S/H} \omega \sin \omega d\omega = (H/n\pi)^2 \left\{ -\omega \cos \omega \right\}_0^{n\pi S/H} \\
&\quad + \int_{\omega=0}^{n\pi S/H} \cos \omega d\omega \left\{ \right.
\end{aligned}$$

Using the above relationships and evaluating the integrals,

$$A_n = \frac{2 V_0 H/S}{I_0 (n\pi R/H)} \times \frac{\sin (n\pi S/H)}{(n\pi)^2}$$

$$\begin{aligned}
\phi(r, z) = V_0 z / H + 2 V_0 H / (\pi^2 S) \sum_{n=1}^{\infty} \left\{ \frac{\sin (n\pi S/H)}{n^2 I_0 (n\pi R/H)} \times \right. \\
\left. \sin (n\pi z/H) I_0 (n\pi r/H) \right\}
\end{aligned}$$

$$\text{Now } E = -\nabla \phi = -\frac{\partial \phi}{\partial r} \vec{r} - \frac{\partial \phi}{\partial z} \vec{z}$$

$$\vec{E}r = -\frac{2 V_0}{S} \sum_{n=1}^{\infty} \frac{\sin (n\pi S/H)}{n I_0 (n\pi R/H)} I_0' (n\pi r/H) \sin (n\pi z/H)$$

$$\vec{E}z = -V_0 / H - \frac{2 V_0}{\pi S} \sum_{n=1}^{\infty} \frac{\sin (n\pi S/H)}{n I_0 (n\pi R/H)} \times I_0 (n\pi r/H) \cos (n\pi z/H)$$

Appendix (4)

Intersection probability

Conversion electrons, which are released as a result of the interaction of gamma radiation with the wall, enter the cell space producing ionization in the gas. These ionization tracks have various length depending on their orientation. Consider two infinite long concentric cylinders with inner radius $\bar{r}$, and outer radius R as shown in the figure.

Field lines are continuous in the volume within $\bar{r}$, and discontinuous between $\bar{r}$ and R . An ionization track has to intersect the inner cylinder in order to produce any extractable ionization electrons. The intersection probability as a function of $\bar{r}/R$ is derived in the following. The projections of the intersection of an ionization track with the inner cylinder along the x -axis is given by

$$r_x = R \tan \theta \cos \phi \leq |\vec{r}|$$

$$r_x = R (1 - \mu^2)^{\frac{1}{2}} \cos \phi / \mu, \quad \mu = \cos \theta$$

$$\lim_{\phi \rightarrow 0} \phi_0 = \cos^{-1} \left[\bar{r} \mu / R (1 - \mu^2)^{1/2} \right]$$

Now consider the limit of θ

$$\lim_{\theta \rightarrow 0} \theta_0 = \tan^{-1} (\bar{r} / R)$$

Therefore

$$\mu_0 = \cos \theta_0 = \left[1 + (\bar{r} / R)^2 \right]^{-1/2}$$

Now the intersection probability $P_{\text{int}}(\mu, \phi)$ is given by

$$\begin{aligned} P_{\text{int}}(\mu, \phi) &= 4 \int_0^{\mu_0} \int_{\phi_0}^{\pi/2} \left(\frac{\mu}{\pi} \right) d\mu d\phi \\ &= \frac{4}{\pi} \int_0^{\mu_0} \mu d\mu \left\{ \frac{\pi}{2} - \cos^{-1} \left[\bar{r} \mu / R (1 - \mu^2)^{1/2} \right] \right\} \\ &= 1 - \frac{4}{\pi} \int_0^{\mu_0} \mu \cos^{-1} \left[\bar{r} \mu / R (1 - \mu^2)^{1/2} \right] d\mu \end{aligned}$$

Appendix (5)

Input impedance for "cooled" damping

In order to reduce the noise generated by a dissipative element at the input of an amplifier, a circuit was designed to produce a non-dissipative input impedance. Such an amplifier used for delay line signal readout maintains a proper termination, but with a reduction in the noise level. A simplified circuit used to illustrate the principle is shown in the figure. Two elements, R_o , C_o , are added to the forward gain path of a charge-sensitive amplifier with feedback arrangement.

Using the parameters given in the figure,

$$Z_F = R_F / (1 + j\omega R_F C_F)$$

$$Z_O = R_O / (1 + j\omega R_O C_O)$$

$$v_{in} - v_o = i_f Z_F \dots\dots\dots(1)$$

$$(i_o + i_f) Z_O = v_o \dots\dots\dots(2)$$

$$i_o = -g_m v_i \dots\dots\dots(3)$$

From eq. (2)

$$-g_m v_i Z_O + i_f Z_O = v_o \dots\dots\dots(4)$$

From eq. (2) & eq. (4)

$$v_{in} (1 + g_m Z_0) = i_f (Z_0 + Z_f)$$

$$Z_{in} = v_{in} / i_f = (Z_0 + Z_f) / (1 + g_m Z_0)$$

Therefore, the input impedance is given by

$$Z_{in} = \frac{R_0 / (1 + j\omega R_0 C_0) + R_f / (1 + j\omega R_f C_f)}{1 + g_m R_0 / (1 + j\omega R_0 C_0)}$$

Consider $R_0, R_f \gg 1$

$$\begin{aligned} Z_{in} &\approx \frac{2 R_0 + j\omega R_0^2 C_f + j\omega R_0^2 C_0}{(j\omega R_0 C_0 + g_m R_0) (j\omega R_0 C_f)} \\ &\approx \frac{(R_0 + R_f) + j\omega R_0 R_f (C_f + C_0)}{(j\omega R_0 C_0 + g_m R_0) (j\omega R_f C_f)} \end{aligned}$$

For a large transconductance, g_m , ($> 10^5$ mho)

$$Z_{in} \approx (C_0 + C_f) / g_m C_f$$

Appendix 6A. A schematic diagram of the timing discriminator.

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