MASTER

ACOUSTICALLY INDUCED VIBRATION OF CIRCULAR CYLINDRICAL RODS

by

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BASE TECHNOLOGY

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ABSTRACT

An analysis is presented for the steady-state response of a circular cylindrical rod subjected to a plane wave; rod displacement natural frequency including fluid inertia effect, and radiation damping, are obtained in closed form. It is shown that the added mass coefficient depends only on the nondimensional wave number α and the radiation damping ζ_r depends on α and mass ratio β . The rod displacement is proportional to wave amplitude and rod radius, and depends on α and β .

I. INTRODUCTION

The essential structural components of a Liquid Metal Fast Breeder Reactor (LMFBR) are designed for a life span of about 30 years. Acoustically induced vibration can be a problem if the components are subjected to a high intensity sound. Some reactor components such as heat exchanger tubes are long, slender members, arranged in bundles and immersed in a flowing fluid. Therefore, they are susceptible to acoustically induced vibration. The excitation may be associated with the pulsation of the pump, pressure fluctuations in the flow, or sodium-water reaction in steam generators.

Understanding and modeling of sound/structure interaction in tube bundles is one of the basic requirements in the development of the methods of analysis and guidelines for designing components that are free from unstable motion. As a first step toward this requirement, in this paper, a circular tube subjected to plane waves is studied. The basic knowledge for a single tube will provide some insights into the general characteristics of more general cases of multiple tubes.

The scattering and radiation by circular cylinders have been considered extensively; those include single cylinder [1], two parallel cylinders [2], and multiple cylinders [3]. The responses of circular cylindrical structures to sound excitation also have been studied. Hine [4,5] studies the bending vibration of slender rods excited by an acoustic wave parallel to the rod axis in a cylindrical duct. Geers [6] presents an analytical study of a plane acoustic step-wave impinging on an infinite circular cylindrical shell. Baley and Fahy [7,8] study the response of a circular beam to acoustic excitation; they obtained the mean square velocity of structures. However, it appears that an exact solution for a circular cylinder to an acoustic excitation has not been reported.

II. ANALYSIS

Consider a uniform rod of radius a and mass per unit length m submerged in a fluid. The purpose is to find the response of the rod subjected to a plane wave traveling in x -direction as shown in Figure 1. The equation of motion of the rod is

$$m \frac{\partial^2 x}{\partial t^2} + \bar{c} \frac{\partial x}{\partial t} + \bar{k} x = P(t), \quad (1)$$

where $\bar{c}$ is the structural damping coefficient, $\bar{k}$ is the spring constant, t is the time, and P is the resultant force acting on the rod. P is obtained by

$$P(t) = - \int_0^{2\pi} p a \cos \theta d\theta, \quad (2)$$

where p is the fluid pressure on the rod surface. The incident plane wave is

$$p_i = p_o e^{ik(r \cos \theta - ct)}, \quad (3)$$

where p_o is the amplitude, k is the wave number which is related to circular frequency ω and wave speed c by $k = \omega/c$. Equation (3) can be written in terms of cylindrical waves:

$$p_i = p_o \sum_{m=0}^{\infty} \epsilon_m (i)^m J_m(kr) \cos(m\theta) e^{-i\omega t}, \quad (4)$$

and

$$\epsilon_m = \begin{cases} 1, & m = 0 \\ 2, & m \neq 0 \end{cases}.$$

Here $i = \sqrt{-1}$, and $J_m(kr)$ is Bessel function of the first kind with the order m . The radial velocity u_i corresponding to p_i is

$$u_i = \left(\frac{p_0}{\rho c}\right) \{i J_1(kr) + \sum_{m=1}^{\infty} i^{m+1} [J_{m+1}(kr) - J_{m-1}(kr)] \cos(m\theta)\} e^{-i\omega t}, \quad (5)$$

where ρ is fluid density. Since the rod is submersed in an infinite medium, there exists an out-going wave p_r ;

$$p_r = \sum_{m=0}^{\infty} A_m H_m^{(1)}(kr) \cos(m\theta) e^{-i\omega t}, \quad (6)$$

where $H_m^{(1)}(kr)$ is Hankel function of the first kind with order m and A_m is the coefficient to be determined. The radial velocity u_r corresponding to p_r is

$$u_r = \left(\frac{1}{\rho c}\right) \{A_0 H_1^{(1)}(kr) + \sum_{m=1}^{\infty} \frac{A_m}{2} [H_{m+1}^{(1)}(kr) - H_{m-1}^{(1)}(kr)] \cos(m\theta)\} e^{-i\omega t}. \quad (7)$$

The total field intensity is given by the sum of incident wave, Eq. (4) and the scattered and radiated wave, Eq. (6); i.e.,

$$p = p_i + p_r = [p_0 \sum_{m=0}^{\infty} \epsilon_m i^m J_m(kr) + \sum_{m=0}^{\infty} A_m H_m^{(1)}(kr)] \cos(m\theta) e^{-i\omega t}. \quad (8)$$

Equation (8) must satisfy the boundary condition imposed by the structure, i.e., the radial fluid velocity at the interface of fluid and solid must equal the normal velocity of the structure and $P(t)$ is equal to the resultant fluid pressure. These boundary conditions can be stated mathematically as

$$(u_i + u_r) \Big|_{r=a} = \frac{\partial x}{\partial t} \cos\theta, \quad (9)$$

and

$$P(t) = -\int_0^{2\pi} (p_i + p_r) \Big|_{r=a} a \cos\theta \, d\theta . \quad (10)$$

Substituting Eqs. (5) and (7) into (9) yields

$$\begin{aligned} & \{i p_o J_1(ka) + \phi_o \sum_{m=1}^{\infty} i^{m+1} [J_{m+1}(ka) - J_{m-1}(ka)] \cos(m\theta) + i A_o H_1^{(1)}(ka) \\ & + i \sum_{m=1}^{\infty} \frac{A_m}{2} [H_{m+1}^{(1)}(ka) - H_{m-1}^{(1)}(ka)] \cos(m\theta)\} \left(\frac{1}{\rho c}\right) e^{-i\omega t} = \frac{\partial x}{\partial t} \cos\theta . \end{aligned} \quad (11)$$

The steady-state solution of the rod displacement can be expressed as

$$x = \bar{x} e^{-i\omega t} . \quad (12)$$

Substituting Eq. (12) into (1) yields

$$(-\omega_m^2 - i\omega \bar{c} + \bar{k}) \bar{x} e^{-i\omega t} = -\int_0^{2\pi} (p_i + p_r) \Big|_{r=a} a \cos\theta \, d\theta . \quad (13)$$

Substituting Eq. (8) into (13) gives

$$\bar{x} = \frac{-i 2\pi a p_o J_1(ka) - A_1 \pi a H_1^{(1)}(ka)}{\bar{k} - i\omega \bar{c} - \omega_m^2} . \quad (14)$$

The coefficient A_1 in Eq. (14) can be determined by multiplying both sides of Eq. (11) by $\cos\theta$ and then integrating from 0 to 2π ; thus

$$A_1 = -2 \frac{i p_o [J_2(ka) - J_0(ka)] + \rho c \omega \bar{x}}{H_2^{(1)}(ka) - H_0^{(1)}(ka)} . \quad (15)$$

Note that Eq. (14) contains coefficient A_1 only. This is due to the fact that a single rod subjected to a plane wave can excite only one component of scattered and radiated wave; therefore, all the coefficients except A_1 are vanished. From Eqs. (14) and (15) it follows that

$$\left\{ \bar{k} - i\omega\bar{c} - \omega_m^2 - 2\pi a p c \omega \left[\frac{H_1^{(1)}(ka)}{H_2^{(1)}(ka) - H_0^{(1)}(ka)} \right] \right\} \bar{x} = i2\pi a p_o \left\{ -J_1(ka) + [J_2(ka) - J_0(ka)] \left[\frac{H_1^{(1)}(ka)}{H_2^{(1)}(ka) - H_0^{(1)}(ka)} \right] \right\}. \quad (16)$$

Since

$$\frac{H_1^{(1)}(ka)}{H_2^{(1)}(ka) - H_0^{(1)}(ka)} = \frac{J_1(ka) + i Y_1(ka)}{[J_2(ka) + i Y_2(ka)] - [J_0(ka) + i Y_0(ka)]} = \frac{\Delta_1 + i \Delta_2}{\Delta}, \quad (17)$$

where $Y_m(ka)$ is Bessel function of the second kind with order m , and

$$\begin{aligned} \alpha &= ka, \\ \Delta &= [J_2(\alpha) - J_0(\alpha)]^2 + [Y_2(\alpha) - Y_0(\alpha)]^2, \\ \Delta_1 &= J_1(\alpha)[J_2(\alpha) - J_0(\alpha)] + Y_1(\alpha)[Y_2(\alpha) - Y_0(\alpha)], \\ \Delta_2 &= Y_1(\alpha)[J_2(\alpha) - J_0(\alpha)] - J_1(\alpha)[Y_2(\alpha) - Y_0(\alpha)], \end{aligned} \quad (18)$$

Eq. (14) becomes

$$\bar{x} = \left(\frac{2\pi a p_o}{m' \omega_o^2} \right) \left\{ \frac{J_2(\alpha) - J_0(\alpha) \frac{\Delta_2}{\Delta} + i[J_1(\alpha) - (J_2(\alpha) - J_0(\alpha)) \frac{\Delta_1}{\Delta}]}{\left(\frac{\omega}{\omega_o} \right)^2 + i 2 \left(\frac{\omega}{\omega_o} \right) (\zeta + \zeta_r) - 1} \right\}, \quad (19)$$

where

$$m' = m + C_M(\alpha)M = \text{virtual mass,}$$

$$M = \rho \pi a^2 = \text{displaced mass of fluid,}$$

$$C_M(\alpha) = \frac{2}{\alpha} \frac{\Delta_1}{\Delta} = \text{added mass coefficient,}$$

$$\omega_o = \sqrt{\frac{k}{m'}} = \text{undamped natural frequency,} \quad (20)$$

$$\zeta = \frac{\bar{c}}{2 \omega_o m'} = \text{damping ratio,}$$

$$\zeta_r = \left(\frac{\omega}{\omega_o}\right) \left(\frac{\beta \Delta_2}{\alpha \Delta + 2\beta \Delta_1}\right) = \text{radiation damping,}$$

$$\beta = \frac{M}{m} = \text{mass ratio.}$$

Eqs. (12) and (19) provide the complete solution for the response of a circular cylindrical rod subjected to a plane wave excitation in an acoustic medium. Substituting Eq. (19) into (12) and taking the real part gives

$$x = B \cos(\omega t - \psi) , \quad (21)$$

where

$$B = \frac{A \left\{ \left[b \left(\frac{\omega^2}{\omega_o^2} - 1 \right) + 2d \eta \left(\frac{\omega}{\omega_o} \right) \right]^2 + \left[d \left(\frac{\omega^2}{\omega_o^2} - 1 \right) - 2b \eta \left(\frac{\omega}{\omega_o} \right) \right]^2 \right\}^{1/2}}{\left[\left(\frac{\omega}{\omega_o} \right)^2 - 1 \right]^2 + 4\eta^2 \left(\frac{\omega}{\omega_o} \right)^2} ,$$

$$\psi = \tan^{-1} \left[\frac{d \left(\frac{\omega^2}{\omega_o^2} - 1 \right) - 2b \eta \left(\frac{\omega}{\omega_o} \right)}{b \left(\frac{\omega^2}{\omega_o^2} - 1 \right) + 2d \eta \left(\frac{\omega}{\omega_o} \right)} \right] , \quad (22)$$

$$A = \frac{2\pi \rho a^2 \omega_o}{m \omega_o^2} , \quad d = J_1(\alpha) - [J_2(\alpha) - J_0(\alpha)] \frac{\Delta_1}{\Delta} ,$$

$$b = [J_2(\alpha) - J_0(\alpha)] \frac{\Delta_2}{\Delta} , \quad \eta = \zeta + \zeta_r .$$

Since

$$\omega_o = \sqrt{\frac{\bar{k}}{m'}} = (1 + \beta C_M)^{-1/2} \omega_1, \quad (23)$$

where

$$\omega_1 = \sqrt{\frac{\bar{k}}{m}},$$

the normalized amplitude in Eq. (21) becomes

$$\frac{Bm\omega_1^2}{2\pi a p_o} = \frac{\left\{ \left[b \left(\frac{\omega^2}{\omega_o^2} - 1 \right) + 2d\eta \left(\frac{\omega}{\omega_o} \right) \right]^2 + \left[d \left(\frac{\omega^2}{\omega_o^2} - 1 \right) - 2b\eta \left(\frac{\omega}{\omega_o} \right) \right]^2 \right\}^{1/2}}{\left[\left(\frac{\omega}{\omega_o} \right)^2 - 1 \right]^2 + 4\eta^2 \left(\frac{\omega}{\omega_o} \right)^2}. \quad (24)$$

At resonance, $\omega = \omega_o$, Eq. (24) reduces to

$$\frac{Bm\omega_1^2}{2\pi a p_o} = (1 + \beta C_M) \left(\frac{\alpha_o}{2\beta} \right) \left(\frac{\zeta_r}{\eta} \right) \Delta^{1/2}, \quad (25)$$

and

$$\psi = \tan^{-1} \left[\frac{J_2(\alpha) - J_0(\alpha)}{Y_2(\alpha) - Y_0(\alpha)} \right] - \frac{\pi}{2}.$$

Note that all variables ζ_r , C_M , and Δ appearing in Eq. (25) are evaluated at $\alpha = \alpha_o = \omega_o a/c$. Therefore, the right-hand side in Eq. (25) is a function of mass ratio β only.

III. NUMERICAL RESULTS AND DISCUSSIONS

The "added mass coefficient" is a function of α only, i.e., for a given system, C_M is dependent only on the wave number k . Figure 2 shows the variation of C_M versus α . Note that for $\alpha \rightarrow 0$, $C_M \rightarrow 1$, and $C_M \rightarrow 0$ when $\alpha \rightarrow \infty$. The maximum C_M occurs at $\alpha \approx 0.45$ and then exponentially decays to zero. For $\alpha > 4$, the fluid inertia effect on structural vibration is almost negligible. At resonance, the radiation damping is a function of α and β . Figure 3 shows the relationship between ζ_r and α for various values of β . It can be seen that $\zeta_r \rightarrow 0$ for both $\alpha \rightarrow 0$ and $\alpha \rightarrow \infty$. The peak value of ζ_r occurs at α slightly greater than 1. The larger the values of β , the bigger the values of ζ_r . This is understandable because the sound radiation is more prominent in the dense medium than in light-density medium. Therefore, in a lightly damped system, acoustic radiation can be an important factor of removing the energy from a vibratory system; it is also responsible for the eventual decay of free vibration and is of primary importance in controlling steady-state responses.

A normalized plot of amplitude B versus β at resonance is shown in Fig. 4 for $\alpha_0 = 1$ and various values of internal damping ζ . For small ζ , the response in dense medium is much smaller than in light fluid. Again, this is due to the energy dissipation associated with acoustic radiation. For larger ζ , the response does not have much difference whether it is in light or in dense fluid because ζ now is as important as ζ_r . It can also be seen from the figure that in a dense medium (for $\beta > 1.0$) the response does not depend much on the internal damping; thus radiation damping plays a major role under such circumstances. Figures 5 and 6 show the response for $\alpha_0 = 0.1$ and 0.01 respectively. It can be said that for a given system at

small wave number such as $\alpha_0 = 0.01$, the response does not depend on the density of the acoustic medium.

To illustrate the influence of frequency on the steady-state response, for a given system, let $\alpha_1 = (\omega_1/c)a$. Since

$$\alpha = \frac{\omega}{c} a = \alpha_1 \left(\frac{\omega}{\omega_1} \right), \quad (26)$$

and

$$\frac{\omega}{\omega_0} = (1 + \beta C_M)^{1/2} \left(\frac{\omega}{\omega_1} \right),$$

for given values of α_1 and β , we can find the frequency ratio (ω/ω_0) corresponding to various values of frequency ratio (ω/ω_1) in vacuo. A plot of normalized amplitude versus frequency ratio (ω/ω_0) is shown in Fig. 7 for $\alpha_1 = 0.01$ and $\beta = 1.0$. It is clear that at resonance, the response is smaller for larger values of structural damping, however, for the other values of frequency ratio far away from resonance, the response is not influenced by the structural damping. This phenomena is common for the structural response in vacuo.

V. CONCLUSIONS

An analysis is presented for the steady-state response of a circular cylindrical rod subjected to a plane wave; rod response amplitude, natural frequency including fluid inertia effect, and radiation damping, are obtained in closed form. Several general conclusions can be drawn from the results: (1) The added mass factor depends only on the nondimensional wave number α ; it is close to one at small α and becomes very small at larger values of α ; (2) Radiation damping ζ_r depends on α and mass ratio β ; for a given β , there is an α at which ζ_r is a maximum, and for a given α , ζ_r increases with increasing β ; (3) Rod displacement is proportional to wave amplitude and rod radius, and depends on α and β .

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Figure 1. Schematic of acoustically induced vibration of a rod.

Figure 2. Added mass coefficients.

Figure 3. Radiation damping at resonance.

Figure 4. Normalized amplitude versus β at resonance. ($\alpha_o = 1.0$)

Figure 5. Normalized amplitude versus β at resonance. ($\alpha_0 = 0.1$)

Figure 6. Normalized amplitude versus β at resonance. ($\alpha_0 = 0.01$)

Figure 7. Normalized amplitude versus frequency ratio. ($\alpha_1 = 0.01$, $\beta = 1.0$)