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Informal Report

EFFECTS OF RANDOM ERRORS IN COIL LOCATIONS IN A HIGH-FIELD
SUPERCONDUCTING ACCELERATOR

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N O T I C E

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Abstract

The effects due to magnetic field perturbations caused by random errors in the coil positions in the bending magnets of a high-field superconducting accelerator are investigated. An error in the position of a particular current block in a bending magnet with a cosine current distribution will introduce multipole terms in the median plane magnetic field. Consideration of the random dipole term and its effect on the central orbit leads to a tolerance on the rms error in the location of the current blocks of about 2.5×10^{-3} in. The effects of the random quadrupole term and the nonlinear terms are also considered.

I. Introduction

Errors in the positions of current-carrying coils of a bending magnet will produce perturbing magnetic fields in the median plane. These perturbing fields can move the central orbit, change the ν -value and, if the perturbing fields are nonlinear, they can excite harmful nonlinear resonances.

For the purpose of this study, the current distribution is assumed to be made up of about five current blocks in a quadrant, where in each current block, the current varies as the cosine of the azimuthal angle. This cosine bending magnet is shown in Fig. 1 for the case of four current blocks. Error fields will be considered which are due to a displacement of each of these current blocks as a whole. The current blocks are made up of many ribbons or conductors, but one may expect¹ that the more severe tolerances will be found when the current block moves as a whole.

The considerations given below lead to a rms tolerance on coil locations of about

$$\epsilon \leq 2.5 \times 10^{-3} \text{ in. ,}$$

for a case where the inside radius of the current distribution is $R_c = 1.5$ in.,

1. T.C. Randle, Rutherford Laboratory Report RHEL/R-197 (1970).

Figure 1

and the blocks are 5/8 in. thick. This tolerance results from considering the displacement of the central orbit, and would give rise to a dipole field error in each magnet of $\Delta B/B = 0.5 \times 10^{-3}$.

The results found here are based on analytic results for the error field due to a displacement of a current block. These analytic results may be in error because of the nonzero thickness of the blocks and other complicating geometric factors, and consequently the results given below need to be corrected by taking the exact geometry into account.

II. Analysis of the Field Errors

The displacement of a single block of current will produce a perturbing field in the median plane which can be broken down into field multipoles. All multipoles will be generated, and the perturbing field will have a radial component as well as a vertical component in the median plane. This analysis of the perturbing field produced by a displacement of a current block has been carried out by the writer; the details of the calculation were presented in a previous paper.² The main results of this calculation are listed below.

Let us write the perturbing field in the median plane as

$$\Delta B_{\theta} = \sum_{\ell=0}^{\infty} \Delta B_{\theta, \ell} r^{\ell} \quad , \quad (2.1a)$$

$$\Delta B_r = \sum_{\ell=0}^{\infty} \Delta B_{r, \ell} r^{\ell} \quad . \quad (2.1b)$$

Then for a radial displacement ΔR of the current block which is centered around $\theta = \theta_n$, one finds that

2. M. Month and G. Parzen, Particle Accelerators 2, 227 (1971).

$$\begin{bmatrix} \Delta B_{\theta, \ell} / B_o \\ \Delta B_{r, \ell} / B_o \end{bmatrix} = \frac{4}{2\pi} \frac{I_b}{I_t} \frac{\ell}{R_c^{\ell+1}} \Delta R \begin{bmatrix} \cos (\ell + 1) \theta_n \\ \sin (\ell + 1) \theta_n \end{bmatrix} . \quad (2.2)$$

I_b is the current carried by the displaced block, I_t is the total current carried by the unperturbed dipole magnet, B_o is the corresponding magnetic field, and R_c is the average radius of the coils.

In deriving Eq. (2.2), it has been assumed that no iron shield is present, and the current distribution is a thin sheet at the radius R_c . The multipole ℓ is low enough that $2\pi/\ell \gg \delta\theta$, the width of the current block.

The effect of the iron shield can be included if one assumes infinite permeability. One finds that the above result is about right for the lower multipoles and overestimates the higher multipoles by a factor of order 2. One can then use the results found in the absence of the iron shield, with the understanding that they are somewhat on the pessimistic side when they are used to compute tolerances.

For an azimuthal displacement $\Delta\theta$ of the current block which is centered around $\theta = \theta_n$, one finds that

$$\begin{bmatrix} \Delta B_{\theta, \ell} / B_o \\ \Delta B_{r, \ell} / B_o \end{bmatrix} = \frac{4}{2\pi} \frac{I_b}{I_t} \frac{\ell+1}{R_c^{\ell+1}} R \Delta\theta \begin{bmatrix} \sin (\ell + 1) \theta_n \\ \cos (\ell + 1) \theta_n \end{bmatrix} . \quad (2.3)$$

Using Eqs. (2.2) and (2.3) for the field error produced by a radial displacement, ΔR , and an azimuthal displacement, $\Delta\theta$, of a current block, one can find the rms field error in each magnet due to random errors in all the current blocks. The rms ΔB_ℓ generated in each component of the magnetic field in each magnet is

$$\Delta B_\ell / B_o = \sqrt{\frac{2}{N_b}} \frac{\ell+1}{R_c^{\ell+1}} \epsilon , \quad (2.4)$$

where N_b is the total number of current blocks in the magnet, and ϵ is the rms displacement of a current block in either the radial or azimuthal direction.

The random magnetic field errors in each bending magnet given by Eq. (2.4) will give rise to field harmonics. If there are M similar magnets around the accelerator ring, the random field errors will generate all harmonics smaller than M roughly equally. The rms harmonic so generated is given by $1/\sqrt{M}$ times the field error in each dipole if we neglect the presence of straight sections in the ring. Thus the contribution of each field multipole to the n^{th} harmonic is given by

$$\frac{\Delta B_{\ell,n}}{B_0} = \frac{1}{\sqrt{M}} \sqrt{\frac{2}{N_b}} \frac{\ell+1}{R_c^{\ell+1}} \epsilon r^\ell, \quad (2.5)$$

and the result is roughly valid for $n < M$ and for either the B_r or B_θ field in the median plane.

Displacement of the Central Orbit

An rms error in the position of current blocks will produce an rms dipole field, $\Delta B_0/B_0$, in the radial component of the magnetic field which causes a vertical displacement of the central orbit and which often gives rise to one of the more severe tolerances.

If one assumes that orbit studies³ show that the central orbit displacement gives a tolerance on the rms error dipole field in each magnet of

$$\Delta B_0/B_0 < 0.5 \times 10^{-3}$$

then one finds a tolerance on the current block position from Eq. (2.4) of

$$\epsilon \leq 2.5 \times 10^{-3} \text{ in.},$$

where one assumed that $N_b = 20$, $R_c = 1.8$ in.

Random Nonlinear Magnetic Fields

Random errors in the location of the current blocks will introduce nonlinear terms like ΔB_2 , ΔB_4 into the magnetic field which are different from magnet to magnet. These random ΔB_2 , ΔB_4 ... can be estimated from Eq. (2.4) using

3. E.D. Courant, R. Chasman, M. Month, and A. van Steenbergen, "Synchrotron Lattice and Orbits," in Design Study of a Cold Magnet Synchrotron (CMS), Brookhaven National Laboratory, Accelerator Dept., BNL 15430 (1970).

$\epsilon = 2.5 \times 10^{-3}$ in. rms error in the coil positions, and one finds

$$\begin{aligned}\Delta B_1/B_0 &= 0.49 \times 10^{-3}/\text{in.} \\ \Delta B_2/B_0 &= 0.41 \times 10^{-3}/\text{in.}^2 \\ \Delta B_3/B_0 &= 0.34 \times 10^{-3}/\text{in.}^3 \\ \Delta B_4/B_0 &= 0.13 \times 10^{-3}/\text{in.}^4 \\ \Delta B_6/B_0 &= 0.056 \times 10^{-3}/\text{in.}^6 \\ \Delta B_8/B_0 &= 0.022 \times 10^{-3}/\text{in.}^8.\end{aligned}$$

These random nonlinearities can be compared with the assumed tolerances⁴ for the consistent nonlinear fields which are the same from magnet to magnet and which may be due to saturation of the iron, or to deviations from a pure cosine distribution in the current distribution. The tolerances for the consistent nonlinear terms $B_2, B_4, \dots$ may be written as

$$\begin{aligned}B_2/B_0 &< 0.5 \times 10^{-3}/\text{in.}^2 \\ B_4/B_0 &< 0.08 \times 10^{-3}/\text{in.}^2 \\ B_6/B_0 &< 0.024 \times 10^{-3}/\text{in.}^6 \\ B_8/B_0 &< 0.018 \times 10^{-3}/\text{in.}^8.\end{aligned}$$

These tolerances result from assuming that one requires that $B''/B < 1 \times 10^{-3}/\text{in.}^2$ for $x \leq 1$ in. for each multipole term separately.

One sees that for $\epsilon = 2.5 \times 10^{-3}$ in. the random $\Delta B_2, \Delta B_4 \dots$ are roughly equal to the suggested tolerances for the consistent $B_2, B_4 \dots$. However, the azimuthal harmonics due to the random ΔB_ℓ will be smaller than those due to the consistent B_ℓ by the factor $\sqrt{M}$, where M is the number of magnets in the ring, which factor is about 15. In spite of the smaller harmonics, the effects of the random ΔB_ℓ may still be worrisome if the ν -value of the accelerator is too close to some nonlinear resonance. The random ΔB_ℓ , unlike the consistent B_ℓ , will give rise to essentially all the lower harmonics and can excite nonlinear resonances close to the ν -value of the accelerator. This happens, for example,² when one

4. G. Parzen, Brookhaven National Laboratory, Accelerator Dept. Report AADD-183 (1971).

wants to use a slow extraction scheme based on using a one-third resonance. If some nonlinear resonance driven by the random ΔB_ℓ causes difficulties, then one can reduce the particular harmonics which are causing trouble by exciting sextupoles and octupoles around the ring.

Random Gradients

Random errors in the location of the current blocks will also introduce random quadrupole terms, ΔB_1 , which are different from magnet to magnet. The random rms ΔB_1 can be estimated from Eq. (2.4) using $\epsilon = 2.5 \times 10^{-3}$ in., and one finds

$$\Delta B_1/B_0 = 0.49 \times 10^{-3}/\text{in.} \quad .$$

The random ΔB_1 can introduce small shifts in the ν -value and narrow stopbands at half-integer ν -values. The ν shift and the stopband width may be roughly estimated by finding the azimuthal harmonics introduced by ΔB_1 according to Eq. (2.5), and one finds

$$\frac{\Delta B'_{1,n}}{B_0} = 3.2 \times 10^{-5}/\text{in.} \quad .$$

The ν shift and stopband width is very roughly given by

$$\Delta \nu = \frac{1}{2\nu} \frac{R}{B_0} \frac{\Delta B'_{1,n}}{\rho} \quad , \quad (2.6)$$

where R is the average radius of the accelerator and ρ is the radius of curvature of the orbit in the magnetic field B_0 , and it is assumed that the accelerator has no long straight sections. Using $R = 220$ m, $R/\rho = 1.5$, one finds

$$\Delta \nu = 0.01 \quad .$$

The presence of long straight sections can increase this result for $\Delta \nu$ by a factor of 2 or 3.

The nonlinear terms will also introduce changes in gradient and thus shifts in the ν -value. These may be estimated by finding $\Delta B'_{\ell,n}/B_0$ from Eq. (2.5).

One finds at $x = 1$ in.,

$$\Delta B'_{1,n}/B_o = 3.2 \times 10^{-5}/\text{in.}$$

$$\Delta B'_{2,n}/B_o = 5.4 \times 10^{-5}/\text{in.}$$

$$\Delta B'_{3,n}/B_o = 6.8 \times 10^{-5}/\text{in.}$$

$$\Delta B'_{4,n}/B_o = 3.5 \times 10^{-5}/\text{in.} \quad ,$$

with similar results for the other multipoles. By comparison with $\Delta B'_{1,n}/B_o$, one sees that the gradients due to the nonlinear terms can each produce small ν shifts of the order of 0.01 at $x = 1$ in.

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