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FOKN: A RELATIVISTIC FOKKER PLANCK CODE WITH LARGE ANGLE SCATTERING AND
RADIATION LOSSES

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I. INTRODUCTION

FOKN is a computer code which employs a relativistic Fokker-Planck algorithm to evolve the distribution functions of the various mutually interacting components of a multi-species plasma forward in time, with the optional addition of high angle, large energy and momentum transfer interactions between the various charged species of the plasma. As a computational expediency, the latter processes are handled by transfer matrices which are generated separately by another code, RNUX, so that once specific transfer matrices are generated, they can be used over and over by FOKN provided the group structures are compatible.

Before going further, several caveats should be explicitly issued:

1) The relativistic Fokker-Planck equation used in FOKN is applicable only to an infinite isotropic medium, i.e., the equation is one-dimensional in velocity space; 2) All charged particles are assumed to remain in the medium, while uncharged particles escape at birth without subsequent interaction. Outputs of the code are separated into numerical outputs and graphical plots. The numerical outputs indicate the number density, energy density, temperature, distribution function of the various components of the simulated plasmas as a function of time. The graphical plots show the distribution functions in phase space as a function of time.

FOKN has a wide range of applications. As examples the following types of problems have been studied using FOKN. First is the study of the deviation of the ion and electron velocity distributions from thermal equilibrium during the nuclear burn of light isotopes under conditions characteristic of those likely to be encountered in classical magnetic confinement CTR machines¹⁻⁴. Second is the study of the fusion energy multiplication factors that can be obtained when beams of light ions are

injected into a relatively cold target plasma ("wet-wood burner" reactors)⁵⁻⁶. Third is the study of energy transfer between electrons and ions under relativistic conditions which might arise from astro-physical studies⁷. Fourth is the study of radiation losses such as cyclotron radiation for non-Maxwellian electron distribution confined by magnetic fields⁸.

FOKN was developed by George Zimmerman, Ted Scharlemann, Lowell Wood, Tom Weaver, Terry Chu, and Geoffrey Lee.

The following sections summarize the basic equations of FOKN and the procedure to run it. Section II describes the relativistic Fokker-Planck equation for an infinite isotropic medium. Section III describes the numerical techniques employed in FOKN. Sections IV and V describe the source and loss terms, respectively. Section VI describes the shapes of initial distribution functions. Section VII describes how large angle collisions and nuclear reactions are handled in FOKN. Section VIII describes bremsstrahlung and cyclotron radiation losses. Section IX describes the procedure to run FOKN. The Appendix contains the names of the latest version of FOKN and the available transfer matrices.

II. BASIC EQUATIONS (FOKKER-PLANCK)

Let $f_a(u)$ be the distribution function for ion species "a" in an infinite isotropic medium where $u = \gamma\beta = \frac{p}{mc}$, $\beta = v/c$, $\gamma = (1 - \beta^2)^{-1/2}$, and v is the velocity magnitude. The relativistic Fokker-Planck equation for species "a" in an infinite isotropic medium is then

$$\frac{\partial f_a}{\partial t} = \frac{1}{u^2} \frac{\partial}{\partial u} \left\{ \frac{\gamma}{u} \left[\frac{\partial}{\partial u} \left(\frac{\gamma}{u} J_a(u) f_a(u) \right) + I_a(u) f_a(u) \right] \right\} \quad (1.1)$$

where

$$\begin{aligned} J_a(u) = & \left(\frac{u}{\gamma} \right) (2u^2 + 5) \int_0^u \left(u'^2 - \frac{u'}{\gamma'} \sinh^{-1} u' \right) \sum_b \frac{\pi_{ab}}{m_a} f_b(u') du' \\ & + \left(u^2 - \frac{u}{\gamma} \sinh^{-1} u \right) \int_u^\infty \left(\frac{u'}{\gamma'} \right) (2u'^2 + 5) \sum_b \frac{\pi_{ab}}{m_a} f_b(u') du' \\ & - \left(2 \frac{u}{\gamma} \right) \int_0^u \left(\frac{u'^3}{\gamma'} \sinh^{-1} u' \right) \sum_b \frac{\pi_{ab}}{m_a} f_b(u') du' \\ & - \left(\frac{u^3}{\gamma} \sinh^{-1} u \right) \int_u^\infty \left(2 \frac{u'}{\gamma'} \right) \sum_b \frac{\pi_{ab}}{m_a} f_b(u') du' \end{aligned} \quad (1.2)$$

$$\begin{aligned}
 I_a(u) = & \left(\frac{u}{\gamma}\right)(2u^2+5) \int_0^u \left(2 \frac{u'^2}{\gamma'}\right) \sum_b \frac{\pi_{ab}}{m_b} f_b(u') du' \\
 & - \left(2 \frac{u^2}{\gamma}\right) \int_u^\infty \left(\frac{u'}{\gamma'}\right)(2u'^2+5) \sum_b \frac{\pi_{ab}}{m_a} f_b(u') du' \\
 & - \left(2 \frac{u}{\gamma}\right) \int_0^u \left(\frac{u'^2}{\gamma'} + 2u' \sinh^{-1} u'\right) \sum_b \frac{\pi_{ab}}{m_b} f_b(u') du' \\
 & + \left(\frac{u^2}{\gamma} + 2u \sinh^{-1} u\right) \int_u^\infty \left(2 \frac{u'}{\gamma'}\right) \sum_b \frac{\pi_{ab}}{m_a} f_b(u') du' \\
 & + \left(u^2 - \frac{u}{\gamma} \sinh^{-1} u\right) \int_u^\infty (4u') \sum_b \frac{\pi_{ab}}{m_b} f_b(u') du' \\
 & - (4u) \int_0^u \left(u'^2 - \frac{u'}{\gamma'} \sinh^{-1} u'\right) \sum_b \frac{\pi_{ab}}{m_a} f_b(u') du'
 \end{aligned} \tag{1.3}$$

$$\pi_{ab} = \frac{4\pi^2 e_a^2 e_b^2 \ln \Lambda_{ab}}{c^3 m_a} \tag{1.4}$$

Here e_a is the charge for species "a", m_a is the mass, c is the velocity of light, and $\ln \Lambda_{ab}$ is the Coulomb logarithm (see FOKN User's Manual). The density n_a is defined by

$$n_a = 4\pi \int_0^\infty f_a(u) u^2 du \tag{1.5}$$

the total energy E_a by

$$E_a = 4\pi m_a c^2 \int_0^\infty f_a(u) \gamma u^2 du \tag{1.6}$$

and the kinetic energy by

$$(KE)_a = E_a - n_a m_a c^2 \quad (1.7)$$

Eq. (1.1) is analogous to the relativistic kinetic equation derived by Beliaev and Budker.⁹

III. NUMERICAL METHODS

The numerical procedure in FOKN follows closely to the difference scheme developed by Chang and Cooper.¹⁰

Using finite difference method, Eq. (1.1) is written as

$$\frac{\Delta f_k^a}{\Delta t} = \frac{1}{\delta_k} (F_{k+1/2} - F_{k-1/2}) \quad , \quad k = 1, \dots, N \quad (2.1)$$

with boundard conditions

$$F_{1/2} = F_{n+1/2} = 0 \quad (2.2)$$

where

$$F_{k+1/2} = \frac{1}{\Delta Y_{k+1/2}} f_{k+1}^a [Y_{k+1} + SS_{k+1}] - f_k^a [Y_k] \quad (2.3)$$

$$\begin{aligned}
 SS_{k+1} = \frac{1}{2} \sum_b \left\{ \frac{\pi_{ab}}{m_a} \left[\delta_{k+1}^3 \left(\sum_{j=k+1}^N \left(\delta_j^5 f_j^b + \sum_{j=k+z}^N \delta_j^5 f_j^b \right) \right. \right. \right. \\
 - \delta_{k+1}^2 \left(\sum_{j=1}^k \delta_j^9 f_j^b + \sum_{j=1}^{k+1} \delta_j^9 f_j^b \right) - \delta_{k+1}^6 \left(\sum_{j=k+1}^N \delta_j^7 f_j^b + \sum_{j=k+z}^N \delta_j^7 f_j^b \right) \Big] \\
 + \frac{\pi_{ab}}{m_b} \left[\delta_{k+1}^7 \left(\sum_{j=1}^k \delta_j^6 f_j^b + \sum_{j=1}^{k+1} \delta_j^6 f_j^b \right) - \delta_{k+1}^5 \left(\sum_{j=1}^k \delta_j^3 f_j^b \right. \right. \\
 \left. \left. + \sum_{j=1}^{k+1} \delta_j^3 f_j^b \right) + \delta_{k+1}^9 \left(\sum_{j=k+1}^N \delta_j^2 f_j^b + \sum_{j=k+2}^N \delta_j^2 f_j^b \right) \right] \Big\} \quad (2.4)
 \end{aligned}$$

$$Y_k = \frac{e^{-x_k}}{1 - e^{-x_k}} R_k \quad (2.5)$$

$$R_k = \Delta Y_{k+1/2} \sum_b \frac{\pi_{ab}}{m_b} \left\{ \rho_k^7 \sum_{j=1}^k \delta_j^6 f_j^b - \rho_k^5 \sum_{j=1}^k \delta_j^3 f_j^b + \rho_k^9 \sum_{j=k+1}^N \delta_j^2 f_j^b \right\} \quad (2.6)$$

$$x_k = \frac{R_k}{\left(\frac{Y}{u} J^a \right)_k} \quad (2.7)$$

$$\begin{aligned}
 \left(\frac{Y}{u} J^a \right)_k = \sum_b \frac{\pi_{ab}}{m_a} \left\{ \rho_k^7 \sum_{j=1}^k \delta_j f_j^b + \rho_k^9 \sum_{j=k+1}^N \delta_j^7 f_j^b \right. \\
 \left. - \rho_k^5 \sum_{j=1}^k \delta_j^{10} f_j^b - \rho_k^{10} \sum_{j=k+1}^N \delta_j^5 f_j^b \right\} \quad (2.8)
 \end{aligned}$$

$$\Delta \gamma_{k+1/2} = \frac{\delta_{k+1}^8}{\delta_{k+1}^1} - \frac{\delta_k^8}{\delta_k^1} \quad (2.9)$$

$$\delta_k^1 = \int_{u_{k-1}}^{u_k} u^2 du \quad (2.10)$$

$$\delta_k^2 = \int_{u_{k-1}}^{u_k} 4u du \quad (2.11)$$

$$\delta_k^3 = \int_{u_{k-1}}^{u_k} \left(\frac{u^2}{\gamma} + 2u \sinh^{-1} u \right) du \quad (2.12)$$

$$\delta_k^5 = \int_{u_{k-1}}^{u_k} 2 \frac{u}{\gamma} du \quad (2.13)$$

$$\delta_k^6 = \int_{u_{k-1}}^{u_k} 2 \frac{u^2}{\gamma} du \quad (2.14)$$

$$\delta_k^7 = \int_{u_{k-1}}^{u_k} \frac{u}{\gamma} (2u^2 + 5) du \quad (2.15)$$

$$\delta_k^8 = \int_{u_{k-1}}^{u_k} (\gamma - 1) u^2 du \quad (2.16)$$

$$\delta_k^9 = \int_{u_{k-1}}^{u_k} \left(u^2 - \frac{u}{\gamma} \sinh^{-1} u \right) du \quad (2.17)$$

$$\delta_k^{10} = \frac{u_k}{u_{k-1}} \frac{u^3}{\gamma} \sinh^{-1} u \, du \quad (2.18)$$

$$\delta_k^m = g_k^m - g_{k-1}^m \quad m=1,2,\dots,10 \quad (2.19)$$

$$\rho_k^5 = 2 \quad (2.20)$$

$$\rho_k^7 = 2u_k^2 + 5 = g_k^2 + 5 \quad (2.21)$$

$$\rho_k^9 = u_k \gamma_k - \sinh^{-1} u_k = g_k^6 \quad (2.22)$$

$$\rho_k^{10} = u_k^2 \sinh^{-1} u_k = g_k^3 \quad (2.23)$$

$$g^1 = \frac{1}{3} u^3 \quad (2.24)$$

$$g^2 = 2u^2 \quad (2.25)$$

$$g^3 = u^2 \sinh^{-1} u \approx u^3 - \frac{1}{6} u^5 + \frac{3}{40} u^7 - \frac{5}{112} u^9, \quad u < .02 \quad (2.26)$$

$$g^5 = 2(\gamma-1) \approx u^2 - \frac{1}{4} u^4 + \frac{1}{8} u^6 - \frac{5}{64} u^8, \quad u < .05 \quad (2.27)$$

$$g^6 = u\gamma - \sinh^{-1} u \approx \frac{2}{3} u^3 - \frac{1}{5} u^5 + \frac{3}{28} u^7 - \frac{5}{72} u^9, \quad u < .05 \quad (2.28)$$

$$g^7 = \frac{1}{3} [2\gamma u^2 + 11(\gamma-1)] \approx \frac{5}{2} u^2 - \frac{1}{8} u^4 + \frac{7}{48} u^6 - \frac{13}{128} u^8, \quad u < .05 \quad (2.29)$$

$$g^8 = \frac{1}{8} [u_Y(2u^2 + 1) - \sinh^{-1} u] \approx \frac{1}{10} u^5 - \frac{1}{56} u^7 + \frac{1}{144} u^9 - \frac{1}{256} u^{11}, u < .08 \quad (2.30)$$

$$g^9 = \frac{1}{3} u^3 + u - \gamma \sinh^{-1} u \approx \frac{2}{15} u^5 - \frac{8}{105} u^7 + \frac{16}{315} u^9 - \frac{128}{3465} u^{11}, u < .08 \quad (2.31)$$

$$g^{10} = \frac{u}{9} (6-u^2) + \frac{\gamma}{3} (u^2-2) \sinh^{-1} u \approx \frac{1}{5} u^5 - \frac{2}{21} u^7 + \frac{8}{135} u^9 - \frac{16}{385} u^{11}, u < .08 \quad (2.32)$$

When writing out these equations, f is assumed flat across bin and u is defined on bin boundary. Eq. (2.1) is solved following Chang and Cooper's technique by first determining the parameter which forces the f 's to quasi-equilibrium, and then by using the "back substitution" algorithm.

IV. SOURCE TERMS

The particle sources are represented by $S_a(u,t)$ which is added to the R.H.S. of Eq. (1.1)

$$\frac{df_a}{dt} = \dots + S_a(u,t) \quad (3.1)$$

$S_a(u,t)$ is of the form

$$S_a(u,t) = \sum_{\ell} J_a^{\ell} T(t-t_0) \quad (3.2)$$

where J_a^{ℓ} is a constant current for group ℓ and $T(t-t_0)$ is a step function such that

$$T(t-t_0) = \begin{cases} 1 & , t < t_0 \\ 0 & , t \geq t_0 \end{cases} \quad (3.3)$$

where t_0 is a specified time.

V. LOSS TERMS

The particle losses are represented by f_a/τ_a which is subtracted from the R.H.S. of Eq. (1.1).

$$\frac{df_a}{dt} = \dots - \frac{f_a}{\tau_a} \quad (4.1)$$

where τ_a is usually referred to as the particle confinement time.

VI. INITIAL DISTRIBUTIONS

The initial value of $f_a(u)$ is given by

$$f_a(u) = K (\gamma-1)^\alpha e^{-[(\gamma-1) - \alpha'] m_a c^2 / kT_a} \quad (5.1)$$

where K is computed so that f_a has the correct initial density n_a and kT_a is the initial temperature. Clearly α and α' must be zero to give a Maxwellian distribution.

VII. LARGE ANGLE COLLISIONS AND NUCLEAR REACTIONS

The nuclear reactions and large angle elastic collisions are handled explicitly with a set of three-dimensional transfer matrices. The use of these transfer matrices can be shown by considering the case of alpha particles in a DT plasma.

$$\begin{aligned} \dot{f}(u_\alpha) = & \int_0^\infty f_D(u_D) \int_0^\infty f_T(u_T) \int_{DT}^\alpha (u_\alpha, u_D, u_T) du_T du_D \\ & + \sum_{i=D,T} \int_0^\infty f_i(u_i) \int_0^\infty f(u'_\alpha) \int_{i\alpha}^\alpha (u_\alpha, u_i, u'_\alpha) du'_\alpha du_i \end{aligned}$$

$$- f_{\alpha}(u_{\alpha}) \sum_{i=D,T} \int_0^{\infty} f_i(u_i) L_{i\alpha}(u_{\alpha}, u_i) du_i \quad (6.1)$$

The change in the alpha distribution is considered to be due to $D(t, n) \text{ He}^4$ reactions, alpha-deuteron and alpha-triton elastic collisions. For every alpha u the source terms in the distribution are computed as a double integral over the distribution functions of the colliding particles. The loss term is a single integral over the distribution function of the particle with which the alpha particle at this u collides.

The transfer matrices need be determined only once since they are functions only of the u 's of the particles involved in the collisions.

The transfer matrices are generated by the computer code RNUX3 (see FOKN User's Manual). To obtain each transfer matrix, experimental differential cross-section data is processed by Monte-Carlo integration over all variables not explicitly carried in the isotropic model. The Monte-Carlo code (RNUX3) uses the same group structure that is to be used by the plasma evolution code (FOKN). For each pair of u -group for the colliding particles, it chooses random u 's within the group and a random relative angle. Transforming coordinates and choosing an angle for one of the emitted particles at random, it evaluates the probability of collision (reaction) and tallies that probability as a function of the u 's of the emitted particles in the laboratory frame. Except for modification by phase space factors this probability distribution for each pair of colliding particle u 's is the desired transfer matrix.

VIII. RADIATION LOSSES

a. Bremsstrahlung

Relativistic electron-ion bremsstrahlung is done explicitly by transferring electrons in momentum space according to the Bethe-Heitler¹¹ formula

$$\dot{f}_e(u) = 4\pi c \sum_i (4\pi \int_0^\infty f_i(x) x^2 dx) \left\{ \int_u^\infty \frac{f_e(x)}{x \left[\frac{u^2}{x^2} - 1 \right]} \phi(x;u) dx - \int_0^u \frac{x}{u} \left[\frac{f_e(u)}{u^2 - x^2} \right] \phi(u;x) dx \right\} \quad (7.1)$$

where ϕ is the Bethe-Heitler cross-section for bremsstrahlung.

$$\phi(x;u) = \left(\frac{Z_i^2 r_o^2}{137} \right) \left(\frac{u}{k \sqrt{x^2+1}} \right) \left\{ \frac{4}{3} - 2[(x^2+1)(u^2+1)]^{1/2} \left(\frac{u^2+x^2}{u^2 x^2} \right) + \frac{\epsilon_0 \sqrt{u^2+1}}{x^3} + \frac{\epsilon \sqrt{x^2+1}}{u^3} - \frac{\epsilon \epsilon_0}{ux} + L \left[\frac{8}{3} \frac{[(x^2+1)(u^2+1)]^{1/2}}{ux} + \frac{k^2}{x^3 u^3} \left((x^2+1)(u^2+1) + x^2 u^2 \right) + \frac{k}{2xu} \left(\frac{[(x^2+1)(u^2+1)]^{1/2} + x^2}{x^3} \epsilon_0 - \frac{[(x^2+1)(u^2+1)]^{1/2} + u^2}{u^3} \epsilon + \frac{2k[(x^2+1)(u^2+1)]^{1/2}}{u^2 x^2} \right] \right\} \quad (7.2)$$

where

$$k = \sqrt{x^2+1} - \sqrt{u^2+1} \quad (7.3)$$

$$\epsilon_0 = 2 \log (\sqrt{x^2+1} + x) \quad (7.4)$$

$$\epsilon = 2 \log (\sqrt{u^2+1} + u) \quad (7.5)$$

$$L = 2 \log \left\{ \frac{[(x^2+1)(u^2+1)]^{1/2}}{k} + xu - 1 \right\} \quad (7.6)$$

and r_0 is the classical electron radius.

$\phi(u;x)$ is the same as $\phi(x;u)$ with the x and u interchanged. The e-e bremsstrahlung is handled by a fudge factor to the e-i bremsstrahlung. Instead of using the fudge factor, the e-e bremsstrahlung can also be handled by feeding the differential cross-sections to RNUX3 to generate a e-e bremsstrahlung transfer matrix. However, this technique is not advisable because the process of using the e-e transfer matrix in FOKN would take up too much computer time. Once the bremsstrahlung is produced, it is assumed lost to the plasma without further interaction.

b. Cyclotron Radiation

Transfer of electrons in momentum space under the influence of cyclotron radiation is handled by the following equation.

$$\frac{\partial f_e(u)}{\partial t} = \frac{1}{u^2} \frac{\partial}{\partial u} \left\{ \frac{1}{\beta} \left[\frac{\partial}{\partial u} \frac{g(u)}{\beta} f_e(u) + g(u) f_e(u) \right] \right\} \quad (7.7)$$

where

$$g(u) = \frac{2\pi^2 u^2}{m^2 c^2} \int_0^\infty \frac{\epsilon \omega}{\omega^2} \eta_\omega(u) \delta(\omega - n\omega_g) d\omega; \quad n=1,2,\dots \quad (7.8)$$

$$\begin{aligned} \tau(u) = & \frac{u^2}{mc^2} \int_0^\infty \eta_\omega(u) \delta(\omega - n\omega_g) d\omega \\ & - \frac{2\pi^2}{m^2 c^2} \frac{\partial}{\partial u} \left(\frac{u^2}{\beta} \int_0^\infty \frac{\epsilon \omega}{\omega^2} \eta_\omega(u) \delta(\omega - n\omega_g) d\omega \right); \quad n=1,2,\dots \end{aligned} \quad (7.9)$$

with ω as the frequency and ω_g as the cyclotron frequency of the electron.

$$\epsilon_\omega = \frac{\int_0^\infty \eta_\omega(u) f_e(u) 4\pi u^2 du}{\alpha_\omega c \left(1 + \frac{3}{4\tau} (1-R)\right)} \quad (7.10)$$

where R is the reflectivity of the containing wall, η_ω is the emissivity, α_ω is the absorption coefficient, and τ is the optical depth.

$$\tau = \alpha_\omega \ell \quad (7.11)$$

where ℓ is the scale length of the plasma.

$$\begin{aligned} \eta_\omega = & \frac{e^2 \omega_g^2 \gamma}{c} (n+1) (2n\beta)^{2n} \left[\frac{n!}{(2n+1)!} \right]^2 \\ & \left[\log\left(\frac{1+\beta}{1-\beta}\right) - 2\beta \right]^{-1} \left[\beta^2 - \left(1 - \frac{n\omega_g^2}{\gamma\omega}\right)^2 \right] \end{aligned} \quad (7.12)$$

$$\alpha_\omega = - \frac{8\pi^3}{m\omega^2} \int \eta_\omega(u) \frac{\partial}{\partial u} f(u) \gamma^2 dy \quad (7.13)$$

The emissivity as given by Eq. (7.12) is valid only when $n\beta \ll 1$. Hence, for hot electrons an alternate form must be used. More work will be done on cyclotron radiation in the future to make it more satisfactory.

IX. FOKN USER'S MANUAL

A. FOKN input cards (problem generator file).

Each card has at most 12 input words per line of at most 10 characters each.

All input formats are field free.

First card of the input file must be an identification card.

1. Identification card (required, must be the first card).

BOX_^---_^-----

First word is "BOX".

Second word is the box number (e.g. W98).

Third word is any name for identification up to 10 characters, but only the first 4 characters appear on FROG DD80 output (e.g. WWDT1).

2. Comment cards (optional).

C_^-----

First word must be "C".

The rest could be anything up to 11 words.

3. Material cards (required, must be ≤ 8 kinds of materials).

M 1 5 18.37E-24

First word must be "M".

Second word is species number from 1-8, could be integer or floating point number.

Third word is the charge number, Z, of the corresponding species.

Use -1 for electrons and 1.E-30 for neutrons. The word could be integer or floating point but never zero.

Fourth word is the mass of the corresponding species in grams.

4. Group structure cards (required, 2 different kinds).

Group structure must be the same as in transfer matrix generator in order to use the transfer matrices.

There are two kinds of group structure cards.

One (ULIN cards) uses linear interpolation to set up the group structure.

The other (ULOG cards) uses logarithmic interpolation to set up the group structure.

First kind:

```
ULIN  1    2    .00242056    .00342319
```

First word must be "ULIN".

Second word is the lower group number of the range considered.

Third word is the upper group number of the range considered.

Fourth word is the u-upper boundary corresponding to the group in the second word.

Fifth word is the u-upper boundary corresponding to the group in the third word.

$u \equiv \gamma\beta$

By definition u-lower boundary for group #1 is 0.

Second kind:

```
ULOG  5    35    .00541253    .085
```

First word must be "ULOG".

Second to fifth words have the same meaning as those in the ULIN card.

5. Plasma background card (required, at most 1000/7 cards).

Background distribution is assumed to be

$$f = \kappa(\gamma-1)^\alpha e^{-[(\gamma-1)-\delta]MC^2/kT}$$

κ is automatically normalized to the range of u wished to be considered by user.

$\delta = 0$ and

$\alpha = 0$ force f to become a Maxwellian

$$kT = 0 \Rightarrow e^{-(\gamma-1)MC^2/kT} \equiv 1$$

F card is of the form:

F 1 1.E14 40. 0. 0. 0. 0.

First word must be "F".

Second word is the species number corresponding to that in the M card.

Third word is the background particle density of this species in $\#/cm^3$.

Fourth word is the background temperature, kT , of this species in keV.

Fifth word (optional, default is 0.) is the lower boundary of the range of u wished to be considered by user.

Sixth word (optional, default is 0.) is the upper boundary of the range of u wished to be considered by user.

(Note: Defaults used in both the fifth and sixth words imply full range of u is to be used to set up the background distribution.)

Seventh word (optional, default is 0.) is " δ " as appeared in the above equation.

Eighth word (optional, default is 0.) is " α " as appeared in the above equation.

(Note: If words 5-8 are all zero, background distribution is a Maxwellian.)

Note: The distribution for one species can be described by using several F cards. The resultant distribution is the sum of all the distributions described in the several F cards.

6. Parameter cards (21 parameters to date, optional).

P 1 1.E-5

First word is "0".

Second word is the parameter number.

Third word is the value of the parameter.

The following is a list of the different parameters with defaults, modes, and units.

<u>Number</u>	<u>Default</u>	<u>Mode</u>	<u>Unit</u>
1	1.E-20	Float	sec
2	0	float	sec
3	100	int	---
4	0.1	float	---
5	1.1	float	---
6	1.E+20	float	sec
7	1.E+20	float	sec
8	100000	int	---
9	1000	int	---
10	1.E-8	float	---
11	100	int	---
12	10	float	---
13	0	float	---
14	0	float	---
15	0	float	---
16	1.E-30	float	---
17	1.0	float	---
18	1.0	float	---
19	0.4	float	---
20	2	int	---
21	1.E+20		

Parameter 1 is the initial time step.

Parameter 2 is the time between edits.

Parameter 3 is the number of cycles between edits.

Parameter 4 is the maximum fractional change in distributions per cycle.

Parameter 5 is the $\max \Delta t_{j+1} \text{ (new)}/\Delta t_j \text{ (old)}$

Parameter 6 is the $\max \Delta t$.

Parameter 7 is the stop time.

Parameter 8 is the maximum number of cycles.

Parameter 9 is the number of cycles between restart dumps.

Parameter 10 is the convergence criteria for stopping iterations.

Parameter 11 is the maximum number of iterations.

Parameter 12 is the optimal number of iterations.

Parameter 13 is the e-i bremsstrahlung multiplier.

Parameter 14 is the e-e bremsstrahlung multiplier.

Parameter 15 is the cyclotron radiation multiplier.

Parameter 16 is the max fractional change in population per group.

Parameter 17 is the ion over-relaxation parameter $1 \leq P17 \leq 2$.

Parameter 18 is the electron over-relaxation parameter $1 \leq P18 \leq 2$.

Parameter 19 makes the photon groups for cyclotron radiation more concentrated around the harmonics of the gyrofrequency.

Parameter 20 centers the photon groups of cyclotron radiation around the gyrofrequency.

Parameter 21 is the source injection stopping time.

Note: All 21 parameters can be changed while the program is running.

See section on how to run the program. If parameters 13-15 are zero, the subroutines BREM and CYCLO will not be used so that there is no bremsstrahlung and cyclotron radiation.

7. Reaction enhancement cards (MULT cards, 2 kinds, optional)

First kind is multiplier of transfer matrix

MULT 30 2.0

First word must be "MULT".

Second word is the reaction number or the transfer matrix number.

Third word is the value of the multiplier, i.e. multiplier for the nuclear cross-sections.

Second kind is multiplier for the Fokker Planck scattering.

MULT 1 2 2.0

First word must be "MULT".

Second word is the scattered species number. This species is affected by the multiplier.

Third word is the scattering species number. This species is not affected by the multiplier.

Fourth word is the multiplier value.

Note: To conserve energy while using the second kind of "MULT" card, both permutations must be used.

8. Transfer matrix card (optional).

NUC T2DTNA ---

First word must be "NUC".

The rest are transfer matrices. Up to 10 transfer matrices can be used.

9. Source cards (optional, at most 10 source cards).

S 1 30 1.E+15

First word must be "S".

Second word is species number.

Third word is the group number which is the intended energy for injection.

Fourth word is the injection rate in particles/cm³-sec.

10. Loss cards (optional).

The loss term is described by the following equation

$$\frac{df}{dt} = - f \left[\frac{c}{r} R / \left[(\tau_p + \tau_f \cdot \frac{r}{v}) (\gamma-1)^p + f_L \right] \right]$$

c = speed of light, r = radius of plasma, v = volume of plasma
default of v = 1 cm³, default of r = 0.62 cm.

L 1 2.1E-11 0. 1. 0. 0.

First word must be "L".

Second word is the species number.

Third word is the value of R.

Fourth word (optional) is value of p, default is 0.

Fifth word (optional) is value of τ_p , default is 1.

Sixth word (optional) is value of τ_f , default is 0.

Seventh word (optional) is value of f_L , default is 0.

Note: The term inside the big bracket is usually referred to as the particle confinement time.

11. New time, volume, and electron temp cards (optional, up to twenty cards).

TV 1. 2. 10.

First word must be "TV".

Second word is the value of time in seconds at which the volume of the plasma and/or electron temperature would be changed.

Third word is the value of the new volume of plasma.

Fourth word (optional, default is 0) is the value of electron temperature. If electron temperature as specified here $\neq 0$, then the electron distribution would remain a Maxwellian at this specified temperature, i.e., the electron distribution would not be evolved by the Fokker-Planck operator.

Note: If there are more than one TV card, logarithmic interpolation is used between cards.

12. DTBURN card (optional).

DTBURN 1 2

First word must be "DTBURN".

Second word is deuterium species number.

Third word is tritium species number.

Note: If this card is used, then transfer matrices or "NUC" cards should not be used. The function of this card is to list the number of $d(t,n)\alpha$ reactions only, so that the α particles will not deposit energy to the plasma.

13. Cyclotron radiation card (optional).

B 10000. 0. 1. 0.5 25

First word must be "B".

Second word is the magnetic field strength in gauss.

Third word is the scale length in cm and can be zero, in which case the volume is used to determine the size.

Fourth word is the multiplier for the initial cyclotron radiation density, with respect to its equilibrium value.

Fifth word is the reflectivity of the wall.

Sixth word is the number of harmonics of the gyrofrequency to be considered. If left blank the default is ten.

14. Coulomb logarithm card (required when scattering transfer matrix is used).

CLOG 1 2 3.00

First word must be "CLOG".

Second and third words are species numbers involved in the collision.

The order of these words is not important.

Fourth word is the value of $\tan \psi$. See explanation below.

Coulomb logarithm (Ref: Spitzer, p. 122 and 127).

If two particles collide with a relative speed u and an impact parameter p , then the angular deflection, χ , of each particle is given by

$$\chi = \pi - 2\psi$$

where

$$\tan \psi = \frac{Mpu^2}{Z_1 Z_2 e^2}$$

where M is the reduced mass.

Coulomb logarithm, $\ln \Lambda$, is defined by

$$\ln \Lambda = n \left(\frac{p_m}{p_0} \right)$$

where $p_0 = \frac{Z_1 Z_2 e^2}{m_1 w_1}$ is the impact parameter for the lighter particle

to be deflected through 90° in the center of mass system with m_1 as the mass of the lighter particle and w_1 its speed in the center of mass system and p_m is the impact parameter beyond which the Coulomb field is screened. p_m is usually taken as the Debye length

$$p_m = \left(\frac{kT}{4\pi n_e e^2} \right)^{1/2} = 6.90 \left(\frac{T}{n_e} \right)^{1/2}$$

Now suppose that one wishes to do the scattering explicitly above a certain angle, θ_c , then

$$\ln \Lambda = \ln \left(\frac{p_m}{p(\theta_c)} \frac{p(\theta_c)}{p_0} \right)$$

and so the $\ln \Lambda$ one wants to use in doing implicit Fokker Planck scattering is

$$\ln \Lambda = \ln \left(\frac{p_m}{p(\theta_c)} \right) = \ln \Lambda - \ln \left(\frac{p(\theta_c)}{p_0} \right)$$

Now $\theta_c = \pi - 24$ so that

$$\tan \psi = \tan \frac{\pi - \theta_c}{2}$$

So for $\theta_c = 20^\circ$, $\tan \psi = \tan 80^\circ = 5.67$.

Thus if the explicit consideration of elastic scattering between particle species 1 and 2 is cutoff at 200° the CLOG card has the form

CLOG 1 2 5.67

In fact, in order to avoid numerical problems, the explicit high angle scattering is not cut off sharply at θ_c but rolled off by the factor

$$\frac{1}{(1 - \cos \theta)^2 + x_0^2}$$

where $x_0 = 1 - \cos \theta_c$.

This factor is used in generating Rutherford cross-sections for inputs of transfer matrix generator. In particular x_0 is the 2nd parameter of the "SET" statement, discussed in the RNUXN input file. For $\theta_c = 20^\circ$, $x_0 = 0.060307379$ and using 9 Legendre polynomials involved in the fit, the "SET" card is

```
SET    9    0.060307379
```

When the integral is done to get the total cross-section, the effects of a cut-off and a roll-off are identical for the Rutherford component.

Note: The CLOG card must be consistent with the SET card when scattering transfer matrices are used. There should be no CLOG card if there is no scattering transfer matrices used.

B. HOW TO USE FOKN

Suppose the latest compiled version of FOKN is called RFOC4 and the problem one wishes to run is called WWD1. Before execution of FOKN, one needs to have the problem generator file (FOKN input file) for WWD1. Usually the name of the problem generator file is the problem name with a "G" added at the end. In this case the problem generator file is WWD1G. Also all the transfer matrices to be used in the problem must be on line. To execute FOKN, the execute line is:

```
RFOC4    WWD1    WWD1G    options / TV
```

First symbol is the latest version of FOKN.

Second symbol is the name of the problem to be run.

Third symbol is the name of the problem generator or G.

A list of options follows (discussed in the following section).

"T" and "V" have the usual meaning of time and value as used in the LLL computer system.

Note: In order to insure the program to run, "T" must be ≥ 1 minute. Program will stop when time left is about one minute.

Options for the execute line.

If there is not list of options in the execute line, all outputs go immediately to FR80 (microfiche) when the program stops.

1. S - Suspends program and waits for instructions from teletype.
2. K - Keeps numerical output so that it will not be sent by the program when the program stops running. Numerical output files have names like HX ----- A for HSP and FXRAD----- for FR80.
3. P - Sends numerical output to on-line printer rather than FR80.
4. H - Sends numerical output to High Speed Printer (HSP) rather than FR80.
5. KD - Keeps DD80 (graphics) output so that it will not be sent when program stops running. DD80 output files have names like DX-----AY.
6. DD - Send DD80 output to white paper output rather than microfiche.
7. Decimal number L (e.g. 40000L) - Specifies the size (number without the L) of storage of the program so that the program may run more efficiently. Default is 40000B.

8. C - Will display output on TV when the program is finished.
The two blanks are for putting in the TV channel number.
9. NOFOK - Will not use the relativistic Fokker-Planck subroutine
(RFOKP).

To restart a problem:

There are two ways to restart a problem. Suppose one wishes to restart a problem named WWD1, one needs either the drop file or the Z file. If the drop file is available (i.e. +WWD1), then the problem can be restarted by the line

+WWD1 / TV.

If the Z file (i.e. WWD1Z) is available, then the problem can be restarted by the line

RFOC4 WWD1 / TV.

Suppose the program is finished due to no time left in the machine, the program can be restarted by typing first

RFOC4 WWD1 S / TV.

and then change parameter 7 to G by typing

G.

TTY options while running FOKN:

1. T linefeed - Reports cycle number, time, time-step, species number and group which limit the time step, and number of iterations.

2. S linefeed - Suspends program.
S linefeed (again) - Program will run one more cycle and suspends.
S number linefeed - Program will run the specified number of cycles before suspension.
3. G linefeed - Gets the program out of suspension and lets it run.
4. ED linefeed - Asks the program to make an edit.
5. FIN linefeed - Terminates the program.
6. D linefeed - Asks the program to make a dump.
7. P parameter number value linefeed - Will change the specified parameter to the specified value.
8. Species number linefeed - Outputs the energy density, number density, and temperature of the specified species on TTY.
9. Species number group number linefeed - Outputs the number density of the specified group and species on TTY.

C. HOW TO PREPARE TRANSFER MATRICES FOR FOKN.

The latest version of the program which generates transfer matrices for FOKN is RNUX3.

Suppose the input data file is I2DDEL. To execute RNUX3, type

```
RNUX3 I2DDEL options / T V
```

First word is RNUX3 which is the latest version.

Second word is the input data file.

A list of options follows (discussed in the following section).

"T" and "V" have the usual meaning of time and value as used in the LLL computer system.

Execution options.

If there is not list of options in the execute line, default is FR80 so that all outputs are in microfiche.

1. K - Keeps numerical output.
2. KD - Keeps DD80 graphical output.
3. DD - Sets up DD80 graphical output for white paper output.
4. P - Sets up numerical output for the on-line printer.
5. H - Sets up numerical output for the high-speed printer (HSP).

Note: While RNUX3 is executing, type "linefeed" on the teletype will print out the pair of group numbers being processed.

To restart, one must use the drop file (file ended with a "+").
For example, to restart I2DDEL, simply type

I2DDEL+ options/ T V

RNUX3 input file.

Each card has at most 12 input words per line of at most 10 characters each.

All input formats are field free.

First card of the input file must be an identification card.

1. Identification card (required, must be the first card)

BOX ^---^-----

First word must be "BOX".

Second word is the box number.

Third word is any name for identification.

2. Comment cards (optional).

C^-----

First word must be "C".

The rest could be anything up to 11 words.

3. Nuclear reaction identification card (required).

NUC 30 0. 10000 T2DDEL EEBREM

First word must be "NUC".

Second word is the identification number (S99) for the nuclear reaction, i.e. D-D elastic scattering in this case.

Third word is the Q-value of the reaction in MeV.

Fourth word is the number of MONTE CARLO particles per pair of groups.

Fifth word is the name of the transfer matrix for the reaction.

Sixth word (optional) can be anything but zero to indicate e-e bremsstrahlung is being done. Default is zero.

4. Material card (required).

There must be at least 4 material cards. The first two materials are reactants with the first one as the beam and the second one as the target. The last two materials are products with the first as the one whose center of mass angle is specified on θ (THET) cards.

M 1 1.67E-24 1 35 NODEP

First word must be "M".

Second word is the charge number of the species.

Third word is the mass of the species in grams.

Fourth and fifth words are the first and last groups, respectively, to be set up by the ULIN and ULOG cards.

Sixth word (optional) can be anything but zero to indicate that the species is not deposited in plasma. Default is zero.

5. Group structure cards (required).

ULIN and ULOG cards: They are the same as those for FOKN. See section on FOKN input file.

(Note: The group structure as inputted here must be compatible with that set up for FOKN in order for FOKN to use the transfer matrices generated under this group structure.)

6. Cross-section fit card (required, must precede THETN and SIGE cards).

SET 9 .060307379

First word must be "SET".

Second word is the number of Legendre polynomials used for fitting the cross sections. "9" is the maximum number of Legendre polynomials allowed.

Third word is the Rutherford roll over parameter, x_0 .

$$x_0 = 1 - \cos \theta_c$$

where θ_c is the cut-off angle. See section on CLOG cards for FOKN input file. θ_c as used in SET card must be consistent with θ_c as used in CLOG card for FOKN.

7. Scattering angle card (required, must precede SIGE cards).

THETN 0. 10. 20.

First word must be "THETN".

The rest are scattering angles in center of mass system for the first product in degrees. If there are many angles, THETN card could be continued by

THET 70. 80.

Note: Continuation cards for THETN cards start with THET instead of THETN.

8. Differential cross-section cards.

The differential cross-sections corresponding to the angles in the THETN cards at different energies are inputted by the SIGE cards.

SIGE 0.300 31647. 31437

First word must be "SIGE".

Second word is the beam energy in lab system in MeV.

The rest are differential cross-sections at the specified energy corresponding to the angles in the THETN cards in mb/ster. The SIGE cards can be continued by

SIG 9325.

Note: Continuation cards for SIGE cards start with SIG instead of SIGF.

9. Total cross-section cards (SIGN cards).

THETN 0.

SIGN .094 .05 1.

First word must be "SIGN".

Second word is the beam energy in MeV in lab system.

Third word is the total cross-section in mb at the specified energy.

Fourth word (cross-section at 0 degrees) with a value of 1. signifies that the cross-section is isotropic and to be normalized to total cross-section.

Note: If the reaction is not isotropic, then the relative importance at different angles must be indicated and the result would be automatically normalized to total cross-section. For example:

THETN	0.					
THET	30.	60.	90.	120.	150.	180.
SIGN	10.	44.	1.			
SIG	.41	.44	.27	.25	.43	.56

10. Resonance well card (required if the reaction has 3 products):

Q2 2.9 1.1

First word must be "Q2".

Second word is the position of the resonance well in MeV in lab system of the secondary product state.

Third word is the half-width in MeV.

APPENDIX

The latest version of FOKN is RFOT4 and the latest version of RNUX is RNUX3. The following transfer matrices are now available. The first column gives the names of the transfer matrices with the corresponding nuclear reactions given in the second column.

T2ABPC14	- $B''(\alpha, p)C^{14}$
T2ALDE	- $\alpha(d, d)\alpha$ elastic collision
T2ALTE	- $\alpha(t, t)\alpha$ elastic collision
T2BAEL	- $B''(\alpha, \alpha)B''$ elastic collision
T2BANN14	- $B''(\alpha, n)N^{14}$
T2BPEL20	- $B''(p, p)B''$ elastic collision with 20° cutoff angle
T2BPEL40	- $B''(p, p)B''$ elastic collision with 40° cutoff angle
T2DDEL	- $d(d, d)d$ elastic collision
T2DDNH	- $d(d, n)He^3$
T2DDPT	- $d(d, p)t$
T2DHPA	- $d(He^3, p)\alpha$
T2DTE	- $d(t, t)d$ elastic collision (older version)
T2DTEL	- $d(t, t)d$
T2DTNA	- $d(t, n)\alpha$
T2EERAD	} e-e bremsstrahlung
T2EERAD2	
T2PAEL	- $p(\alpha, \alpha)p$ elastic collision
T2PB3A	- $p(B'', \alpha)2\alpha$
T2PPEL	- $p(p, p)p$ elastic collision

The input files for the above transfer matrices are named correspondingly with T2 changed to I2, e.g., I2DDEL is the input file for generating T2DDEL. In general, these transfer matrices were produced to cover the range of interest in a particular study. Users should therefore check the input files for the transfer matrices to see if the cross-sections are accurately represented over the energy and/or angular range of interest to them. All of the above files are available in photostore in directory .944075:FOKPUB, and can be accessed directly with ELF.

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